Transmission Lines & E. M. Waves
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Lecture – 57
Problems on Uniform Plane Wave

In this session let us solve some problems based on the concepts which we have developed for uniform plane wave. So we will try to find out for examples the electric and magnetic field or the phase velocity of the wave or the polarization of the wave in this session. So let us consider a problem here: a uniform plane wave at frequency of 300 megahertz travels in vacuum along plus y direction. The electric field of the wave at some instant is given as e equal to 3x plus 5z where x cap and z cap are the unit vectors. Find the phase constant of the wave and also the vector magnetic field.

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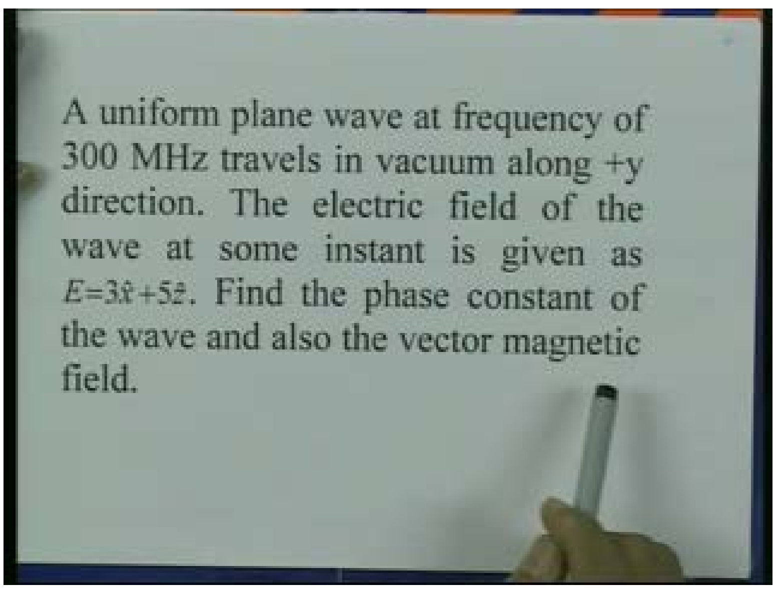

So in this problem the electric field vector electric field is given at some instant of time and we are asked to find out the phase constant of the wave and also the vector magnetic field. now since the wave is traveling in vacuum the wave travels with velocity of light in vacuum that is given as 1 upon square root mu epsilon so we get the velocity of the wave v that is equal to 1 upon square root mu 0 epsilon 0 where mu 0 and epsilon 0 are permeability and permittivity of the medium respectively and that is equal to 3 into 10 to the power 8 meters per second.

So phase constant as we know is given of the wave which is beta that is equal to omega square root mu 0 epsilon 0 so that will be equal to... the frequency is 300 megahertz so this is 300 into 10 to the power 6 into 2pi divided by this quantity which is 3 into 10 to the power 8 meters per second. So from here we get the phase constant to the wave which is 2pi radians per meter.

Now since the wave is traveling in y direction and for uniform plane wave the electric and magnetic fields lie in a plane perpendicular to direction of wave propagation the e and h both vectors must lie in a plane which is perpendicular to the direction of wave propagation which is plus y direction so that means the electric and magnetic field vectors both lie in the xz plane; the xz plane is a plane which is perpendicular to the plus y, x is... the e which is given here has components in the x and z direction so this vector lies in the xz plane, the magnetic field vector also would lie in the xz plane.

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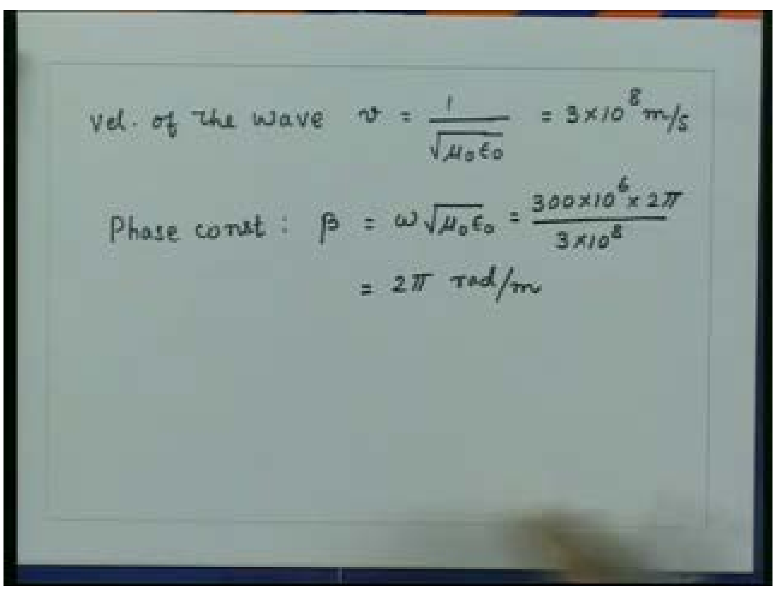

So let us assume that the magnetic field vector h now which lies in the xz plane is given as A x cap plus B z cap. So now we have electric and magnetic field vectors which lie in the xz plane and we know from the property of the uniform plane wave that the electric and magnetic field vectors also must be perpendicular to each other and the ratio of the amplitude of electric and magnetic field is equal to the intrinsic impedance of the medium. So we can essentially make use of that property to say that E dot H that is equal to 0 and also for uniform plane wave mod of E divided by mod of H that is equal to the intrinsic impedance of the medium and in this case the medium is vacuum so this is equal to eta 0 which is nothing but equal to 120pi for the vector.

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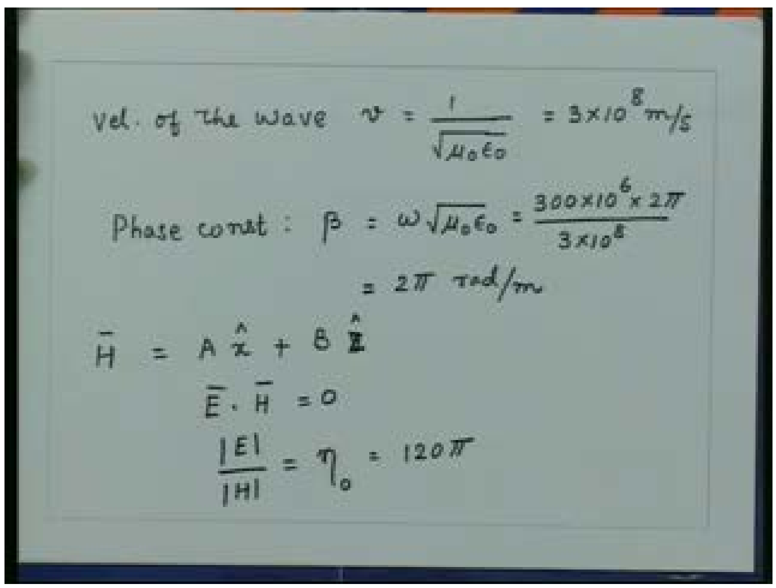

So now we can write down the electric field is given which is 3x cap plus 5z cap so I can write here and H is given by this (Refer Slide Time: 6:44) so we can write the dot product which is E dot H that is equal to 3 A plus 5 B that is equal to 0; this is one equation.

The second equation which we get is the magnitude of the magnetic field is mod of E divided by eta 0, from here we get mod of H equal to square root of A square plus B square that is equal to mod of E divided by eta 0 that is equal to square root of mod of E which is 3 square plus 5 square. So this is 9 plus 25 divided by 120pi. So this is equal to the mod of H is equal to 15.48 into to n to the power minus 3 Ampere's per meter.

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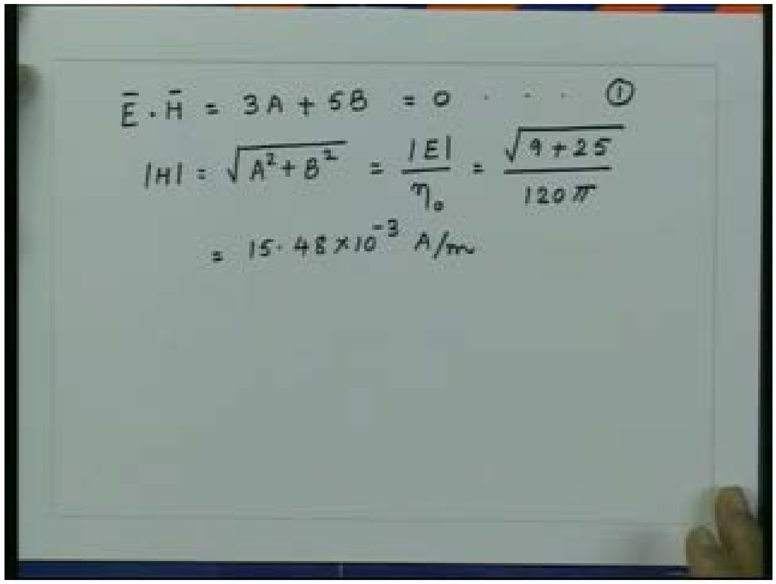

Now knowing this quantity this is the second equation which we have which relates A and B so from these two equations one can solve for A and B. So we get A is equal to plus minus 5 upon square root 120 pi and b is equal to minus plus 3 upon square root 120pi. So the magnetic field vector H now, H is equal to plus minus 5x cap minus 3z cap upon square root 120pi.

So what we did first we took the the electric field, argued that for uniform plane wave the dot product for the electric and magnetic field is zero so we assumed the magnetic field which lies in the xz plane which has to be perpendicular to direction of wave propagation which is plus y direction, find out the dot product of E and H that should be equal to 0 because E and H should be perpendicular to each other for uniform plane wave and also the magnitude of the magnetic field is equal to the magnitude of the electric field divided by the intrinsic impedance of the medium and then solving these two equations essentially we get the vector magnetic field.

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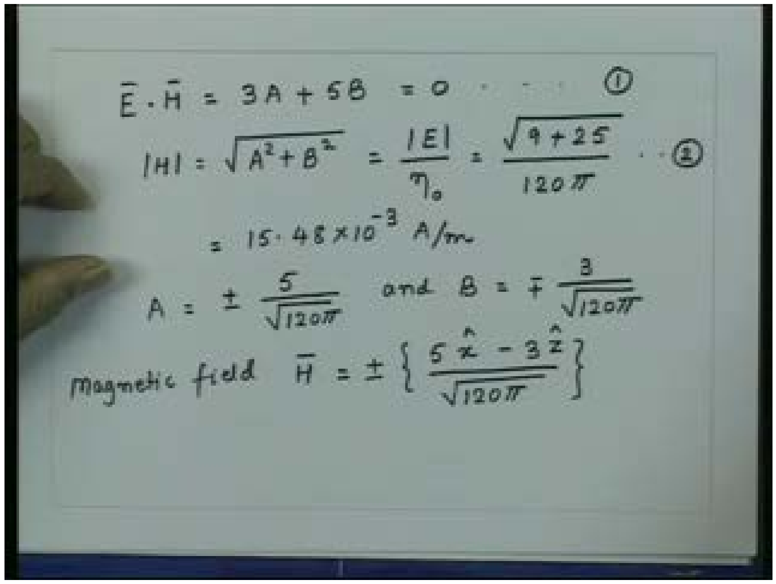

So this is one of the simple problems where the electric field was given and we were asked to find out what is the corresponding magnetic field of the wave.

Let us take another problem which is related to the polarization of the electromagnetic wave.

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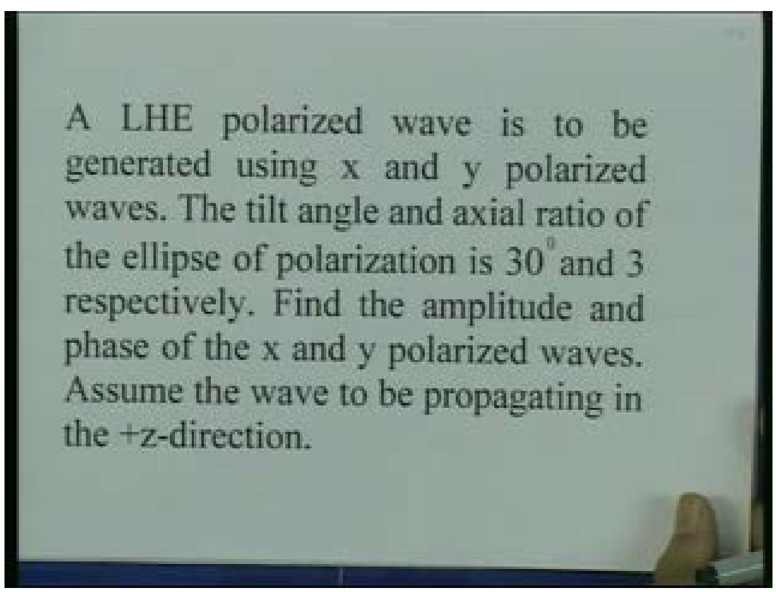

So let us say that we have to generate a left handed elliptically polarized wave. The problem is as follows: A left handed elliptically polarized wave is to be generated using x and y polarized waves. The tilt angle and the axial ratio of the ellipse of polarization is 30 degrees and 3 respectively. Find the amplitude and phase of the x and y polarized waves. Assume the wave to be propagating in the positive z direction.

So this problem again is a very straightforward problem. Essentially what we are saying is that if we have two orthogonal polarization which are x and y in what proportion they should be combined together to generate an elliptically polarized wave which is left handed sense of rotation and as a tilt angle of 30 degrees and axial ratio of 3.

So, for this problem if we go to the notation which we used in our analysis essentially we are given the quantities the tau the tilt angle that is equal to 30 degrees so equal to pi by 6 radians and epsilon which we have defined as cot inverse of axial ratio and in this case the axial ratio is 3 which has a sense of rotation which is left handed and we have used the convention that the sense of rotation left handed means positive sign so axial ratio for left handed elliptically polarized wave is plus 3 that is equal to 18.4 degrees.

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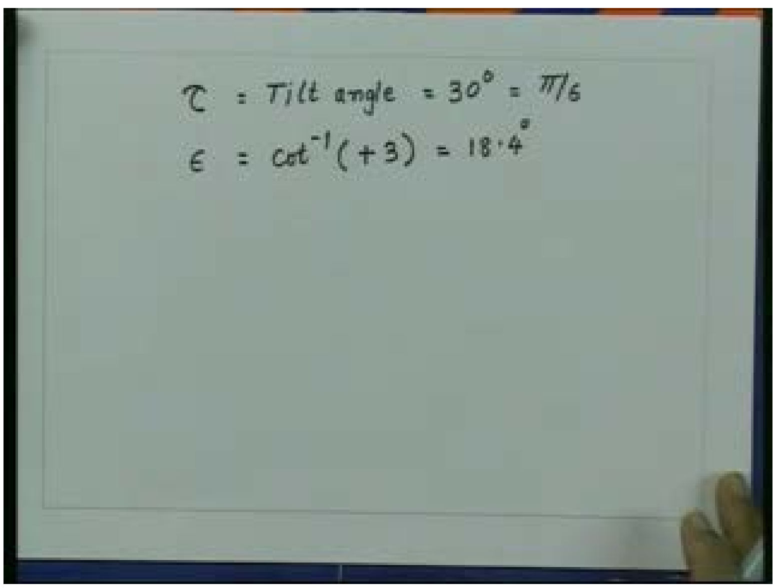

Now, using the expression which we have got for conversion from the ellipse parameter to the electrical parameter which is the ratio of the amplitude of x and y component and the phase difference between them we can get these quantities what is called gamma and phi.

So we have defined this quantity gamma, so we know gamma which is equal to tan inverse of E y upon E x and the angle phi is equal to angle E y minus angle E x and knowing the transformation relation between these parameters we we get cos of 2 gamma which is equal to cos of 2 epsilon cos of 2 tau; substituting for epsilon equal to 18.4 degrees and tau 30 degrees we get cos of 2 gamma and from there we can solve for gamma you get gamma is equal to 33.2 degrees. So the ratio E 2 or E y upon E x E y upon E x that is equal to tan of gamma that which is equal to 0.65.

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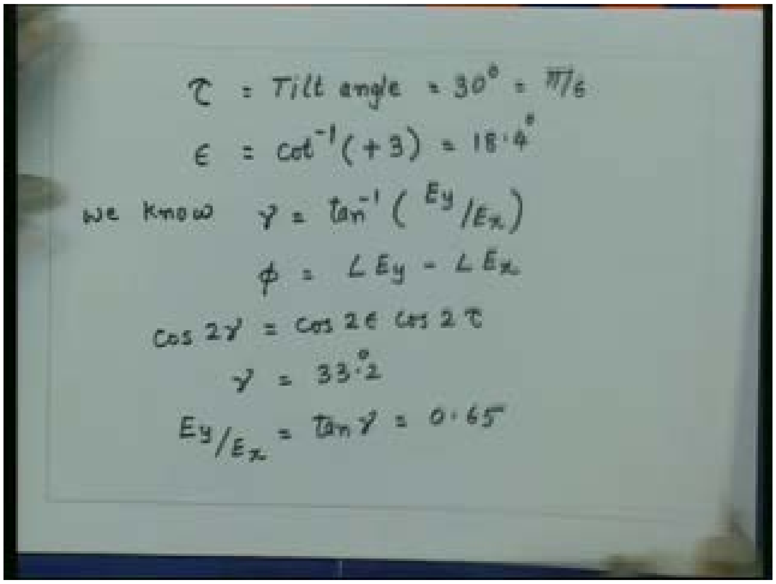

The phase angle phi can be obtained by using this relation the tan phi is equal to tan of 2 epsilon divided by sine of 2 tau that is equal to 0.8658 which gives angle phi that is equal to 40.88 degrees. So what essentially we get now is that if I take a coordinate system which is let us say this is x, this is y (Refer Slide Time: 16:02) and now if I go put my right hand rule the x to y if I if I do then z must come like this so this is the direction which is z. So if I take the ratio of the electric field in the x and y direction which is 0.65 and if we excite these fields with a E y component leading the x component by 40.88 degrees then we will generate an elliptically polarized wave with the axial ratio of 30 degrees axial ratio of 3 and tilt angle of 30 degrees so this will generate essentially the wave which will look something like this (Refer Slide Time: 16:52) this to this ratio if we calculate maximum if I take this is equal to 1 this will be equal to 0.65 so we get from here the E x component which is let us say amplitude 1 cosine of omega t, its E y component will be 0.65 cosine of omega t plus 40.88 degrees.

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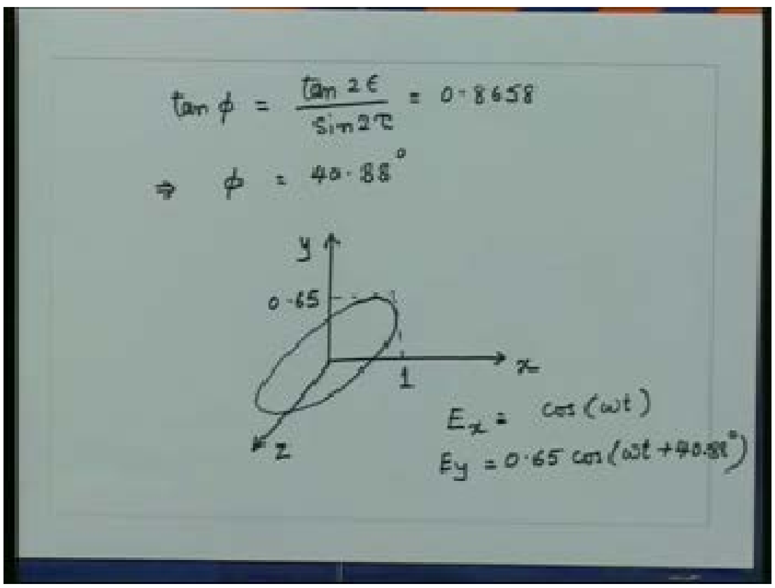

So, by exciting these two fields simultaneously in superposition of these two would generate a wave which will give the elliptically polarized wave of left handed sense and since the wave is going in this direction the rotation which is like this will be the right handed rotation we are going we are seeing in this direction so the opposite to that that will give the wave which will be the left handed polarized wave.

The next problem which we can now take is generation of a linearly polarized wave by a combination of two circularly polarized waves.

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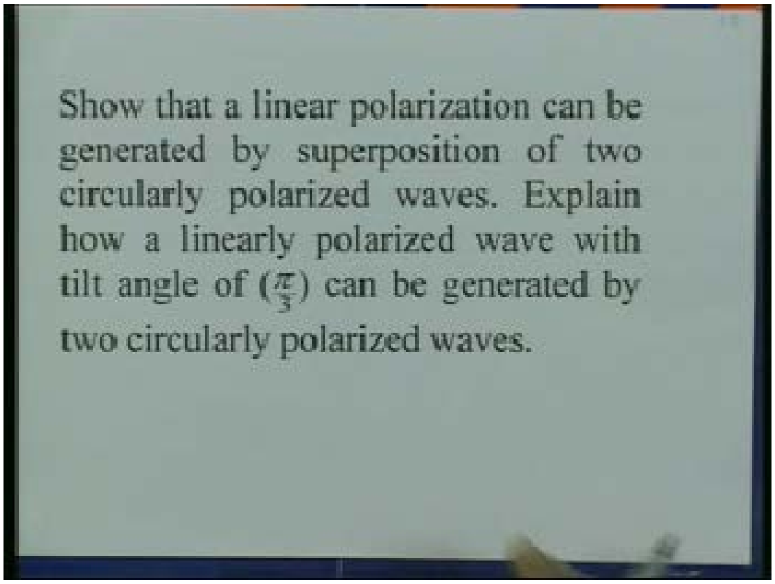

By discussing the polarization we have seen that an arbitrary state of polarization can be generated by a superposition of two orthogonally polarized waves. In the analysis we took two polarizations which were x and y oriented polarizations and they were orthogonal to each other and then we showed that by a combination of these two essentially we can generate a general elliptically polarized wave. So, by eliminating the time parameter essentially we got the equation of ellipse representing that the electric field vector would draw a shape which will be an ellipse.

One can ask a question if I have to now generate a state of polarization by using some other orthogonal state or a pair of some orthogonal states; how would I proceed. An immediate thing which can come to your mind is that if we have two circularly polarized antennas... so if I have two circularly polarized wave with opposite sense there how do I generate an arbitrary state of polarization. So let us take a problem here.

Show that a linear polarization can be generated by a superposition of two circularly polarized waves. Explain how a linearly polarized wave with a tilt angle of pi by 3 can be generated by two circularly polarized waves. So essentially now we want to represent a linearly polarized wave in combination of two circularly polarized waves.

Now in this case first the circularly polarized wave is given to you or two set two circularly polarized waves are given to you of opposite sense of polarization and now we want to find out when I combine this how a linearly polarized wave will be generated. So, as we did earlier when we were writing the electric field in terms of x and y components we represented the field in two components which had x and y. Precisely the same thing I can do, I can take the electric field and represent in the two components which are now left and right handed polarized wave. So, for that, first we have to define what are called the unit vectors corresponding to left and right handed polarized waves.

So let us say that the wave...we take a coordinate system which is let us say like that so this is x this is y and this is z (Refer Slide Time: 21:14) and assume that the wave is traveling in positive z direction so a unit left handed polarized wave would represent a vector whose amplitude remains 1 and it rotates in the left handed side. So since the wave is coming out of the paper this will represent this rotation will represent the left handed rotation so I have an electric field which is now rotating in the clockwise direction which will represent a left handed polarized wave.

Similarly, if I consider a wave of unit amplitude whose vector is rotating in the anticlockwise direction that will represent a circularly polarized wave with unit amplitude with right handed sense. So this is the right handed circularly polarized wave and this is the left handed circularly polarized wave.

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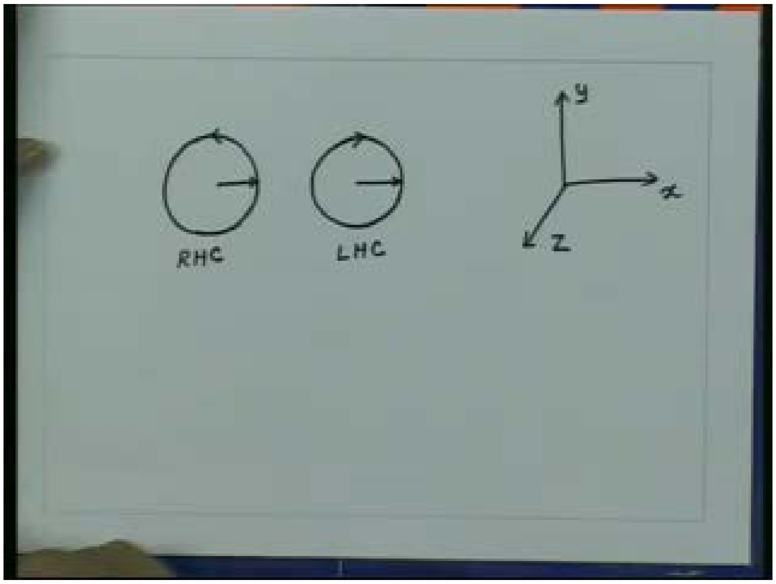

So if I take the amplitude of this circle the radius of this circle as 1 then that represents a unit left handed polarized wave and this would represent the unit amplitude right handed polarized wave. So this one then I can denote as the unit vector for circularly polarized wave so let me call that as R cap and this one let us call as L cap.

Now we know that a left handed polarized wave can be generated by two unit amplitude x and y polarized wave with a phase difference between E y and A x of plus 90 degrees and the right handed polarized wave can be generated by combination of two unit vectors x and y with E y lagging by 90 degrees with respect to E x. So in this case as we know that if I consider E x equal to 1 and E y is equal to 1 angle plus pi by 2 we will generate a left handed circular circularly polarized wave of unit amplitude. Similarly, for right handed if we take E x is equal to 1 and E y is equal to 1 angle minus pi by 2 we generate a unit amplitude right handed circular polarized wave so that means this L vector unit vector then we represented as x plus 90 degree phase shift into y so we get here the L unit vector we can write as the unit vector x plus j into unit vector y.

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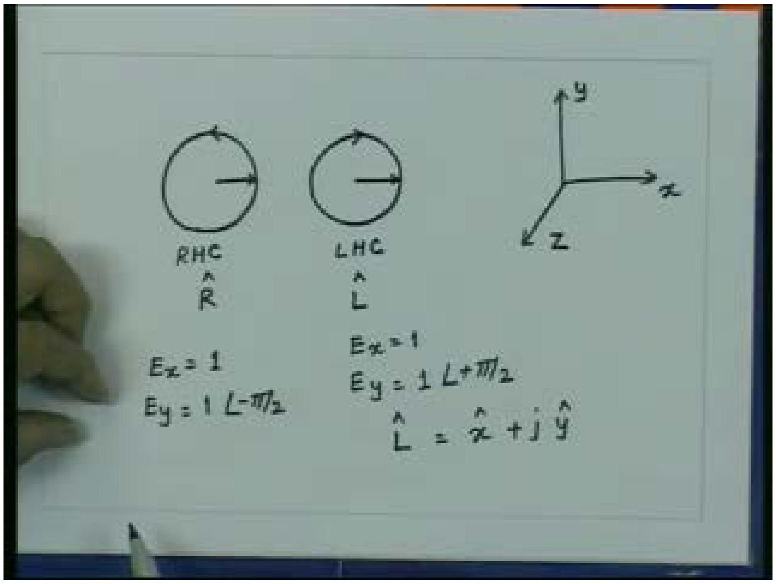

Similarly, for the right handed case we get R unit vector that is equal to x minus j into y. So now essentially we represented the unit right handed and left handed vectors in terms of the unit x and y vectors. By solving these two essentially we can get of the x unit vector is L plus R upon 2 and y unit vector that is equal to l minus r upon 2 j.

Now if we consider a electric field we can represent the electric field either by the unit vector which are R and L or we can represent the unit vector in terms of x and y. So any electric field now can be represented as E x unit vector x plus E y unit vector y or this is equivalently represented by unit vectors L and R. So let us say the amplitude for that is E L and the amplitude for the right handed component is E R unit vector R.

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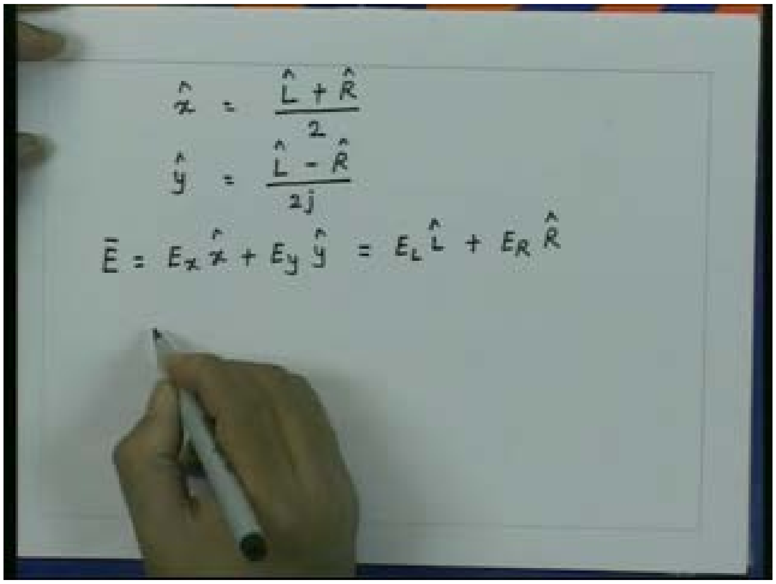

Substituting now for x and y from from here, essentially this is equal to E x x component which is L plus R upon 2 plus E y l minus r upon 2j. So separating the component corresponding to L and R essentially we get E l is E x minus j E y upon 2 and E r equal to E x plus j E y upon 2.

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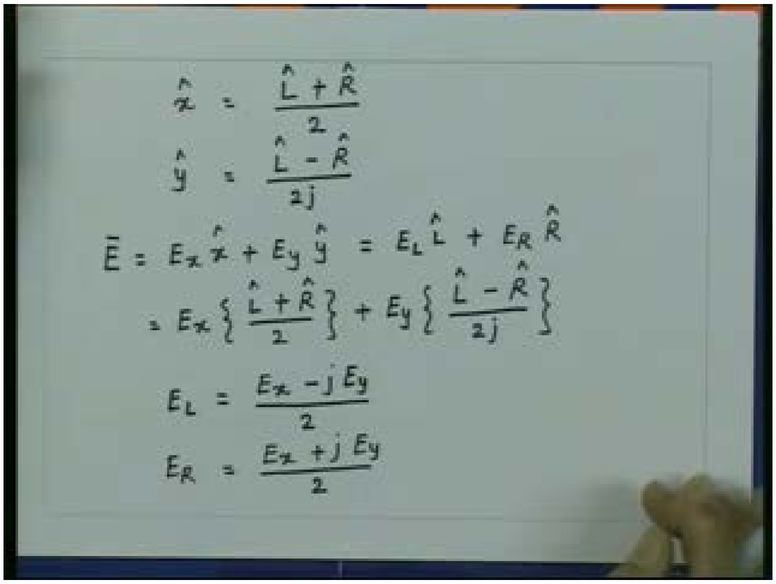

Now we want to represent... the problem is we want to represent a linearly polarized wave by the superposition of the two circularly polarized waves. So essentially the field electric field which we are representing here (Refer Slide Time: 27:48) this field is linearly polarized and we know that the linearly polarized wave is generated when E x and E y are in phase; there is a phase difference between E x and E y that gives you a polarization which is linear polarization. So without losing generality we can we can take E x and E y to be real and then one can write this as half square root of this amplitude E x square plus E y square and angle will be tan inverse of minus of E y upon E x. Similarly this amplitude would be half square root of E x square plus E y square angle tan inverse of E y upon E x.

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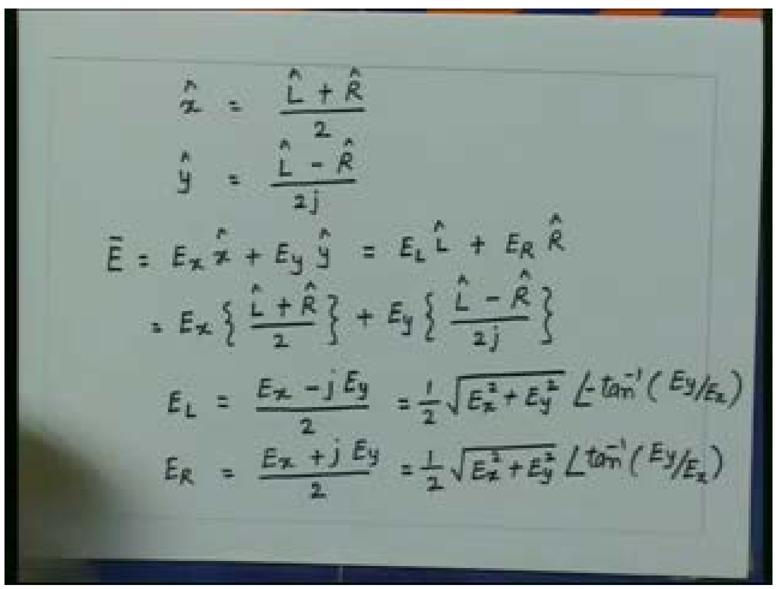

Now this quantity (Refer Slide Time: 29:03) if we note here tan inverse E y upon E x that is the quantity which gives the inclination of the line which the linearly polarized wave draws. So if I consider a xy plane this is xm this is y, a line linearly polarized wave will look like that and this angle would be nothing but tan inverse of E y upon E x.

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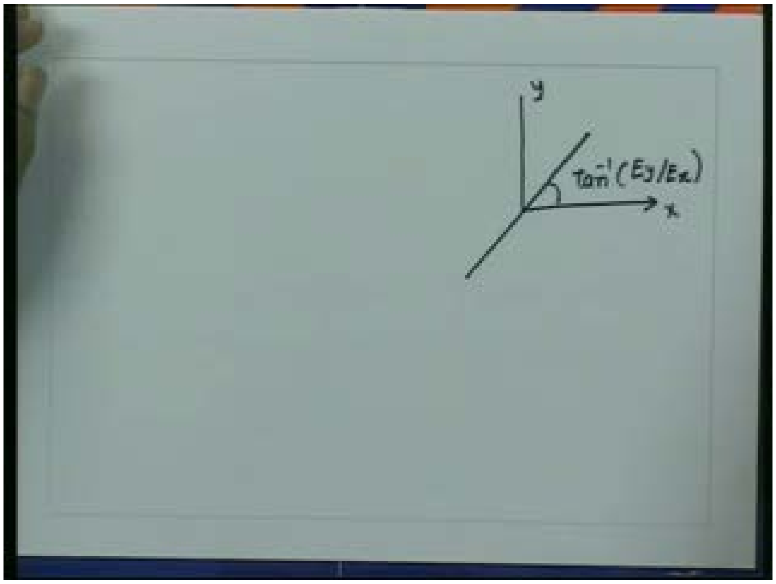

So what we see from here is that the tilt angle for the linearly polarized wave will be nothing but the phase of this component E r or negative of the phase which is this quantity. So if I... so what that means essentially is that the tilt angle tau is equal to the angle of el is equal to minus angle of E r and for linearly polarized wave these two quantities are same so the magnitude of E l and E r should be equal and tilt angle is this and mod E l should be equal to mod E r.

So what that means is if we consider the two circularly polarized wave or left and right handed and if their amplitudes are same and if they have a phase then the tilt angle will correspond to the phase of the left handed polarized wave and the right handed polarized wave will be having the negative phase.

Now what is the meaning of phase for a vector which is rotating in the right handed or left handed side?

So if we consider now a left handed wave a rotation of the vector in the direction of rotation that will correspond to the positive angle whereas if I take the rotation which is op or angle which is opposite to the rotation of the vector then that angle will be considered as a negative angle.

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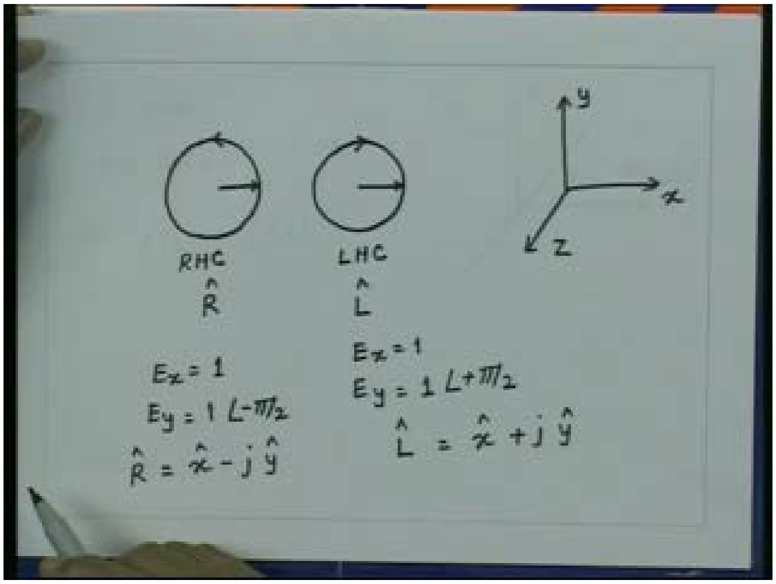

So what essentially that means is that some instant of time you launch the wave and they are rotating in the opposite directions, they will meet in the direction which will correspond to this angle which is E y upon E x. So essentially what we what we have, if we take the two waves one is a left handed and right handed let us say this is left handed (Refer Slide Time: 32:16) this is right handed and if I consider now a vector at some instant of time which is like this and like that; if I consider this as some difference direction at that instant of time the two vectors were like this. Now this vector will rotate by an angle and this vector also will rotate by an angle so when this rotate... so this angle if we take will start with as 2 tau, the two vectors will meet half way so that will give me the direction of the resultant vector which will be at an angle tau and that time the two vectors will align to give you the maximum field. So if these two are equal and if we start with the vector position which is like this so the difference between the two vectors is 2 tau then you will get a linearly polarized wave with an angle tau, so this angle is tau (Refer Slide Time: 33:25).

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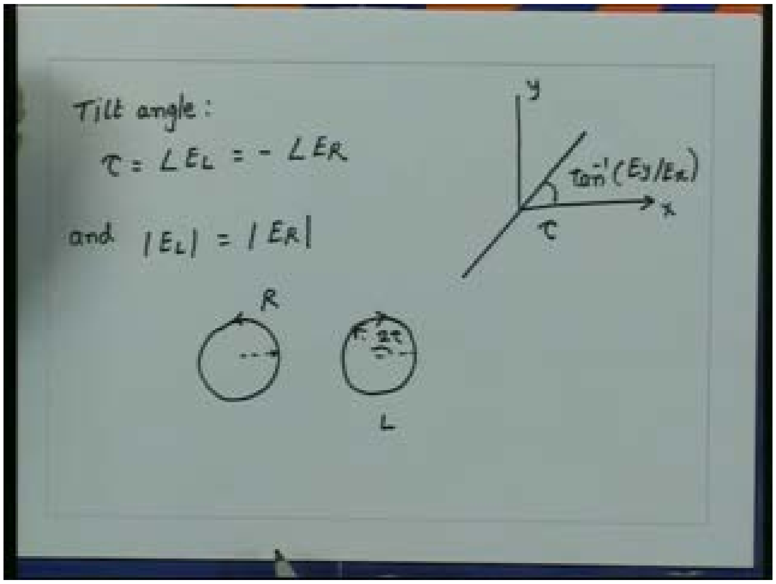

So for generation of a linearly polarized wave by combination of two circularly polarized waves essentially is that you should have two circularly polarized waves which have equal magnitudes and then at the instant of time when these two waves start propagating their vector should be oriented such that the the angle between these two vectors is equal to 2 tau and then they will meet half way so you will get a linear polarization which will be inclined at an angle of tau.

One can verify this that when these two vectors are these two amplitudes are equal we will always get a linearly polarized wave.

So consider now situation like this. This is your one vector which is rotating (Refer Slide Time: 34:22) and there is another vector which rotates in the opposite direction. So this is my left handed and you have another vector which is of same amplitude; for the clarity reason let us put this little inside this dotted line, this is the vector which is rotating in the opposite direction. So the two vectors meet; this is the direction which is x so these two vectors meet in this direction to get a double value, this angle is is tau.

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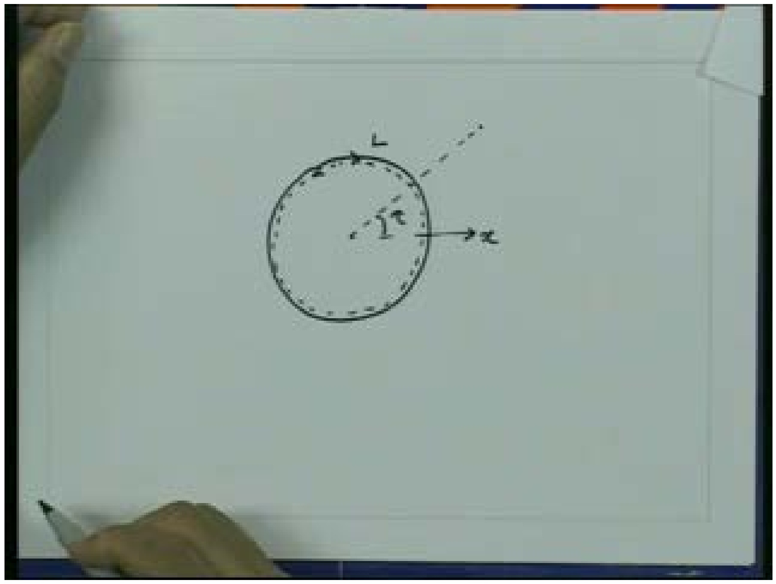

Now, since these two vectors are going to make always equal angle with respect to this line as they as they rotate this vector will rotate this way, (Refer Slide Time: 35:12) this vector will rotate that way, the component of the vector which is perpendicular to this line will be of equal magnitude and are in opposite direction so it will always cancel out and as the vector rotates the component which is in direction of this line will vary so when the two vectors are in this direction you get a maximum value which is this, when the two vectors are let us say in this direction: one is this, other one is that (Refer Slide Time: 35:42) the resultant of these two vectors again would lie corresponding to this so it will become like that the value will be this and when the two vectors become perpendicular to each other that time they will cancel and the amplitude will go to zero and when the vector rotates now further essentially the quantity will move like that and after half cycle the two vectors will come exactly in the opposite direction here, they will again add and you will get an amplitude which is double. So you will get a polarization of the wave which essentially will draw like that so this amplitude now will be double of the amplitude of the individual circularly polarized wave.

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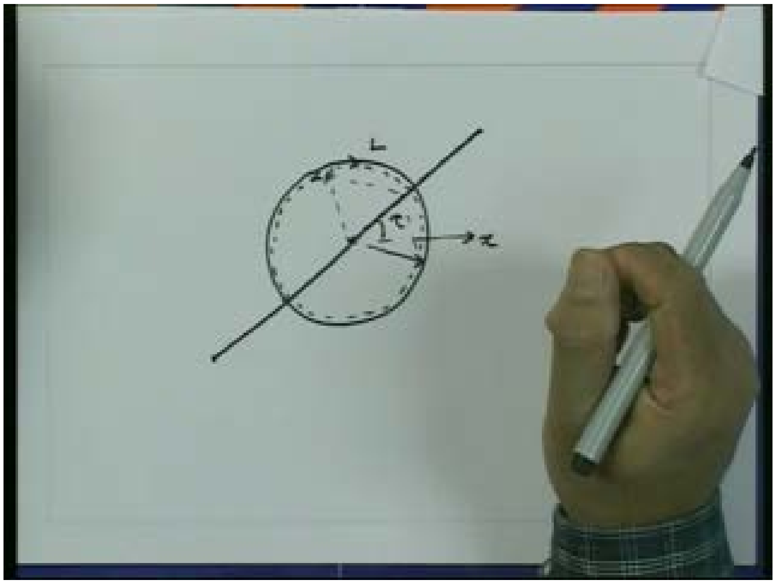

So this problem essentially explains how a linear polarization can be generated by two orthogonal states which are two circular polarized waves. The concept can be extended now to any other polarization, the analysis will be exactly on similar lines so you can write down the E x E y and you can write down E l and E r equate the component and then from there you can find out the relation between the E l and E r.

Let us take now a problem on the dielectric and conducting property of the material.

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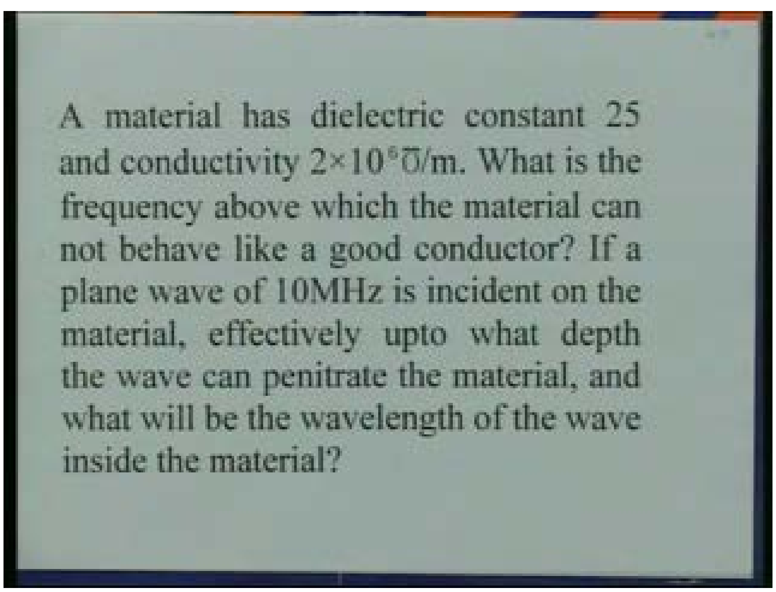

So here is the problem: A material has dielectric constant of 25 and conductivity of 2 into 10 to the power 0 mohs per meter. What is the frequency above which the material cannot behave like a good conductor? If a plane wave of ten megahertz is incident on the material, effectively up to what depth the wave can penetrate the material, and what will be the wavelength of the wave inside the material?

So now in this case we are talking about a composite material which is neither ideal conductor nor ideal dielectric. But the conductivity is large here so this is more like a metal. But still we want to find out when the material can be treated like a dielectric and when the material can be treated like a good conductor.

So, as we have seen that this is decided essentially by the contribution which the material gets from the conduction current and from the displacement current. So we say the material is a good conductor... for a good conductor we should have sigma much much greater compared to omega epsilon 0 into epsilon r.

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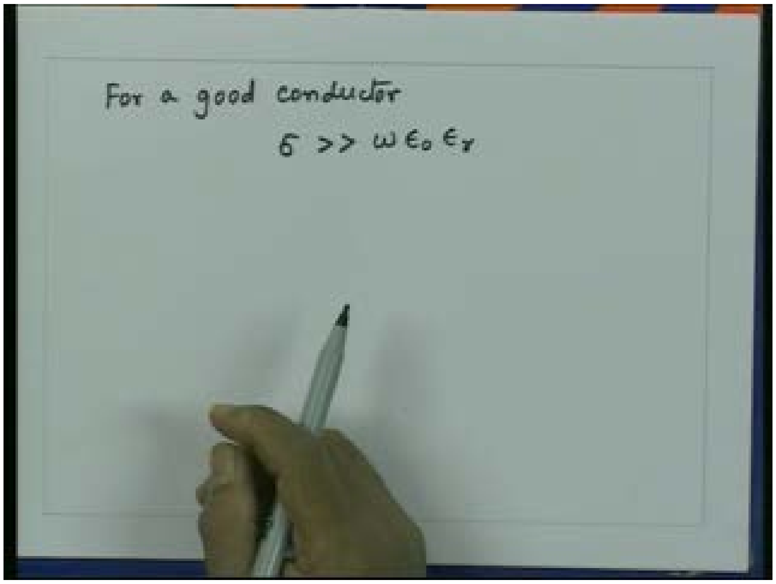

So let us say that this condition is much much greater means if this quantity is ten times this quantity just an arbitrary number then we say the material will behave like a good conductor. So let us say that the sigma should be greater than or equal to ten times omega epsilon 0 into epsilon r. From here then I can I can find the frequency which is omega that is equal to sigma upon 10 times epsilon 0 epsilon r.

So, if the frequency is less than this then the conductor material will behave like a good conductor, if the frequency is more than this then we will say it is no more a good conductor. So I can substitute now the values for sigma and epsilon r given here (Refer Slide Time: 40:15); sigma is 2 into 10 to the power 6 Ohms per meter and the dielectric constant which is epsilon r that is 25.

We can write here 2 into 10 to the power 6 divided by 10 into epsilon 0 which is 1 upon 36pi into 10 to the power of minus 9 multiplied by 25 which is dielectric constant.

Solving this essentially we get frequency, so omega which is equal to 2pi into f we get a frequency f which is 1.44 into 10 to the power 14 hertz. So what that means is that when the frequency is above 10 to the power 14 hertz the material will stop behaving like a conductor good conductor and this frequency if you recall is the frequency which is of the visible light somewhere here so that means by the time we reach to the visible light frequencies the material which would have the conductivity of 2 into 10 to the power 6 and that dielectric constant would no longer start move would no longer work like a conductor so they may start behaving more like dielectric.

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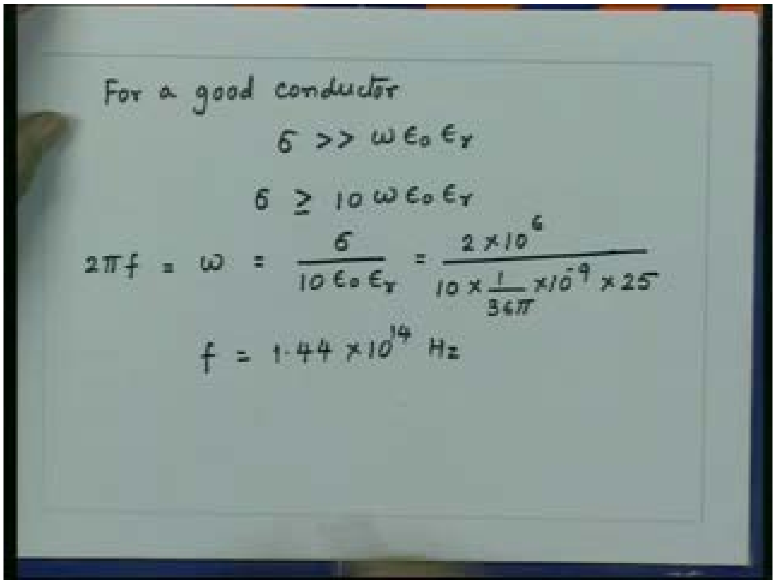

Now the second thing which we have to find out in this problem is the effective depth up to which the wave will propagate and that we know is essentially given by what is called the skin depth. So we get the skin depth this delta that is equal to 1 upon square root pi f mu 0 into sigma. So substituting now for these quantities which is delta the delta is equal to this is 1 upon square root pi, the frequency is 10 megahertz which is 10 to the power 7 into mu 0 which is 4pi 10 to the power minus 7 into sigma which is conductivity which is 2 into 10 to the power of 6.

So substituting and solving this now we get the skin depth which is 112.54 micro meter...

and recall the skin depth essentially is the distance over which the wave amplitude reduces to 1 over e of its initial value. So in this material over a distance of 112.54 microns the wave and amplitude would reduce to 1 over e of its initial value.

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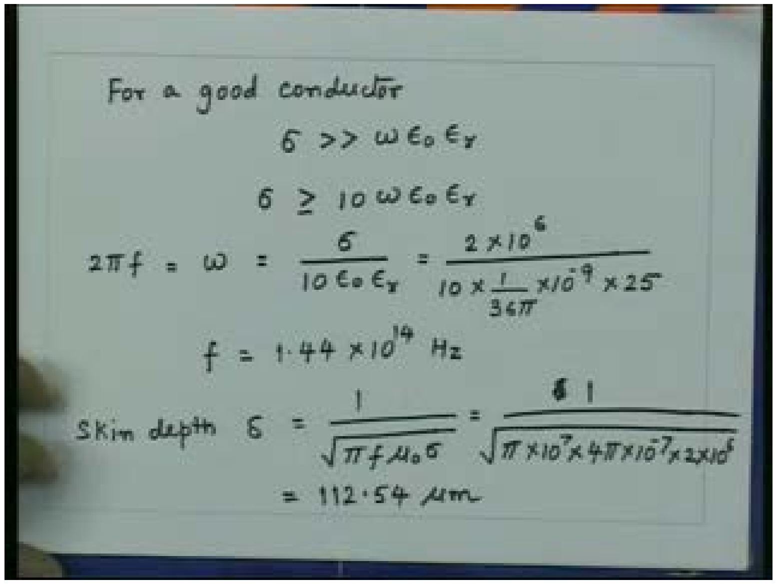

The next thing which is asked in this problem is what will be the wavelength of the wave inside this material. And as we know the wavelength is related to the phase constant of the material so we first find out what is the phase constant beta which is square root of pi f mu 0 into sigma and that if we calculate we essentially get equal to 8885.5 radians per meter and therefore we get the wavelength lambda which is 2pi divided by beta is equal to 2pi divided by this quantity 88885.5 per meter so that gives you approximately 0.707 micro meter millimeter.

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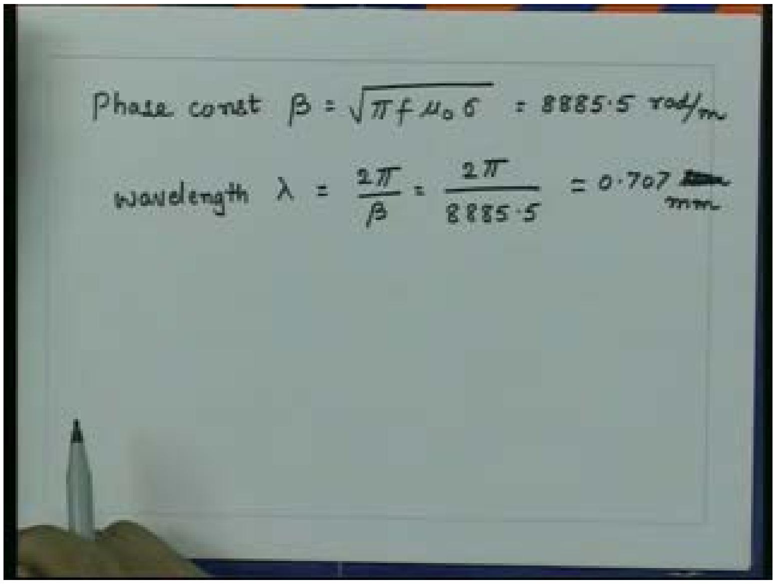

So, at 10 megahertz wave when is when it goes in a material which is having a conductivity reasonably large about 2 into 10 to the power of 6 mhos per meter the wavelength becomes as small as 0.7 millimeter. Note: when the same wave goes 10 megahertz into into the free space the wavelength for this is 30 meters. So in the free space this wave will have a wavelength of 30 meters whereas when this wave goes into a material which is a highly conducting material like this then its wavelength will reduce to as small as 0.7 millimeter. This conductivity in fact is very close to the conductivity of copper, copper has a conductivity of 5.6. So if we take a good conductor that means in practice what... we notionally have good conductors that is like copper or silver or brass then the conductivity of those things will be typically in this range and then the wavelength of the wave will reduce to few millimeters when the wave penetrates this material.

Let us take one more problem. Let us say some instant and some location inside a lossy dielectric material the measured peak electric field of a wave is 10 volts per meter. The material has relative permittivity of 8 and conductivity of 100 mhos per meter. Find the average power density of wave at that location. Also find the power density at a distance of 1 centimeter in the direction of the wave propagation inside the material. The frequency of the wave is given to be 300 megahertz.

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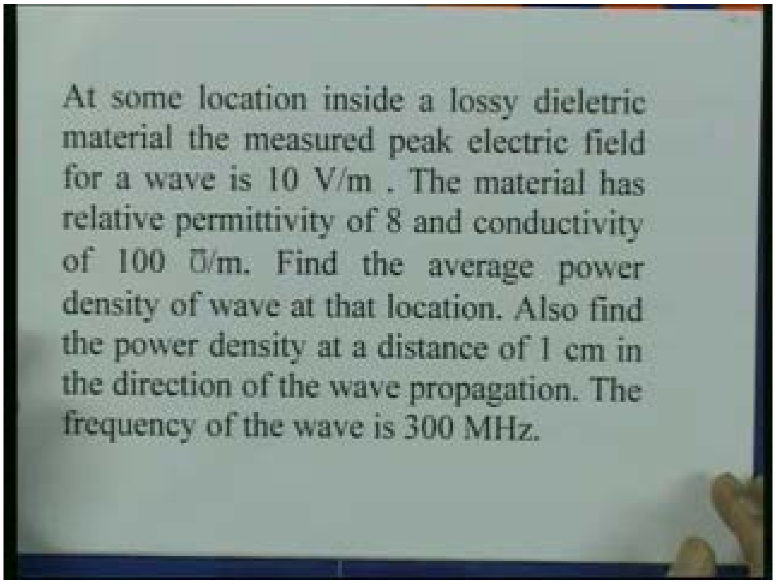

So since the electric field is given and the medium properties are given we can find out what is called the pointing vector which gives me the power density at that location. again since the medium properties are given we can find out the propagation constant and there we can find out what will be the amplitude of the electric field at a distance of 1 centimeter in the direction of wave propagation and then again we can calculate the pointing vector at that location.

So the frequency which is given is 300 megahertz; say in this case omega is equal to 2pi into 300 into 10 to the power 6 radians per second. And the intrinsic impedance of the medium eta for this that is equal to square root of j omega mu 0 divided by sigma plus j omega epsilon 0 epsilon r. So, substituting now this value that the conductivity sigma is 100 ohm mhos per meter and then permittivity is 8 we get here sigma is equal to 100 mhos per meter and epsilon r is equal to 8. We get the intrinsic impedance of the medium eta that is equal to 3.44 plus j 3.44.

Once I know the intrinsic impedance of the medium and the electric field which is 10 volts per meter then we can find out the average pointing vector p average that is equal to mod of e square upon 2 and here the peak electric field is given which is 10 volts per meter so this 2 factor will come because of this rms power this is the real power of 1 upon eta conjugate.

Substituting now for eta from here and the electric field which is 10 volts per meter peak amplitude we get power density which is 7.27 watts per meter square.

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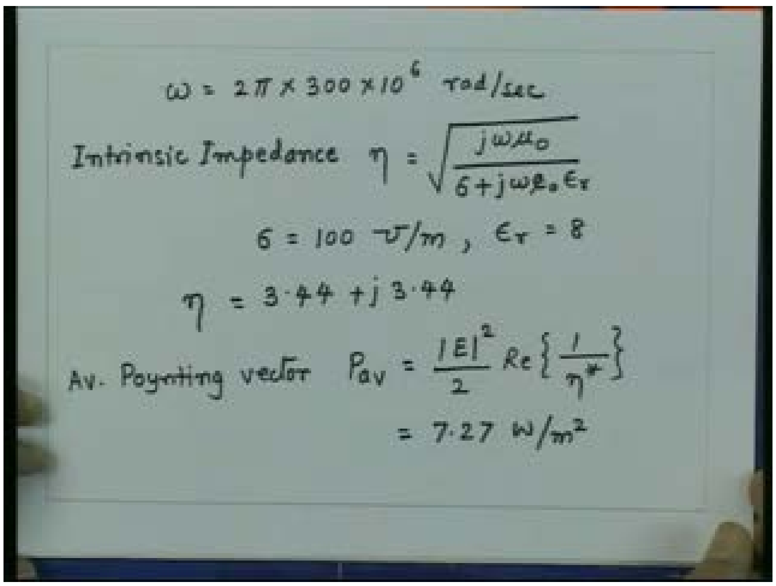

To find now the electric field at a distance of 1 centimeter we require the propagation constant so we get propagation constant gamma which is alpha plus j beta and that is equal to square root of j omega mu 0 sigma plus j omega epsilon 0 epsilon r. I can substitute in the values for sigma and epsilon r we get the values which are 343.9 plus j 344.6 per meter. So the attenuation constant alpha for this would be 343.9 nepers per meter. So the power density at 1 centimeter would be p average at 1 centimeter from the initial location that will be equal to the initial p average we got e to the power minus 2 alpha into distance which is 0.01 that is 1 centimeter.

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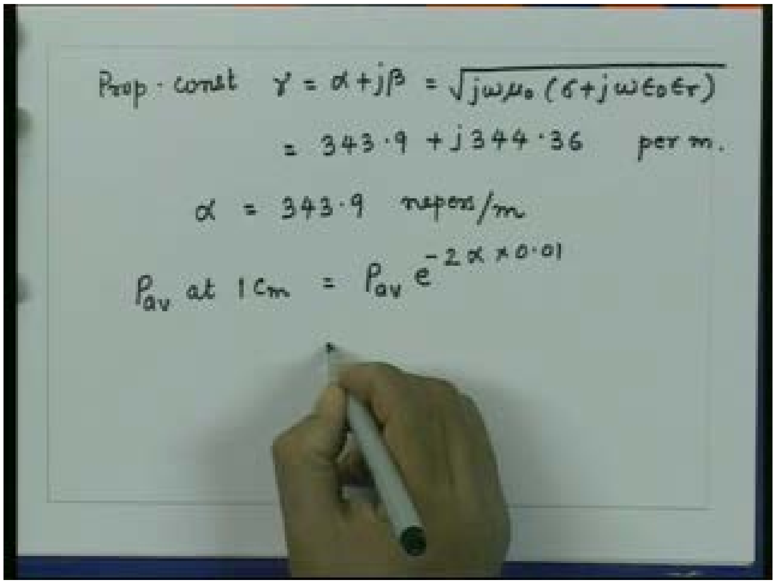

So substituting for value for alpha from here we get the power density at 1 centimeter that will be equal to 7.49 milliwatts per meter square. So essentially by systematically following the theory which we have developed for uniform plane wave we can... using the medium properties essentially we can find out the electric and magnetic fields or the power density or the state of polarization of the electromagnetic wave. So these problems essentially give some hand on experience in using the theory which we have developed for the uniform plane waves.