Transmission Lines and E.M. Waves
Prof R.K. Shevgaonkar
Department of Electrical Engineering
Indian Institute of Technology Bombay

Lecture-56

Welcome, we are discussing an important topic called Array Synthesis. In the last lecture we investigated the arrays whose radiation pattern is specified in terms of its nulls. We saw that there are applications where we would like to place some nulls in the specific directions to avoid the interference coming from those directions if the radiation pattern is specified in terms of the nulls of the radiation pattern then we can synthesize an array by using the circle diagram that is what we essentially investigated in the last lecture.

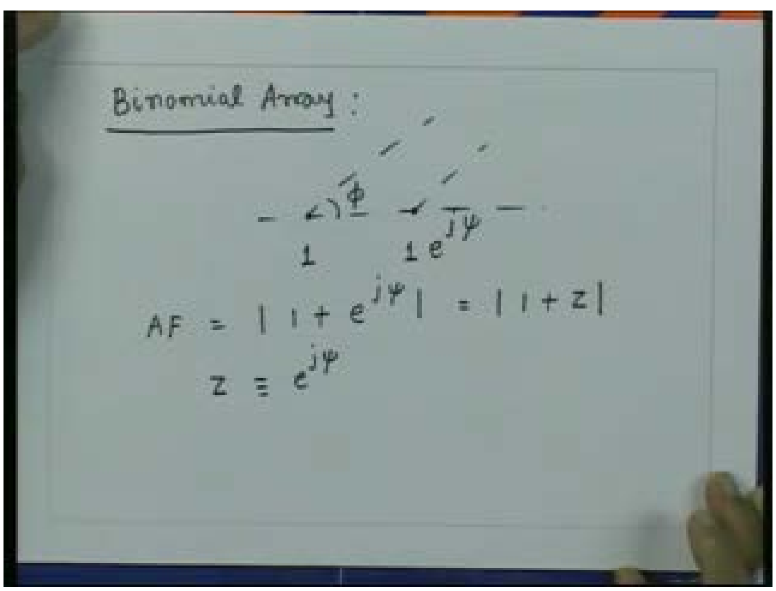
(Refer Slide Time: 02:20 min)

Today we investigate one more interesting array called the binomial array and later on we will go to the general array synthesis. As we have seen when the number of elements in an array increase the number of nulls also increase in the radiation pattern that means there are directions in which there is no radiation now we increase the size of the array because we want to increase the directivity as we have seen as the size of the array increases the beam width of the radiation pattern decreases and consequently the directivity of the array increases.

So if we consider the uniform array then by increasing the length of the array or by increasing number of elements in the array we can increase the directivity but at the same time we also get more nulls in the radiation pattern. There maybe some applications where these nulls are undesirable we do not want to increase the nulls in the radiation pattern but at the same time we want to increase the directivity of the array. Precisely that is what is realize by an array what is called the binomial array.

The concept is vary simple, let us consider a two element array to start with and as we have seen the radiation pattern for this can be given in terms of the z which we defined as a complex parameter last time so this one is having amplitude 1 and this one also has let us say amplitude 1 and it has a phase ψ into e to the power j ψ where the ψ is βd cosø so the angle ø is measured from the axis of the array so this angle is ø. So the array factor as we saw last time is equal to mod of one plus e to the power j ψ which we wrote as 1 + z where z is e to the power jψ so we defined this complex parameter z which is equal to e to the power j ψ.

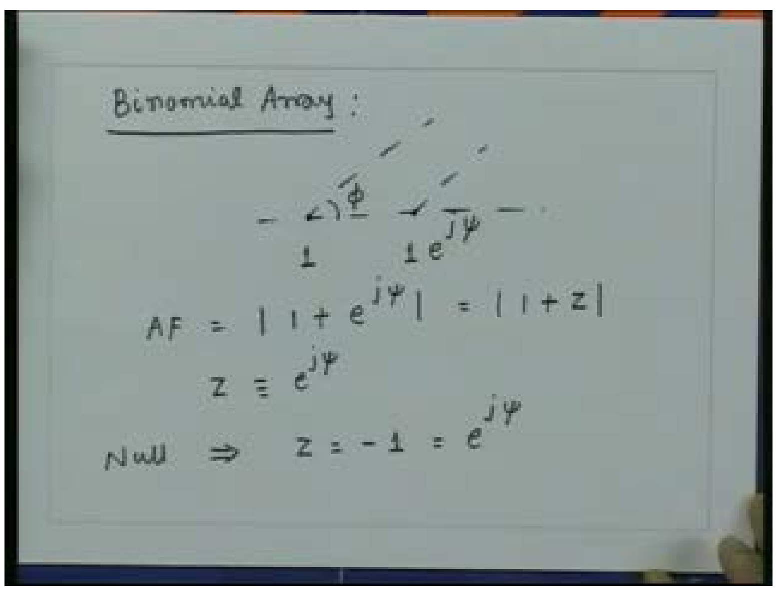
(Refer Slide Time: 05:00 min)



Now for this array we have a null when z is equal to -1 so when this quantity goes to zero we have nulls. So for this array the null correspond to z is equal to -1 that is e to the power j ψ is equal to -1. So whenever this condition is satisfied you get the null in the radiation pattern.

Now if you want that the directivity of the array should be increased but the nulls should not change in the radiation pattern one of the simplest way is that if I consider this as a polynomial and if I take N th power of this polynomial then the nulls of the function will not be change we will have again the nulls which will correspond to z equal to -1 and by expanding this polynomial then we will get N +1 terms which will represent an array. So as we have seen in the last lecture a polynomial in z can represent an array and an array can be considered as a polynomial.

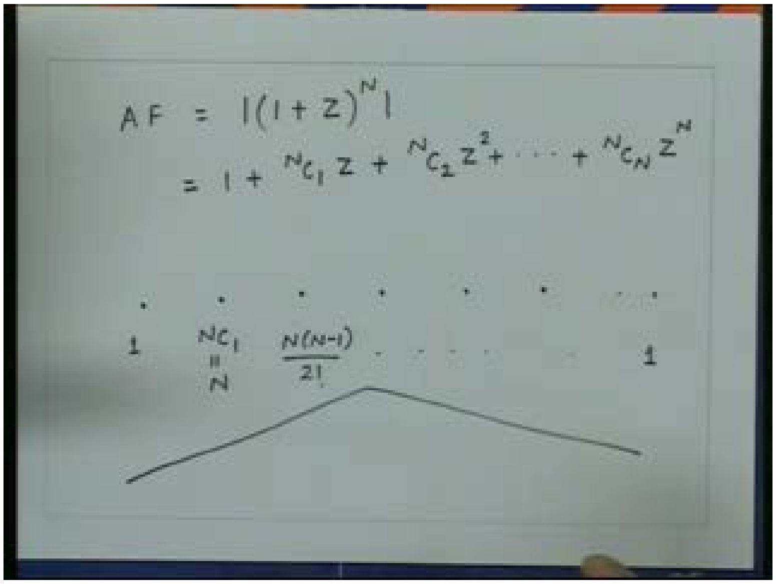
(Refer Slide Time: 06:13 min)



So if I consider an array factor which is one plus z to the power N we get the new array factor which is one plus z to the power N so if I expand this now binomially I will get this is one plus Nc_1 z plus Nc_2 z square plus and so on with plus Nc_N z to the power N. As we discussed in the last lecture if I take the coefficients of z they essentially give me the current excitation so now this array factor corresponds to an array which have (N + 1) elements because there are N + 1 terms in this polynomial and the excitation coefficient would correspond to Nc_1 Nc_2 up to Nc_N. So if I consider an array of N + 1 elements with the coefficient as one, Nc_1 which is equal to N, Nc_2 which would be N(N – 1) upon 2! and so on till I go to the last element which is Nc_N which is again equal to one.

If I have the current excitation which are given by this then the radiation pattern for this array would have the nulls only which are given by z equal to -1 so the nulls will not change but the length of the array is increased now there are N + 1 elements in the array and if I look at the current distribution it will start from one there will be n and then the current will increase up to the center of the array and again it will decrease we have a symmetric function for the current so essentially the current distribution if I look at the array it will it will look something like that and this will give me the radiation pattern which will have the same nulls which would correspond to z equal to -1.

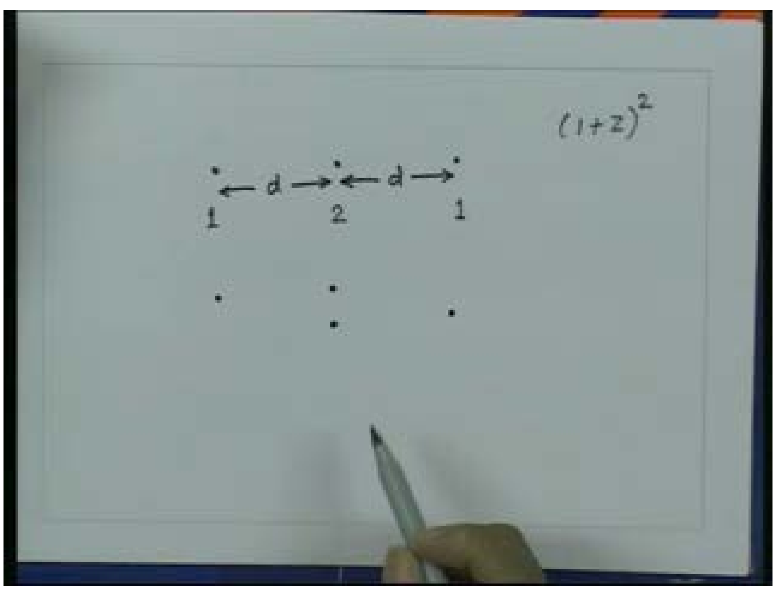
(Refer Slide Time: 09:01 min)



We can look at this in a little different way from our basic the analysis of the array. Now let us consider a three element array which is having binomial coefficients so let us say we have three elements which are having three elements binomial array so this would correspond to essentially one plus z square that is what the polynomial would be so the coefficient for this would correspond to one two and one these are the binomial coefficient for one plus z square.

Now if I consider a radiation pattern which would be given by one plus z square null would correspond to z = -1 and expansion of this will give me three terms so you have a three element array and the binomial coefficient should be one two and one. Now let us say this distance between these elements is d and this is also d. What we can think is this array is now equivalent to having array which is current like this and a current like this so this element which is having current amplitude two I can say is some of these two elements one and another one so this array of three element is can think of as the two arrays which is one array and this is another array.

(Refer Slide Time: 10:52 min)

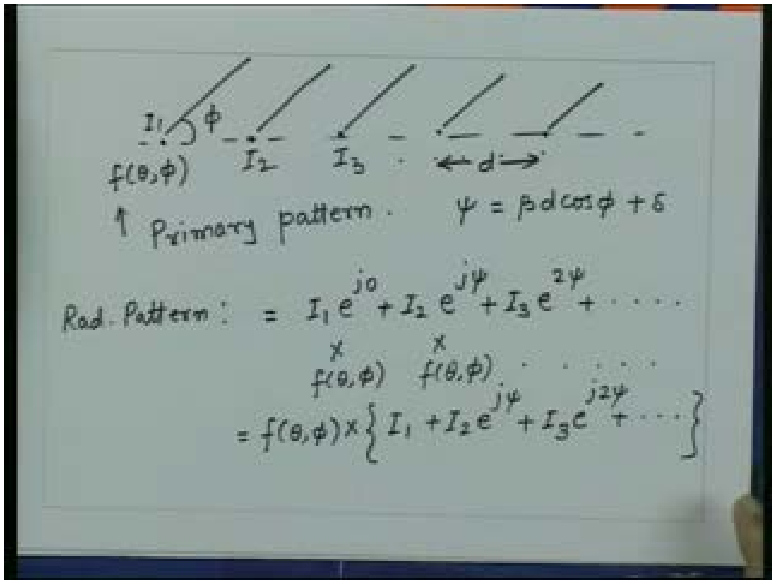


Up till now we have consider the radiation pattern of the individual element which is isotropic. Before we investigate this array one can ask a question if the array elements where not isotropic if they had some intrinsic radiation pattern how would the array analysis change. So the question we are asking before we investigate this is if we consider an array of elements which are no more isotropic but have a radiation pattern which is given by the primary pattern which is given by f(θ. ø) that means if I go to some direction theta and phi now the fields will not be uniform be because of the individual elements they will be weighted by this function what we call as the primary pattern of the antenna element. So we call this function as the primary pattern.

Now if I consider a direction which is ø with respect to the axis of the array as we have taken in our analysis let us say this angle is ø the radiation now will not be proportional to simply I but will also be proportional to what is this quantity f(θ, ø) which is the primary radiation pattern. So in a given direction the radiation is now the superposition of the fields because of the individual elements weighted by the primary radiation pattern. So if we consider that all the elements are identical so they have their primary radiation patterns which are identical all the radiations from different elements will be weighted by the same factor in a given direction. So I can write down the radiation pattern for this array which will be let us say the currents are given by I_1, I_2, I_3 and so on and if the spacing between the elements is d and if the progressive phase shift between the elements is delta then we have we defined this quantity ψ which is β d cosø plus delta and the array pattern as we discussed is equal to I_1 e to the power j0 plus I_2 e to the power j ψ plus I_3 e to the power j 2ψ plus and so on.

Each of this radiation is now weighted by this quantity f(θ, ø) so earlier we had the field at a far away point which was proportional to only current and the phase was coming because of the propagational delay as well as the propagational phase shift. Now we say that the amplitude is going to be weighted by this quantity which is the primary radiation pattern. So essentially each of this term is to be multiplied by f(θ, ø) we will be multiply this by f(θ, ø) f(θ, ø) and so on which is nothing but saying that we have this pattern f(θ, ø) which is multiplying the pattern I_1 plus I_2 e to the power jψ plus I_3 e to the power j 2ψ plus and so on.

(Refer Slide Time: 15:04 min)



So this quantity represents the radiation pattern which we would get assuming the elements to be isotropic. So we simply call this quantity as the array factor and this is the primary radiation pattern of your element so the total radiation pattern of an antenna array is equal to the primary radiation pattern multiplied by the array factor. So even if you have the antenna elements which do not have isotropic radiation pattern but the elements are identical that means their primary radiation patterns are same.

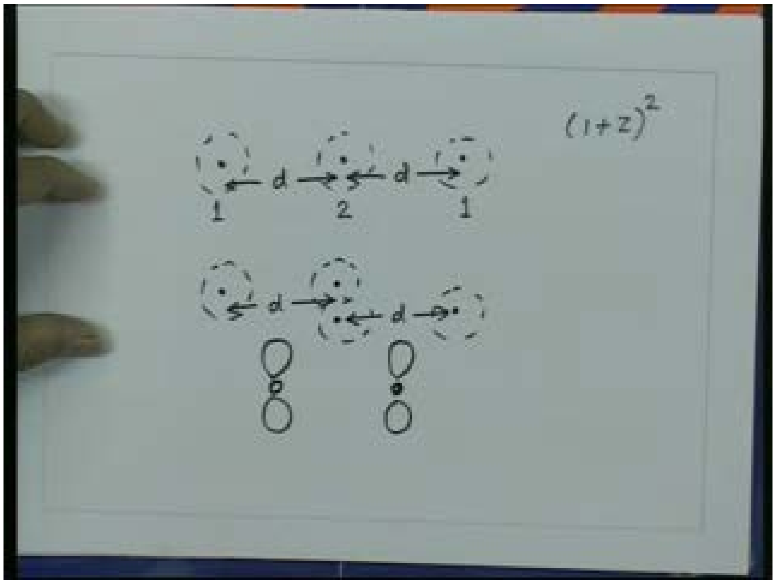
We can still treat the array to be isotropic element array and whatever radiation pattern we get that that we call as the array factor finally we multiply that array factor by the primary radiation pattern of the antenna element. So the array analysis does not change whenever we have an array of non isotropic elements first we replace those elements by isotropic sources find out the radiation pattern what we call as array factor and then that array factor we multiply by the primary radiation pattern of the antenna element and we will get the total radiation pattern.

Now this concept which is multiplication of the radiation pattern can be applied in variety of situation. It is possible that intrinsically the antenna element does not have a radiation pattern it is isotropic but I can combine some of the elements and I can treat the combination as a basic element which will then have radiation pattern which I can call as the primary radiation pattern and so on. Precisely that is what we will do in the analysis of this binomial array.

So here initially we have the elements which are isotropic elements and now we have constrain that we have a three element array which has a current distribution one two and one and then we say this is equivalent to saying that there are two elements here which are having the amplitudes one and one so essentially we are considering now this three element array as a combination of two arrays of two elements so if this spacing is d, this spacing is d and this spacing is d. So now we are saying that this is equivalent to the two arrays or two elements so this two combined together now I can call an another element which is effectively located at the center of these two I can call another element which is located at the center of these two. But now this element is not isotropic because this element is representing combination of these two elements.

So initially these elements are isotropic radiators but if I think of this as a combination of two arrays and I say this array is equivalent to this radiating element then now this one is having a radiation pattern which is a radiation pattern of these two element array. So initially the radiation patterns of they were isotropic that combination of these two are now isotropic but when we come here they are no more isotropic they have a radiation pattern which will look something like that.

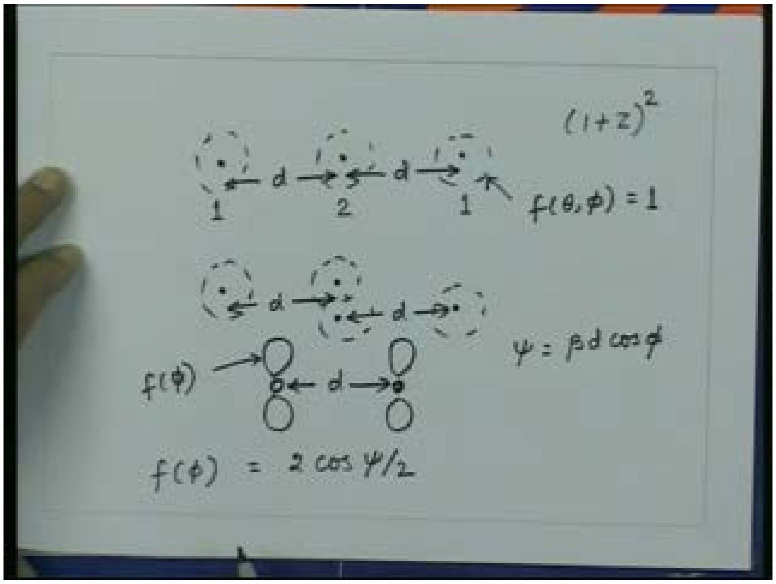
(Refer Slide Time: 19:24 min)



So in this case your f(θ, ø) is equal to one because the radiation pattern is a isotropic whereas now in this case you have a pattern which is f(ø). So essentially what we can do is first you can find out the radiation pattern of these two elements which is f(ø) then we find out the radiation pattern of this product of the two will give the radiation pattern which is the radiation pattern of this three element array. So for this in the spacing is d we can write here the ψ which is β d cosø so for these two elements when we combine we have a primary radiation pattern f(ø) which is the array factor for these two elements that would be given as f(ø) would be equal to two times cosψ/2.

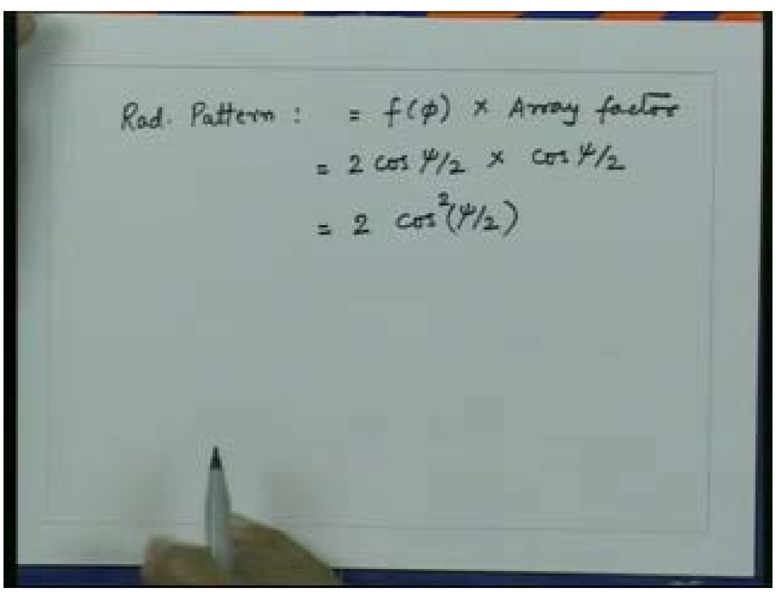
Now we if we consider the two elements which are again separated by a distance d the array factor for these two elements also is given by cosψ/2 because d is same. So the total radiation pattern is nothing but product of this with the radiation pattern of these two elements which is again cosψ/2.

(Refer Slide Time: 20:51 min)



So the total pattern which we get for this binomial array will be equal to f(ø) multiplied by the array factor which is equal to two times cosψ/2 multiplied by again the array factor which is cosψ/2 so we get a radiation pattern which is two times cosine square of ψ/2.

(Refer Slide Time: 21:44 min)

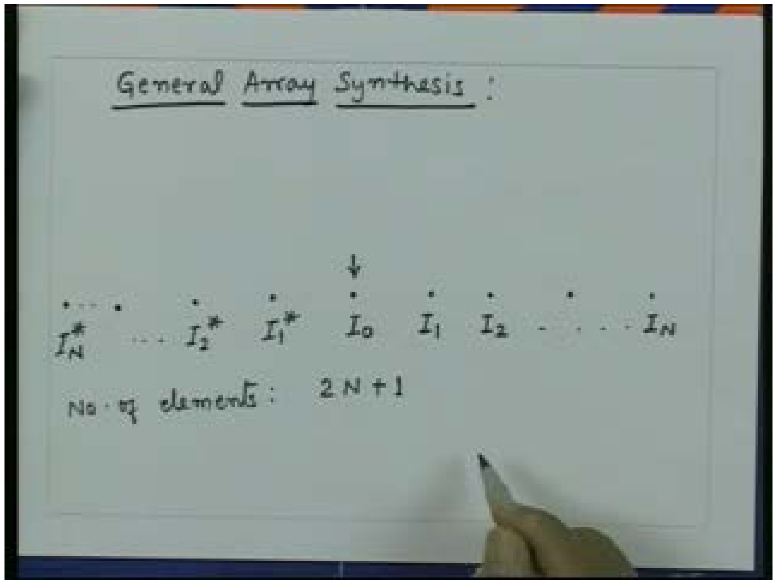


So note here that whatever were the nulls in the radiation pattern the nulls are not altered and nulls correspond again to the nulls of the two element array which are given by this quantity $\cos\psi/2$ going to zero so we got a radiation pattern which is square of this and this concept can be extended now to any number of elements. So if we are having a binomial array of n element you can think of that as a combination of doublets so we can split this into a pairs of two and then we can combine them again together to find that the radiation pattern is simply N th power of the radiation pattern of the two element array. So this array is an important array because it can increase the directivity of the array without increasing the number of nulls in the radiation pattern.

Having understood this, now we can come to the general synthesis of the array that is if somebody specifies the radiation pattern then we would like to know what are the excitation of the currents to realize that arbitrary radiation pattern. So let us now discuss the general array synthesis.

Without losing generality let us say we now have the antenna elements which are isotropic and as we have seen even if they are not isotropic we can always multiply finally by the primary radiation pattern of the antenna. So as far as the array analysis is concerned we can treat the array element to be isotropic and let us say there are 2N + 1 elements in the array so we have number of elements which are 2N + 1 and let us assume that these elements are excited with the currents which are having conjugate symmetry. So let us say we have this center element this one for which the current is given by I_0 and then we have the currents which around this I which is I_1 I_2 and so on up to I_N and on this side we have the currents which will be I_1 conjugate I_2 conjugate and so on up to I_N conjugate.

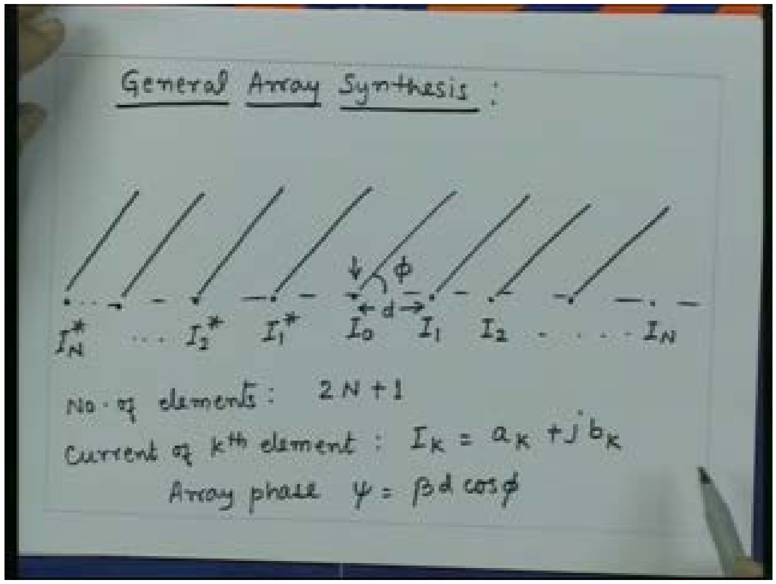
(Refer Slide Time: 24:46 min)



We are assuming essentially there is an array which is having odd number of elements and there is symmetry in the current distribution in the sense that the amplitude of the currents is symmetric and the phases of the currents are antisymmetric. So if this phase is delta this will be minus delta and so on. Now this quantity is here I so I can write down in terms of real and imaginary parts. So explicitly we can say that current of let us say K th element is I_K is given by $a_K + jb_K$.

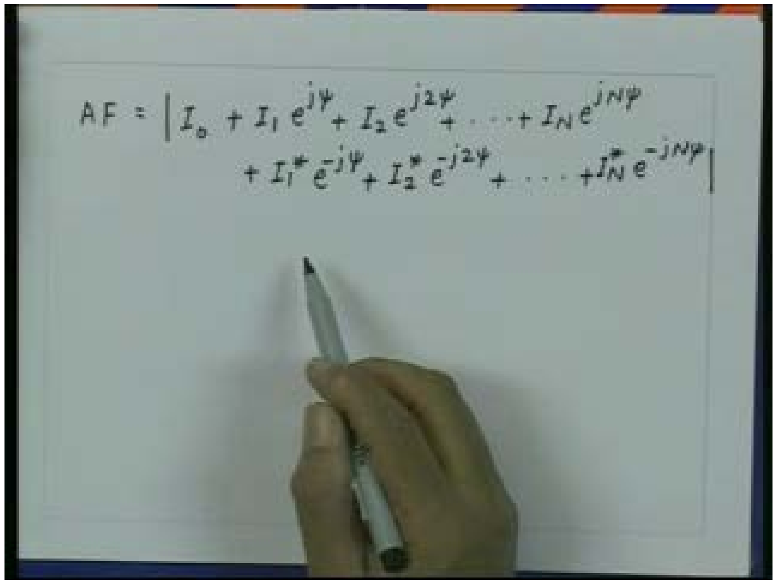
We can now do the array analysis essentially on these lines similar to what we have done earlier we consider a point which is very far away from the array in a direction let us say ø. So this is the axis of the array let us say this angle is ø and let us say we measure the distances from this reference element which is a center element which is having current I_0 so distance between the elements is d and we can define this quantity now ψ the array phase ψ which is equal to βd into cosø, we are not considering here the progressive phase shift explicitly because that can be observed into the phase of this current because we are considering this current which are complex in nature so whatever progressive phase shift we want to put on the array can be observed into this complex currents therefore we can define the array phase which is simply βd into cosø.

(Refer Slide Time: 27:06 min)



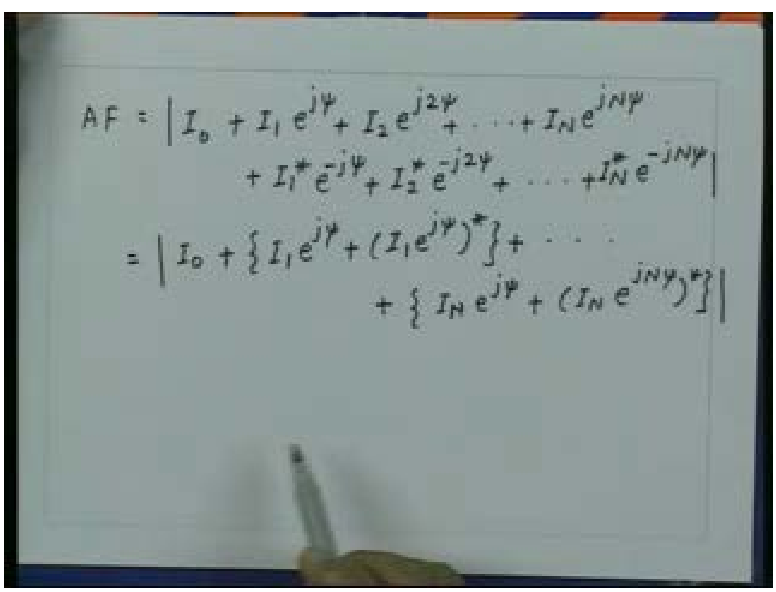
Once we do this then now we can write down the array factor for this by simply summing up the field which is induced by the individual antennas so applying simply superposition we can write down the array factor the current which is I_0 for which we are considering as the difference element so the phase for this will be zero then this will be having a phase which will correspond to this ψ so I_1 e to the power j ψ, I_2 e to the power j 2ψ and so on and same thing we can do for the things on the other side. So we can write here I_0 plus I_1 e to the power j ψ plus I_2 e to the power j 2ψ plus up to the N th element on the right side which is I_N e to the power j Nψ and then I take the element which are on this side so this will be I_1 conjugate e to the power minus j ψ because this will be now lagging with respect to this element by phase ψ so you get here plus I_1 conjugate e to the power minus j ψ plus I_2 conjugate e to the power minus j 2ψ plus so on till in conjugate e to the power minus j Nψ.

(Refer Slide Time: 29:16 min)



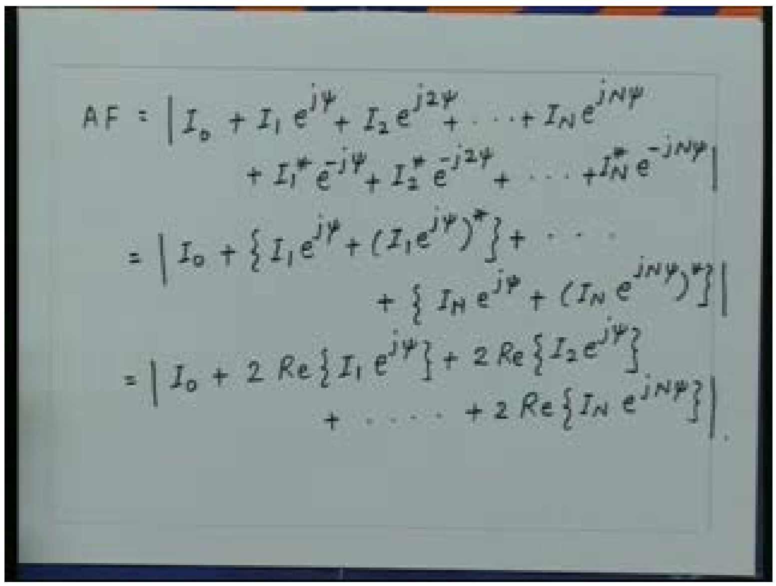
Now we can combine the terms of this and noting that this I_1 e to the power j ψ and I_1 conjugate e to the power -j ψ are complex conjugate of each other we can write this array factor as I_0 plus I_1 e to the power j ψ plus I_1 e to the power j ψ conjugate this gives me I_1 conjugate e to the power -j ψ plus so on and the last term would correspond to I_N e to the power j ψ plus I_N e to the power j Nψ conjugate.

(Refer Slide Time: 30:26 min)



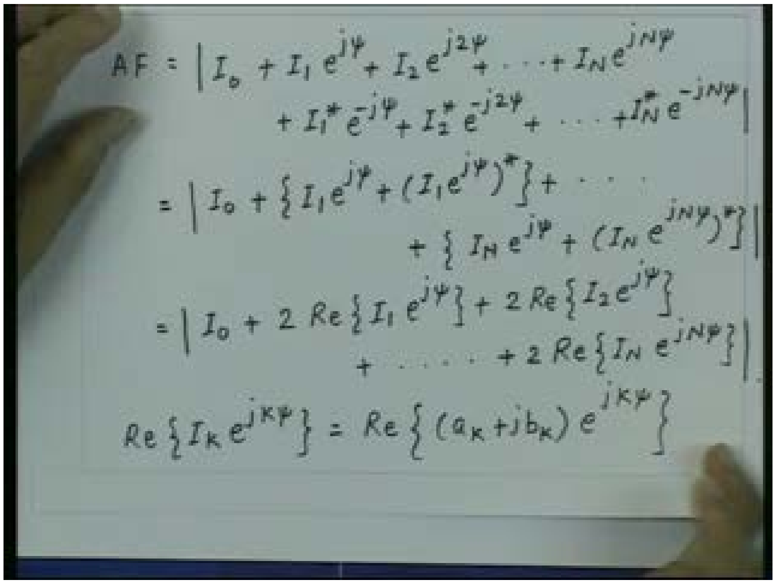
Now this thing can be combined together this is the quantity which is I_1 e to the power j ψ and the conjugate of conjugate of that. So this essentially gives the real part of this quantity two times the real part of this so we can write down the array factor as I_1 plus two times real part of I_1 e to the power j ψ plus two times real part of I_2 e to the power j ψ plus and so on two times real part of I_N e to the power j Nψ.

(Refer Slide Time: 31:20 min)



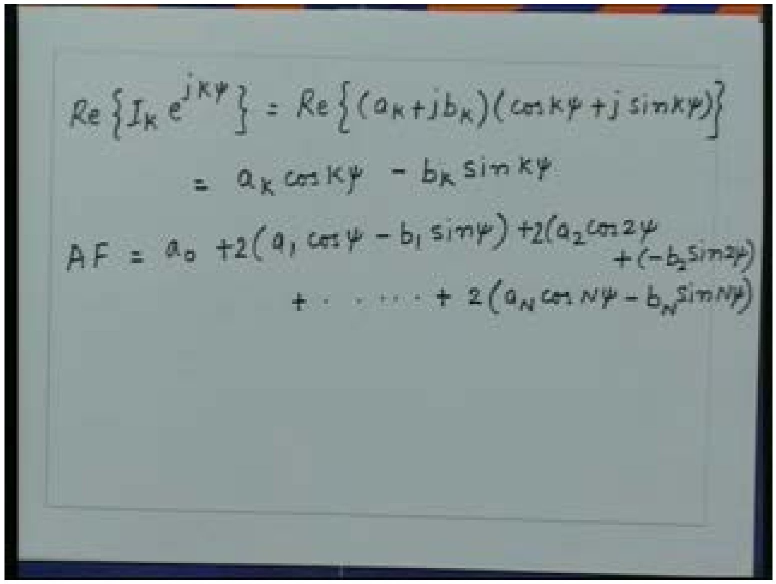
Now noting that this current I_K we can write in terms of this real and imaginary part as $a_K + jb_K$ we can substitute into this and that gives the real part of I_K e to the power jK ψ that is equal to real part of $a_K + jb_K$ e to the power jKψ.

(Refer Slide Time: 32:09 min)



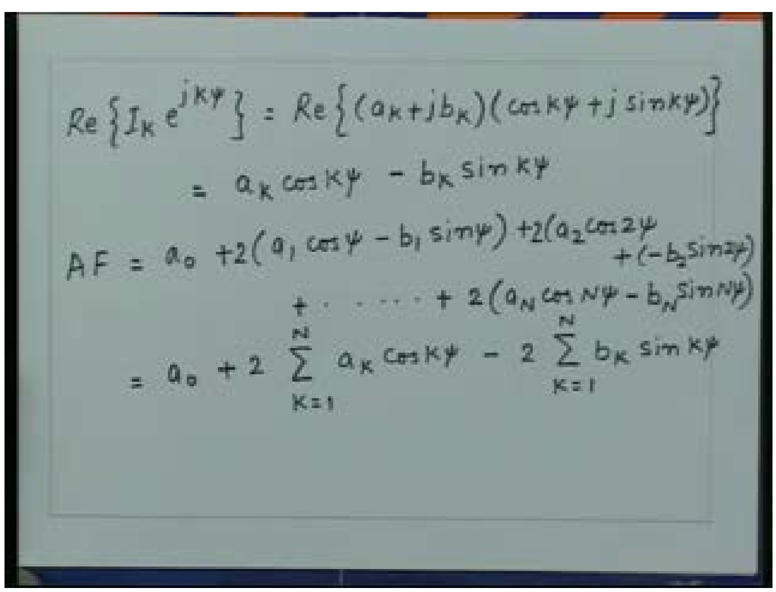
Now if I take this then I can expand as e to the power jKψ is equal to cosine of K ψ plus j sine of Kψ so from here we can get this quantity real part of I_K e to the power jKψ that is equal to real part of a_K + jb_K into cosKψ plus j sinKψ so that is equal to a_K cosKψ - b_K sinKψ. So now I can substitute in this expression for the array factor so this quantity will be two times a_1 cosine of ψ minus b_1 sine of ψ and so on. So we can write the array factor of this that is equal to I_0 and from this I_0 will be a_0 + jb_0 and since we are saying there is a conjugate symmetry the b_0 has to be identically zero so we get from this expression has a_0 + 2(a_1 cos ψ - b_1 sinψ) so two times this quantity plus two times a_2 cos of 2ψ plus (–b_2 sine of 2ψ plus and so on till you go to the last term this two times a_N cosine of Nψ minus b_N sine of Nψ.

(Refer Slide Time: 35:11 min)



We can write down this term on cosine in one summation and sine in another that will give me a_0 plus two times summation K is equal to one to N a_K cosine of Kψ minus two times K is equal to one to N b_K sine of Kψ.

(Refer Slide Time: 35:53 min)



Now this is nothing but the Fourier series of this quantity array factor so essentially the coefficient which we have here represents the real and imaginary parts of the current excitation they are nothing but the Fourier coefficients of the array factor which is nothing but the radiation pattern of the antenna.

Now this relationship we had seen earlier in general relationship which was the Fourier transform relationship between the current distribution and the radiation pattern, the same thing now however we are seeing now in terms of the discreet current distribution and also since we are having a periodicity in ψ as we know that the radiation pattern is periodic over ψ we get instead of Fourier transform essentially we get a Fourier series in this case so if we know the radiation pattern which is the array factor and if Fourier expanded then the coefficient of Fourier expansion essentially represent the complex current excitation of the elements.

So as we know that the Fourier coefficients can be obtained essentially multiplying these things by cosine of $K\psi$ and integrating over a period essentially we can get these quantities a_K and b_K. So now what we see from here is that the radiation pattern essentially is given by the Fourier series expansion of this and larger the number of coefficients in this Fourier series better will be the approximation to the array factor in fact we can say that if we take the infinite terms in the Fourier series then this array factor which is periodic over ψ can be exactly represented. So what that means is that increasing the number of elements in the array we can increase the accuracy of the realization of a given radiation pattern.

So now while designing the arrays we can have two constraints, either we can design an array which says that the desired radiation pattern should be realize within certain error. In that case then we will take increase the number of elements in the array so that we get the error between the two which is less than the specified error. So in this situation the error between the desired radiation pattern and the realized radiation pattern is specified and there is no constraint on the number of elements. The other possibility could be that depending upon the physical situation somebody may specify the number of elements and then say get the best possible radiation pattern from this which is close to the desired radiation pattern.

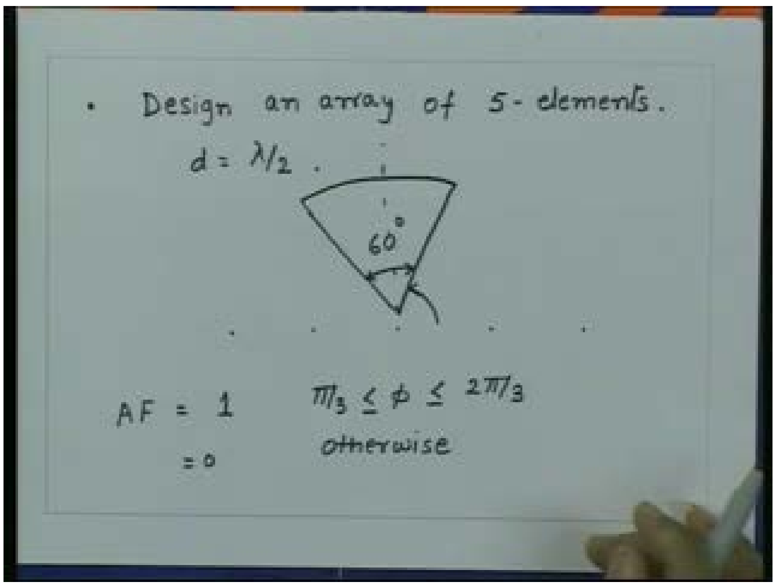
So in that case essentially we find out the radiation pattern which is closest to the desired radiation pattern in the least square sense. So, essentially the Fourier expansion when we do this it gives the radiation pattern which is best fit to the desired radiation pattern in the least square sense. Precisely that is what essentially we do now when we design the arrays that first we express the radiation pattern in terms of ψ then we Fourier expand that function of ψ which is periodic over 2π and then the coefficient of the Fourier series gives you the current distribution or the excitation of the antenna elements.

Let us look at a problem and this will become little clearer. Let us say we want to design an array of five elements and the array is broadside in nature and its radiation pattern is a sectorial radiation pattern with sixty degree sector angle. So let us consider the problem design an array of five elements. Now as we have seen earlier the spacing between the antenna elements should be equal to $\lambda/2$ if we do not want overlap of the range of ψ, also we have seen if the spacing was taken less than $\lambda/2$ then the range of psi is less than 2π and then the full range of ψ is not covered by the visible range and therefore it is optimum to take the spacing between the antenna elements to be $\lambda/2$.

So let us say we take in this case also the distance $d = \lambda/2$ and let us say now the radiation pattern which we want to realize by this is a sectorial radiation pattern in broad side so it is having an angle which is sixty degrees this angle is sixty degrees. We are now looking for a radiation pattern which is uniform over an sixty degree sector around broad side and no radiation beyond the sector. In practice you might find an application where you would like to eliminate the certain zone in the radiation region. For example if we have a satellite and I want to put an antenna beam over a certain current and we do not want to create interference near by regions we would like to create a beam which is uniform over a certain sector and it should be zero ideally beyond the range of their sector.

So we have a requirement where now the ø essentially from here to here is zero here ø is going from here to here the radiation pattern is zero and it is one over this sector. So essentially the specification is that your array factor of the radiation pattern is equal to one for ø less than π/3 greater than π/3 less than or equal to 2π/3 and it should be zero otherwise.

(Refer Slide Time: 43:23 min)

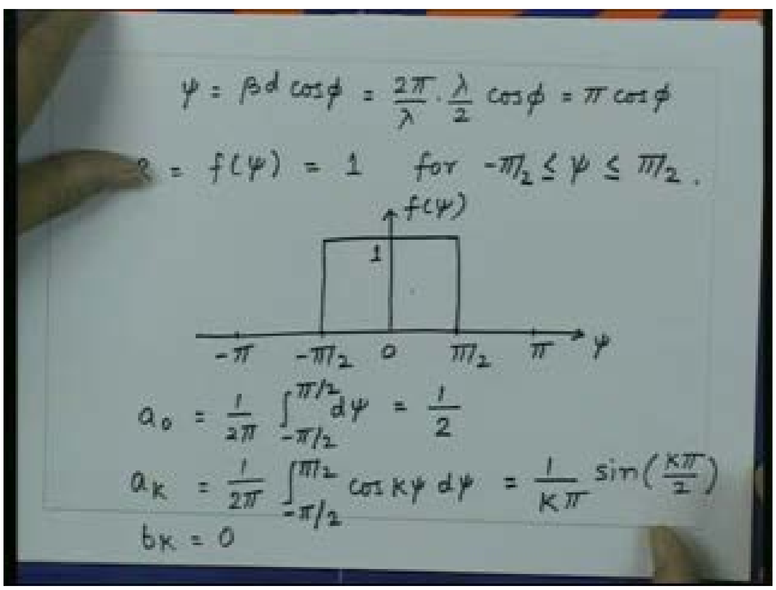


So we want to find out the excitation of the antenna element which would produce this radiation pattern. Now as we have seen earlier first we have to find out the radiation pattern in the ψ domain so ψ will be β d cosø and since we are taking d as λ/2 this is 2π divided by λ into λ/2 cosine of ø which is π cosine of ø. So now the array factor in terms of ψ which is f(ø) will be equal to one for this range of ø and if I substitute that in this I get the values of ψ in between -π/2 to π/2.

So now essentially we are looking for a function radiation pattern which is a function of ψ which is one over range of -π/2 to π/2.and is zero elsewhere. If I plot now the radiation pattern in ψ domain this is f(ψ) this is -π/2 this is +π/2 this is zero and this is π this is -π so as we mentioned the periodicity is over 2π so the function which we want the Fourier expand is functional this value is one and this is zero in this range zero in this range. Firstly we will note for this that this function is symmetric as a function ψ so the coefficient b_K will be identically zero for this expansion and we will get a_0 which is one over 2π integral -π to π amplitude is unity d ψ that is equal to 1/2 and we can get here this quantity a_K that is equal to 1/2π.

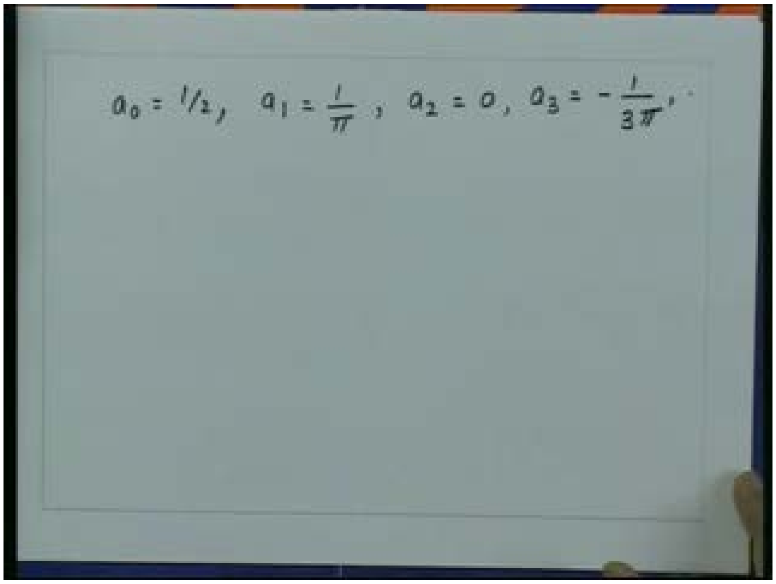
Now note here this function is one only from -π/2 to +π/2 that is why you get this quantity which is 1/2. Similarly this one is from -π but this quantity is only one over -π/2 to +π/2 so we can as well put the limits as -π/2 to +π/2 cosine of Kψ dψ that will be equal to one upon Kπ sine of Kπ/2 and b_K's are identically zero because this function is a symmetric function in ψ.

(Refer Slide Time: 47:23 min)



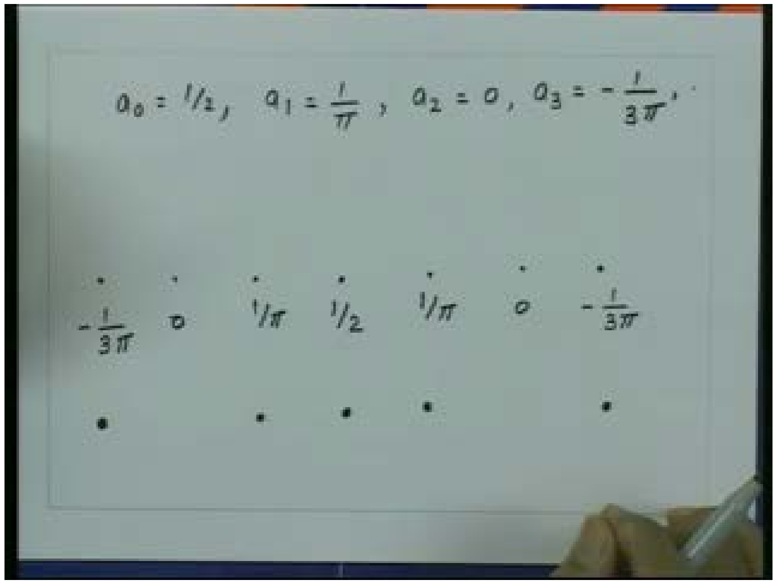
So now essentially we get the current coefficients for this which would be a_0 which is 1/2 then we have a one which is if I put K = 1 in this, this will be one upon π so a_1 will be one upon π, $a_2$will be when I put K = 2 this will become sinπ which is zero so a_2 will be zero, a_3 is equal to -1/3π when I put K = 3 and so on these are the coefficients which we essentially get.

(Refer Slide Time: 48:17 min)



So now if I look at the current excitation for this array as we have seen these are conjugatively symmetrically excited so this quantity is one upon two this excitation, the a_1 is having excitation of $1/\pi$ and since this quantity is coming real in this case this also will be $1/\pi$ that is the currents, the second quantity which correspond to this element is zero and since we are permitted to take five elements we want to design an array which has five elements since this current is zero we can take the next current this current is zero this element is not there and for which the amplitude would be $-1/3\pi$, similarly this current will be zero and next current will be $-1/3\pi$ so the array essentially which we get which is five element will be this, this is one element this is another element this is another element then this element and this element. So though we started initially with a array which is uniform array that spacing between the adjacent element was same which was $\lambda/2$ in synthesis process we essentially end up into an array which is non uniform because this current is now zero so we may say that we have five elements but the spacing between the elements is not same this spacing is double of the spacing between these two elements. So essentially now if we excite this array with these coefficients then we can get a radiation pattern which will be the sectorial radiation pattern of this nature.

(Refer Slide Time: 50:43 min)



So essentially the array synthesis problem is rather straight forward problem once the array pattern is specified in ψ domain and if we take the inter element spacing to be λ/2 then the visible range of ψ is equal to 2π which is the periodicity of ψ. So the steps involved in the synthesis are as follows, first you take the radiation pattern which is specified in the physical angle ø convert this radiation pattern in ø to the radiation pattern in ψ which is the array angle ψ so you get now the function which is periodic over ψ equal to ±π then you find out the Fourier coefficients for this function and the coefficient of the Fourier series essentially will give you the current distribution.

So by doing this and by increasing the number of elements to any arbitrary number essentially one can realize the radiation pattern which could be as accurate as possible to the desired radiation pattern. So as we mentioned there are two possibilities one is that the number of elements are specified then we get a radiation pattern which is the best fit to the given radiation pattern, if the number of elements are not specified but the error is specified then one can get the number of elements which would give the error between the desired and the synthesis radiation pattern less than the specified error.

So this essentially completes the discussion on the array analysis and synthesis that is if the antenna array is given to you we can find out the radiation pattern but in practice the problem is opposite at somebody specifies the radiation pattern and we have to find the current distribution. So array synthesis essentially provides you the formulation which can give you the excitation of the currents for a given radiation pattern.

Thank you.