Transmission Lines and E.M. Waves
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Lecture-55

Welcome, in last couple of lectures we investigated the problem what is called the array analysis that means if the array configuration is given, if the current excitation is given on each of the array element then we can find the radiation pattern for that array.

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In practice, however the problem is opposite a user specifies a radiation pattern and he is interested in finding out what array configuration are and excitation would give that radiation pattern, this problem is called the Array Synthesis problem.

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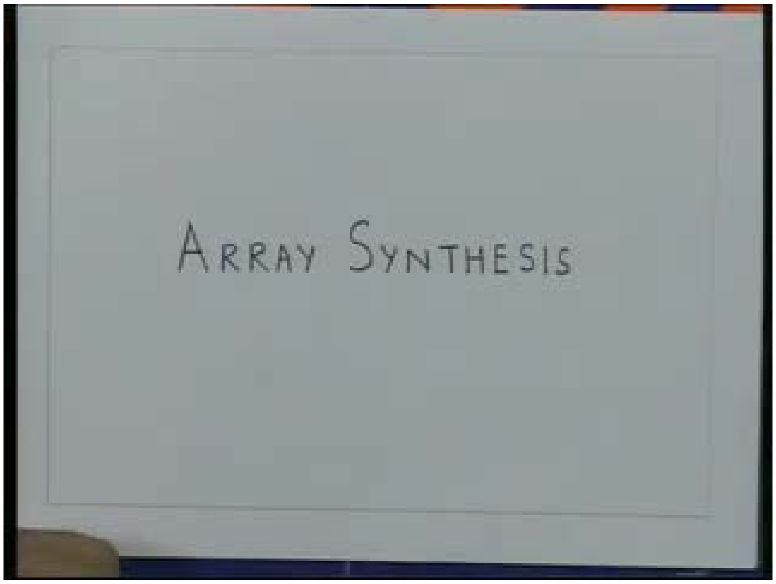

So here the radiation pattern is specified and we have to find the parameters of the array like the number of elements required in the array the spacing between the elements of the array and also the current excitation of the antenna elements of the array.

Just to get an example there maybe an application where we want to have a reception of some low intensity signal but there is a strong interfering source located somewhere in a specified direction. So while getting the reception we would like to eliminate the radiation coming from the interfering source. The simplest way of doing that is we can place a null of the radiation pattern in the direction of the interfering source and then that radiation coming from interfering source will be killed. So by designing the radiation pattern appropriately if we place a null in the direction of the interfering source then the low intensity signal can be received effectively without modifying your receiving system.

Many applications we would like to illuminate a certain angular zone for reception or for transmission and we want that the power should be wasted in some other direction so we may interested in getting a radiation pattern which looks like an angular sector. In practice, we have a variety of radiation patterns depending upon the application and now we want to find out what would be the array configurations and the excitations which would give the flexibility of designing that radiation pattern.

So today essentially we discuss the problem called a Array Synthesis problem and this problem is not as straight forward as the array analysis problem because in analysis the array was given excitation was given so we could simply apply supper position of the fields and we can find the variation of the electric field as a function of angle and we get the radiation pattern.

In this case the problem is opposite, of course we know that there is a relationship between the radiation pattern and the current excitation and that is there is a Fourier transform relationship between these two but we also have seen that there is no uniqueness coming from radiation pattern to the current distribution, there was ambiguity in that and that is the reason the error synthesis is a rather difficult task. So in this lecture essentially we develop the theoretical basis for synthesizing a radiation pattern of an array and then we will take some special cases of radiation patterns like having the radiation patterns specified only by the nulls or specified by a sectoral zone or something like that.

So in general let us say we have a N element array and now we have want to use essentially all degrees of freedom so we want to say we can the number of elements, we can change the inter element spacing and we can change the current excitation also. However, in practice the inter element spacing is not varied we keep the inter element spacing d same for the array and try to manipulate the radiation pattern only by controlling the complex current excitation on the antenna. So normally in practice the number of elements are given to you, also the inter element spacing is given to you or the maximum size of the antenna array is given to you from there you can find out what is the maximum inter element spacing and then one can ask a question what should be the current excitation for the antenna elements so that we get the radiation pattern as close to the desired radiation pattern as possible.

So without losing generality let us say we have antennas which are (N + 1) antennas we have somewhere a element which is element of symmetry and we have these elements which are (N + 1).

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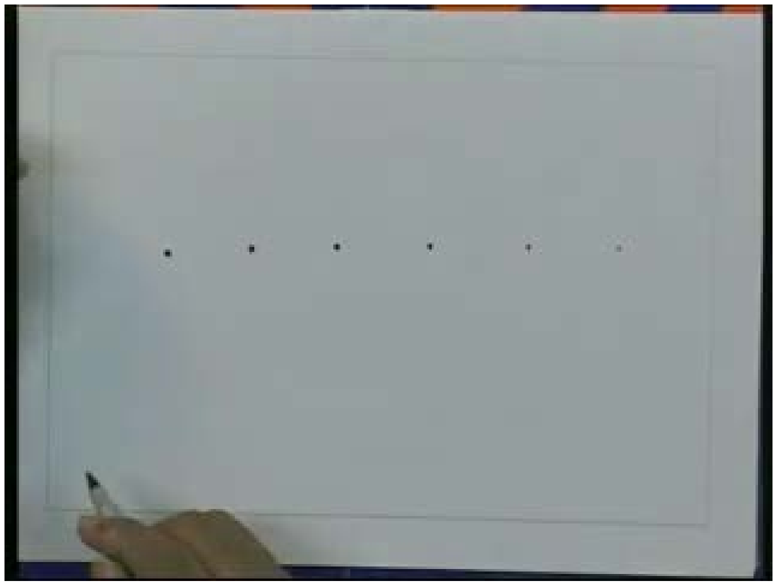

And the currents are excited in such a way where the amplitude of the (N + 1) of the element that is unity with zero phase so without loosing generality let us say that we have (N + 1) elements and we also say that the current excitation of (N + 1) element is unity so the current which you have will be I_N that is equal to one angle zero. So we excite this current element let us say this element we start with 0, 1, 2, 3 and so on up to N so we have (N + 1) elements and the currents in this are given *by* I_0 I_1 I_2 I_3 upto I_N where I_N we have chosen to be amplitude unity and the phase angle zero.

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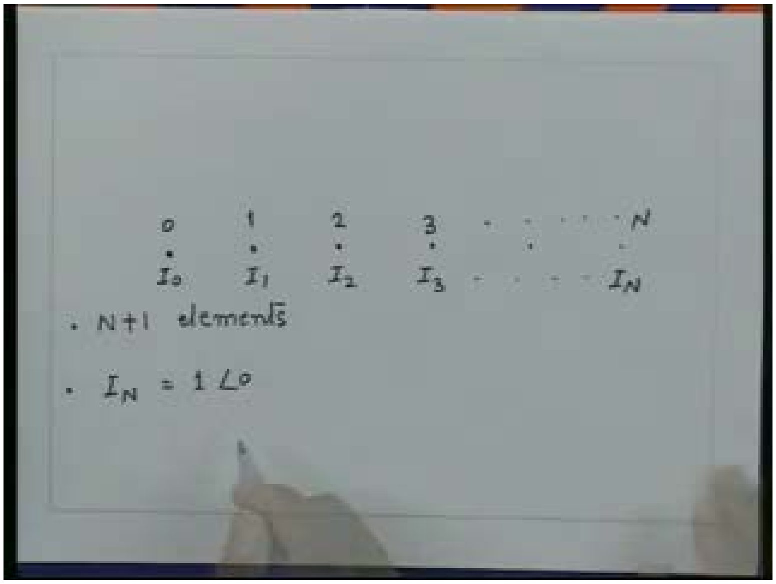

Then since the enter element spacing is constant and since this currents are now complex we say that the progressive phase shift which we used to talk in terms of uniform array can now be absorbed into the face of this currents so we can now defined the phase of the array which is ψ which is simply βd into cosø. So again we take a direction which is the angle measure with the axis of the array this angle is ø and we defined this quantity now the phase ψ that is equal to βd into cosø.

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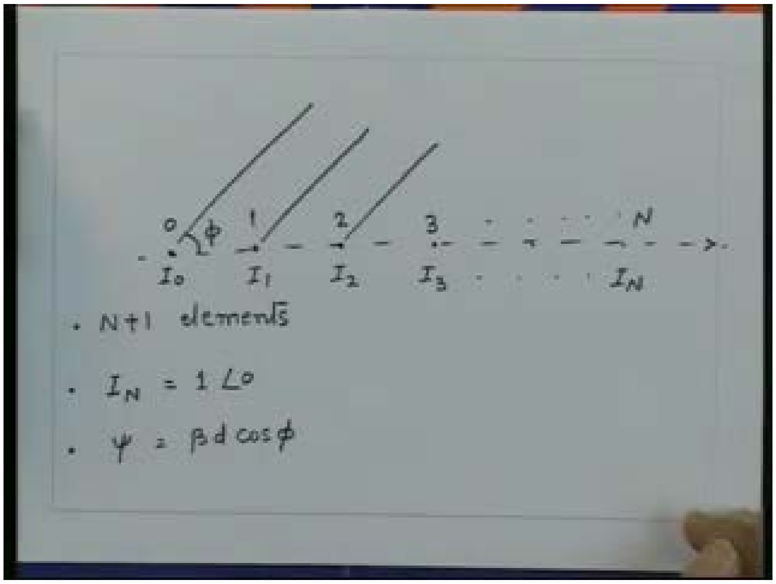

Recall incase of uniform array we had defined this quantity ψ which was βd into cosø plus delta which was the electrical un progressive phase shift, however, that delta now we can absorb into this excitation currents because this I_0 I_1 I_2 I_3 upto I_N all these currents in general are complex currents. So we also say here now in general this I_0, I_1 and so on are complex quantities.

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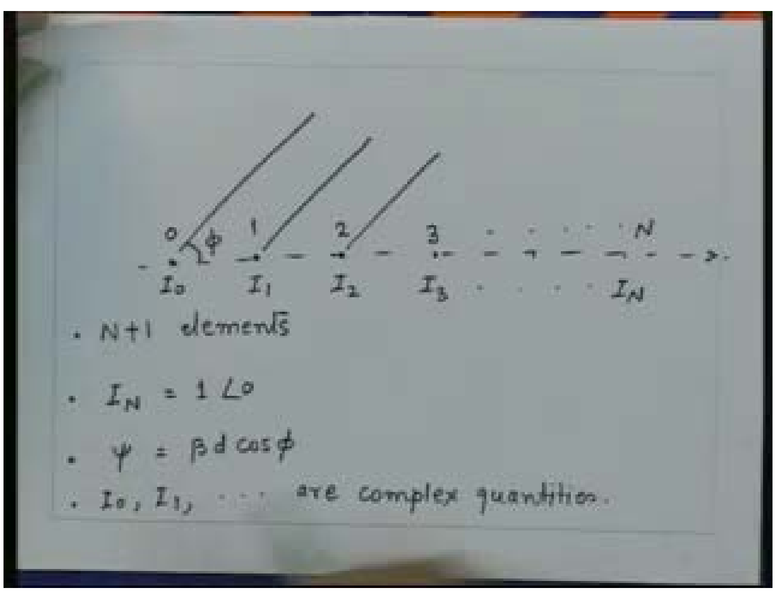

Once we say that then the array factor or radiation pattern of this array is straight forward, the field which is produced because of the current at a vary far away point will be proportional to the current here and there will be a phase which will be corresponding to the space phase which will be this ψ. So now we can write down the array pattern for this and let we call that as array factor AF that is equal to magnitude of I_0 plus I_1 e to the power jψ plus I_2 e to the power j2 ψ plus and so on will go to I_N which is equal to one so this will be e to the power j Nψ magnitude.

Now let us define this quantity e to the power jψ which represent a complex number with unit amplitude. So let us define some quantity z that is e to the power jψ so that means mod of z is equal to unity. So as the value of angle changes the ø changes essentially the phase of this quantity z would change so in complex gamma plane this point would lie on the unit circle because magnitude of z is equal to one. So the array factor can be written now in terms of the z so we have array factor which I_0 plus I_1 z plus I_2 z square plus and so on z to the power N.

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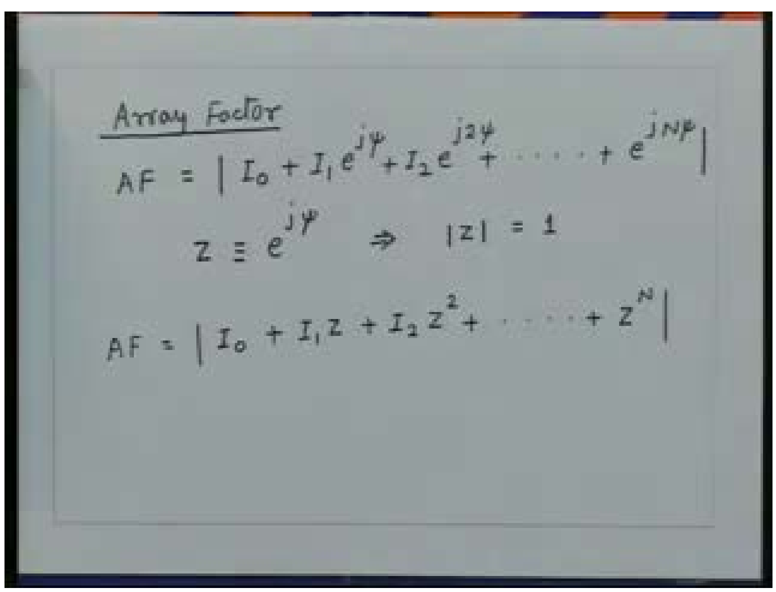

So the array factor now can be represented by a polynomial of this quantity z which is simply a phase function the magnitude of z is equal to unity. So there are theorems due to ((12:18)) which say that an array factor can be represented by a polynomial of z or even conversely say every polynomial can be taught of as an array. Now in general then we can say that the radiation pattern of an antenna for which the inter elements spacing is same can be represented by polynomial object.

Now as we know the polynomial of N th degree has N roots in the complex plane so essentially this polynomial will have N th roots in the complex plane so this can be written as z minus zeta one z minus zeta two and so on z minus zeta N.

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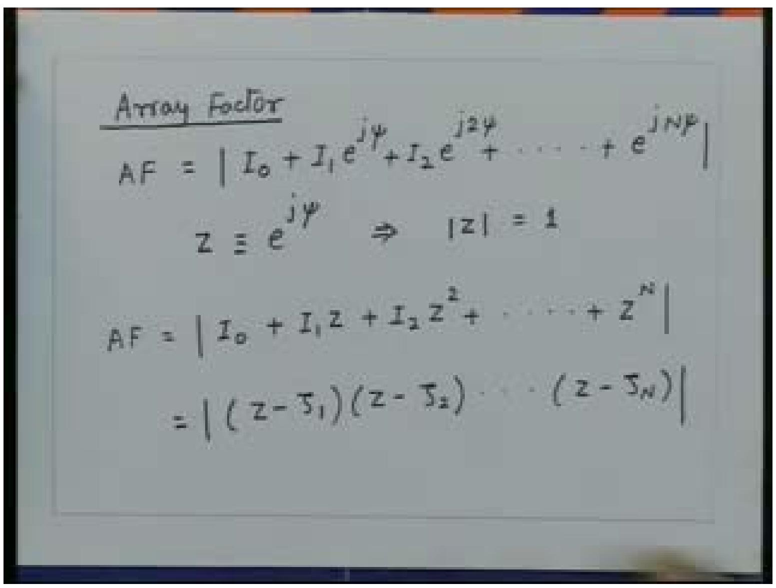

So essentially a polynomial is now represented by the product of this doublet which is z minus zeta one z minus zeta two and so on and zeta one zeta two zeta n are the roots of the polynomial which is represented by this. So these are the roots of the polynomial that means at this z equal to zeta one or z equal to zeta two the array factor would go to zero that means the radiation pattern would have a null at that location indicated by zeta one zeta two and zeta N

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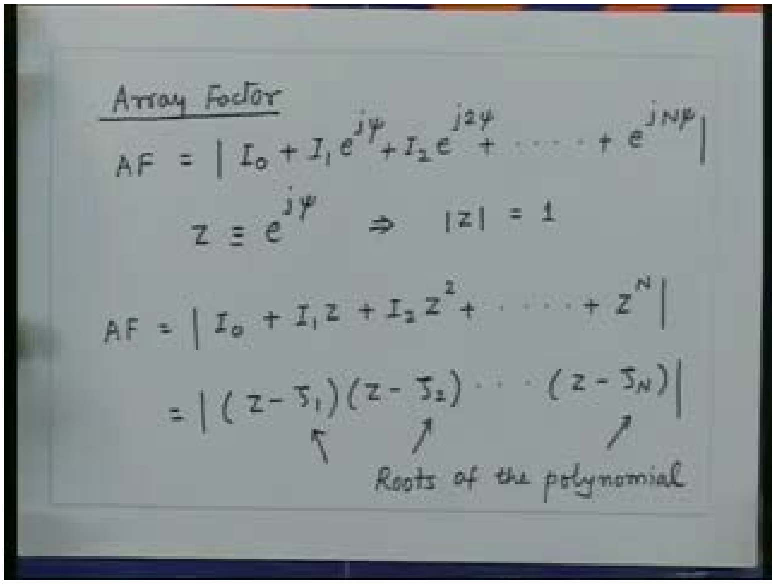

So, essentially the roots of this polynomial correspond to the nulls in the radiation pattern. So zeta one direction corresponding to this, so now if I take z which is equal to e to the power j ψ that is equal to e to the power j β d cosø and if that is equal to zeta one zeta two zeta N and if I invert this and find out what is the corresponding ø that essentially indicates the direction of the nulls in the radiation pattern. So one thing is immediately clear from here that since this quantity is representing a phase function that means that I change the value of ø the magnitude of z always lies is equal to unity that means this point represented by this lies on the unit circle in the complex z plane if the magnitude of zeta one zeta two is equal to unity then only this point will lie on the unit circle and then there will be corresponding angle which would represent the direction of the null. So what we see from here is that in the complex z plane you have a unit circle and zeta one and zeta two might lie somewhere here this might be zeta one, this might be zeta two, this might be zeta three and so on zeta N, only those zeta's which lie on the unit circle might correspond to the physical nulls we might have angle phi corresponding to only those zeta's when this point do not lie on the unit circle they will not represent the corresponding value of ø because the magnitude of this is not equal to unity and therefore they will not represent a null in the physical space.

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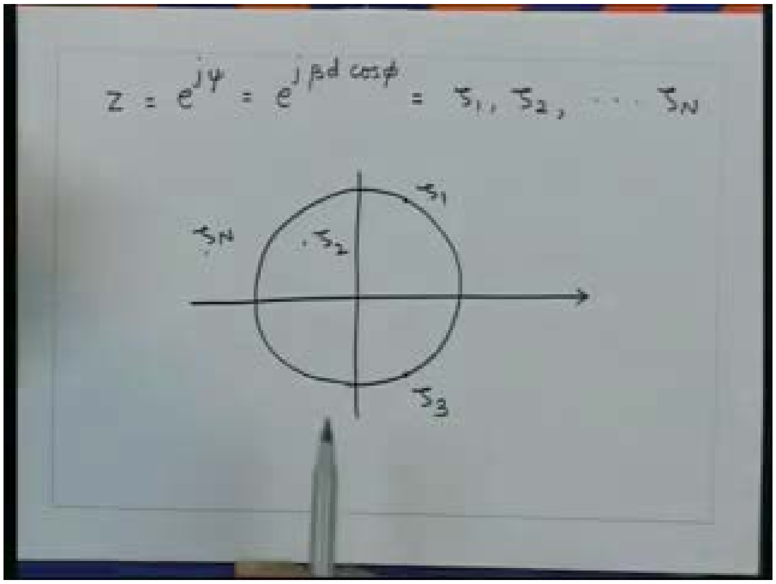

Now although the theorem says that every polynomial can be thought of an array and every array represents essentially radiation pattern and a polynomial the root of polynomial not necessarily represents the null of the radiation pattern because for the null to represent the point should lie on the unit circle and if they do not lie on the unit circle there will not be any corresponding value of ø which will which will give me the null in the physical space. So though we are having a degree of polynomial which is N we will always have nulls which are less than or equal to N if all the roots lie on the unit circle then all nulls might be visible if some of them do not lie on the unit circle then those nulls will not be visible in the physical space.

So first thing which we can observe from here is that when a polynomial is given to you you can find out the roots of this polynomial we can find that this polynomial are doublets and this will give me the location of the nulls and if the points lie on the unit circle then those nulls might be corresponding to the physical nulls in the radiation pattern. Why do I say might be? if the point lies on the unit circle does not it necessarily mean that there will be a null in the radiation pattern, the answer is no. Let us look at is this in little more detail let us say we have this quantity ψ which is given by βd cosø.

We have this quantity ψ which is equal to βd cosø and ø varies from zero to π. So the range of ø we have is greater than equal to zero less than or equal to π that means as the ø changes from 0 to π the value of ψ would change from βd to -βd so that means the range of ψ essentially goes from βd to –βd, this is the range of ψ we called as the visible range of ψ because that corresponds to the visible angle physical angle the radiation pattern which is the ø which goes from zero to π. So this range from βd to -βd is called the visible range of ψ.

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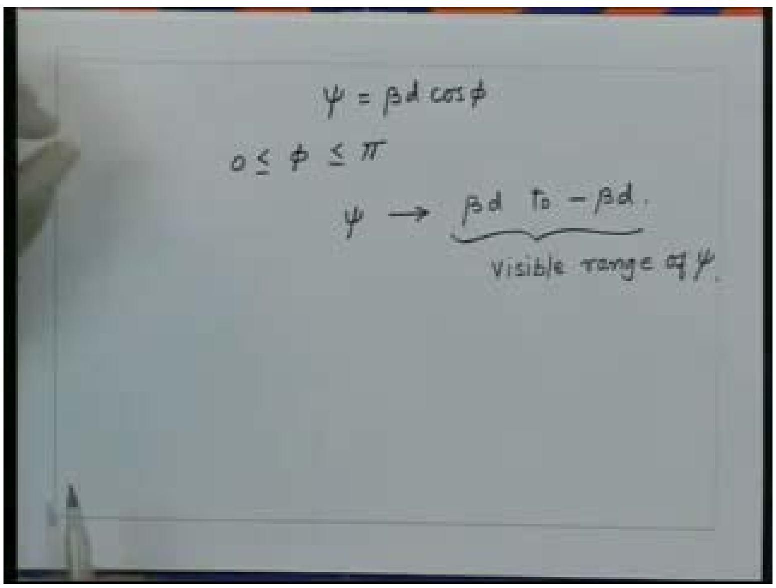

Now depending upon the value of d we will have coverage of angle ψ which might go from +π to -π or it may go over only a portion of the total angle from –π to +π so if let us say we take d as λ/4 in case one then the range of ψ would correspond to βd so that will give me βd is equal to 2π/λ into λ/4 so that will be equal to π/2 so the range of ψ would correspond to π/2 to -π/2.

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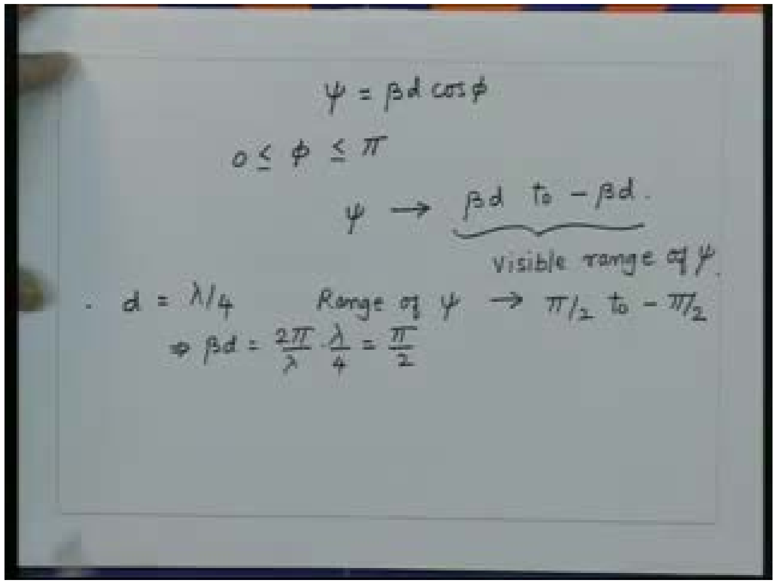

If I take d = λ/2 that would correspond to βd = π so range of ψ would be π to -π and if I take d > λ/2 then the range will be the largest. So if I take this d = λ let us say that would correspond to βd = 2π so the range of ψ would be 2π to -2π.

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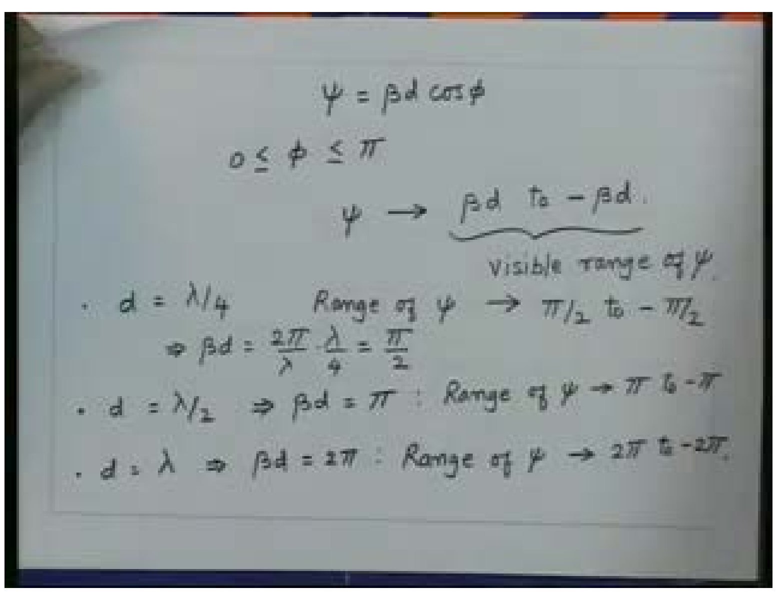

Let us go back to the unit circle in the complex plane and then I ask for different values of d what is the angular sector covered by the physical range of ψ so the point moves lies on this circle and then I have changed the value of ø from zero to π the value will change from βd to -βd passing through zero so if I take d equal to λ/4 in this case the range of ψ is from π/2 to -π/2.

So I can write this circle if I take d = λ/4 this would correspond to π/2 which is corresponds to +βd for d = λ/4 so this will correspond to ø = 0 and then the range of ψ will be like that and at this point this will be corresponding to ø = π. So we got a visible range of ψ from here to here so this is the visible range of ψ that means now if I have a inter element spacing of the array which is λ/4 and if the root of the polynomial lies somewhere here in this region though the root is lying on the unit circle still it will not represent the physical null because the physical null correspond to only the range of ψ which lies here from +π/2 to –π/2 this is ψ.

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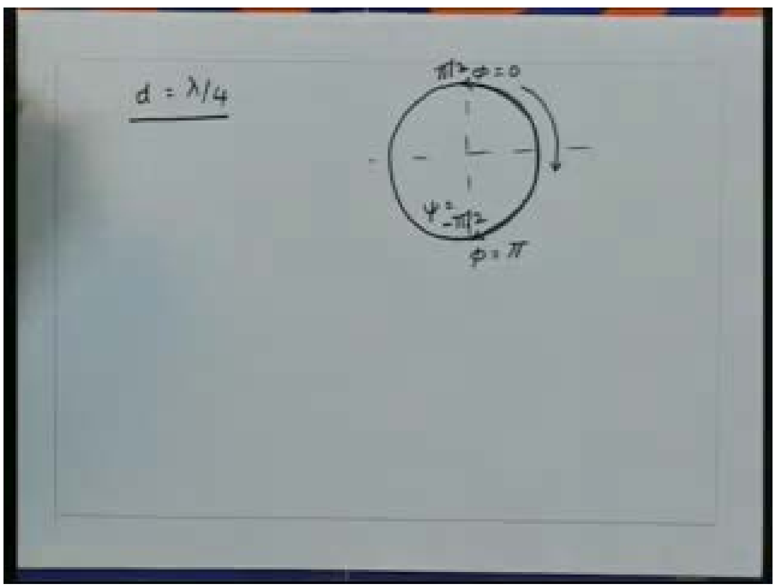

Now two things must happen for nulls to be visible in the radiation pattern, one is the root must lie on the unit circle, second is the root must lie in the visible range of ψ then and then only we can see the nulls in the radiation pattern. Any of the things not happening will not represent a null in the physical space. So in this case we essentially can place the nulls or the roots of the polynomial in this region without significantly affecting the nulls of the radiation pattern in the physical space. So that we will investigate later when we synthesis the array which is defined only by the directions of the nulls that time essentially this phenomena can be used more effectively.

As we saw if we take $d = \lambda/2$ then the range of ψ is from $+\pi$ to $-\pi$ so this is $+\pi$, this is $-\pi$, this is zero so the ψ essentially is going to go from $+\pi$ all the way to $-\pi$ that is your range of ψ. So essentially it covers all the points on the circle when $d = \lambda/2$.

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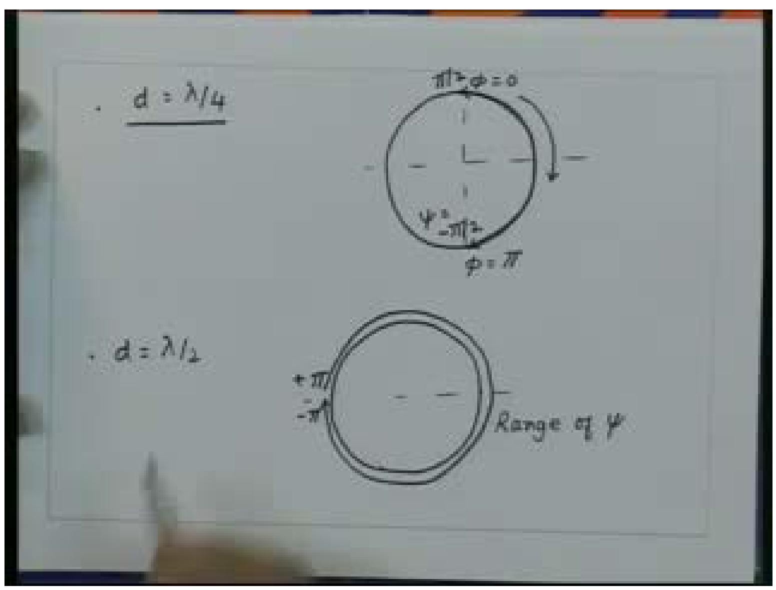

If I increase the d beyond $\lambda/2$ then obviously the range of the ψ will overlap with itself and as we saw in the in the case when $d = \lambda$ the range goes from 2π to -2π so here it will start with 2π which is 2π here it will go all the way from here to zero and then again it will go -2π so it will come up to this point so you will have to start with this and the range of ψ will go like that is equal to -2π.

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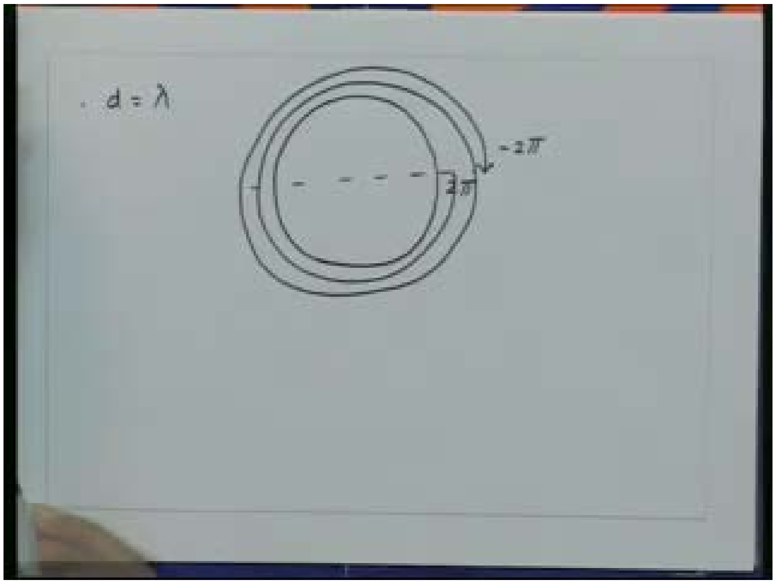

So now if I look at these three cases here in this case what we see is that the total unit circle is not covered by the visible rage of ψ. So we have a portion of the unit circle where if we put the roots of the polynomial they do not affect the null of the radiation pattern, in this case if you put nulls on the unit circle they have corresponding directions for the nulls in the radiation pattern and which are unique. However, in this case if I consider a null let us say somewhere here it would correspondent to two values of ψ. Let us say I take a angle location of the nulls let us say is this. Now this corresponds to the value of ø which would correspond to this value of ψ which is this but you would have also corresponds to this value of ψ which is here, so for a given null on the unit circle now we have two ψ values because the visible range of ψ overlaps with itself but two different values of ψ essentially correspond two different angles in physical space two different angles ø. What that means is if I try to put a null corresponding to this value of ψ automatically there will be a null plane in this direction which correspond to this value of ψ.

So now I do not have a independent control of every point on the radiation pattern as soon as certain point on the radiation pattern is defined the other point on the radiation pattern is automatically defined. So if you have a range which overlaps with itself and that would happen if d is greater than $\lambda/2$ then the independent control of every point on the radiation pattern is lost, as soon as you certain portion of the radiation pattern the other portion of the radiation pattern is automatically defined.

So looking at these three cases now we can make the following conclusion that in this case since the entire range of angle ψ from zero to 2π is not covered by the visible range of ψ there are certain nulls which can be there in the polynomial which may not be physically visible. If I go to this when the range of ψ overlaps with itself then we do not have independent control of every point on the radiation pattern so the optimum inter element spacing is $d = \lambda/2$ because in this case you cover the entire range of ψ which is from zero to 2π or $-\pi$ to $+\pi$ and at the same time you still have independent control of every point on radiation pattern, that is the reason invariably we prefer to use the inter element spacing of an array which is equal to $\lambda/2$.

So generally when we do the array synthesis we prefer to use the inter element spacing which is equal to $\lambda/2$ and then we ask the question what would be the current excitation and the number of elements which would give the desired radiation pattern.

This diagram essentially is called the circle diagram of the array and we can essentially mark the roots of the polynomial that means the direction of the nulls on this diagram. So this thing we call as the circle diagram of the array.

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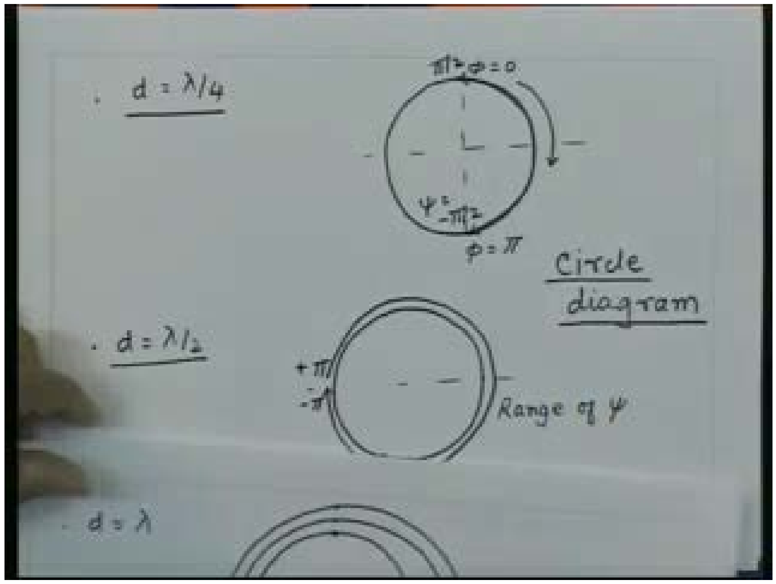

So circle diagram essentially is the visualization of the roots of the polynomial on the unit circle or in general on the complex z plane to see whether these nulls would correspond to the physical nulls in the radiation pattern or not. Once we understand this then synthesizing an array which is specified by the nulls is very straight forward.

So as we said there are certain applications where we would like to place the nulls in some given directions because there is a strong interfering source coming from that direction so if you put a null in that direction then the interference source will be killed.

Now we can synthesis an array based on the directions of the nulls so the simplest of this array synthesis is synthesis of array defined by its nulls. Now the steps involved will be as follows, first specify the directions of the nulls let us say they are given by ϕ_{n1}, ϕ_{n2} and so on up to ϕ_{nN} where n represents the null and 1, 2, 3 up to N represents the number of nulls in the radiation pattern. Then you find out the corresponding values of ψ so you have ψ corresponding to these nulls ψ_{nK} that is equal to $\beta d \cos\phi_{nK}$ where K would be equal to one to n.

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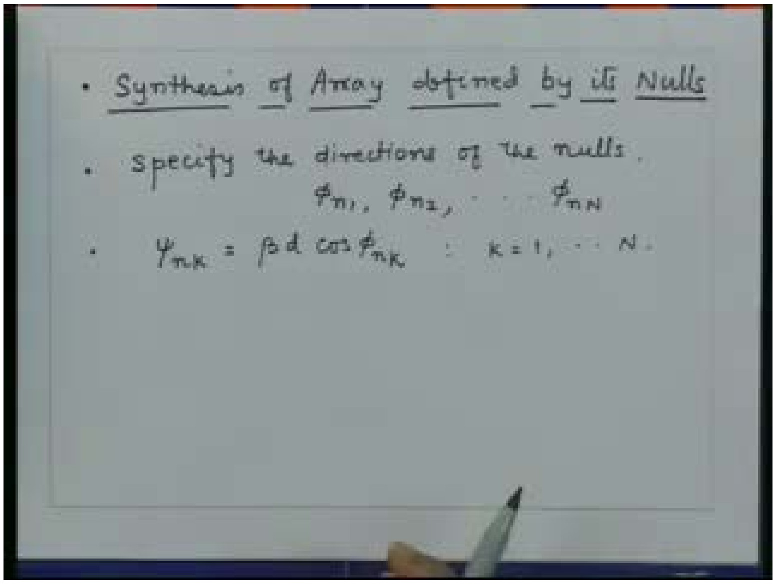

Then you find out the corresponding values of z which is zeta K that will be e to the power $j\psi_{nK}$ and then we can write the polynomials by multiplying the doublets which is z minus the roots of this polynomial so then from here we can get the array factor which is equal to z minus zeta one, z minus zeta two so on up to z minus zeta N. Expand the polynomial and the coefficient of z give you the current excitation of the antenna array get coefficients of z to the power K gives excitation of K th element.

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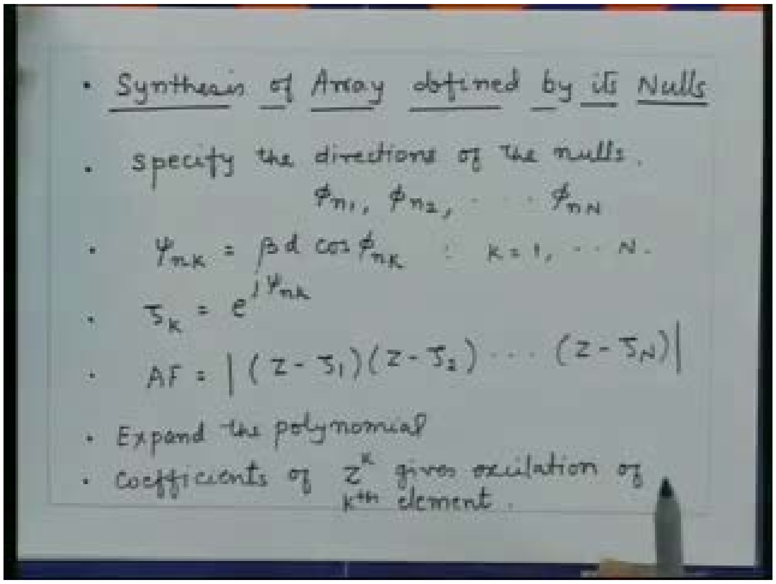

So array which is specified by the nulls can be synthesis in a very straight forward manner. So first you find out the directions of the nulls which are specified by the users which are given by $ø_{n1}$, $ø_{n2}$ upto $ø_{nN}$ find out the corresponding values of ψ's find the root corresponding to these values of ψ's which is defined by e to the power $j\psi_{nK}$ multiplying by the doublets z minus the roots you get the polynomial expand the polynomial you will get now powers of z's get the coefficients of z to the power K that gives you the excitation of K th element. So the synthesis of an array which is defined by its nulls is a rather straight forward problem.

Let us take a simple example to understand how do we define this problem. So let us say we want to design an array which has two nulls one corresponding to $ø = 0$ and other corresponds $ø = \pi/2$. So I do not know how many elements will be required by this but since there are two nulls which are specifying the radiation pattern we require three elements we require a polynomial of degree two. So let us say my requirement is this is the array this is now a three element array I have a null in this direction and we want a null in this direction so this corresponds to $ø = \pi/2$, this corresponds to $ø = 0$. So we have two nulls in the radiation pattern at $ø = 0$, at $ø = \pi/2$.

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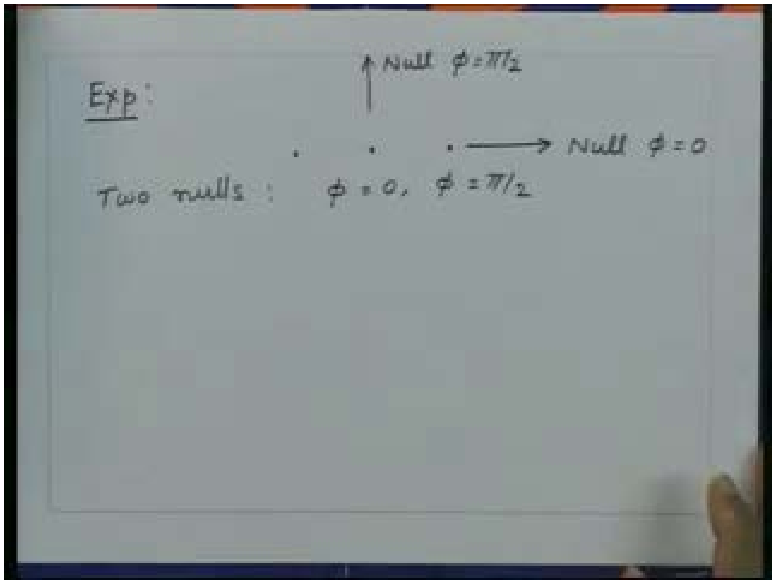

Now taking the inter element spacing d = $\lambda/2$, first we choose spacing take d = $\lambda/2$ giving βd cosø equal to $2\pi/\lambda$ into $\lambda/2$ cosø that is equal to π cosø.

Now we have the size corresponding to the two nulls so we get here ψ_{n1} that is equal to when I put ø = 0 we will get ψn1 which is equal to π and you will get ψ_{n2} that is equal to when ø = $\pi/2$ so this is zero so this quantity will be zero. If I put the circle diagram for this we have two nulls one corresponds to ψ equal to π and other one corresponds to ψ equal to zero so I have one null here and I have another null here so this corresponds to ψ= 0, this corresponds to $\psi = \pi$ or this point corresponds to ø = 0 and this corresponds to ø = $\pi/2$. Once I get this then the roots of the of this polynomial which would define the radiation pattern for this would be zeta one will be e to the power j π that is equal to -1 and zeta two will be e to the power j zero that will be equal to +1.

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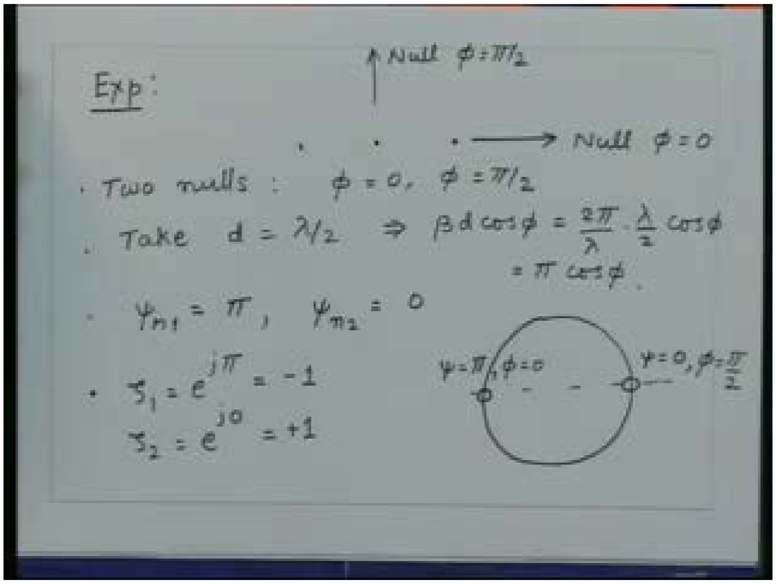

Now if I take the rule which have -1 and +1 then we can essentially define now the polynomial which will be z minus this quantity, z plus this quantity and if I expand their polynomial that would essentially correspond to the excitation of the of the array element. So finally we can get the array factor which will be equal to mod of [z – (-1)] into [z – 1].

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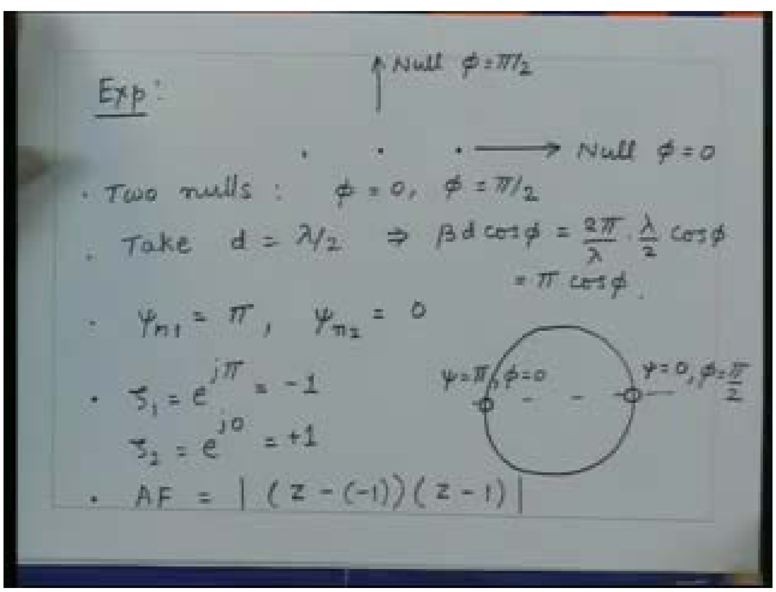

So you get the pattern which is array factor which is equal to mod of (z +1) into (z – 1) which after multiplying and expanding essentially we get the coefficients which will be z square minus one or if I write in the way we wanted to write the polynomial the ascending degrees of z this will be minus one plus z square or we can write here this is minus one plus zero into z to the power one plus z to the power two.

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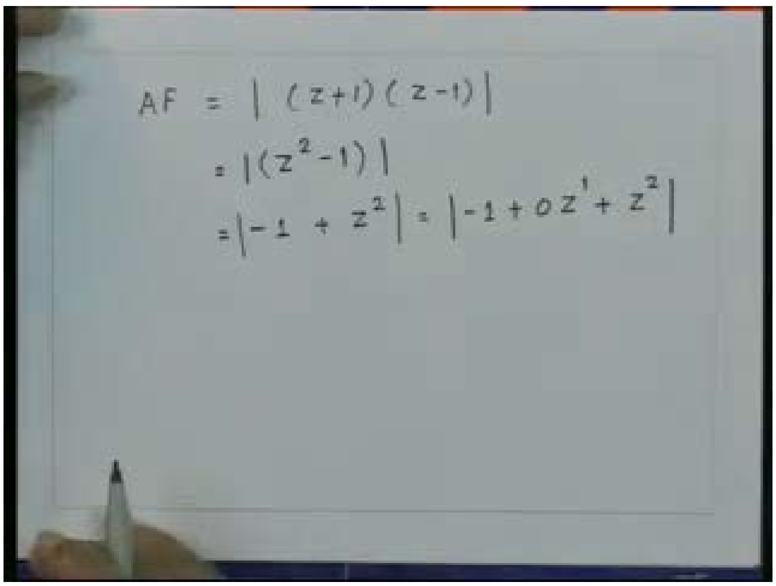

So, the coefficient of null the powers of z that defines the excitation of the current for the array element. So we have these three elements now showing here the inter element spacing we have taken is λ/2. So this current is minus one so here the amplitude is one angle is π that is what -1 could mean, there is no current excitation for this element so this element is zero so this current element is not there and this will have an excitation which will be one angle zero this will be one angle zero. So essentially now we have an array since this current is not excited this current is this element is not existent so essentially we have a array which is this which is having a spacing of λ and they are excited out of phase so this excitation is one angle ø and this excitation is one angle zero.

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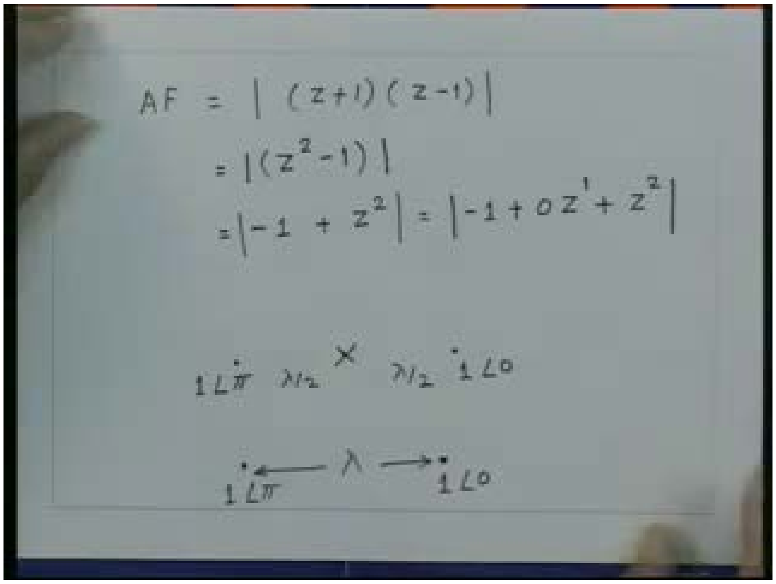

So though we initially started with a three element array because there were two nulls required by synthesizing this in terms of nulls essentially we find that the two elements are the one which are enough to give you a null in the direction of ø = 0 and the direction ø = π/2. Since there are two elements array which we already investigated we can see what the radiation pattern for this would be so the radiation pattern for this array would be equal to we have we have got here the array for which the angle this is ø measure from the axis of the array so we can get from here one e to the power j π current plus angle one e to the power j0 e to the power j βd cosø.

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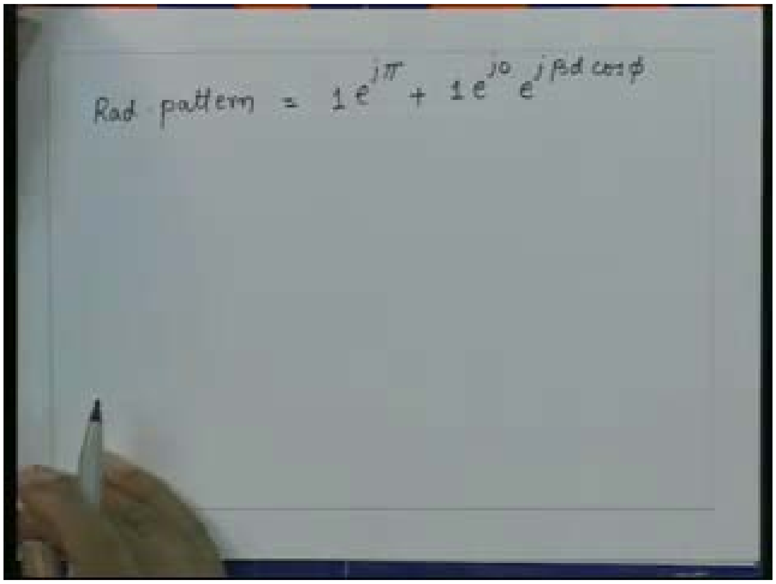

So that is equal to minus one plus one e to the power j βd cosø but we have βd in this case which will be equal to 2π/λ into d which is now λ in this case d which is equal to 2π.

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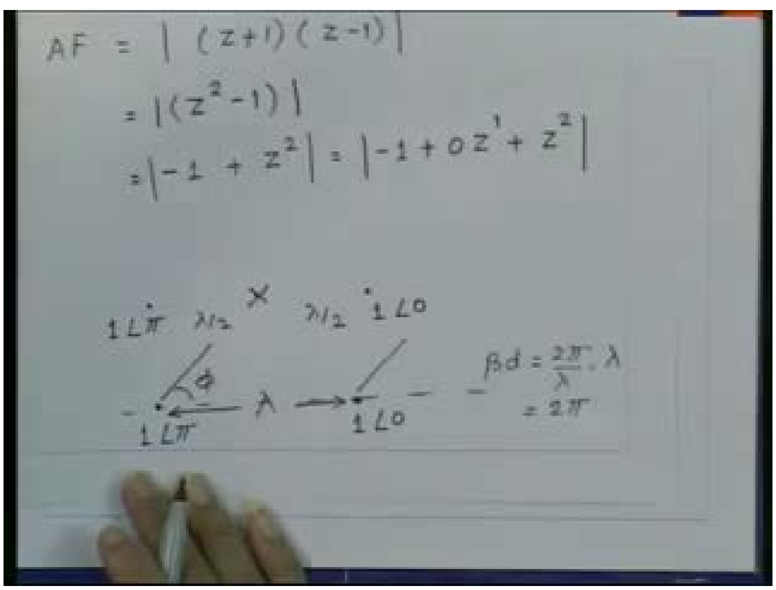

So we can write here this e to the power j 2π into $\cos\phi$. As we have done earlier we can take e to the power j π $\cos\phi$ common from here so this will become equal to minus e to the power -j π $\cos\phi$ plus e to the power j π $\cos\phi$ that multiplied by e to the power j π $\cos\phi$.

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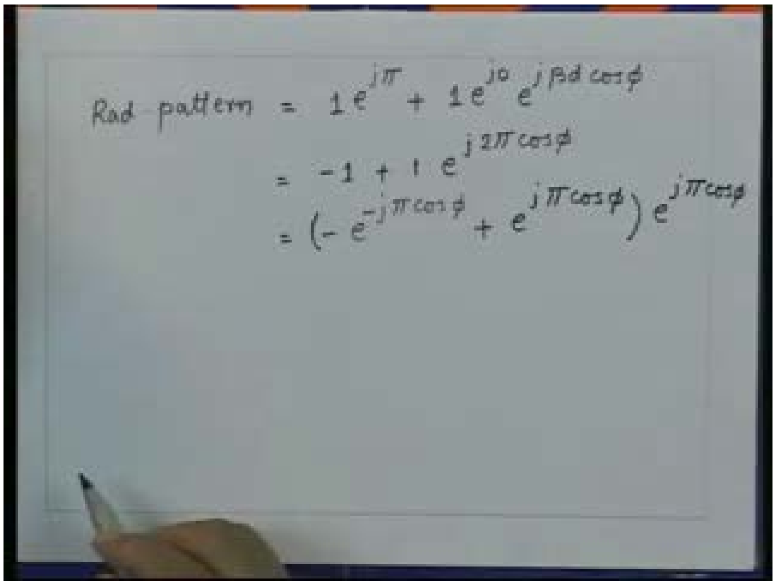

Now since for the radiation pattern we are interested only in the amplitude variation this term does not matter so essentially we get the radiation pattern which will be e to the power j π $\cos\phi$ minus e to the power minus j π $\cos\phi$ that will be equal to two times j $\sin(\pi \cos\phi)$.

So now we can verify that when $\phi = 0$ that time this quantity is one and we get $\sin\pi$ which is zero so we get a null in the direction of $\phi = 0$, when the $\phi = \pi/2$ this quantity is zero so again sine of zero that will give me the null in that direction. So this is the radiation pattern which I get corresponding to this excitation which will have nulls in the direction of $\phi = 0$ and $\phi = \pi/2$. One can also note from this direction on the on the circle diagram that this point essentially corresponds to $\psi = \pi$ which is this point but this point also corresponds to $\psi = -\pi$ and $\psi = -\pi$ corresponds to the angle ϕ which is equal to π. So that means as soon as we put a null here corresponding to ψ equal to π automatically a null has been kept placed in the direction ø = π and which we can see from here that when we put the ø = π this quantity will become -1. So again we will get sin(-π) that will be zero so you will have a null in the direction of ø = π also.

So although we did not want a null we wanted only these two nulls in the radiation pattern automatically a null has been placed in the direction of ø = π also. So this is more like a forced null in the radiation pattern.

So we will have now three nulls for this array, one will be in this direction, one will be in this direction which is π/2 and one null which will be forced in this direction this is the forced null which was not defined by the user but automatically the null will come in this direction. If we did not want null to come in this direction then we could have taken the spacing between the elements smaller than λ/2 and then this point would not correspond to ø = 0 and π the visible range will get shorten and the two points will not coincide and then we can make sure there is only one null exist in this radiation pattern.

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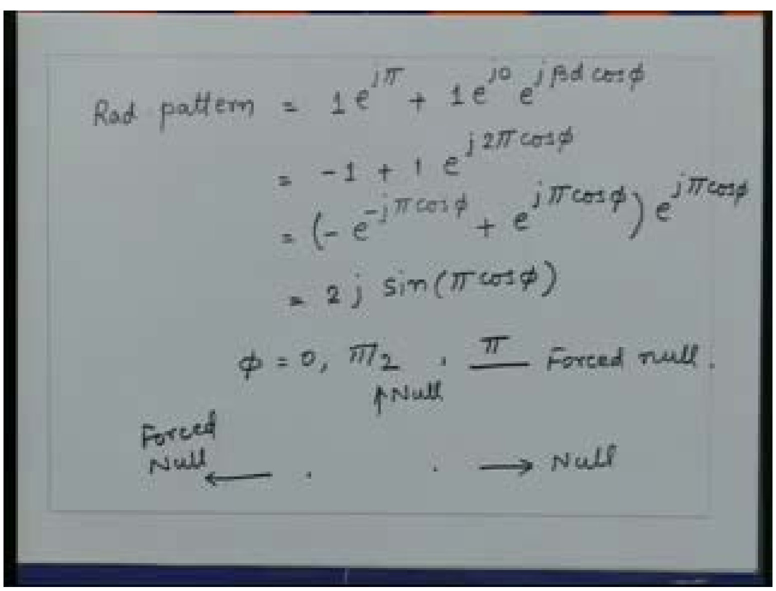

So what we see from this example is that now by defining the nulls of the radiation pattern we can find out the excitation of the current elements of a uniformly spaced array where the inter element spacing is constant and its possible that some of the current excitation coefficient might somewhat turn out to be zero so those current elements can be removed and then the remaining element if they are excited with the proper current excitations then they will they will place a nulls in the desired direction, it is possible in this process some additional nulls also might get introduced as we saw in this case if the visible range of ψ overlaps with itself. In this case only one point was overlapping but in general if the d is greater than $\lambda/2$ then there will be more overlapping of the visible range of ψ and the more nulls will get placed other than where user want to have the nulls.

Another thing to note in this analysis is now that the array pattern is defined purely by the nulls and we do not have control over any other parameter of the radiation pattern, for example once the nulls are defined I now cannot say which direction the maximum radiation would be whatever the direction of the maximum radiation is we have to accept that direction. So we have control over the nulls of the radiation pattern but we do not have control on the radiation pattern as a whole because now I cannot control the direction of maximum radiation I cannot control let us say the level of the side lobes or direction of the side lobes and so on.

So this synthesis based on the nulls of the radiation pattern is useful in practice, however, we would like to have a much better control of the radiation pattern not only the nulls of the radiation pattern and that is what precisely is done by what is called the general array synthesis and that is what we will discuss in the in the following lecture that if the radiation pattern has a whole was defined not only by its nulls then what should be the current excitation to realize that radiation pattern.

Thank you.