**Transmission Lines and E.M. Waves**
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**Lecture-54**

Welcome, we are discussing a very important topic in antennas called antenna arrays, first we saw some broad characteristics of the two element array and later we started investigating a uniform linear array.

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So we saw in the last lecture that we have a uniform array which are excited with equal amplitudes the spacing between adjacent elements is also same and then we have essentially three parameters for this array the total number of elements in the array the inter element spacing we denote by d and a progressive phase shift that is the phase shift between the two adjacent elements that is delta.

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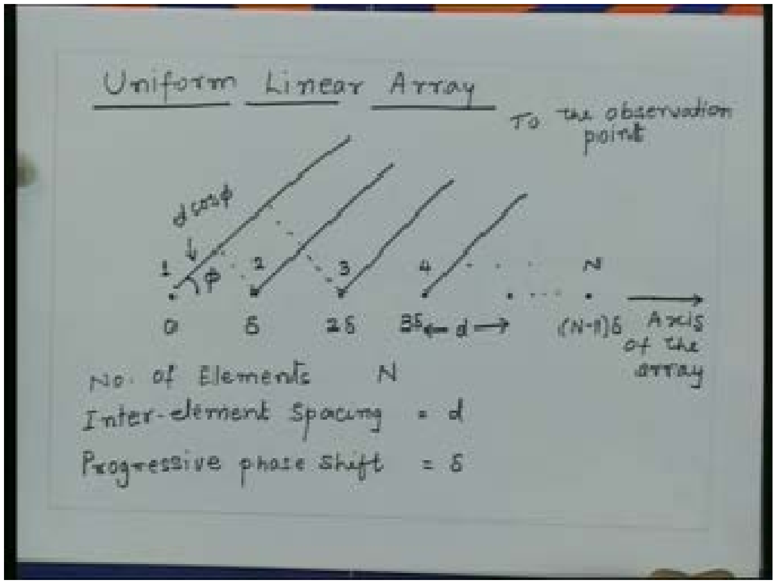


And then we started investigating the characteristics of this array and the effect of these three parameters on the radiation pattern of the array. We defined this quantity the total phase ψ which is β d into cosø where ø is the angle measured from the axis of the array so axis is the line joining the antenna elements and the angle ø is measured from this axis so the angle ψ is defined as β d cosø which is the space phase and the electrical phase which is in the excitation of the currents of different elements so that is the progressive phase shift delta. Then by simply applying the superposition we get the radiation pattern of the linear array of elements and in normalized radiation pattern essentially it is given by this expression and then we investigated the properties of this radiation pattern that is the direction in which the radiation is maximum and we saw that when $\psi = 0$ that time or radiation terms add and we get a maximum radiation so $\psi = 0$ corresponds to maximum radiation and then we also investigated the directions of the nulls that is when the numerator goes to zero that means $N\psi/2$ is equal to zero that time or a zero or multiples of $\pi$ that time we get the nulls in the radiation pattern.

Following further now you would like to know that what the directions of the side lobes are, what is the level of the side lobe and also we will try to investigate what is the directivity of this array and we will also try to see how the directivity changes as the direction of the maximum radiation changes.

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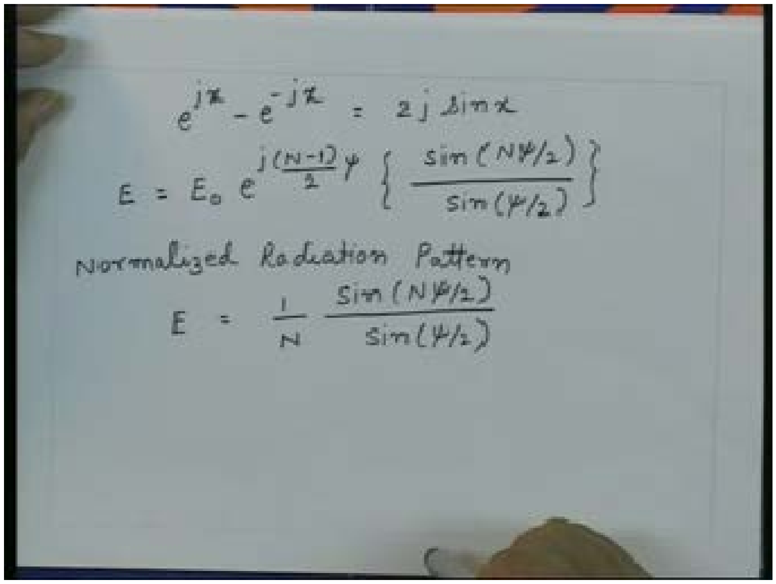


So if I look at this function here the numerator function if I plot as the function of ψ if N is large then this function is a rapidly varying function this function is relatively slowly varying function so if I plot these two functions on the same scale as a function of ψ the functions numerator and denominator would look like that. So here we are plotting the numerator modulus of that and here we are plotting the denominator and we vary the ψ from zero to 2π. So this function is rapidly varying function if N is large and when ever this function goes maximum that time we have a local maxima when this function goes to zero we have null. So essentially these directions where this function is maximum correspond to the directions of the side lobes.

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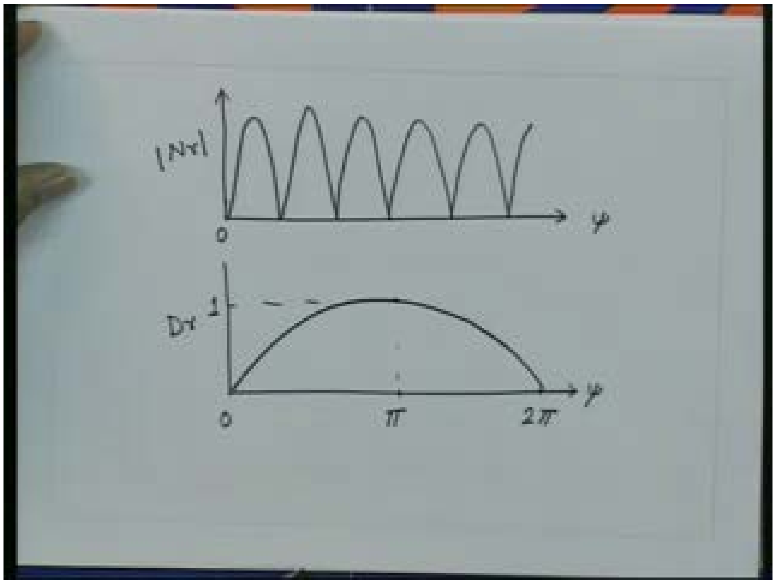


So going back to the expression then one can say when ever this quantity Nψ/2 is maximum and that will be this quantity is one because maximum value of sine is one so when ever this quantity is odd multiples of π/2 that time you will have a maximum for this function and then you will have a side lobe at that location.

So today we see the directions of the side lobes and this side lobe essentially comes in the direction N when Nψ/2 is odd multiples of π/2 this is ± (m +½) π. Now substituting for ø which is βd{cosø - $cosø_{max}$} we essentially now get the directions for the side lobe so from here this gives essentially the ψ which is equal to β d into cosø and let us call this directions as SL representing the side lobes and -$cosø_{max}$ that is equal to ±(m + ½) π, like I bring this two up there so this is 2/N.

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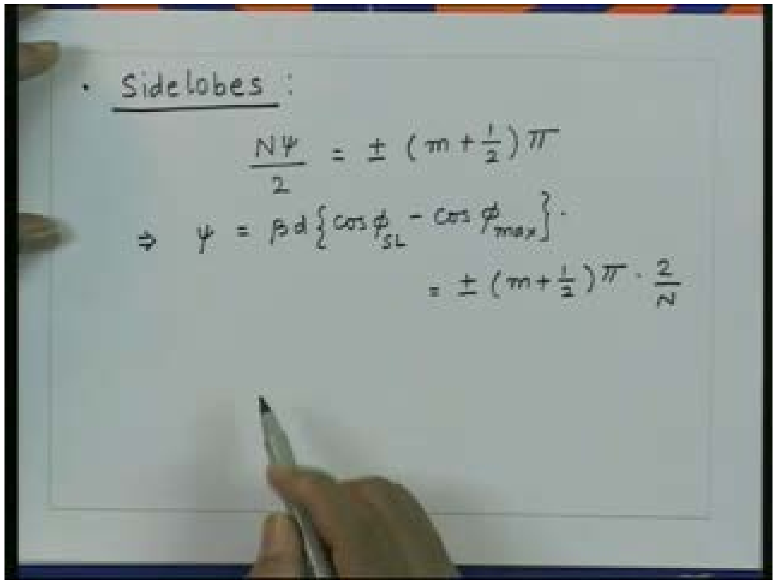


Now the direction of the side lobe by inverting this thing β d on this side essentially we can write down the directions of the side lobes, if I take n equal to one that is the first side lobe then that will be giving me the first level of the side lobe after the main beam then I increase the value of m essentially I get the amplitudes of the various side lobes. So from here I can invert this the relation so from here we get $\cos\text{ø}_{SL}$ that is equal to $\cos\text{ø}_{max} \pm (m + ½) \pi$ imto 2/N divided by β d and β d is $2\pi/\lambda$. So $2\pi$ will gets cancel and we get from here $\cos\text{ø}_{max} \pm (m + ½) \lambda/dN$

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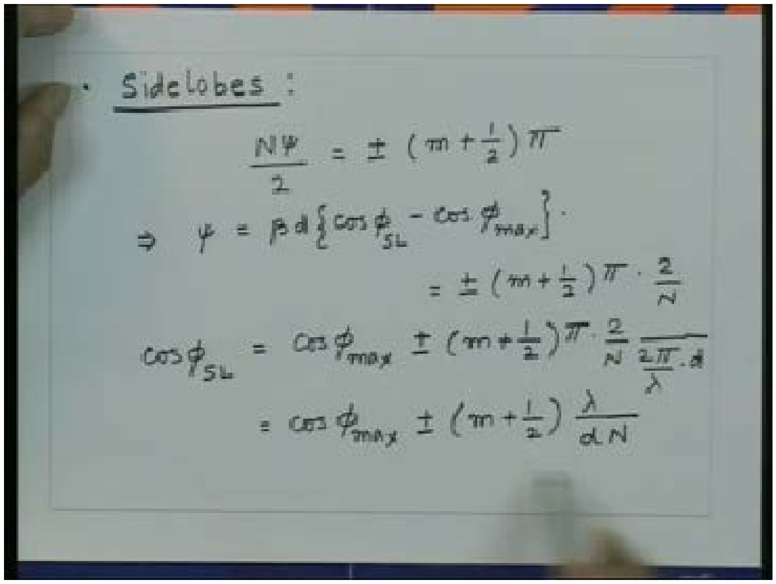


Now again since this is representing $\cos\phi_{SL}$ we have to choose those values of m for which this quantity magnitude of this quantity will be less than one and that essentially will represent the direction of the side lobes. One can also argue from the simple thing as we discussed earlier that is finding out the local maximum requires differentiation of the expression, whereas if you follow a simple logic that there is a maximum between the two zero's of the function we can very easily calculate the directions of the nulls by equating the function to zero. So first we find out the direction of the nulls and then we say somewhere half way between the two nulls there must be maximum for the function so approximately we can calculate the direction half way between the two nulls. So if you are interested in finding out approximate directions for the side lobes then essentially we find out the direction of the nulls and we say the side lobe between any two adjacent nulls and half way is the maximum for the local function. So basically you have a side lobe which is half way between the two nulls.

So by using any of the arguments essentially we can find out approximately the directions of the side lobes. What is important however for the side lobe is not the direction but what is the amplitude of the side lobe because that tells you how much energy is leaked in the direction in which we never intended to send the energy, the array is used to send the energy in the direction of maximum radiation that is what is called the main beam of the antenna, however, because of side lobes the power leaks and that essentially is the wastage of power.

So more important parameter for the side lobe is what is the amplitude of the side lobe compared to the main lobe so in the normalized radiation pattern as we know the main lobe will have a amplitude which is one because when the size equal to zero this quantity would be equal to one so we have the maximum amplitude in the radiation pattern which is equal to unity then one can ask what is the amplitude of the side lobe or what is the amplitude of the highest side lobe. So first let us see if I vary this value of m from one two and three and so on how the amplitude of the side lobes will vary? So as we said earlier when ever this quantity is pi by two I get the first side lobe when ever this quantity is $3\pi/2$ I get second side lobe, $5\pi/2$ third side lobe and so on.

So essentially when we put the quantity here m which goes from 1, 2, 3 and so on so m equal to one correspond to the first side lobe remember here the side lobe is going to come only after first zero is crossed so $m = 0$ would not represent the side lobe the m has to start from one because the first null will occur corresponding to $m = 0$ that will correspond to this quantity when we put $m = 0$. So essentially what we find from this expression is that if I substitute m equal to one which corresponds to the first side lobe then at that location this quantity will be one and sine will be corresponding to $N\psi/2$ equal to $3\lambda/dN$.

So we get from here that for the first side lobe we have $N\psi/2$ that is equal to $3\pi/2$ so you can get from here $\psi/2 = 3\pi/2N$ so the amplitude of the first side lobe will be equal to if I substitute the $\psi/2$ is equal to $3\pi/2N$ that will be equal to one upon N mod of one upon sine of $3\pi/2N$.

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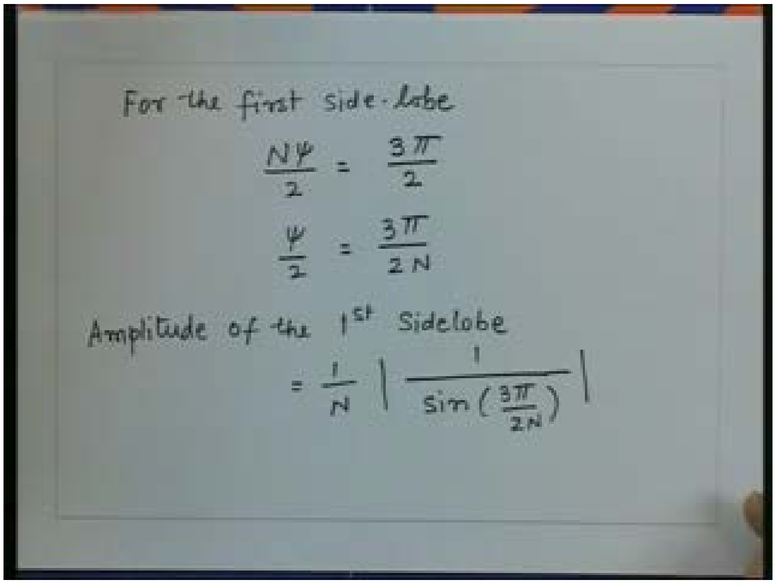


Note here the side lobe the numerator is maximum which is equal to one now we have substituted the value of ψ corresponding to making this quantity 3π/2N or this quantity is equal to one so this is the amplitude of the first side lobe. Now if I consider a array which is large that means N is large this thing can be approximated equal to θ so approximately I can say this is equal to one upon N into mod of 1/(3π/2N) the N would cancel and this is approximately 2/3π so the amplitude of the first side lobe is 2/3π.

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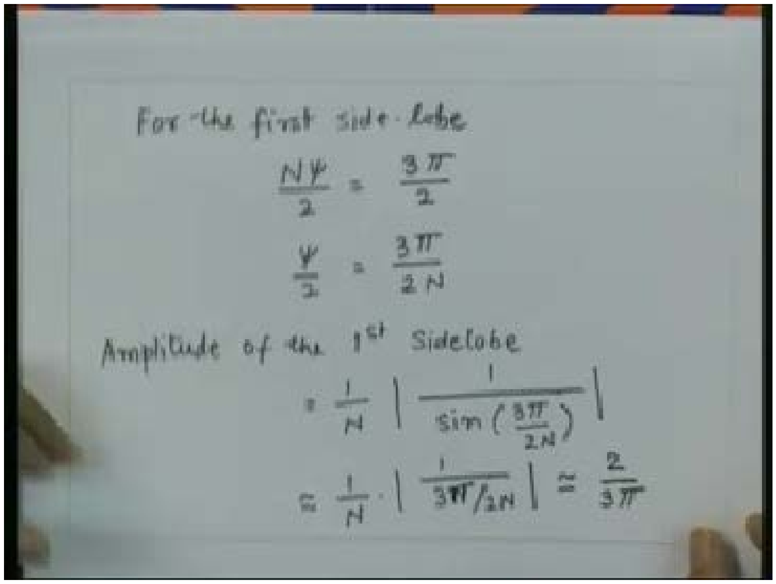


if I go to the second side lobe that would correspond to this angle which is sine 5π/2 so Nψ/2 if I take 5π/2 that will give me the second side lobe, if I take that equal to 7π/2 that will give third side lobe and so on.

So here I have the second side lobe amplitude which is approximately 2/5π, the third side lobe amplitude will be approximately 2/7π and so on so as I go away from the main beam the m increases and the amplitude of the side lobe decreases so the highest side lobe essentially is the one which is the first side lobe which is next to the main beam. So if I look at the radiation pattern if I plot this radiation pattern now the radiation pattern will essentially look like that in the Cartesian coordinate let us say this is the plot of this expression the radiation pattern which we have got which is sine Nψ/2 upon sine ψ/2 as a function of ψ if I put the function of ψ this is the one which corresponds to ψ = 0 which we call as the main beam and these are the locations of the nulls where the function goes to zero these are nulls and these are the locations of the side lobes so we have this is side lobe one this is also side lobe one this is side lobe two and so on.

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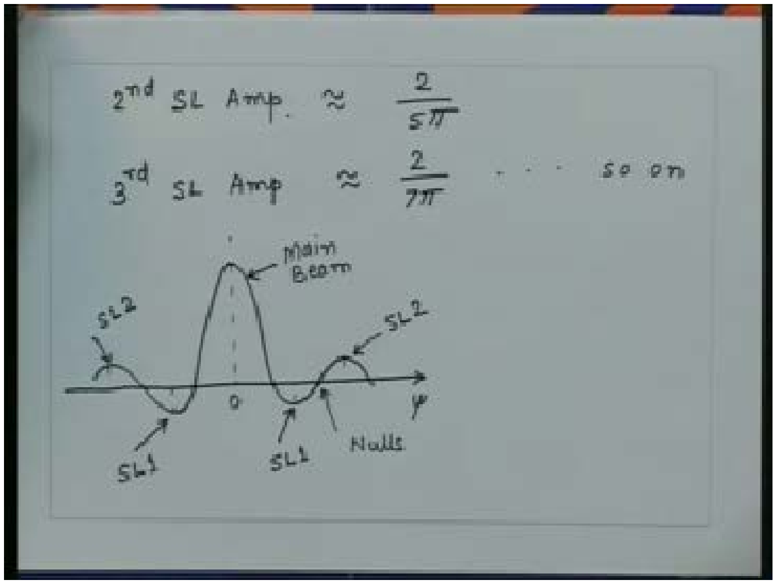


So as we saw this amplitude here is 2/3π this amplitude will be 2/5π so the amplitude of this side lobe is reducing as we go away from the main beam so the highest side lobe is this one which is having an amplitude which is 2/3π which is approximately 21% so this quantity first side lobe which we get is approximately 21% of the main beam amplitude so if I say this is normalized this is one  this will be -0.21, this amplitude which corresponds to second side lobe which is 2/5π will be thirteen percent of the maximum so this value will be 0.13 and so on the same pattern if I put in the polar plot then it will be a maximum beam which corresponds to the main beam then we have a side lobe which is 21% of this which is like that, this will be second side lobe and so on and this is the direction which corresponds to main beam which is $\psi = 0$ so if i see on a polar plot this gives you the unity this will correspond to 21% of the level which is first side lobe.

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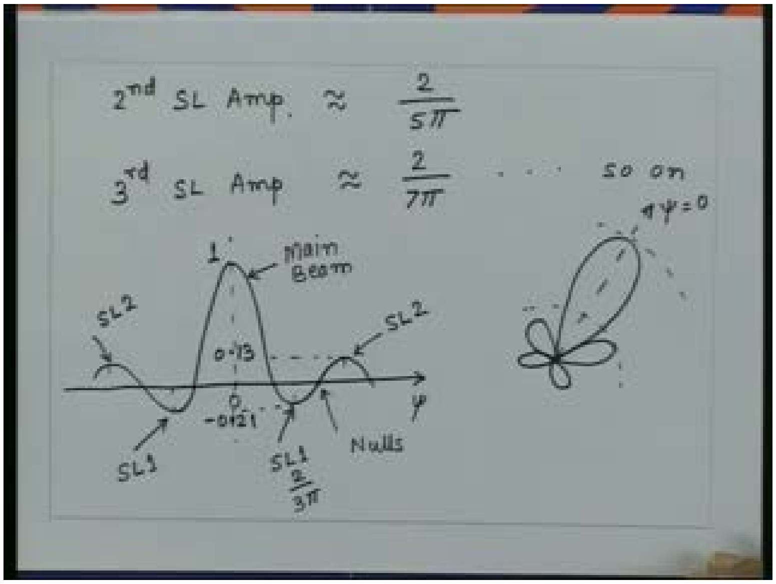


So the interesting thing which we note from this is the level of side lobe is not dependent of any of the array parameters, for example, we had three array parameters which were identified that is the number of elements in the array the progressive phase shift of the array and the inter element spacing of the array where the progressive phase shift decide the direction of the maximum radiation, the number of elements are not coming in the direction of the maximum radiation and the nulls are of course decided by the number of elements in the array and larger the value of N more will be the nulls so the number of side lobes which we will see in the radiation pattern will increase then the number of elements increase in the array but the amplitude of the side lobe is independent of number of elements.

As long as N is large the first side lobe which will be about 21% and by no means you can really control this parameter that means for a uniform array you will always have the directions in which the power will leak and the power will be leaking in the direction which will be substantial because this is about 21% this will be 13% if I sum up all together you will see that the substantial loss of power in the directions which are the side lobe directions compared to the main beam direction. So side lobe as such as is a very undesirable characteristic of radiation pattern because that essentially represents the loss of power in the radiation pattern. However, we cannot do much for uniform array as long as the current distribution is uniform that means if the currents are equally excited and if there is only progressive phase shift we will always get the side lobe level which is 21%.

The next thing that one would like to find out is what the effective angular sector in which the radiation is going for this array is and that thing we measure by parameter by the half power beam width of the array. So once the radiation pattern is given to you the next interest would be what the half power width of this radiation pattern is. So as we defined earlier we have the radiation pattern that we draw the radiation pattern little expanded so this is what the radiation pattern as the function of ψ this is the direction of the maximum radiation and if I take the 3dB points of this radiation pattern that is when the electric field goes to one over root two of its maximum if I take these two points where this is one upon root two we can find these two directions and this angular width would give me the half power beam width. So I can get first the half power beam width inside and from there I can convert the half power beam width into the physical angle which is ø.

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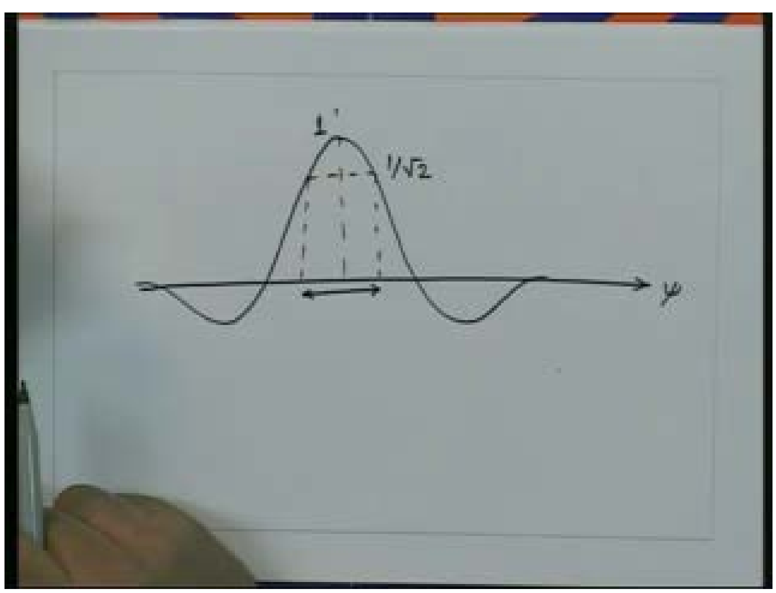

However, in this case again we have to take the radiation pattern equate the radiation pattern to one over root two, solve the expression and then find out the directions from there we can calculate the half power beam width. So essentially what we are saying is if I take this expression for electric field E which is sin(Nψ/2) divided by N sin(ψ/2) and equate that to one over root two, solve numerically this because you cannot solve analytically so solve this numerically find out these two directions where the amplitude will reduce to one over root two of its maximum value and then find out the half power beam width.

However, this process is very tedious because this problem you have to solve numerically so what people normally do is they say if the array is large and the number of elements is large this function is almost like linearly varying from one to zero up to the first nulls so if you make an approximation this function is more or less linearly varying from here to here essentially this width the width between the first nulls around the maximum radiation will be approximately double of the half power beam width if this function is approximated by a linear function and finding this direction of null is much easier than finding this direction where the function reduces to one over root two of its maximum value.

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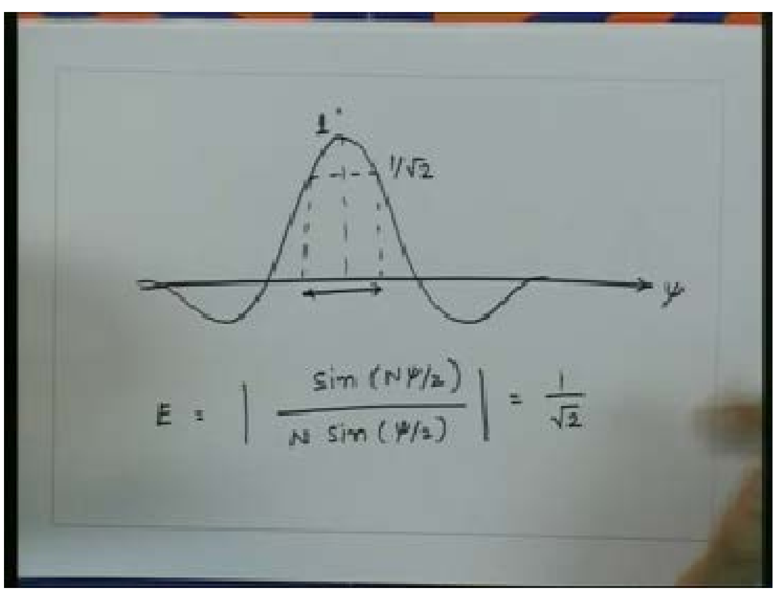

So for a approximate calculation of the half power beam width what we can do is we can find out the directions of the nulls from there we can find out the beam width between the first nulls and then we say the half power beam width is approximately half of the beam width between the first nulls. So we essentially say that approximately the half power beam width is equal to the beam width between first nulls divided by two.

Now if I have a radiation pattern I my job is essentially to find out the directions of the nulls from there I can calculate the beam width between the first nulls and the half of that would be approximately the half power beam width. However it should be kept in mind that depending upon the direction of the maximum radiation the two nulls may not be always visible, what do I mean by that is let us consider a radiation pattern let us say this is the axis of the array and the direction of maximum radiation is somewhere here this is my ø max, now the radiation pattern might look like that other possibility if the direction of maximum radiation is somewhere there let us say this is the direction of maximum radiation I may have a radiation pattern which will look like that.

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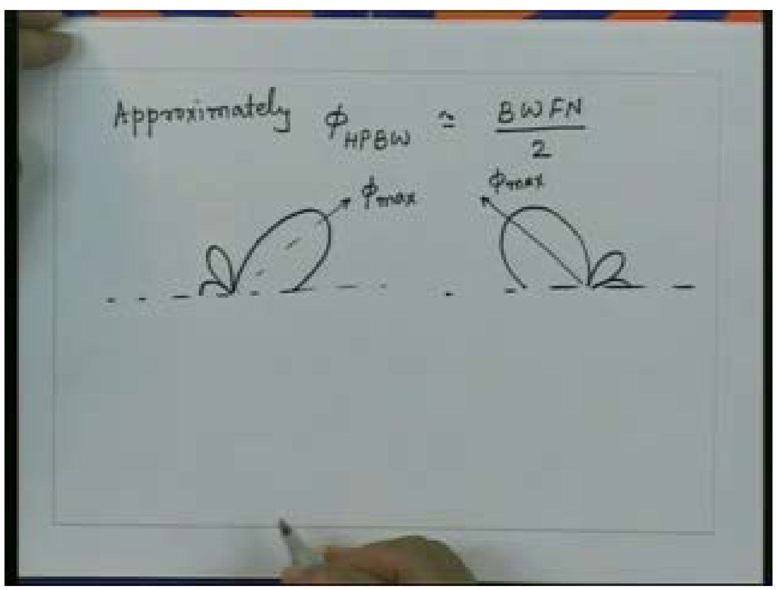


Since the radiation pattern is a figure of revolution around the axis of the array essentially we will get this will be if I draw that it will look like that the same will happen here like this. So since the range of phi is from zero to pi the nulls for this radiation pattern which corresponds to this is visible but the null corresponding to the other side of the main beam is not visible in this case.

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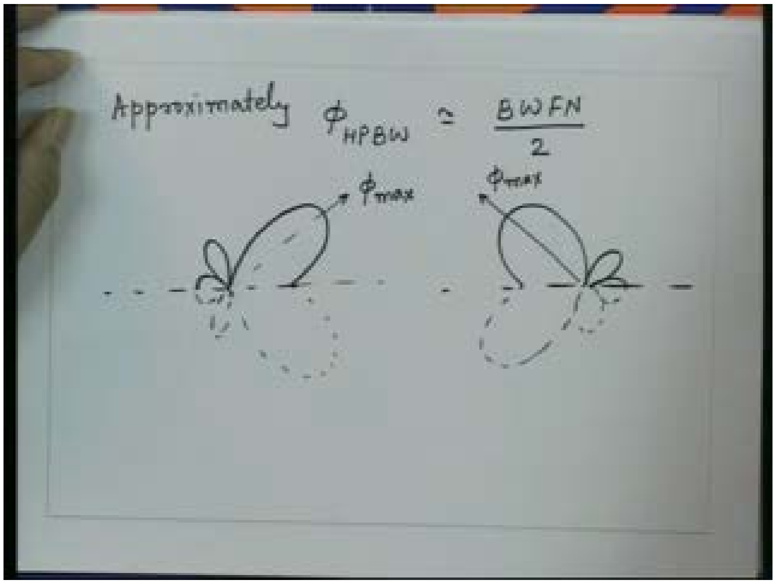


So now if I look at the expression for the side lobe which we have got here we get this quantity plus or minus so if I take the value plus here it represents essentially the nulls which are on this side of the main beam whereas if I take the negative sign and that would correspond to the null which are on this side of the main beam. So in this case you will have a null corresponding to m = +1 and so on, where as in this case the null corresponding to m = +1 is not visible but the null corresponding to m = -1 is this the minus sign. The same is true in this case in this case the negative sign null corresponding to negative sign is not visible but the null corresponding to the positive sign is visible.

So in this situation one can make further approximation and can say that assume that the nulls are symmetrically placed in ø domain, also they are symmetrically placed in ψ and as we mentioned earlier there is a non linear relation between ø and ψ so the nulls need not be equi spaced in ø, however, if you make an approximation that nulls are equi spaced in ø then we can easily find out the direction of the maximum radiation and the null which is half of the beam width between the first nulls which is approximately equal to the half power beam width of the array. So for approximate calculation of half power beam width of the array we can now do this kind of simplification so essentially what we do is we find out a direction of the maximum radiation find the direction of any of the nulls which is visible, find the difference between these two directions that is approximately equal to the half power beam width of the array.

So ideally speaking if these two angles are let us say this is $ø_2$ and this is $ø_1$ and let us say this one corresponds to the null for which the angle is given as $ø_{n1}$ to the power plus where this plus corresponds to the positive sign and this angle corresponds to angle $ø_{n1}$ to the power negative sign. So the half power beam width $ø_{HPBW}$ is equal to $ø_2 – ø_1$ that is the half power beam width that is approximately as we said is $ø_{n1}$ to the power plus minus $ø_{n1}$ to the power minus divided by two and that we said if the direction of the maximum radiation is $ø_{max}$ that we said is approximately equal to $ø_{n1}$ to the power plus minus $ø_{max}$ and that is approximately also equal to $ø_{max}$ minus $ø_{n1}$ to the power minus.

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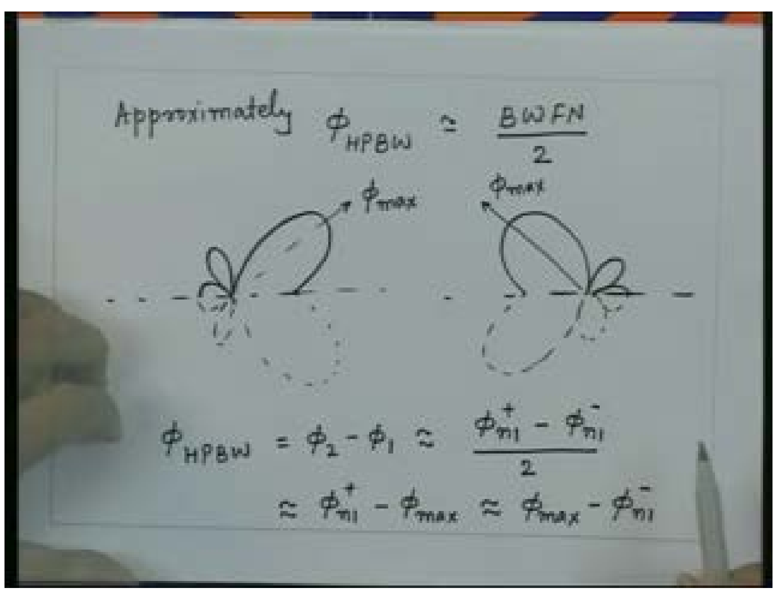

So if I consider any nulls in this case this would correspond to $ø_{n1}$ to the power minus this would correspond to $ø_{n1}$to the power plus so this angle is approximately equal to the half power beam width the same is true here this angle corresponds to the half power beam width.

So if I have a situation like this I will say half power beam width is $ø_{n1}$ to the power plus minus $ø_{max}$, if I have a situation like this and I can say this is $ø_{max}$ minus $ø_{n1}$ to the power minus. If the both nulls are visible then I can calculate these two directions and is $ø_{n1}$ to the power plus minus $ø_{n1}$ to the power minus divided by two will give me approximately the half power beam width.

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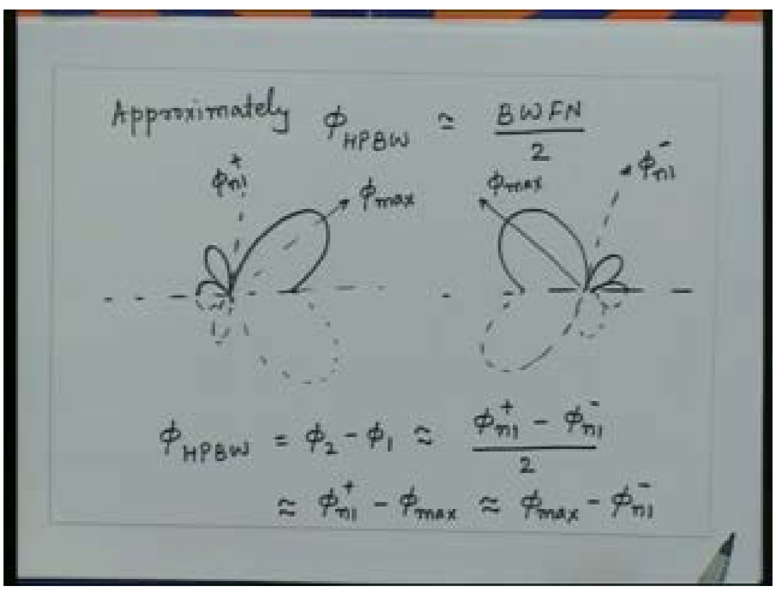


Let us take one of the cases let us consider that I use this expression I have a situation something like this and that gives me approximate direction of the half power beam width. So let us say I use this expression which says the half power beam width is approximately equal to $ø_{n1}$ to the power plus minus $ø_{max}$. Now $ø_{n1}$ to the power plus would correspond to when this m is if you take this plus sign and this is m = 1 so by doing this we get cosine of $ø_{n1}$ to the power plus would be equal to $\cos ø_{max}$ minus $\lambda/dN$

so you can get from here cosine of $ø_{n1}$ to the power plus minus cos $ø_{max}$ is equal to $\lambda/dN$ we can take the sign as positive we can expand this the cosine using the identity so we can get this as two times sine of $ø_{n1}$ to the power plus minus $ø_{max}$ divided by two into sine of $ø_{n1}$ to the power plus minus $ø_{max}$ divided by two that is equal to $\lambda/dN$.

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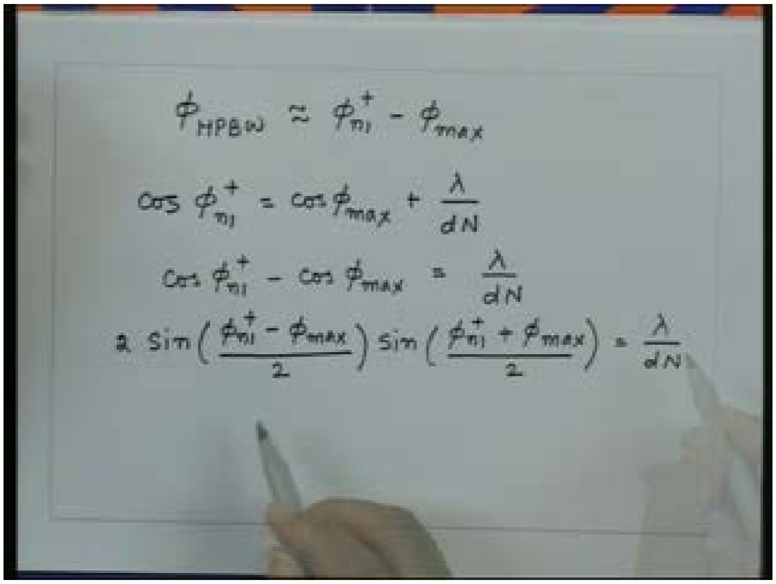


Now this quantity as we said is nothing but half power beam width so I can substitute now for $ø_{n1}$ to the power plus as the half power beam width plus $ø_{max}$ so this one can be written as two times $\sin(ø_{HPBW}/2)$ into if I substitute for this which is $ø_{max}$ plus phi half power beam width it will be $\sin[(2\, ø_{max} + ø_{HPBW})/2]$ that is equal to $\lambda/dN$.

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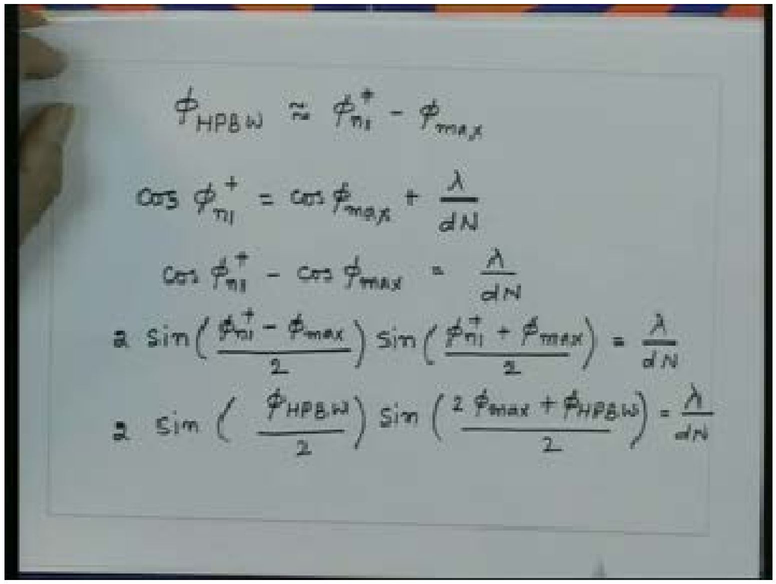


I can expand this thing to get two times sin ($ø_{HPBW}/2$) times the expansion of this which is sin($ø_{max}$) cos ($ø_{HPBW}/2$) plus cos($ø_{max}$) into sin ($ø_{HPBW}/2$) that is equal to $\lambda/dN$.

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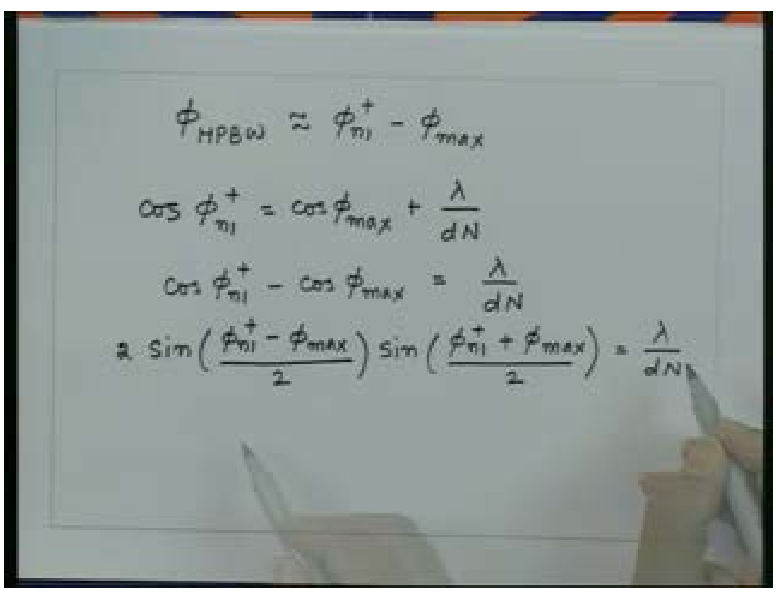


Now if N is very large the half power beam width is much much smaller than one because the beam width between the first nulls is going to be very small. So if I say that if n is much much greater than one for a large array that would mean that ø half power beam width is much much less than one in radians.

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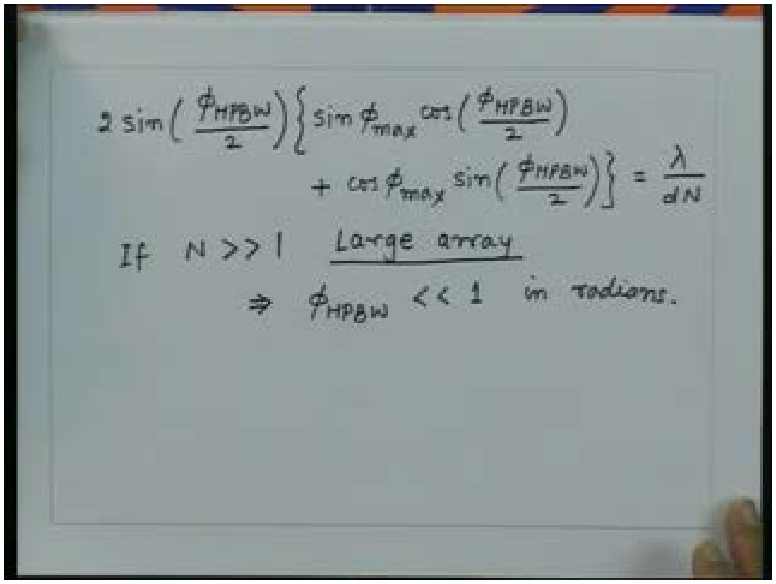


So I can make the approximation that if this condition is satisfied so you know that if x is much much less than one then sin x can be approximated by x and cos x could be approximately one so I can substitute now into this so this quantity here which is very small so I can make this quantity almost equal to one and this quantity is half power beam width divided by two so we can write down this expression approximately it is two times sine of we are approximating this also by x so two times half power beam width upon two sine of ø max this quantity is one plus cos ø max and this is equal to øø half power beam width divided by two so this is phi half power beam width by two that is equal to λ/dN.

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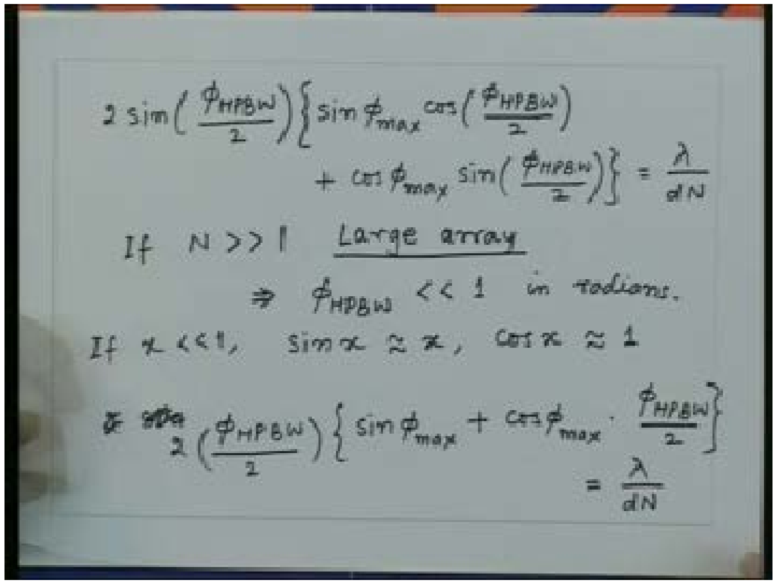


I can simplify this it to further to get the expression which is $ø_{HPBW}$ square into cos $ø_{max}$ + two times sin $ø_{max}$ into $ø_{HPBW}$ that is equal to 2λ/dN. Note here the expression which we have got is from here so this quantity is ø of half power beam width multiplied with this that will give me the ø square half power beam width multiplied by cosine of $ø_x$, second term will be two times sine of phi maximum multiplied by ø of half power beam width that is equal to 2λ/dN.

Now we can take two extreme cases that are when the beam is in the broad side direction and the beam in the N ø direction that means when the $ø_{max}$ is zero that is the end fire direction or when the $ø_{max}$ is π/2 is the broad side direction. First thing we would note that if I solve this equation for the phi half power beam width the phi half power beam width essentially increases as we go from broad side direction to the end fire direction.

So if I numerically solve this for different values of phi max essentially we will see that as the beam direction changes from the broad side to the end fire the half power beam width of this array increases monotonically. So if I take these two extreme cases that when $ø_{max}$ is π/2 that is broad side direction and if I take the case end fire $ø_{max}$ is zero which is the Nø direction, essentially I will get this two extreme cases and one can say that systematically the beam width will be increasing from the broad side direction to the n phi direction.

So for broad side array the $ø_{max}$ is $\pi/2$ so I can substitute here $ø_{max}$ equal to $\pi/2$. So this will go to zero this will be equal to one so I get from here the half power beam width for the broad side array which is approximately equal to $\lambda/dN$, for the end fire array the $ø_{max}$ = 0 and then this quantity is zero this will be equal to one so we will get $ø_{HPBW}$ which will be approximately equal to square root of two $\lambda/dN$.

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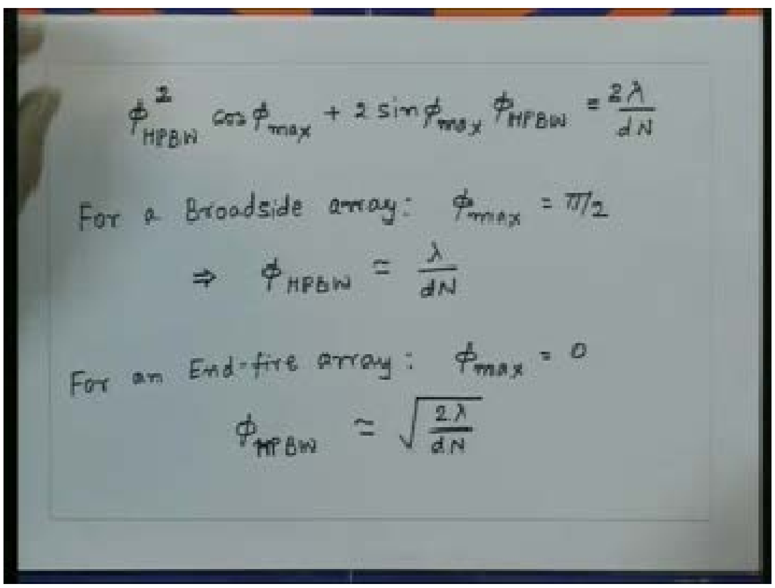


Now for a N element array since the inter element spacing is d the length of the array is d into (N – 1), if N is very large the d into (N – 1) can be approximated like d into N so this quantity d N essentially gives me the length of the array. So then approximately I can say that this is λ divided by the length of that, the same thing I can do here so this is square root of 2λ divided by length of the array.

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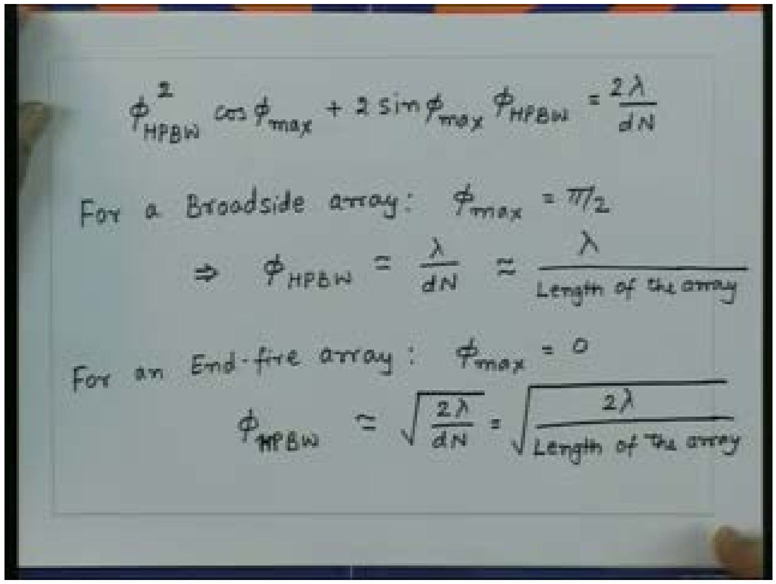


So when the beam is in the broad side direction the half power beam width is inversely proportional to the length of the array or for given inter element spacing it is inversely proportional to the number of elements. So number of elements essentially plays a role in deciding the angular zone in which effectively the energy goes, as the number of elements increase in the array for a given inter element spacing the half power beam width will become smaller and consequently now the radiation will go into a narrower angular zone or in other words, the antenna array now has more focus radiation the radiation does not go in larger sector it goes into a very narrow cone so we get more focusing of the radiation in given direction or in other words it increases directivity of the antenna when the number of elements in the antenna increase for a given inter element spacing. Or if I go in general if I combine d and N into together I can say that as the length of the array increases which could be a combination of the inter element spacing and the number of elements, in general the directivity of the antenna would increase because the half power beam width of the antenna would decrease.

So around the broad side direction since there is inverse relationship between the half power beam width and the length of the array the antenna beams broadens almost linearly as a function of length, however, as I go towards the N ø direction then the dependence is weaker because you are having a square root of the length of the array. So essentially as we go for a given array two things we have to observe now one is if I scan the beam for the phase array let us say this is the array I will get the narrowest beam in this direction and I will get the broadest beam in this direction. So the beam width essentially it increases as we change the direction of the beam from the broad side to the end file. This is very important because when ever we use the antenna array in a environment where we want to scan the beam by changing electronically the phases what is called the phased array antennas the beam width does not remain constant while scanning the antenna.

So purpose of the phase array antenna is that without moving physically the antenna if you can control electronically the phases or if I change progressive phase shift of the antenna array then the direction of the maximum array would change and the beam would scan something like that from horizon to horizon depending upon how much phase gradient you are going to put on the array. While doing this however we do not want the beam shape to be changed significantly but we see here that it will not happen when we see when we have a beam in this direction you will get the beam width which is the narrowest when the beam comes here it will be broader compared to this and the beam comes to the horizon the beam will be the broadest beam.

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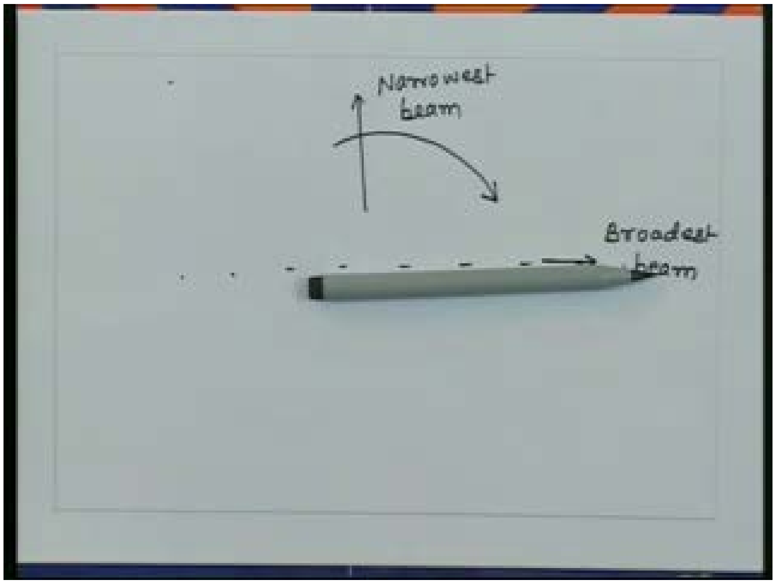


Visualize this radiation pattern as the beam is scanning in the three dimensions and in fact they have totally different appearances it looks like if I consider the planar radiation pattern it will look the beam width will be narrow here something like that when it comes here it will be it will be broader so it will be looking something like this similarly when it comes from the other side it will look broader like that so it looks as if the beam simply is broadening otherwise the pattern practically remains the same that is the appearance you get from the planar radiation pattern.

However as we mentioned earlier we should always look at the radiation pattern which is in three dimension so if I do that then it is radiation pattern is figure of revolution around the axis of the array so for a broad side the array would essentially look like that which is more like a disc something like this, where as when I go to the end fire array the end fire array will be figure of revolution around the axis it will be like that.

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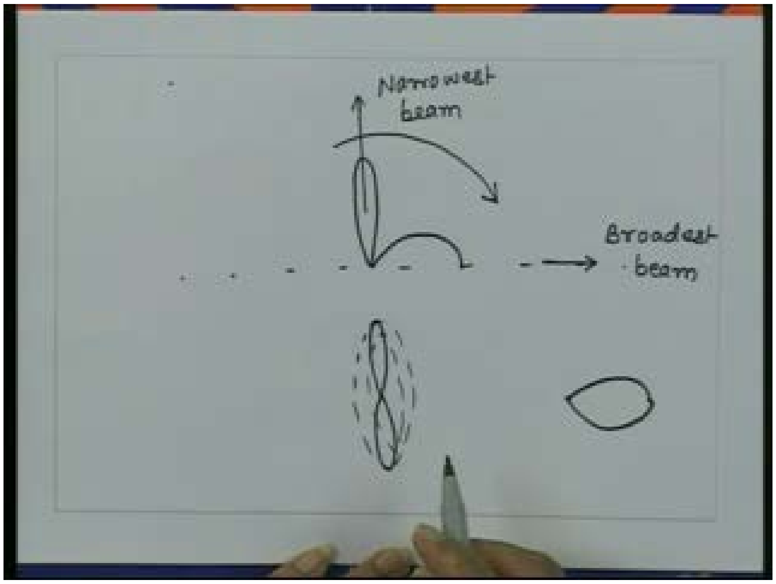


So in one case though the beam in the planar looks only change in shape for change in the half power beam width if I visualize the radiation pattern in three dimension the radiation patterns are quite different in two situations the end fire will look more like a balloon like that where the broad side would look more like a flat disc which is figure of revolution in this direction.

Having understood this then one can go to the calculation of the directivity of these antennas in the two extreme cases which is the broad side and the end fire direction and there is something surprising because looking at this it appears when the beam is scanning from here to here since the beam is broadening the directivity of the antenna is decreasing because the half power beam width increases. However this conclusion may be erroneous if we make only on the basis of planar radiation pattern. So further we will investigate the directivity of uniform array and that understand would develop essentially by getting the three dimensional radiation pattern of this array.

Thank you.