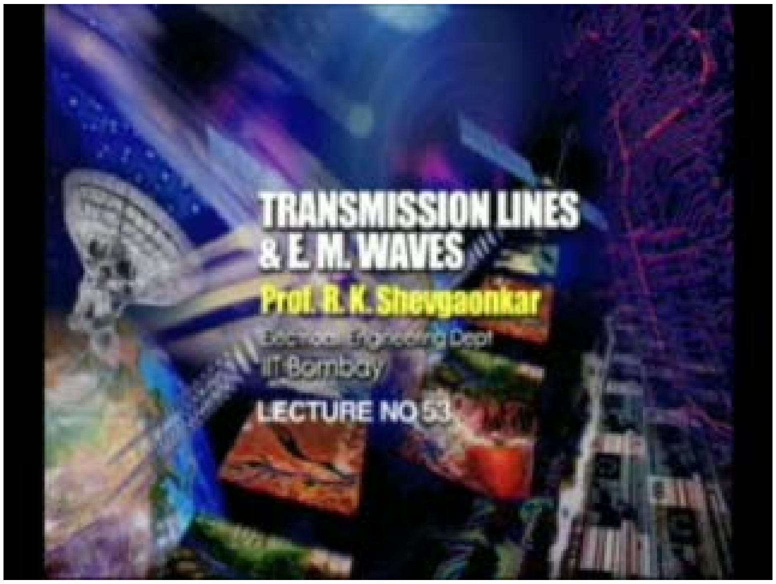
Transmission Lines and E.M. Waves

Prof R.K. Shevgaonkar
Department of Electrical Engineering
Indian Institute of Technology Bombay

Lecture-53

Welcome, we are discussing antenna arrays we found that antenna arrays provide a great deal of flexibility in realizing complex radiation patterns without affecting the terminal characteristics of the antenna. Last time we also saw the characteristics of two element array and we saw there were three parameters in the two element array one was the spacing between the elements the amplitude ratio of the currents exciting the two elements and the phase difference between the two elements.

(Refer Slide Time: 01:55 min)



We saw that each of these parameters have a very distinct effect on the radiation pattern for example, the spacing between the two antenna elements decides the nulls in the radiation pattern and as we increase the spacing between the elements the number of nulls increases in the radiation pattern, the ratio of the amplitudes of the two currents essentially decide the depth of the nulls so if the ratio is equal to one that means the two currents are equal then you have the nulls which are sharp nulls there is a complete translation of the two fields so the direction in which the radiation field goes to zero however if the ratio is not equal to one then we do not have directions in which the field will go to zero.

The third parameter which was the phase difference between the two antenna elements that essentially decides the direction of the maximum radiation. Having understood these basic characteristics of the two arrays now we can go for more complex antenna arrays. Now with these three parameters for the two element array the degrees of freedom which we have are very limited and that is the reason we cannot really manipulate the radiation pattern too much with the two element array, normally since we do not want to send the radiation in only very specified directions the spacing between the antenna element is not such a free parameter because if we increase that parameter then we get more number of nulls that means we get the radiation in zones. So normally the spacing between the two elements is confined between $\lambda/2$ and λ so that we do not have excessive nulls in the radiation pattern so this parameter which is the spacing between the two elements is not that free parameter to choose.

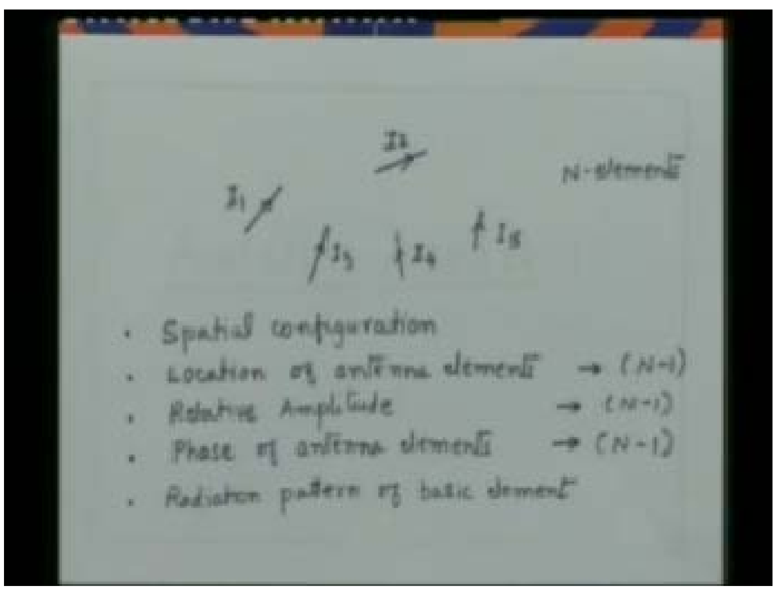
Also, since we want the directions in the radiation pattern where the field should go to zero there should be complete cancellation of the fields we want there should be very deep nulls in the radiation so normally even the amplitude ratio is not chosen very arbitrarily unless we want a complex cliché of radiation pattern. So even the amplitude ratio is not that a free parameter while designing the antenna arrays so the parameter which one controls is the phase difference between the antenna elements with that then obviously we have very little control over the radiation pattern that means we can almost control the direction of the maximum radiation and steer the radiation only in desired direction.

In practice, we want much more flexibility in designing the radiation patterns and that is the reason essentially we increase the number of elements in the array. Obviously as we saw last time that an antenna array if I have a N element array then I have $3(N - 1)$

degrees of freedom one corresponding to the location of the elements other corresponding to the relative amplitude of the elements and the third one corresponding to the phase of the antenna elements.

As we argued just now that location normally is chosen by some other parameters that is normally the separation between the antenna elements which lies between λ/2 and λ and we will see the reason why we do that. The relative amplitude normally is taken constant to these two degrees of freedom essentially we do not use in the simple arrays, also we do not require actually control of each and every antenna element though it gives you much more flexibility it require much more complex electronic circuitry for controlling phase of individual antennas and in many of the applications such a individual control of the antenna is not really required.

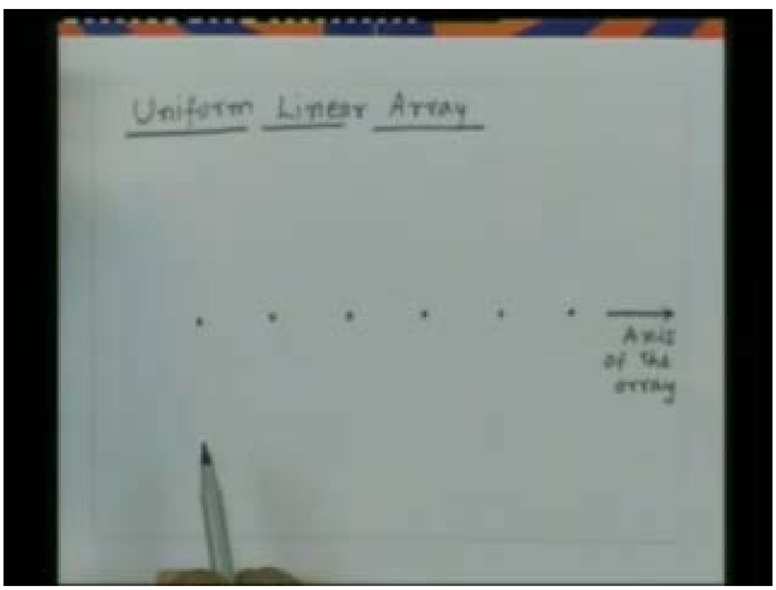
(Refer Slide Time: 06:14 min)



So then what we do essentially is we take a very special case of array called the uniform array in which the spacing between the adjacent elements is same so that all the elements are equi spaced. Now all the elements are excited with equal amplitude so that means the amplitude ratio of the currents different elements is one and the phase difference between two adjacent elements is also same, then this array which is excited with uniform current but with equal phase difference between the adjacent elements is called the uniform array.

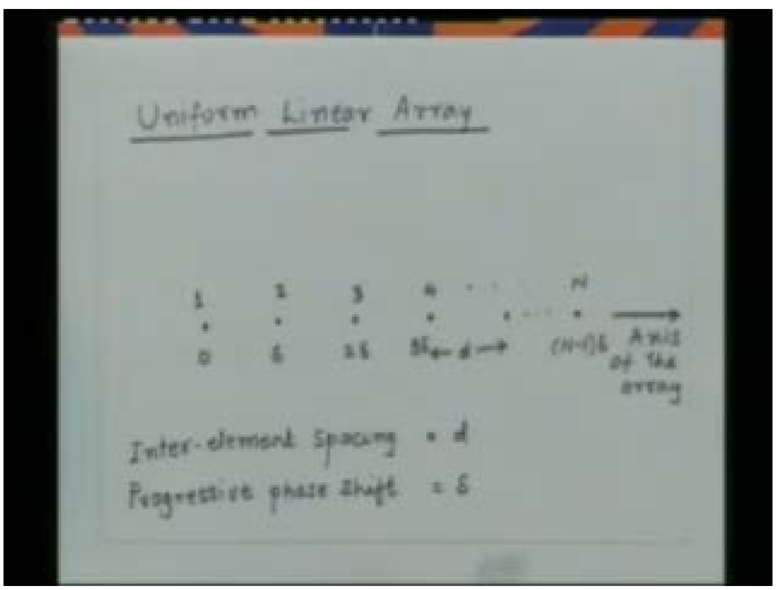
So today we essentially investigate the characteristics of the uniform array and if the antenna elements are arranged in this uniform fashion along a straight line then this array is called a linear uniform array. So in today's lecture essentially we investigate the characteristics of linear uniform array. So let us say we have now the antenna elements which are arranged along a straight line and they are equi spaced so we have this antenna elements which are equi spaced and they are arranged along a straight line again as we define for the two element array the line joining these elements the straight lines we call as the axis of the array. So this line is the axis of the array and as we did earlier essentially we consider all the angles measured from the axis of the array.

(Refer Slide Time: 08:23 min)



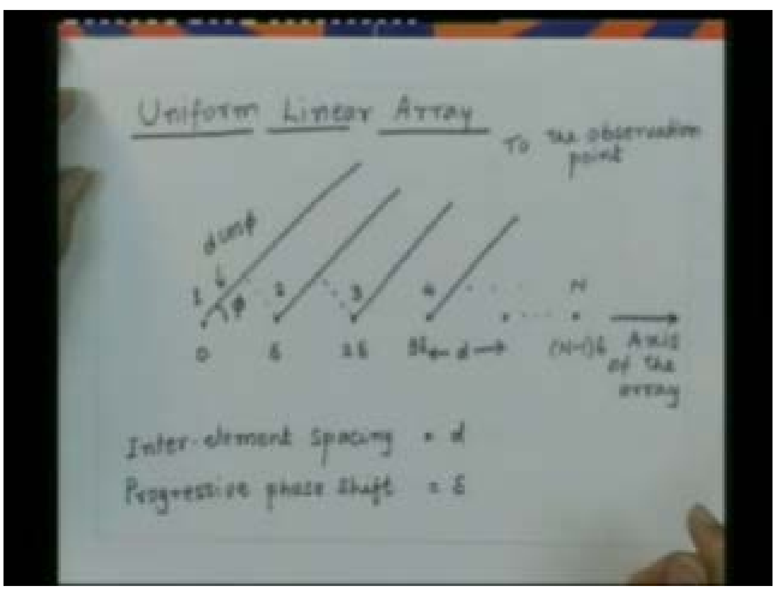
Now as we did in the previous case we again assume that all these antennas are isotropic antennas that means they are radiating equally in all directions and essentially we go to a very far away point from this array and find the superposition of the fields coming from the individual antennas. Now as we mentioned the spacing between these elements is same so let us say this spacing is given by d and we have a parameter for the array which is inter element spacing that is d. Then the amplitudes are same for all of them. So let us say without losing generality I can say the amplitude for each of them is unity and the phase difference between any two adjacent element is same that means if I take this antenna element as the reference element this phase is zero then this phase let us say is delta so this will be two delta this will be three delta four delta and so on that phase then we call as the progressive phase shift of the uniform array we have this quantity the progressive phase shift let us denote that as delta. So the phases for this element this is the phase zero, this will be delta, this will be two delta, this will be three delta and so on if I take the N th element let us say this element is one this is two three four and so on and let us say this element is the n th element so the phase of the N th element will be (N – 1) into delta.

(Refer Slide Time: 10:34 min)



Now the uniform array has three parameters, one is the inter element spacing d not that a free parameter as we said its range will lie between somewhere λ/2 and λ progressive phase shift which can be chosen arbitrarily for any value and third parameter is the number of the elements in the array this quantity N and essentially now we want to find out what way the radiation pattern is controlled by these three parameters the number of elements the inter element spacing and the progressive phase shift. As we did earlier let us take a very far away point so these directions are parallel with that point is the observation point we are going to the observation point which is very far away so the radiation which leaves this antenna would now be leading the radiation coming from here by two quantities one is the phase because of the electrical phase difference between these two antennas the currents are excited with a phase difference which is delta, secondly the phase which is coming because of the space that this radiation is traveling a distance shorter compared to this radiation by this amount. So if you say that this angle is phi so this distance which is d so d cosø will be the phase difference between the two radiations due to the space so this distance is d into cosø, similarly this radiation will be leading with respect to this again by delta because of the current phase difference and again by beta into d cosø because of the space so this is the distance d cosø the β is the phase change per unit distance so the phase difference between these two radiation will be βd into cosø.

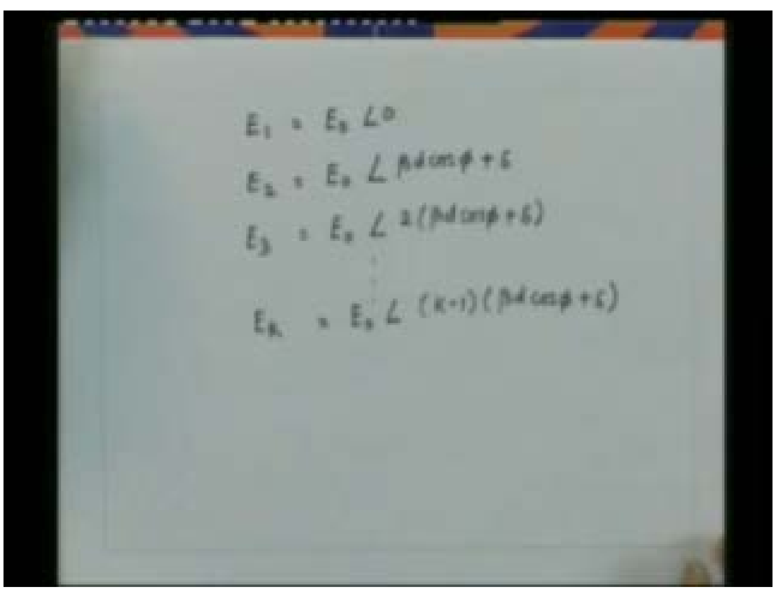
(Refer Slide Time: 13:04 min)



So now we have we take if I take this element as a reference element then let us say the phase of this will be zero this element will be leading in phase compared to this one which is beta d cosø plus delta this will be leading with this element by again β d cosø plus delta so that means with respect to this element it will be leading by two times β d cosø into delta it leads by this phase corresponding to this distance and the phase difference between these two which is equal to two delta.

Now I can say that the electric field which I am going to get because of these different elements that I can write as E_1 which will be some quantity E_0 which is the radiation field which I get because of this element at a very far away distance with the angle θ this is because I have taken the element one as the reference element. The second element will have electric field E_2 will again be E_0 with the phase difference of β d cosø plus delta, the E_3 will be E_0 with angle two times β d cosø plus delta and so on. So the K th element in general will have the electric field that will be E_0 angle (K – 1) into β d cosø plus delta where K essentially will go from one to N so you get the phase for the first element which will be equal to zero and the phase for the N th element which will be (N – 1) into β d cosø plus delta.

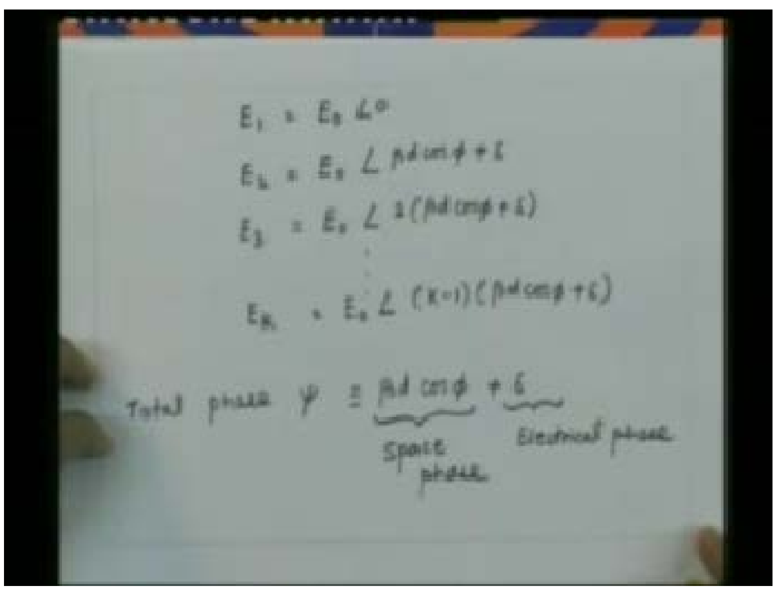
(Refer Slide Time: 15:39 min)



So essentially now we see that the radiation which is coming from different elements has the phase component coming from two reasons, one is the current which is exciting the antenna is having a phase difference which is delta and secondly the radiation traveled by from different elements traveled by different distances and because of that the radiation undergoes different phase changes so we get this quantity here β d cosø which we can call as the phase shift between the two adjacent elements or the phase difference between the radiation due to two adjacent elements that quantity which is β d cosø plus delta.

Just for the brevity reason let us call this is the total phase which the adjacent radiation or the radiation from the adjacent elements undergo. So let me call for brevity reason the total phase ψ is equal to β d cosø plus delta where this quantity is coming because of phase and this quantity is coming because of the electrical phase.

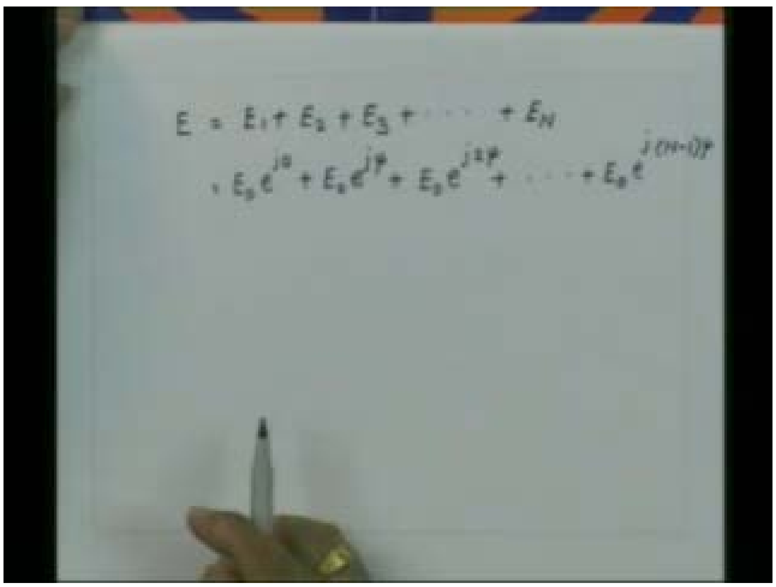
(Refer Slide Time: 17:14 min)



So we have here the space phase and we have the electrical phase once we get that then the problem is straight forward essentially we take these electric fields from different elements and simply superimpose and find the total electric field which is sum of all the electric fields. So we get the sum of total electric field for this array which is E that is equal to E_1 plus E_2 plus E_3 plus so on up to E_N if I substitute for all these things here this will be E_0 angle zero so I can write that there is a form E_0 e to the power j zero this is E_0 the power j βd cosø plus delta and so on.

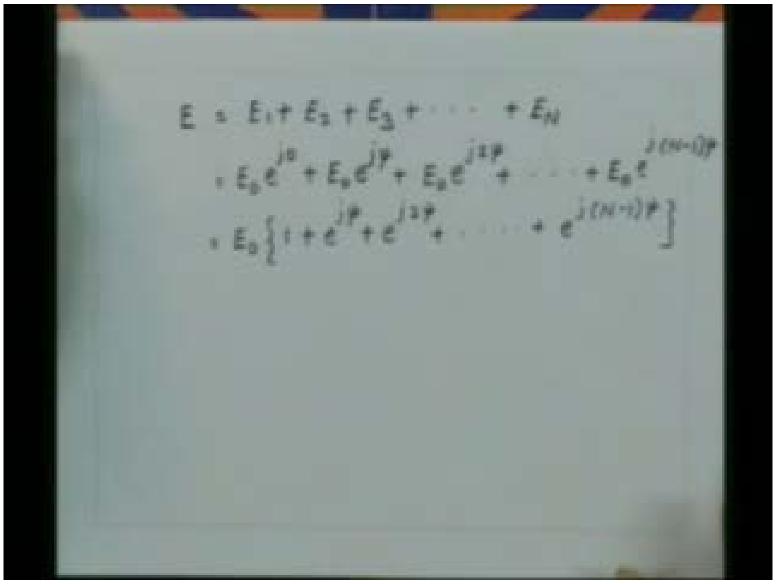
So essentially this quantity will be e to the power j zero plus e two that will be E_0 e to he power j βd cosø plus delta for that we have to find out this quantity what is the total phase ψ so this one we can write as e to the power j ψψ this will be E_0 e to the power j 2ψ plus so on for E_N it will be E_0 e to the power j (N – 1)ψ.

(Refer Slide Time: 19:07 min)



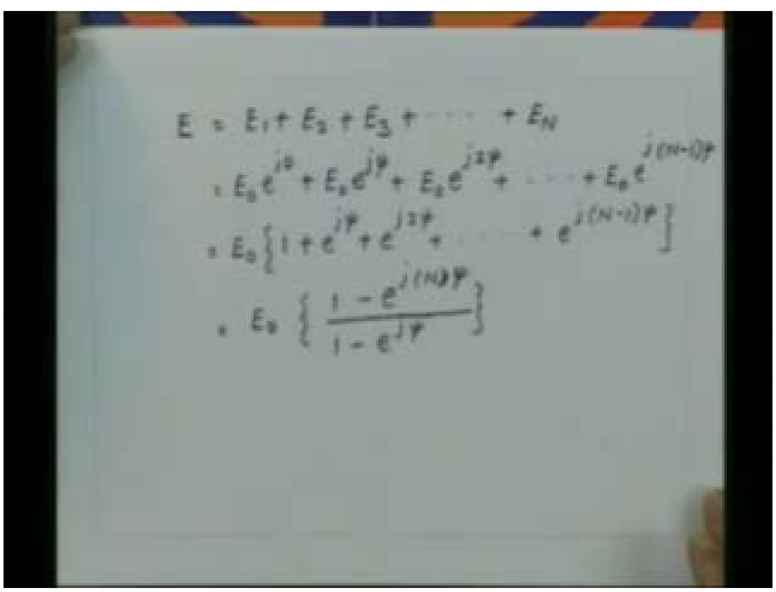
Here E_0 is common so we can take this out this is E_0 e to the power j zero is one so this is one plus e to the power jψ plus e to the power j 2ψ plus so on e to the power j(N – 1)ψ.

(Refer Slide Time: 19:37 min)



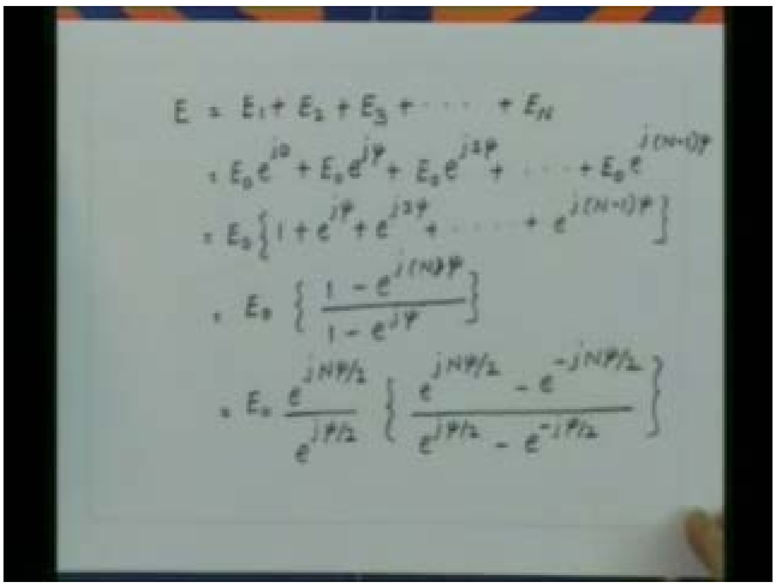
Now this is the geometric series with a progression ratio of e to the power jψ so we can use the expression for the sum of the geometric series so this one we can write as E_0 into one minus e to the power j Nψ divided by one minus e to the power j ψ.

(Refer Slide Time: 20:23 min)



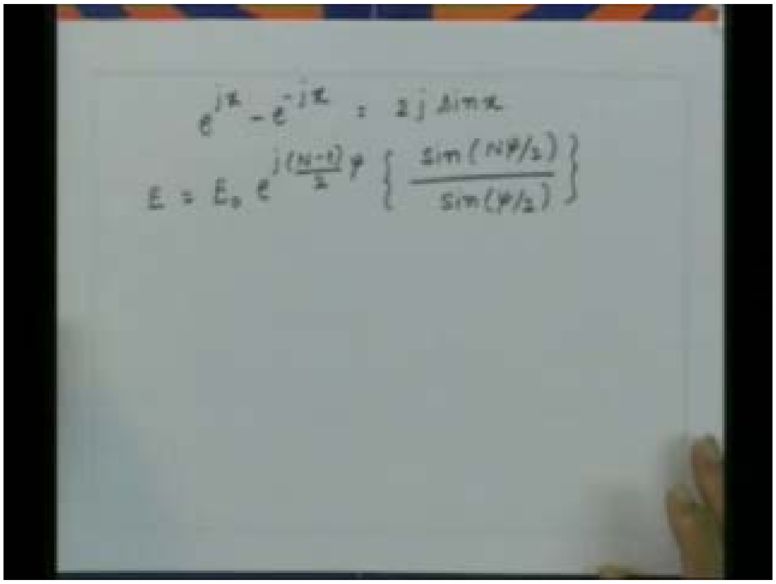
I can simplify this expression by taking e to the power j Nψ by two common from the numerator and e to the power j ψ by two common from the denominator. So this one we can write as E_0e to the power j Nψ/2 and from the denominator e to the power jψ/2 and also taking the minus sign common from numerator and denominator this will be e to the power j Nψ/2 minus e to the power -jNψ/2 upon e to the power j ψ/2 minus e to the power -j ψ/2.

(Refer Slide Time: 21:28 min)



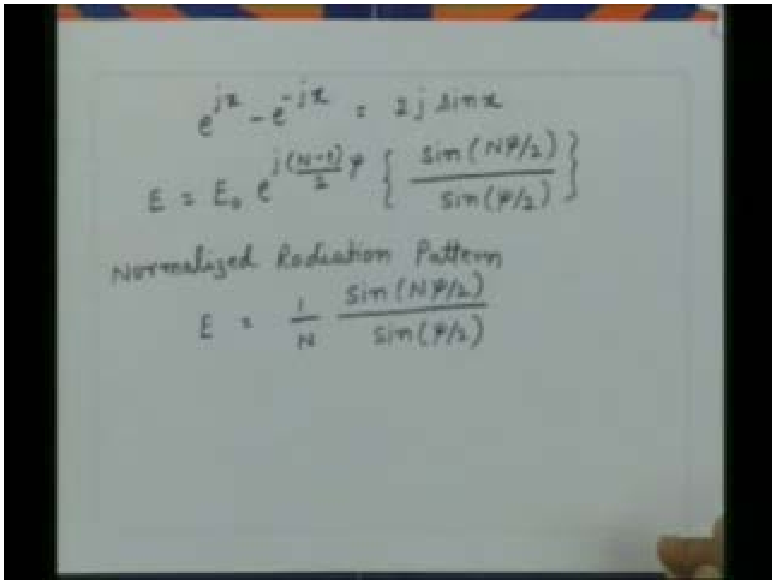
Now note this quantity e to the power jNψ/2 minus e to the power minus -jψ/2 it is two times j sine of Nψ/2. So we know this that e to the power j x minus e to the power -j x that is equal to two times j sin x. So I can substitute from this so we get the electric field due to this N element array that is E_0 e to the power (N - 1) upon 2ψ this quantity Nψ/2 minus ψ/2 multiplied by 2j sine of Nψ/2 and this will be 2j sine of ψ/2 two j will cancel I will get here sine of Nψ/2 divided by sine of ψ/2.

(Refer Slide Time: 22:48 min)



Now again since we are interested only in the radiation pattern and as we know radiation pattern is a normalized pattern so we can divide this electric field by this quantity to normalize it, also we have to find what is the maximum value of this expression so that we can normalize with respect to that quantity and we get the maximum value essentially from the basic series so maximum will occur when $\psi = 0$ that time all this term will add in phase so when $\psi = 0$ all would be E_0 plus E_0 plus E_0 N times so you will get the maximum value of this which will be equal to n. Now normalizing this thing with the maximum value which is equal to N we get the normalized radiation pattern E is equal to you are dividing by the maximum value which is E_0 into N and we are interested only in the amplitude variations so this quantity gives you only phase so this normalized radiation pattern would be $1/N \sin(N\psi/2)$ divided by $\sin(\psi/2)$ so this is the radiation pattern which you get for a uniform linear array.

(Refer Slide Time: 24:44 min)



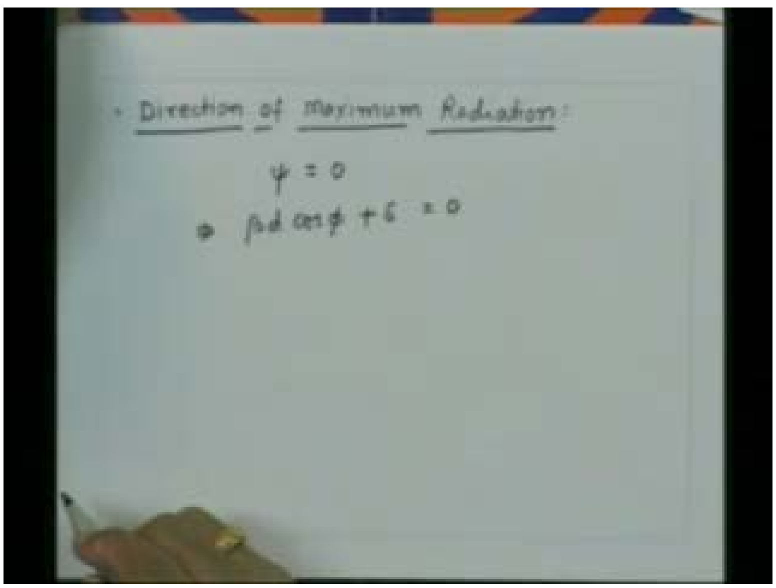
So once I know the quantity which is the progressive phase shift and the spacing between the antennas and the total number of elements I can get the radiation pattern for this array which in normalized terms is given by this expression it is easy to remember this is $\sin(N\psi/2)$ divided by $\sin(\psi/2)$ so it is a very simple expression to remember for an N element uniform array.

Once we get the general radiation pattern then obviously there are certain characteristics we would like to investigate as we did in case of two elements that is what is the condition for the maximum radiation, which direction the radiation will be maximum, what are the directions of the nulls, what is the side lobe level of the radiation pattern and so on. So now essentially we have to ask the questions about the characteristics of the radiation pattern that is direction of maximum radiation, the directions of the nulls and the level of the side lobe which this radiation pattern would have.

So let us say first we investigate the direction of the maximum radiation. Now as we saw in this case that the maximum radiation would be corresponding to when all the terms add in phase that means when this quantity ψ is zero that times this series will be E_0 plus

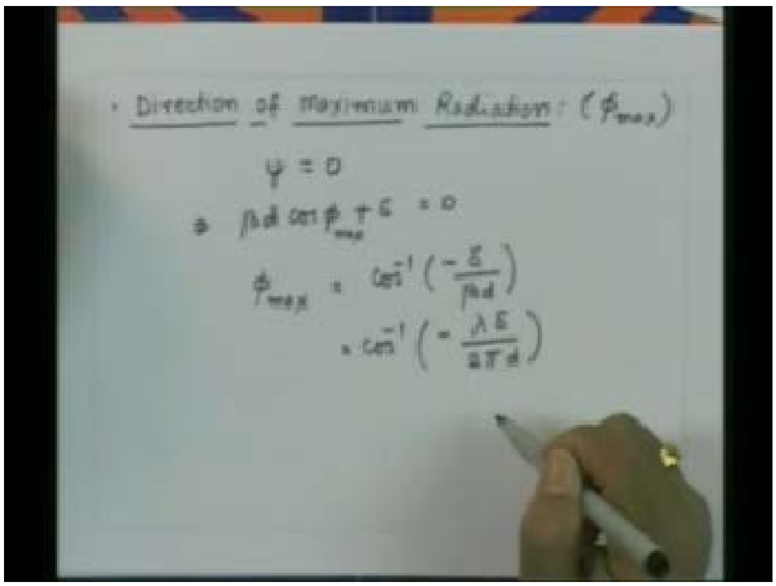
E_0 you will get a maximum amplitude which will be N times E_0, also when ψ is multiples of 2π that time also again the series will give the same value which will be equal to N. times e zero so we get maximum radiation when ever ψ is zero or is multiples of 2π However, we will see later on that the direction which corresponds to ψ = 0 is the direction in which we decide to send the radiation. Normally we do not want the radiation to be sent in two directions or many directions we have a specified direction in which you want to send the radiation so normally we choose ψ = 0 at the direction of the main beam so we say that the radiation will be maximum when this quantity ψ the total phase that goes to zero. So we get from here that maximum radiation which corresponds to ψ = 0 that is your β d cosø plus delta that is again equal to zero.

(Refer Slide Time: 28:00 min)



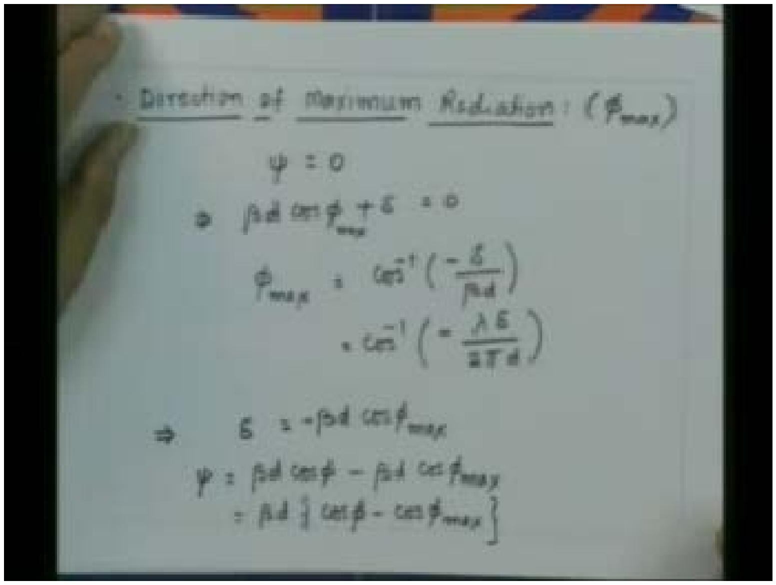
Let me call the direction where the radiation is maximum at ø max so direction of maximum radiation denote that as angle ø max so for that angle the ψ should be equal to zero and from here then I can find out the direction of the maximum radiation which will be the ø max that would be equal to cos inverse of minus delta upon βd and since β is equal to 2π/λ. So this is cos inverse of -λ into delta upon 2π d. So the thing to note here is that the progressive phase shift this quantity delta is the one which controls the direction of the maximum radiation and from this understanding we had obtained from even two element array that the phase difference between the two elements was the parameter which was controlling the direction of the maximum radiation. So in this case again we see the direction of maximum radiation would be governed by quantity delta which we now call as the progressive phase shift of the array.

(Refer Slide Time: 29:50 min)



So by controlling the progressive phase shift of the array essentially the direction of the maximum radiation is changed. In other words, if I take this quantity now I can say that delta is nothing but -β d into cosine of ø maximum so this relation is also tells me that the progressive phase shift delta that is equal to -β d into cos of ø max. So while designing the antenna array we would know the direction ø maximum we would like to design an array so that the radiation goes maximum in certain direction so that is the parameter which the designer has in his hand. So you know this quantity ø max so now first we have to choose inter element spacing once we chose we choose the value d and the direction of maximum radiation then we can find out with what phase shift the array should be excited so that the maximum radiation is in this direction. Once we get this progressive phase shift delta like this then we can write this quantity ψ which is β d cosø plus delta I can substitute for delta now that will be β d cosø - β d cos ø$_{max}$ so now ψ can be defined as λ d(cosø - cos ø$_{max}$).

(Refer Slide Time: 31:52 min)

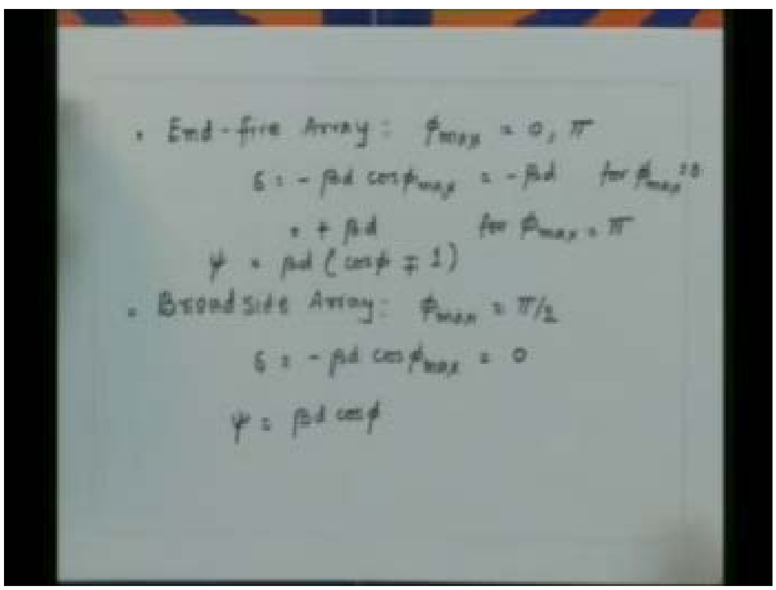


Now the range of ø as we know goes from zero to π if I consider a linear array the angle ø is measured form the axis of the array there is a rotational symmetry around the axis of the array so the ø can transform zero in this direction and can go all the way up to one eighty degrees which will be in this direction so the ø will go from zero to one eighty degrees so the range of ø essentially is from zero to π. So the ø$_{max}$ will also go from zero to π.

Now we consider the two extreme cases and they are the special cases for the antenna array, one is if the radiation goes along the axis of the array either this way or that way we call that array as the end fire array. So if the radiation for this antenna array in this direction that means along the axis or it goes in this direction along the axis then we call this end fire array. On the other hand if the radiation goes perpendicular to the axis it is like this and in this case it is not one direction but anything which is in this plane will be maximum that array then we call as the broad side array.

So for uniform array we have two specific cases, one is the end fire array and the other is the broad side array. Now as the name suggests for the end fire array the radiation is going along the axis let us say the radiation goes along this direction ø equal to zero so for this array the ø$_{max}$ is either zero or π, whereas for the broadside array the radiation goes in the direction perpendicular to the axis so the angle ø is π/2 so in this case ø$_{max}$ is equal to π/2. So the progressive phase shift which we require for the end fire array is delta which is -β d cosø$_{max}$ and in this cos ø$_{max}$ will be zero so cosø will be one or ø$_{max}$ is π the cosø$_{max}$ will be -1 so cosø$_{max}$ will be equal to -β d for ø$_{max}$ equal to zero and that will be equal to +β d for ø$_{max}$ equal to π so the phase ψ which we want for the array is βd into cosø minus plus one where minus sign corresponds to the ø$_{max}$ = 0 and the plus sign would correspond to ø$_{max}$ = π. For the broad side direction the progressive phase shift delta will be -β d cosø$_{max}$ is equal to cosine of pi two which is zero so you have delta equal to zero and therefore ψ would be only β d into cosø.

(Refer Slide Time: 36:33 min)

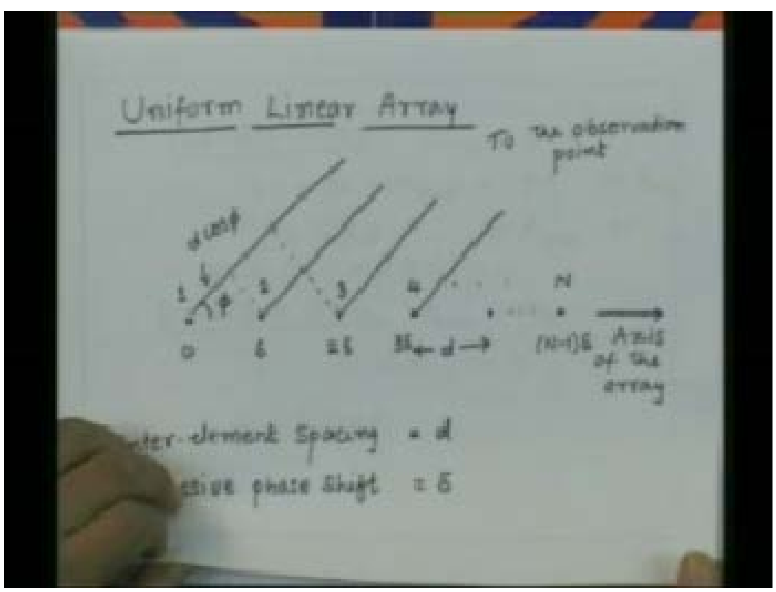


We can see physically here that broad side array essentially the maximum radiation goes in this direction and if delta is equal to zero that means all the currents are excited in phase so if I go in this direction all the radiation traveled by the same distance and they reach in phase in this direction so we expect in the broad side case when all the elements are excited in phase the maximum radiation to occur perpendicular to this. So the two special cases the end fire array and the broad side array are the commonly used configurations of antenna arrays in practice.

Although the end fire array gives you the direction which is $ø_{max}$ is equal to zero so that gives you the direction in which the radiation is maximum and although this thing gives you the direction in which the radiation is maximum for broad side array, there is a small distinction between this two and that is when I say direction of maximum radiation for the end fire array it really is the one direction which is the axis of the array so you will get a beam kind of thing which will be coming in this direction for the end fire array.

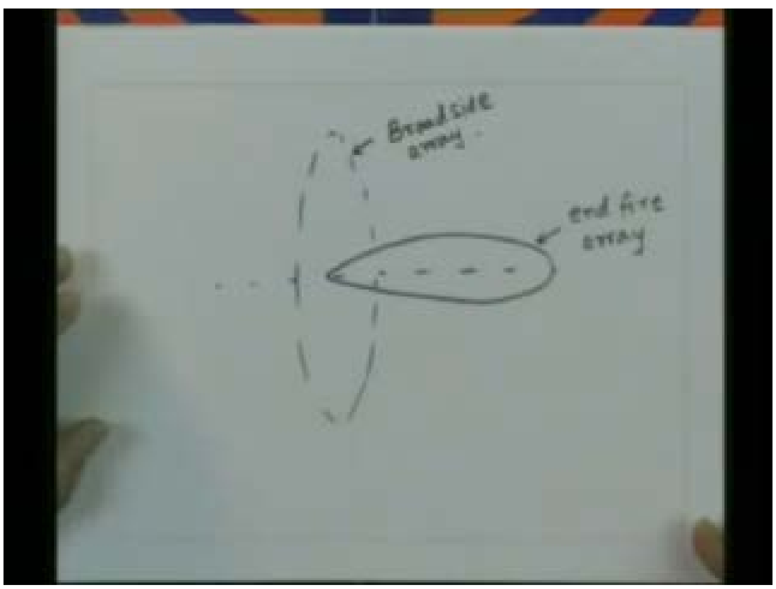
However, if I look at the broad side array the radiation is maximum perpendicular to the array and perpendicular there is no one direction in any direction which lies in this plane is perpendicular to the array so in fact the broad side array when I say the direction of maximum radiation it is in fact this plane which is for maximum radiation, whereas when I say the end fire array the direction really means this direction in which the radiation is maximum.

(Refer Slide Time: 38:33 min)



So again if you recall when you are discussing the radiation properties of the antennas we have said that we should visualize the radiation pattern in three dimension and not only the planar figures, precisely the same thing we should continue here that we should keep visualizing the radiation pattern in three dimensions and then it will be very clear that for the end fire array the direction of maximum radiation means only one direction whereas for a broad side array it would mean a plane which is perpendicular to the axis of the antenna array. So essentially then one can say that if this was the array the radiation will go something like that for the end fire array whereas the radiation would go in the plane perpendicular to this which will be something like that and that is the plane which is the broad side array.

(Refer Slide Time: 39:49 min)

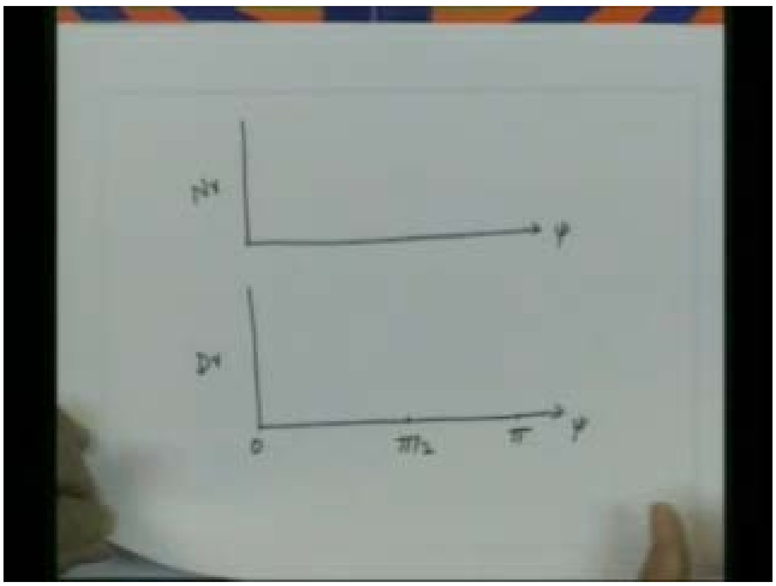


So if I try to visualize these radiation patterns the end fire will look like a elongated balloon whereas this will look like a flattened disc. Having now obtained the direction of the maximum radiation the next thing one can go for is the direction of the nulls. So looking at the expression of the radiation pattern which is this depending upon the value of N, I see the variation of this function the numerator and denominator as a function of ψ

so this function is a slowly varying function as function of ψ, whereas if N is large this function is a very rapidly varying function as a function of ψ.

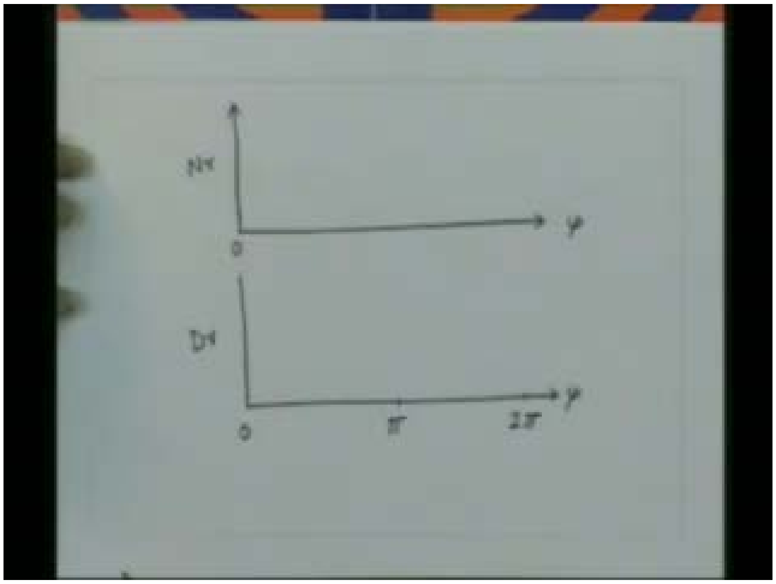
So let us say if I plot the numerator and denominator of this expression so this one we plot as the denominator this one we plot as the numerator and I go from this is the ψ and let us say ψ goes from zero to π so we have here π/2, here is π.

(Refer Slide Time: 41:19 min)



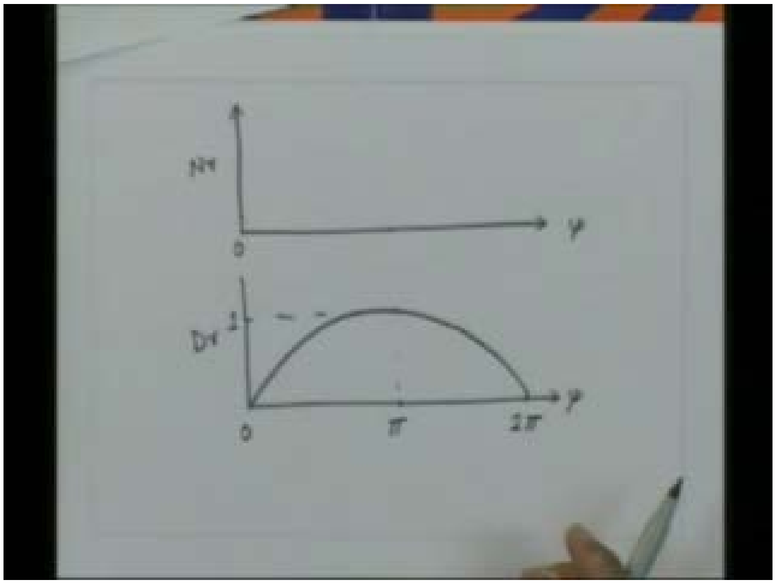
Now let us take the range from zero to 2π. So again this is ψ this is numerator this is denominator lets say this is zero this is π this is 2π.

(Refer Slide Time: 41:55 min)



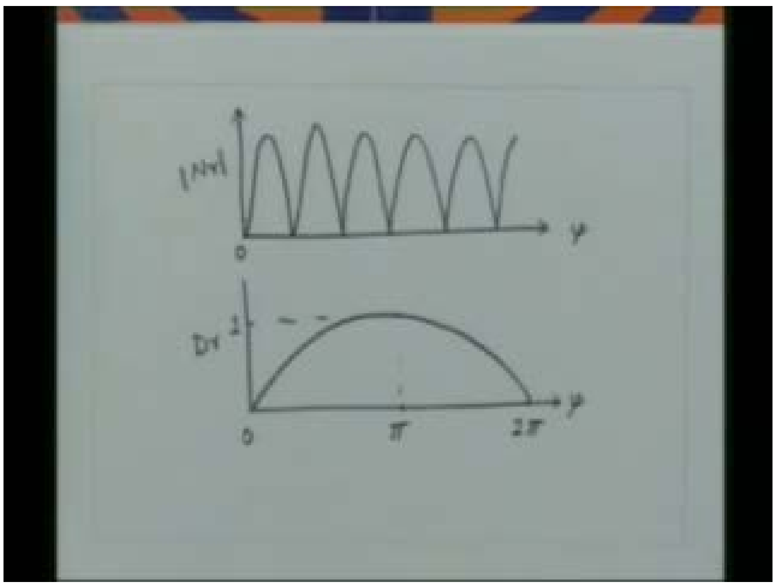
If I look at the denominator here when the ψ is zero this quantity is zero when ψ becomes π that time this will become π/2 so you will get maximum which is equal to one and when this ψ goes to 2π this will be π so again this quantity will go to zero. So I will get a variation for this function which will essentially look like that where there is maximum here which is ((42:33))

(Refer Slide Time: 42:33 min)



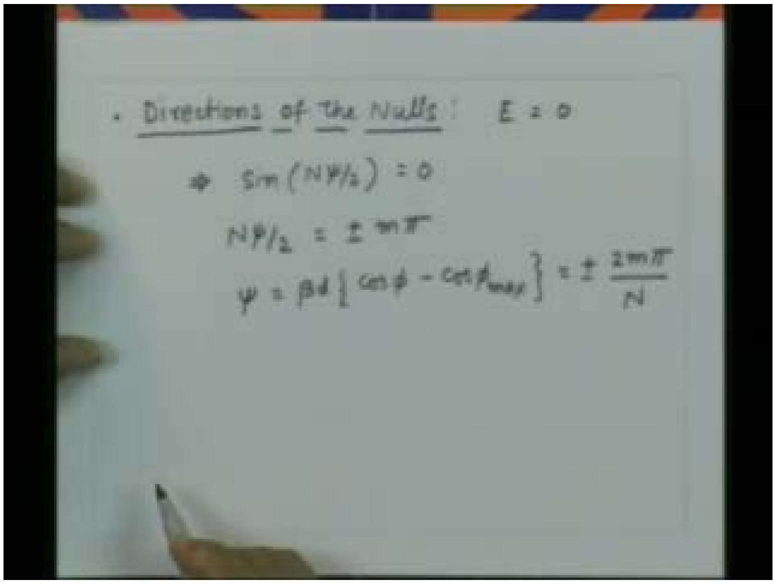
On the other hand if I go to the numerator and if N is large then this function will be varying very rapidly as a function of ψ so you will get if I plot magnitude of this function this function essentially something like that so now this function the denominator is a very slowly varying function and the numerator is a rapidly varying function depending upon the value of N and if N is large over the small region here but this function can be considered almost constant this function would be varying by a large amount.

(Refer Slide Time: 43:24 min)



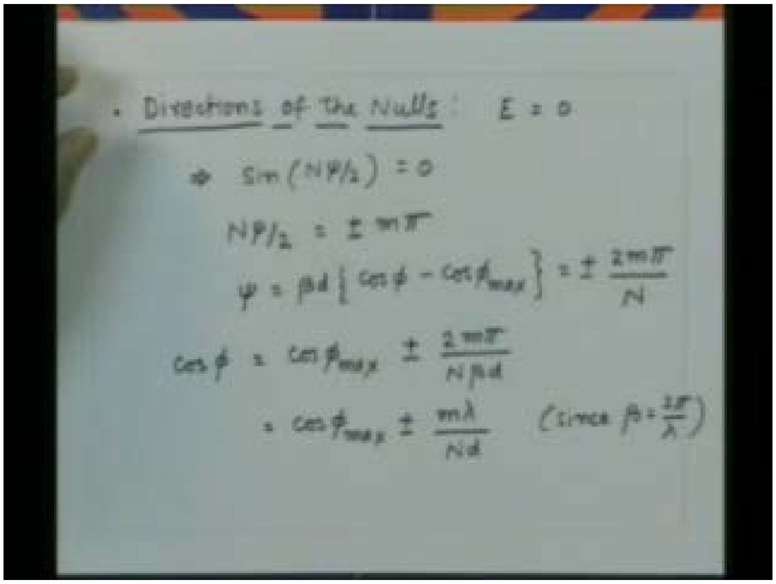
So from this essentially we can find out two things one is when ever the numerator goes to zero that time the total radiation field will go to zero and that will give me the directions of the nulls, when ever the numerator goes to maximum. Since this function is slowly varying function I can say that that would correspond to the local maximum or that would give me the amplitude of the side lobe so the numerator of this function essentially would decide the directions of the nulls as well as the direction of the side lobes. So for directions of nulls if I take this quantity the numerator and set it equal to zero or multiples of π I get the directions of the nulls so I say directions of the nulls I can obtain by equating the numerator of the radiation pattern to zero so that nulls corresponds to the E = 0 that essentially gives the sin(Nψ/2) that is equal to zero or that gives Nψ/2 that is equal to ±m π, simplifying this we get ψ which is β d{cosø - cos$ø_{max}$} that is equal to ±2mπ/N.

(Refer Slide Time: 45:52 min)



You can bring down this βd and we can write for β which is 2π/λ so from here you can get the cos of phi with the directions of the nulls that would be equal to cos$ø_{max}$ ± 2mπ/N into βd and if I substitute for beta its 2π/λ so that will be equal to cos$ø_{max}$ ± mλ/Nd since β equal to 2π/λ.

(Refer Slide Time: 46:52 min)



So once for the array I know the maximum radiation $ø_{max}$ for which the array is designed if I know the inter element spacing and the total number of elements I can find out the directions of nulls keep in mind that all those directions for which this quantity lies between minus one and plus one would give me the physical nulls in the radiation pattern. also one should note that the values of m which we are going to choose is now symmetric for positive and negative sign depending upon the value of ø max you may have some range of m for positive m and some other range for m for the negative sign. so for all possible values of m you have to choose with the positive and negative signs which will give this quantity lying between -1 and +1.

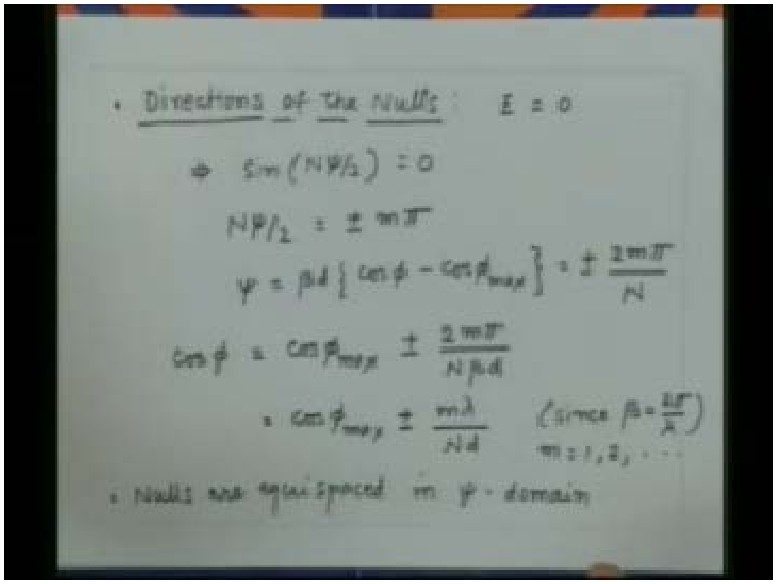
However, one can note immediately that if N is very large there is possibility of having more number of m’s for which this quantity will remain within -1 and +1 that means as the number of elements increase in the array for the same element spacing d the number of nulls increase in the radiation pattern. Recall when we talked about two element radiation pattern the number of nulls was increasing with the spacing between the two antenna elements. In this case we are keeping the spacing same and the spacing for two element array may not give you more nulls but as we increase the number of elements essentially the nulls in the radiation pattern go on increases. And, since we know that between every two nulls there is a local maximum so you have the side lobes in the radiation patterns so the number of side lobes also goes on increasing as the number of elements increase in this array.

So the number of elements in the array play a role in two ways, one is it gives you the number of nulls and also it gives you the side lobes into the radiation pattern or in other words this radiation pattern now has a leakage of power in those direction for which the array was not actually designed for the array was designed to give a radiation only in the direction of phi maximum however we will now get the direction corresponding to the side lobes between the two nulls where there will be leakage of power.

Coming back to the question of that all possible values of m should be chosen which will give the value ±1 let us take a simple example suppose this quantity $ø_{max}$ was zero that means the radiation was going in the N ø direction then this quantity would be equal to one so this cosø will be 1 ± mλ/Nd and in this case m would go from 1,2 and so on, obviously any value of m with positive sign will make this quantity greater than one. So no nulls corresponding to the positive sign would be physically seen, only the nulls corresponding to the negative sign will make this quantity lie between -1 and +1 and that would give us the physical nulls. So in this case when $ø_{max} = 0$ the nulls would correspond to only the negative sign what ever maximum value of m is permitted but there will not be any null corresponding to the positive value of m.

Another thing one can note here from this expression is that now the nulls are given by this ψ which is 2mπ upon m and m is varying from one to what ever maximum value permissible so in ψ domain if I consider this space of total phase ψ then the nulls are equi spaced in ψ domain and the spacing between the two nulls is always 2π/N. So we have something interesting to note here and that is nulls are equi spaced in ψ domain.

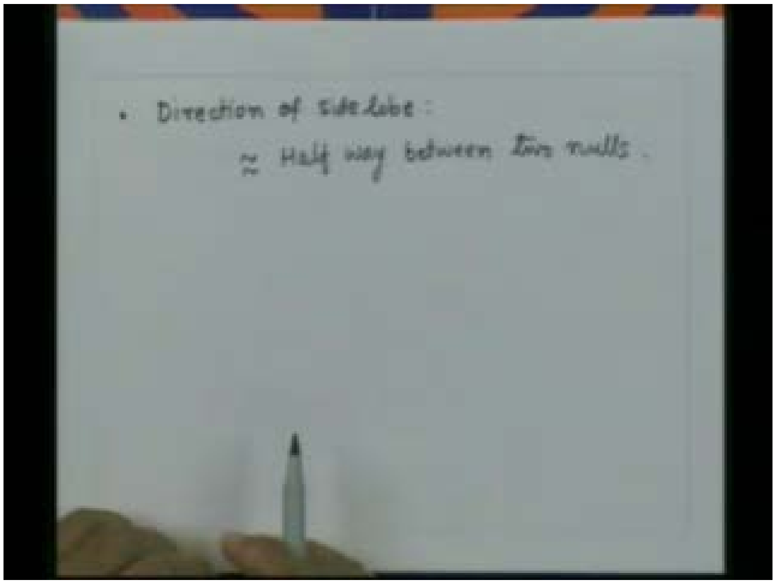
(Refer Slide Time: 52:08 min)



Since there is a non linear transformation between ψ and the physical angle ø the nulls are equi spaced in π they will be having different spacing in the physical space which is ø but as far as this angle ψ is considered this angles are equi spaced so the nulls will be equi spaced in the ψ domain. So if you carry out the analysis of the antenna arrays in terms of the ψ space then all the nulls will essentially be equi spaced with the spacing of 2π/N.

Now we have a side lobe between every two nulls so essentially when we get when ever this quantity Nψ/2 that is odd multiples of π/2 that time essentially we get this quantity equal to one and I will get a side lobe that will be between the two nulls so for every every pair of nulls half way between the ψ domain you will have the maximum radiation so then one can say that we get the direction of side lobe is approximately half way between two nulls.

(Refer Slide Time: 53:58 min)



So essentially we find out first the direction of the nulls because that calculation is straight forward we take the expression for the radiation pattern we equate that to zero and from there we find out the directions of the nulls and then we say approximately half way between the two nulls the radiation must be going the local maximum so we get the direction approximately for the side lobes.

We will continue with this discussion on the uniform arrays and we will see more properties of the uniform array like what will be the amplitude of the side lobe level, what is the highest side lobe we see in the radiation pattern, what are the parameters in the array which affect the side lobe level of the radiation pattern and then we also see by increasing the number of elements how does the focusing capability of the antenna array changes or the directivity of the antenna changes as a function in the number of elements in the array.

Thank you.