Transmission Lines and E.M. Waves
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Lecture-52

Welcome, up till now we studied the radiation characteristics of Hertz Dipole and following that the linear dipoles we saw that the Hertz Dipole has a very broad radiation pattern and consequently it has a very low directivity. For making the directivity higher that means for making the radiation pattern narrower we investigated dipole antennas which are of finite length however while doing this we found that a terminal impedance of antenna gets modified so as we increase the length of the dipole two things happen the directivity of the antenna increases and beam width becomes narrower but the same time we start getting multiple beams that means the radiation starts going in some undesired directions.

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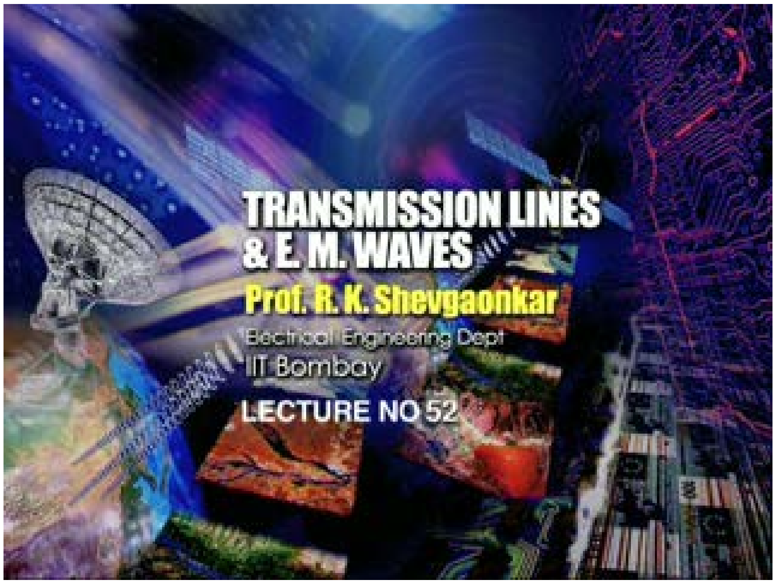

We also develop some nulls that means is the directions in which there is no radiation and at the same time the terminal impedance of the antenna also gets modified. So what we find is that as we try to manipulate the radiation characteristics like radiation pattern the terminal impedance automatically gets modified we do not have a independent control over the terminal impedance against the radiation pattern. In many applications we would like to modify the radiation pattern depending upon the need but the same time we want that the impedance characteristics of the antenna should not get modified. We also saw that for a dipole since the current distribution was sinusoidal we knew the current distribution we could find out the radiation pattern, however, if we take some arbitrary antenna then it is very difficult to find out the current distribution on that but once we get the current distribution on that antenna then finding out the radiation pattern is a straight forward problem but the reverse problem we mentioned earlier that somebody gives you the radiation pattern and says tell us the physical structure which will give me this radiation pattern or giving just the physical structure can we just say half hand what kind of current distribution will be existing or for a given current distribution what should be the physical structure these problems are extremely difficult or sometimes impossible problems.

So what we want is we want to manipulate the current distribution because from our Fourier transform relationship between the current distribution and the radiation pattern we know that if you manipulate the current distribution we can get the desired radiation pattern. So now our requirement is that we must have a mechanism of modifying the current distribution without affecting the terminal characteristics of the antenna and that kind of flexibility is provided by the antenna arrays. So as the name suggests the antenna array is collection of basic antenna elements.

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Now what we do is we essentially have a large number of antennas whose terminal characteristics are decided by the pre designing this antenna and by placing this antenna in the vicinity of each other and exciting them simultaneously we essentially get the superposition of the fields due to each of the antennas and because of the phase space which will be there because of the placement of these antennas the total radiation pattern gets modified. So by the use of the antenna array essentially we decouple the terminal characteristics of the antenna and the radiation pattern, of course when two antennas are brought in the vicinity of each other the terminal character gets modified but this modification is marginal if the spacing between two antennas which are kept in the vicinity of each other are more than $\lambda/2$. So that means if the antenna elements are separated by a distance more than $\lambda/2$ the terminal characteristics of the antenna practically remain unchanged, however, the superposition of the field due to different antennas can give you modification in he radiation pattern.

So in fact by using proper distribution and the excitation of the different antenna elements one can achieve any current distribution and consequently we can realize any arbitrary radiation patterns. So an antenna array essentially provides flexibility in designing the radiation patterns without affecting the terminal or the impedance characteristics. The idea is as follows, what ever frequency we want to work on we first find out a suitable antenna with proper impedance bandwidth characteristics let us say we can take a dipole antenna and match it to what ever data we want to make it to over the bandwidth so this is now the basic element which is the radiating element. Now by reproducing the same element at different locations in space essentially we create an antenna arrays so each antenna now is a best behaved radiating element at that frequency for that bandwidth and superposition of all the radiations from different antennas of the similar type we will get a radiation pattern which will be the desired radiation pattern.

So when we talk about antenna arrays principally there is no necessity of having different antenna elements which are identical you may have different elements one may be dipole other may be parabolic dish third may be something else, however, it does not really give an advantage of using different antenna elements in the array in fact it is more advantageous to use identical antenna elements in the antenna array so that your analysis becomes simpler and the final radiation pattern which we get for the antenna array is essentially decided by the array characteristics rather than by the individual antenna characteristics.

So once you have a basic antenna element we worry only about the terminal characteristics and the radiation characteristics are decided by array and the basic radiation pattern of the antenna elements does not play any role in the final radiation pattern. So with that understanding then one can say we are essentially going to put identical antenna elements in some special configurations excite them with some proper pattern and this is going to create a radiation pattern which will be the desired radiation pattern. So let us say antenna array as we say is collection of antennas so let us say we have dipole antennas which could be like that they are located in different locations and each of them might be excited with different currents which are complex currents so this one may be having a current I_1, this may have a current I_2, this may have current I_3, and this is I_4 I_5 and so on.

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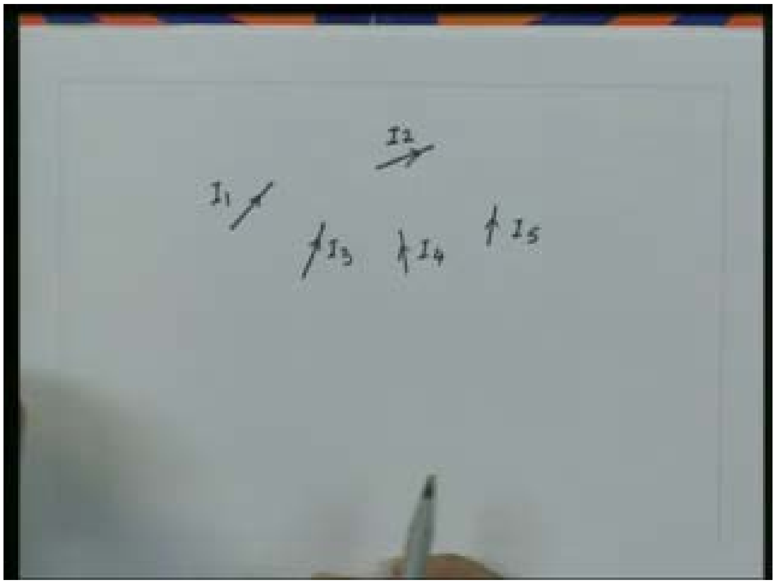

Now, principally these antennas again need not be oriented in same direction, however, if I do not do that then again we have to do the vector addition of the electric field when I go very far away from the antenna it again does not give you any great advantage as far as the radiation pattern is concerned so normally these elements are also oriented in same direction so they have identical radiation characteristics as a function of θ and ø.

So in general we are having the quantities which can control the radiation pattern as follows, one thing is the configuration of the antennas that means the pattern in which the elements are distributed in the space so we say Spatial configuration which one can choose to modify the radiation pattern, second thing is for a given configuration the specific location of the antennas so let us say the configuration could be a line a Linear configuration but the spacing between different elements might be different. Similarly, I may have configuration which may look circular but the antenna which are located in side the circle might have different location. So for a given special configuration the location of the antenna could be again a parameter which is in the hand of the design the location of antenna elements.

Once we get the configuration and the location decided for the antenna then for the antenna excitation we have possibilities that is we can change the relative amplitude between different antennas we have Relative Amplitude which is a controlling parameter then the phase of different antennas so we have Phase of antenna elements and finally we may have some control over the final radiation pattern due to the original radiation pattern of the antenna. For example if I use the basic element of the antenna which is dipole this will have a null along this axis so final radiation pattern will also be having a null along this axis if all elements are oriented in same direction so primary radiation pattern also would have some effect on the final radiation pattern so that radiation pattern we call as the primary radiation pattern of the antenna so we say the radiation pattern of basic element.

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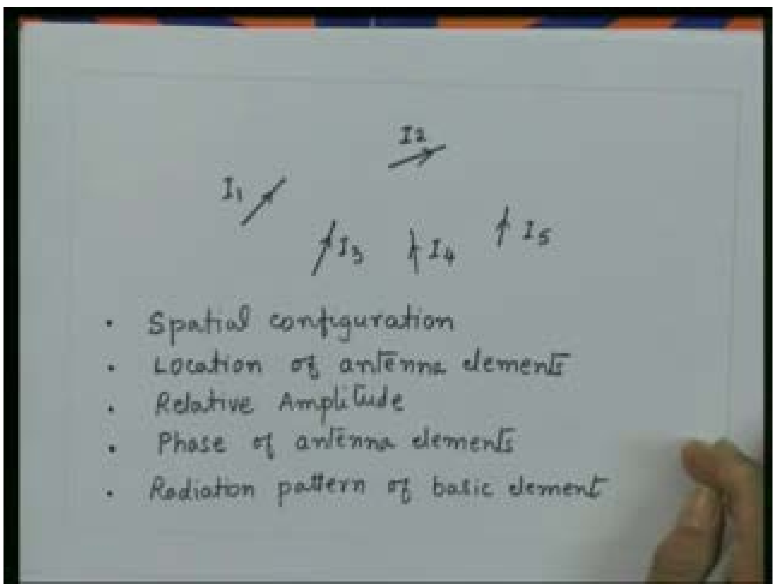

Now for a given configuration and location we have some given number of antennas basic elements so let us say in an array we have total N elements. Now having fixed the configuration and the location of these antennas and choosing the antenna the radiation pattern of the basic element is fixed, special configuration is fixed and the location of the antenna elements are also fixed so I have these two quantities essentially in control or may be even the location i can keep as a free parameter. So let us say first I fix the configuration I choose the basic element so the radiation pattern is fixed then I have got these three quantities which are to be controlled for manipulating the radiation pattern. So the location of the antenna we have essentially degrees of freedom which are (N – 1) if there are N elements the absolute location of the antenna element do not matter what matters is the relative location between different antenna elements because superposition of the electromagnetic waves from different antennas would simply be decided by the contribution relatively getting from the different elements so from the location we get degrees of freedom for choice this will give me a degree of freedom which is (N – 1), similarly, relative amplitude also will have degrees of freedom which is (N – 1) and the phase of the antenna elements also give me a degree of (N – 1).

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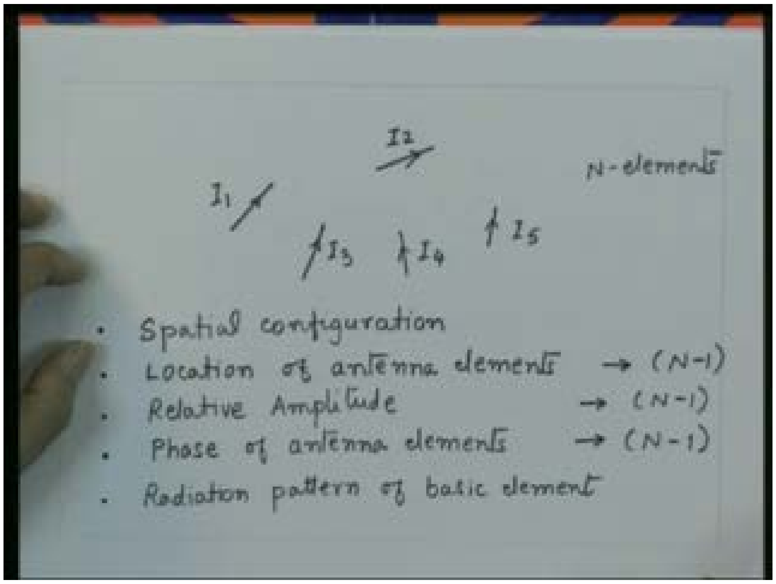

So for a given configuration and basic element chosen essentially in a array we have three into N - 1 degrees of freedom that means we have three into N – 1 parameters to be controlled which can manipulate the final radiation pattern that means we can control three into N – 1 features in the radiation pattern by controlling these three into N – 1 parameters.

In fact if you take an array for which N is reasonably large and typical arrays would have a number which is large we have a very large number of degrees of freedom three N - 1 this number is extremely large. So in practice we do not require that large number of degrees of freedom so essentially we will relax some of this requirements for example when we go for uniform arrays we will say the location of the antenna is chosen so the spacing between the adjacent element is same, the amplitude also is same and the radiation characteristics is controlled only by the phase variation of the antenna elements.

As we go for more complex problem then we will say okay location is fixed but the amplitude and the phase of the antenna are varied and the radiation pattern is manipulated so slowly we can relax this condition but to understand how the antenna array works let us first investigate the simplest possible array that is the array of two elements. In that case we will have degrees of freedom there will be only 1, spacing between them there will be amplitude which will be one and the phase variation will be one.

Now we have three degrees of freedom for a two element array and let us see what the effect of each of this parameter on the radiation pattern of this array is. So our basic array we will investigate is a two element array as the name suggests this consists of two elements that means two basic radiating elements without loosing generality let us say these radiating elements are isotropic that means their basic radiation pattern is like a sphere that is uniform in all direction. Of course, this kind of radiator we will not find in practice because as we have seen even the simplest current element gives us the radiation pattern which is a sinθ radiation pattern which is like an apple. So this isotropic radiation pattern which is same in all directions is an imaginary source this source will never realize in practice.

However, there are sources which can definitely have a radiation pattern which is isotropic in certain planes. For example if I take a Hertz Dipole the radiation pattern is isotropic in H plane, similar is true for the half wave dipole and the other dipoles that means if I consider let us say Hertz Dipole which is perpendicular to the plane of the paper then in the plane of the paper the radiation pattern will be isotropic. So in general of course we do not have radiation pattern which are isotropic but radiation patterns which are isotropic in a plane are certainly realizable in practice. So here without worrying about that let us say we have the two radiators which are having isotropic radiation patterns that means their radiation goes symmetrically in all directions so these are essentially telling you the phase fronts which are originated by these two antennas and as we said there are three degrees of freedom now one is the separation between the antennas the amplitude ratio of the excitation of this two antennas and the phase difference between these two antennas.

So let us say this one is having a current I_1, this one has current I_2 with a phase difference let us say this one is having a phase delta one this one phase delta two, separation between the antennas let it be given by d so I have a degree of freedom which is phase difference between these two which is delta two minus delta one I have a degree of freedom for amplitude relative amplitude which is the ratio of I_2 and I_1 and the distance between the two elements.

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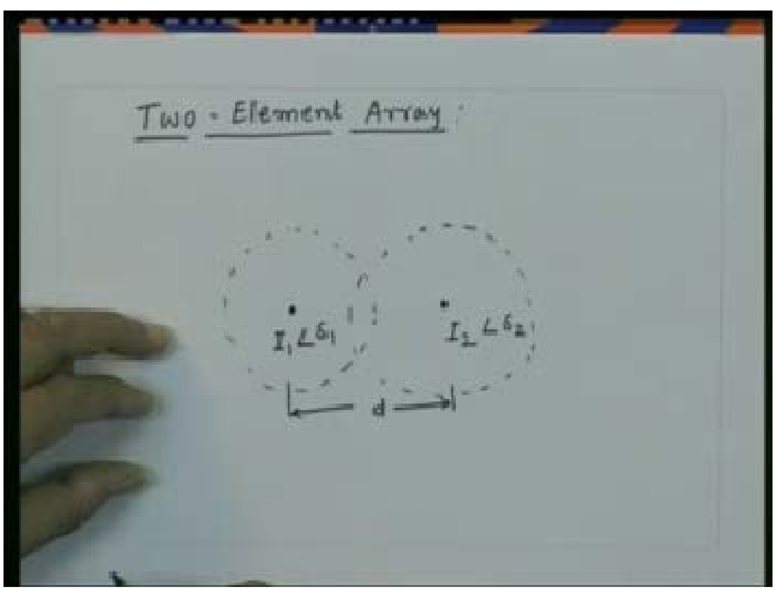

So I have got three parameters to control here the one is d other one is delta two minus delta one and third one is the ratio of the current amplitude I_2 and I_1.

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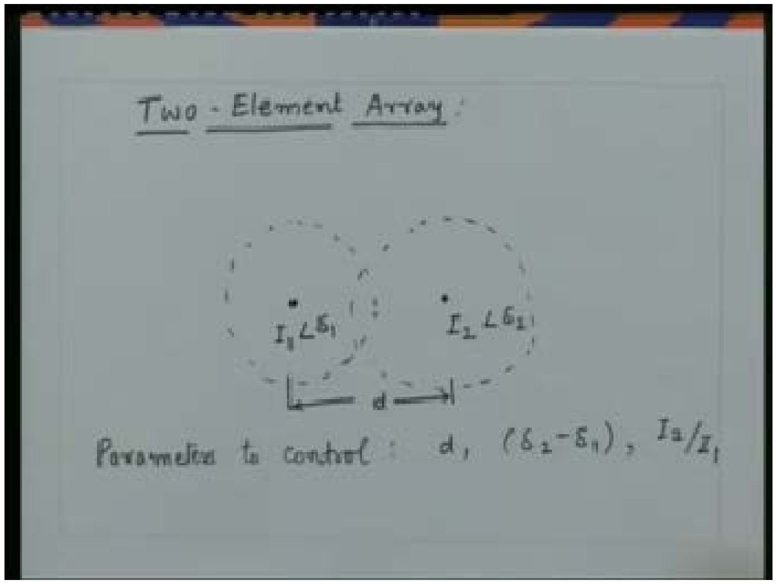

Now the radiation field as we know is proportional to the current of different elements so it will have a phase term which is e to the power -j β r which is a distance from the elements and it will be varying inversely proportional to r. So if I take the field due to this element then in certain direction I will get let us say take some direction which is very far away from this antenna array and let us say we measure the angles with respect to the axis of the array which is this so the axis of the array is the line joining these two elements so this we have axis of the array and let us say the angle we had measured for this direction at which we are measuring the field is given by ø.

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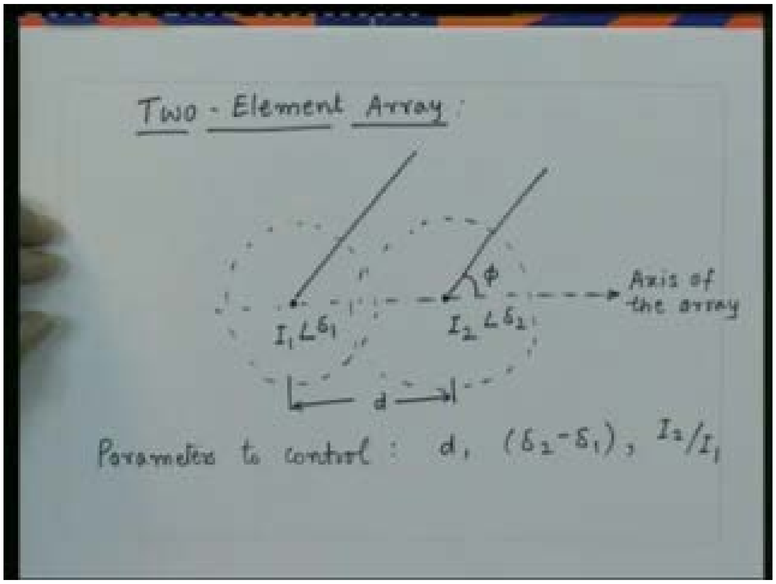

So this is going to some observation point. Since the observation point for radiation field is very far away from the antenna then these angles are almost equal to ø and let us say the distance from this element is given by r so distance of this point from this will be short by this which is d cosø so r - d cosø so this distance will be r - d cosø so the field which you will get because of this will be having a phase e to the power -j β r and it will be varying as 1/r the field because of this we will be having a phase of e to the power -j β r - d cosø and it will be varying 1 over r - d cosø and the fields will be proportional to their respective currents.

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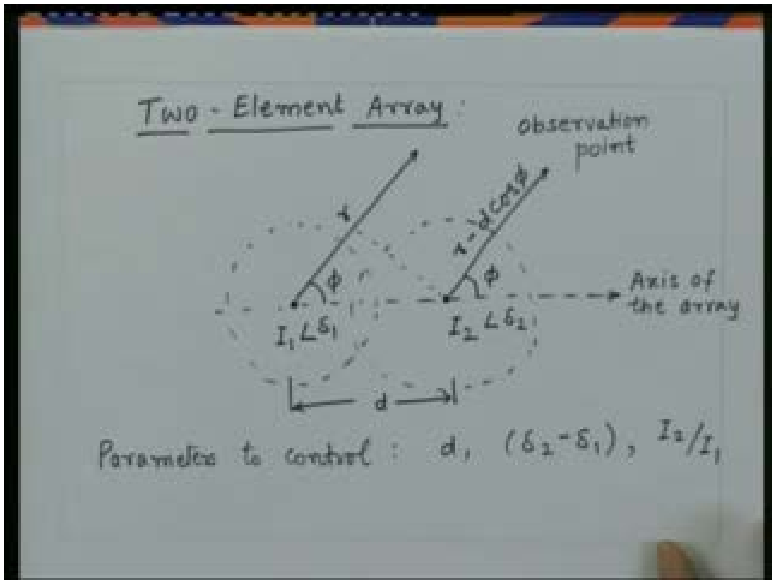

So essentially we can write down the field due to the two antennas let us say these two antennas is one let us say this antenna we call as 1, this antenna we call as 2 so the field due to antenna 1 and 2 can be written E_1 that will be some constant proportional to current I_1 e to the power j delta one as the complex excitation current inversely proportional to r into e to the power -j β r and the field due to second element E_2 will be K I_2 e to the power j delta two into e to the power -j β (r - d cosø) upon r - d cosø.

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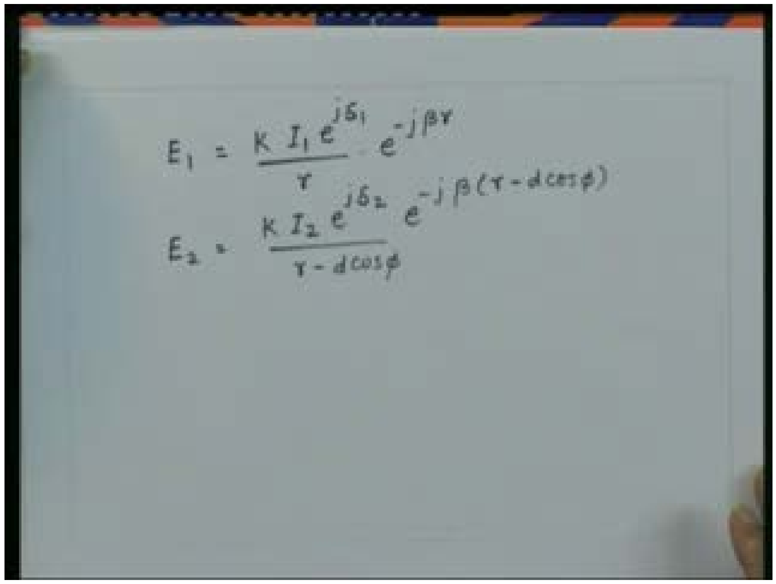

Now as we have done earlier when you are analyzing the dipole if r is very very large this term can be approximated by r in the amplitude term and this term which is phase term we retain this because here this quantity is with respect to the λ so this cannot be neglected however in amplitude terms d is much much smaller compared to r this quantity can be neglected so E_2 as we have said can be approximately written as KI_2 e to the power j delta two e to the power -j β r upon r into e to the power j β d cosø.

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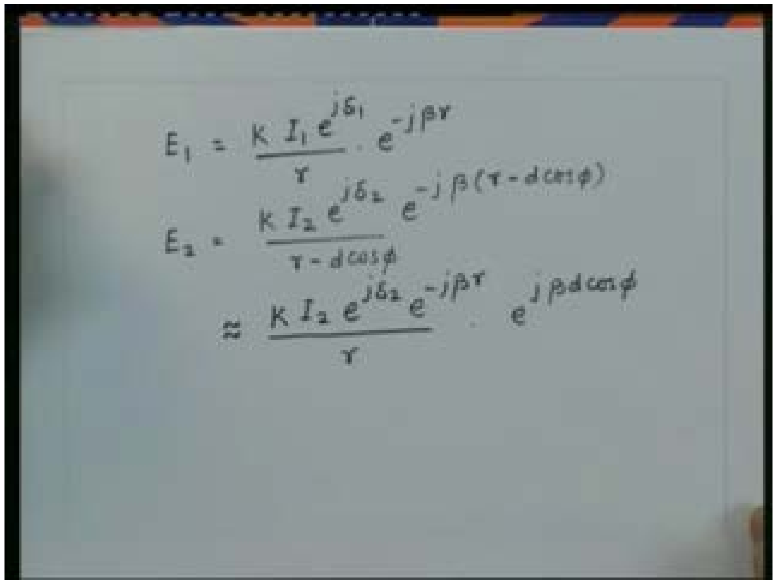

So the total field now which we get is superposition of these two so essentially we are seeing the interference phenomena of the waves which are originated by these two antennas so these phase fronts move essentially they give the interference and that is what is the resultant which you are going to get at any point in the space. Since we are again interested in radiation pattern which is the relative variation of the electric field as a function of angle ø we can absorb this quantity K upon r into e to the power -j β r into some another constant so you can call that some constant K_0 so we can define K e to the power -j β r upon r is equal to some constant let us say K_0. Now the fields can be written as E_1 that will be K_0 I_1 e to the power j delta one and E_2 will be equal to K_0I_2 e to the power j delta two into e to the power j β d cosø.

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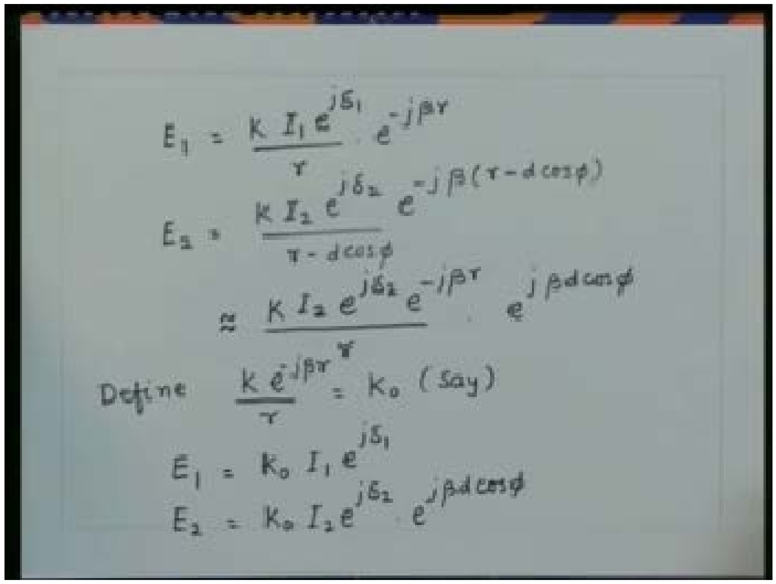

So the total electric field which will be sum of these two superposition of this electric field which is just sum of these two terms so I get the resultant field the total field E that will be $E_1 + E_2$ that is equal to K_0 into I_1 e to the power j delta one plus I_2 e to the power j β d cosø phi plus delta two.

As you have seen we are not interested in again the absolute quantity we are only interested in relative distribution of the electric field so we can take his I_1 e to the power j delta one common so we get the field which is K_0 I_1 e to the power j delta one into one plus the ratio of the two currents I_2/I_1 e to the power j delta two minus delta one and the phase term e to the power j βd cosø.

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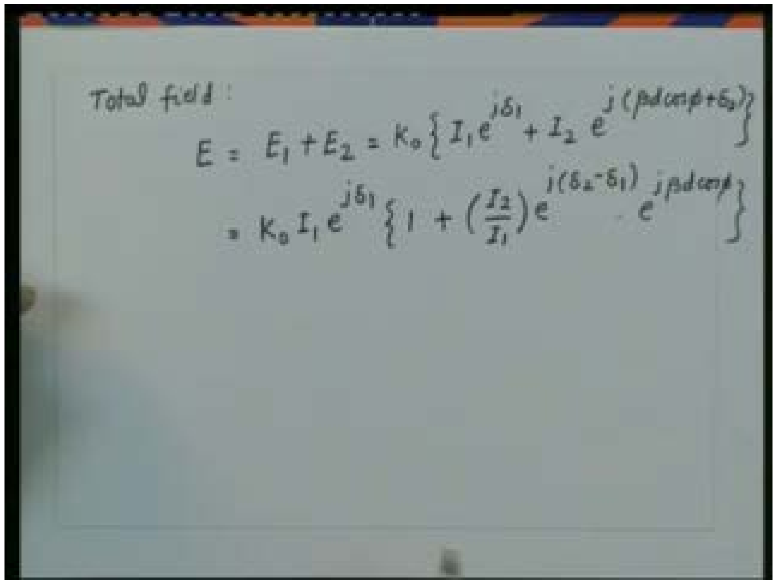

So as we mentioned earlier we are having three parameters to control one is this parameter which is ratio I_2/I_1, another one is the phase difference between the two elements and third one is this quantity d which is the spacing between the two elements. Let us now investigate the effect of each of this quantities the ratio the phase difference and the spacing between the elements on the radiation pattern. So let us say first we want to investigate what is the effect of spacing d on the radiation pattern so without loosing generality let us choose this quantity I_2/I_1 something phase difference delta two minus delta one something I will just ask if I vary d what way the radiation pattern is going to get affected so let us say now we want to find out the effect of variation in d the inter element spacing. So without loosing generality let us say we take two elements which are identically excited that means I_2/I_1 is equal to one and delta two minus delta one will be equal to zero.

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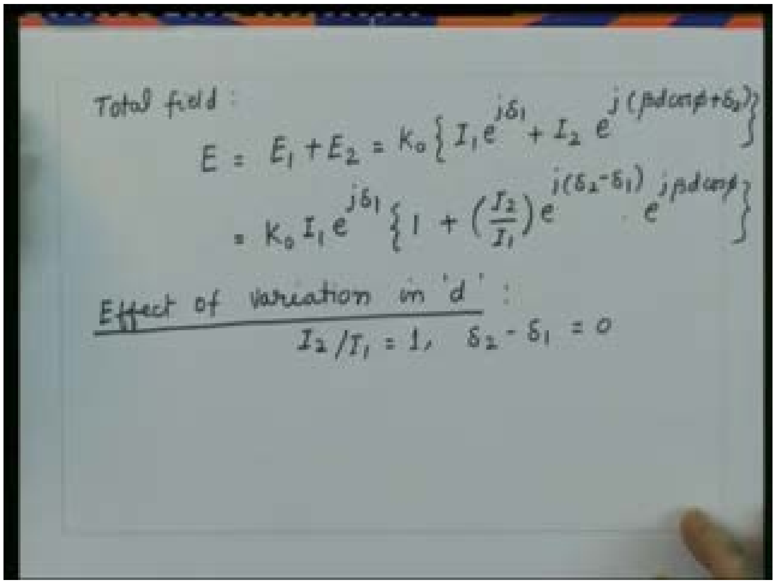

So let us say we excite these two elements by identical currents and now ask a question if I simply change the spacing between the antenna elements what way the radiation pattern is going to get modified. So if I substitute this into this the total electric field or radiation pattern will be this quantity which is K_0 and again without loosing generality I can say I_1 one is one delta one is zero so this quantity will become one so this will be K_0 into one plus e to the power j β d cosø we can take e to the power jβd by two cosø common from here so this is K_0 e to the power j beta d by two cosø so this gives me e to the power -j beta d by two into cosø plus e to the power j β d by two into cosø.

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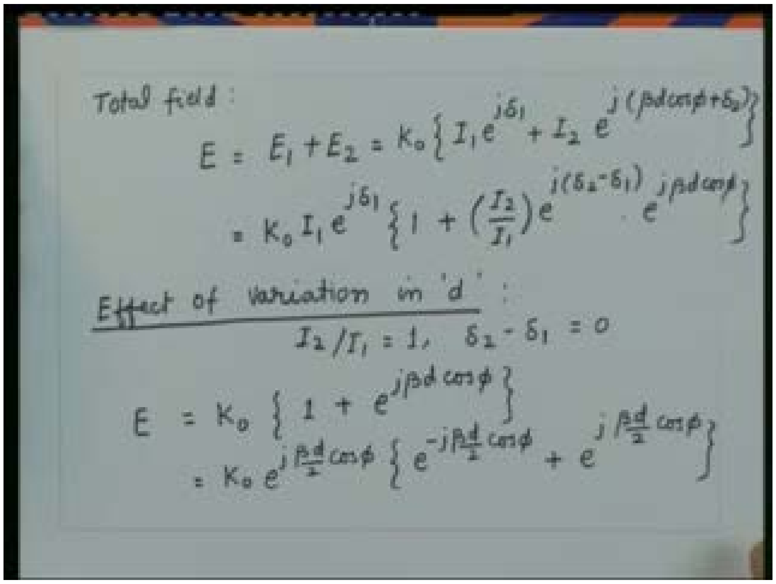

And this quantity is nothing but two times the cosine of this quantity so we can write the final radiation pattern for this that is E equal to K_0 e to the power j β d by two into cosø into two times cosine of β d by two into cosø

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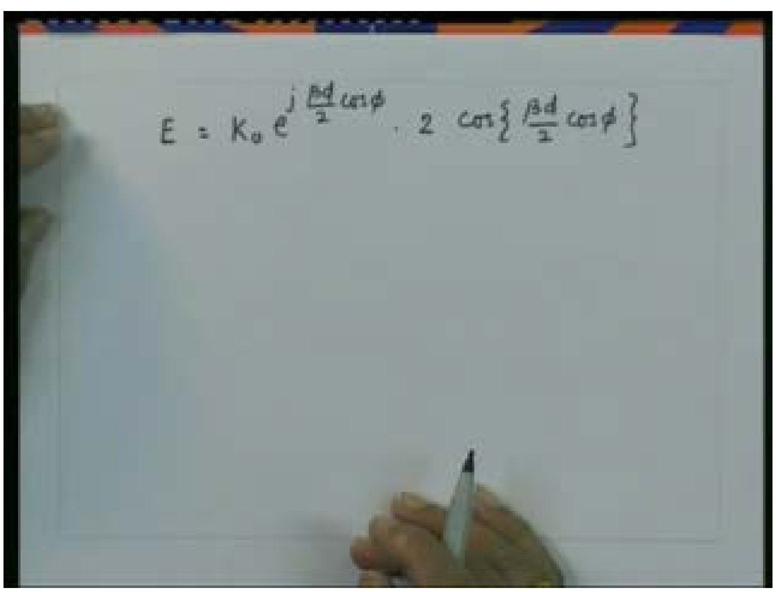

Now K_0 is constant this is the only phase term so the radiation pattern which is the variation of the amplitude as a function of angle phi essentially is given by this quantity even this 2 is a constant which can be absorbed into this so the radiation pattern of a two element array with equal excitation that means I_2/I_1 is one and the phase difference is zero essentially this is given by that. So we can say the radiation pattern and we can say this is maximum value of this is going to be one so we can even say this is normalized radiation pattern that will be equal to cosine of β d by two into cosø.

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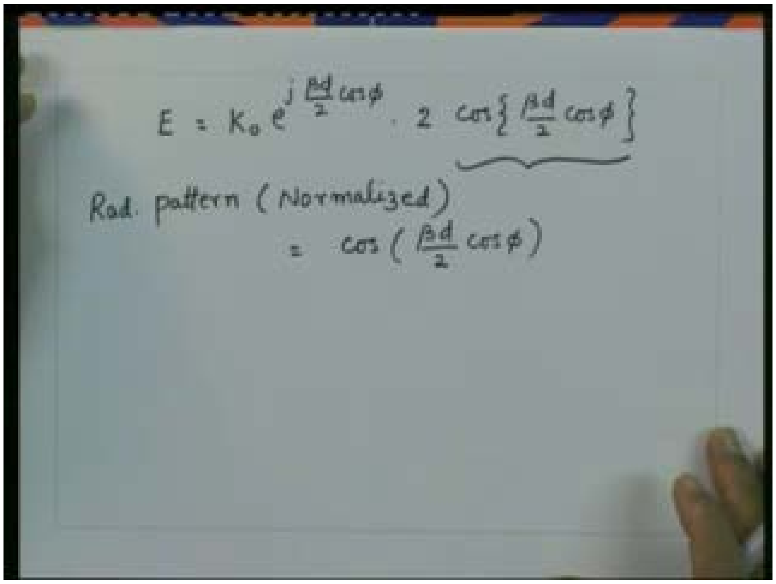

Now one can note here when ø is ninety degrees that means when I go to a direction perpendicular to the axis of the array that time this quantity will be zero and the cosine of zero will be one so you will get a maximum in the radiation when ø is equal to ninety degrees so if I consider this two element array from here the maximum will be in this direction for this case which will correspond to ø equal to ninety degrees because ø is measured from this direction so this angle is π/2 this is the axis.

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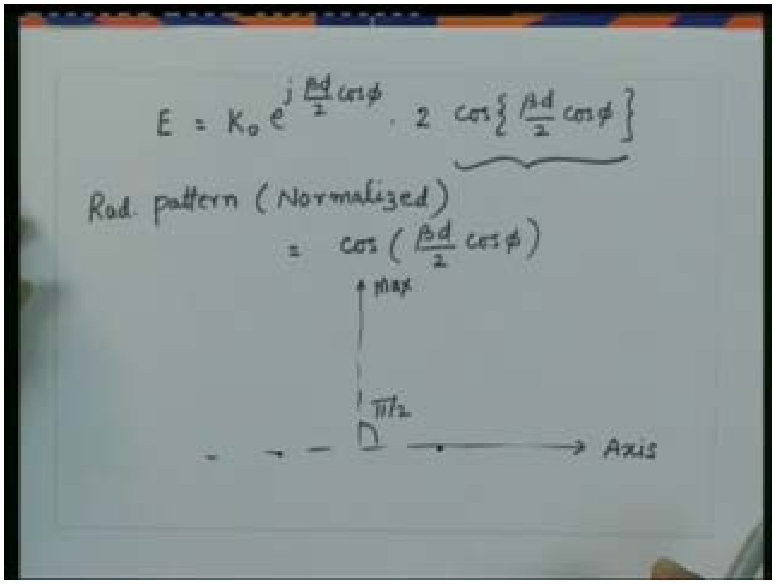

However what we note is that when this quantity is multiples of π/2 at that time this function will be zero and you will have the nulls in the radiation pattern. So you will get from here nulls which will correspond to when βd upon two into cosø is equal to odd multiples of π/2 so π/2 3π/2 5π/2 7π/2 that is the time when this quantity will become zero and you will get the nulls. So we can write down here that this is π/2 π/2 and you can put plus or minus on this, if this condition is satisfied then we get the null in radiation pattern.

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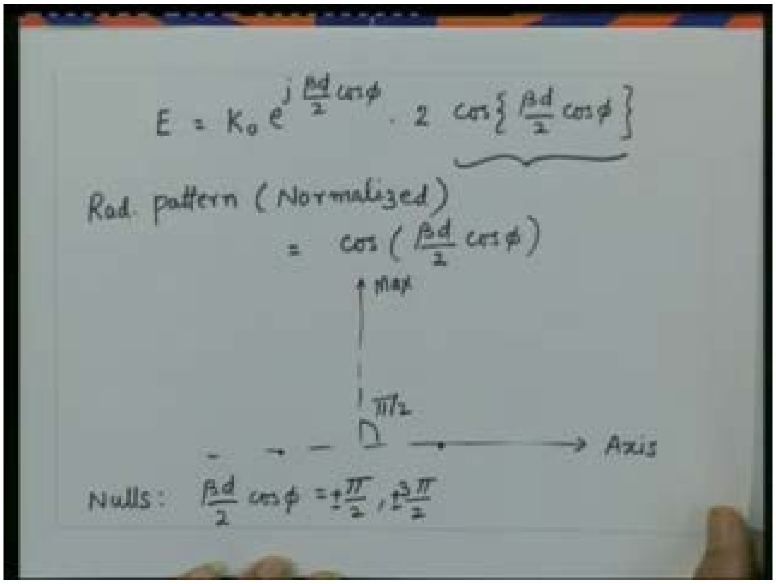

Expanding for β which is 2π/λ we can write that means 2π/λ into d upon two into cosø that is equal to ±π/2, ±3π/2 and so on we get the nulls or by doing this I can this thing π will get cancelled so I can get the angles for a given d so the direction of the nulls cosø will be π cancels with this two will cancel with that so you will get ±λ/2d, ±3λ/2d and so on.

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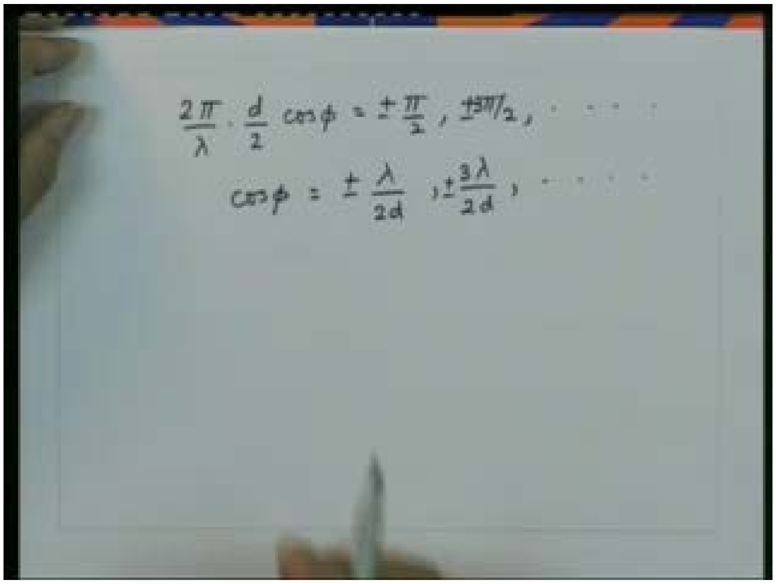

Now when ever this modulus of this quantity will be less than one then we will have physical angle ø in which the radiation will be zero or we will have physical null. If this quantity is greater than one then there will not be any physical angle in which the radiation will go to zero that means the radiation pattern will not have null. However what we see here is as we increase the d for a given value of λ essentially we can have now many more directions at which the field can go to zero, for example suppose I take d less than λ/2 then this quantity will always be greater than one so I will not have any physical null in the radiation pattern, however, if I take let us say d = ø then this is also permissible this is also permissible next one will also be permissible and so on so if I take d < λ/2 then we have no nulls in the radiation pattern if I take d = λ then this λ will get cancelled this will be ±½ so you will get two nulls, d λ this is ±3/2 this will not be possible so we will get two nulls so if d = λ then we get two nulls if I take d = 2λ then these two nulls are possible by putting this d = 2λ these two nulls are possible but the next one which will be 2λ so that will become d = 2λ if I substitute into this the next one will be four five λ/2d and d if I put 2λ then this will become 5/4 which is greater than one so these nulls are not possible so that will give us four nulls and so on.

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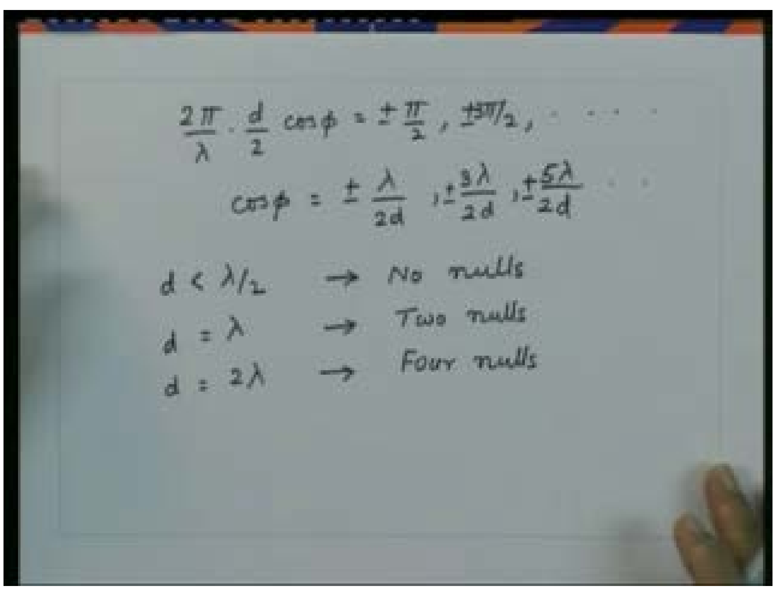

So what we find now is that for given current excitation that means for given amplitude ratio for the currents and the phase difference between the antenna elements as the separation between the elements increases the number of nulls go on increasing so larger the spacing between the antenna elements more will be the nulls in the radiation pattern. So this is the direction of maximum you will get nulls which will be somewhere here here here here and so on and as we argued as earlier when we were discussing the dipole antenna that between two nulls the field must have gone locally maximum so that means between these two nulls the field is maximum which is this again between these two nulls it would have gone maximum and so on, or in other words, now the radiation is not going in the direction in which let us say we wanted to send which was maximum but these zones also the radiation is going because field will going to be locally maximum and in this case the value of the maximum will be exactly same as this maximum which is unity. So as the spacing between the antenna elements increases the radiation pattern sectorised in zones and the radiation now starts going in sectors which are separated by this nulls so the number of nulls increase in the radiation pattern and the radiation starts going into different zones in the space.

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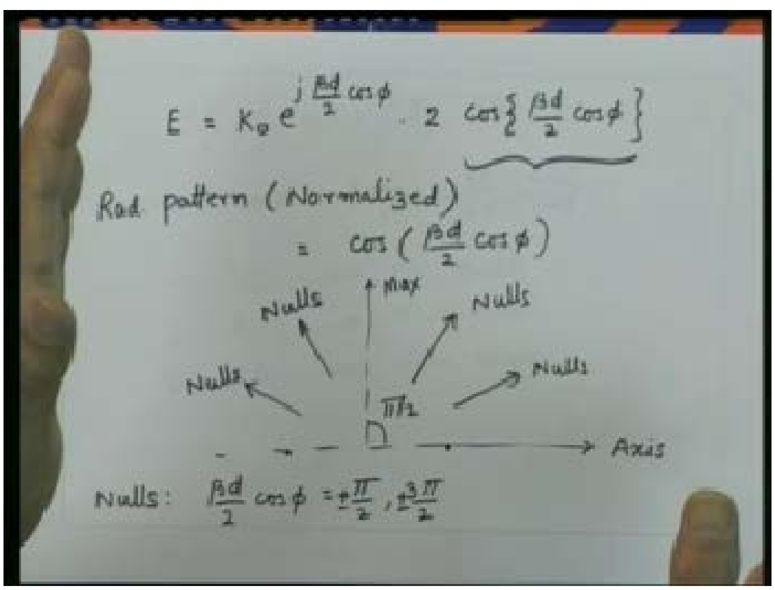

If we want the maximum in the radiation pattern then the spacing between the elements should be $\lambda/2$ so that there are no nulls, however, we will see later on that this choice is may not be very desirable you may get some other constraints from the radiation pattern but as it looks that if we did not want any null in the radiation pattern then the spacing should be less than $\lambda/2$ if the current elements are equally excited.

The second quantity of which we can find out the effect of the radiation pattern is this quantity which is the ratio of the two currents so the second thing is effect of variation in I_2/I_1. Now in this case we can fix some antenna spacing so let us say we fix $d = \lambda$, let us say the phase difference between the two elements is zero let us say delta two minus delta one is equal to zero so for $d = \lambda$ we have β d cosø which is equal to $2\pi/\lambda$ into λ cosø that is 2π cosø.

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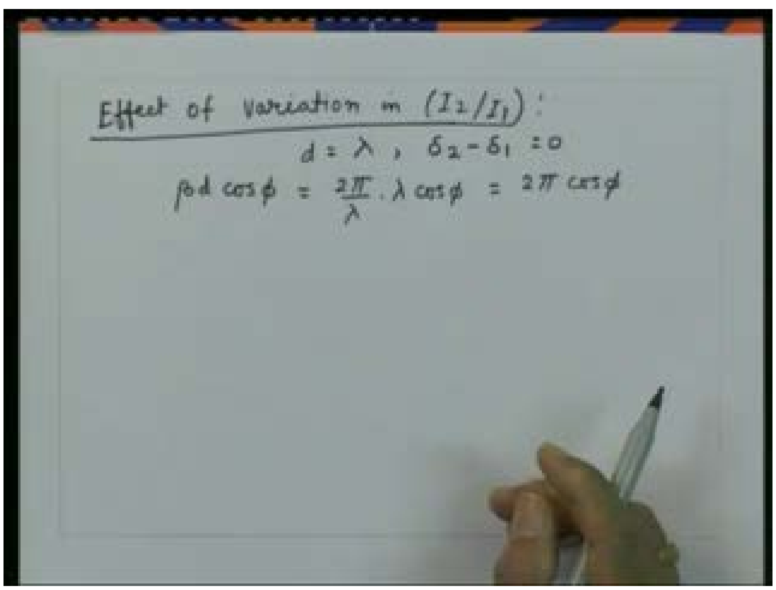

I can substitute into the general expression for the radiation pattern so we get E that is K_0 and again you can assume delta one is equal to zero and I_1 is equal to one without loosing generality. So this quantity then I_1 e to the power j delta one will be one so this is K_0 into one plus this ratio I_2/I_1 where I_1 is one so e to the power j β d cosø will be 2π cosø.

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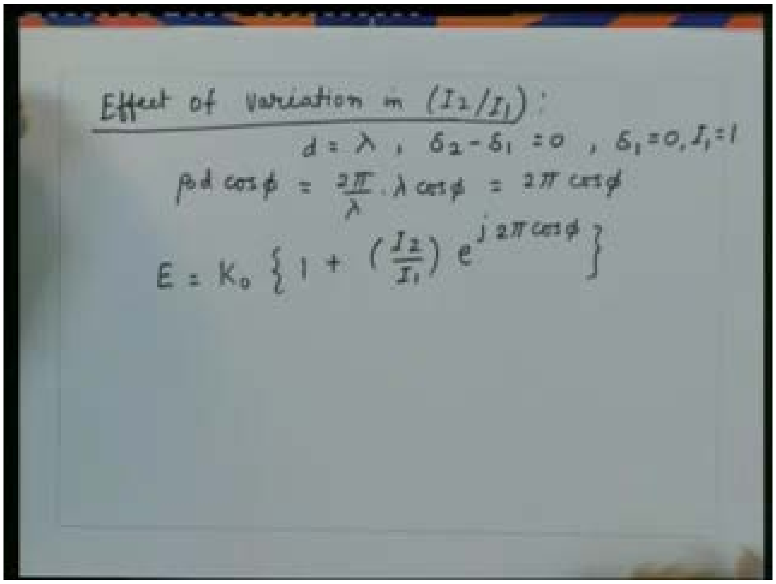

Now as the angle ø varies essentially total phase of this quantity varies and the electric field essentially is the sum of these two vectors one is one other one is having ratio amplitude which is I_2/I_1 and its phase is changing as the angle ø changes. So when ø is equal to zero these two terms will be in phase so this is equal to 2π when this quantity will be $\pi/2$ this will become zero again this will be in phase but if this quantity is one by two then this will be π if this quantity is 1/4 it will be $\pi/2$ and then this will be zero you will have a radiation which is coming because of this plus a ninety degree phase shift between these two. So when this quantity is ½ that time this is e to the power j π which is equal to -1 so these two terms will cancel out each other whereas when ø = 0 that time this is e to the power j 2π that means these two terms will add in phase. So what we now see from here is for ø = 0, two terms add to give the electric field which E equal to K_0 one plus I_2/I_1 for ø = $\pi/3$ if I put ø = $\pi/3$ this quantity will be n by two so this will be π then the two terms cancel each other and we get electric field that will be equal to K_0 into one minus I_2/I_1.

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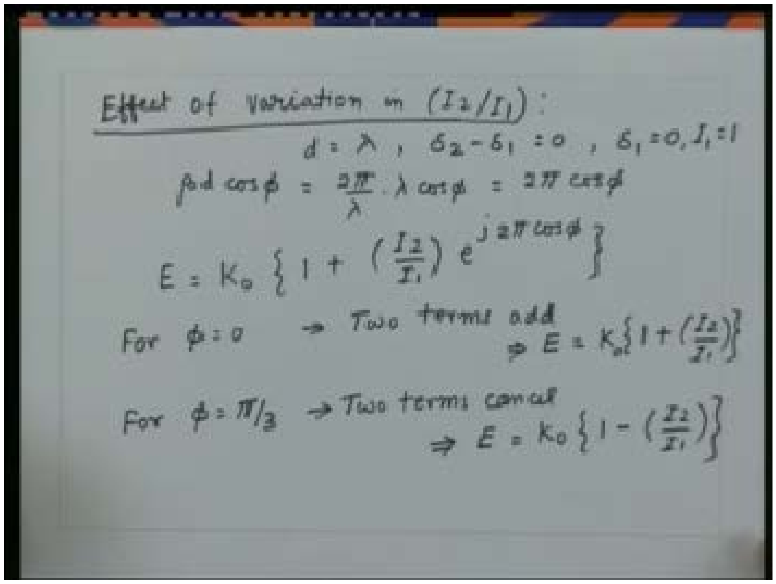

So this is the maximum value I will get for the electric field as I vary the angle phi and this is the minimum value of the electric field which I get in the radiation pattern but the important thing to note here is if the two currents are not equal that means I_2/I_1 is not one then there is no complete cancellation of the two fields that means we do not have completely constructive or completely destructive interference so when the ratio I_2/I_1 is not equal to one that time you will never have a null in the radiation pattern you have maximum in the radiation pattern will die down to some lower value but there will be no direction in which there will be null in the radiation pattern so this thing essentially tells you no null in the pattern.

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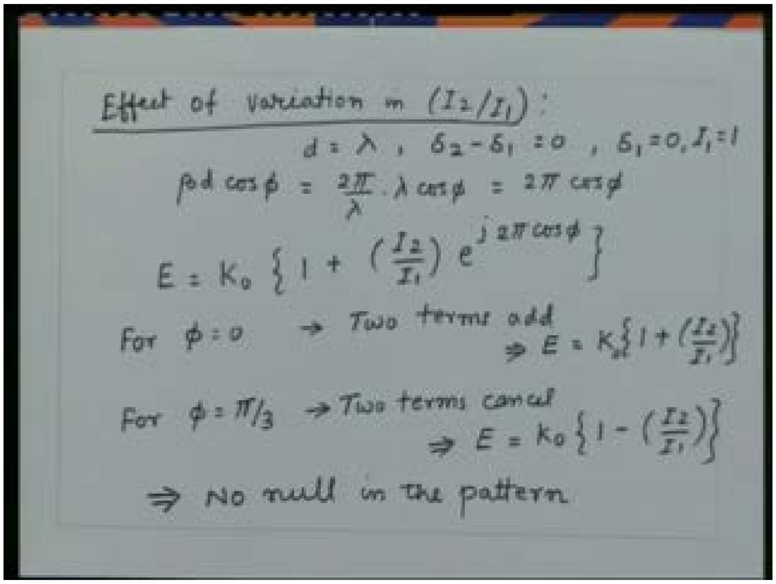

So in general then we can say that the ratio of the current amplitudes essentially control the depth of the nulls in the radiation pattern as I_1 approaches to I_2 and this two become equal the depth is full that means we have a zero field for the null when I_2 approaches I_1 that time it is null when any of these quantities go to zero there is no null rather the radiation pattern becomes isotropic because if I_1 is zero only I_2 radiates for which the radiation pattern is isotropic when I_2 is zero only I_1 radiates for which the radiation pattern is again isotropic which do not have any nulls. So, essentially the depth of the nulls is controlled by this parameter which is the ratio of the two current amplitudes.

Finally we can see the effect of the phase difference delta two minus delta one and let us say that is given by delta. So again without loosing generality let us say we have some distance d and the amplitude ratio now is taken as one so let us say we take a case where there is a full interference so this is equal to one so the electric field for this case E will be given as K_0 into again I can take I_1 equal to one and delta one equal to zero so this will be I_2/I_1 is one, this quantity delta two minus delta one is ø so we can write here this is e to the power j β d cosø plus delta.

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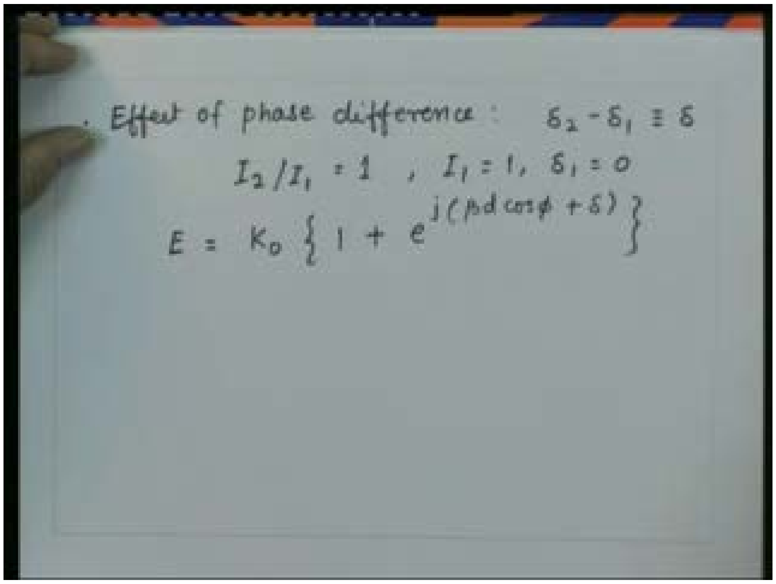

Now as we did in the previous case by taking this quantity common here this term essentially becomes cosine of half of this quantity so on the same lines we can say now that the radiation pattern in this case will be cosine of β d cosø plus delta upon two. Now the maximum radiation you get when ever this quantity is zero that is the angle for which this will become equal to one so we get maximum in the radiation pattern that corresponds to β d cosø plus delta upon two that is equal to zero or it can be multiples of π because when this quantity is π again the magnitude of this will be equal to one so you will get a maximum so this is 0, π, 2π and so on.

Let us say we concentrate only on first condition which is zero so this gives for the maximum radiation your β d cosø plus delta which is equal to zero so the direction for maximum radiation cosø of that will be equal to minus delta upon β d so for maximum radiation let us say if we call this angle as ø max the cosine of ø max is given by minus delta upon β d.

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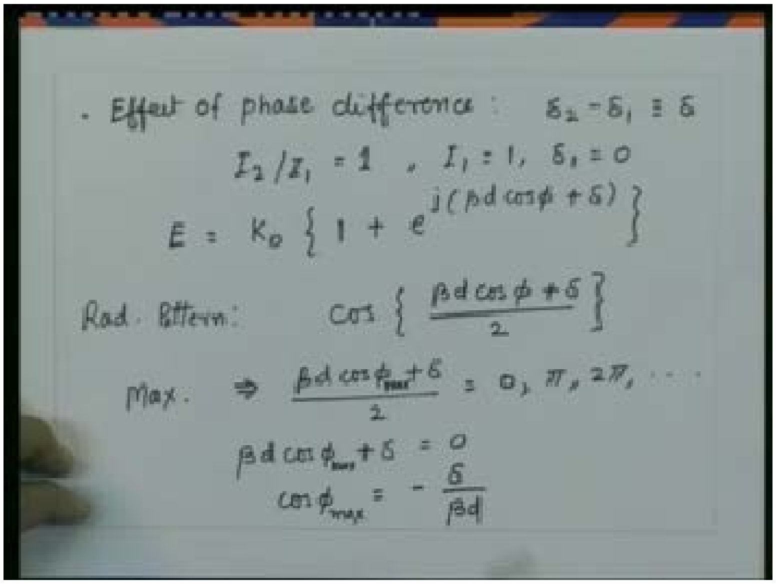

So as we change the value of delta essentially this angle changes for given inter element spacing which is d so direction of the maxima can be changed by changing the phase difference between the two elements so if I put let us say delta equal to zero then cos of ø max that will be equal to zero so the ø max will correspond to π/2 so for delta is equal to zero you get ø max equal to π/2. If I put delta is equal to let us say ±βd then we can get this quantity as ±1 and ø max would correspond to zero or π. So for delta equal to ±βd will give me the ø max which is equal to π or 0.

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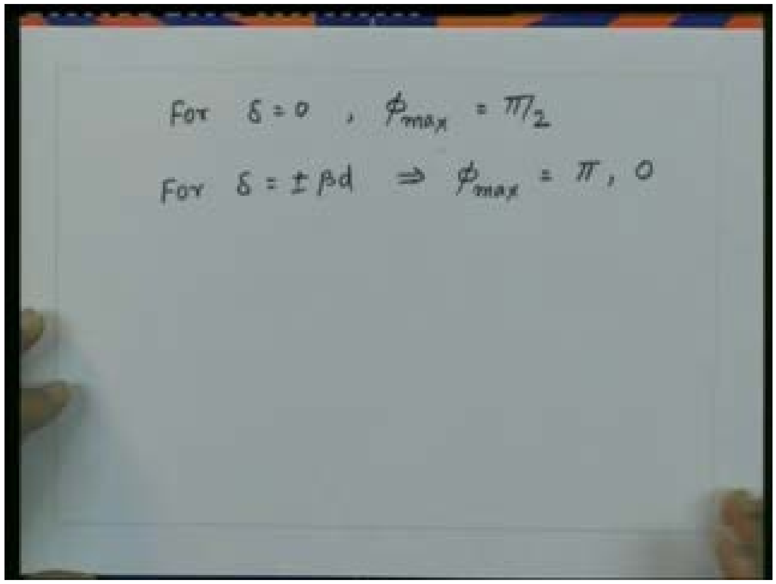

So what we note here is as we change the value of delta the beam maximum direction is changes from zero to π/2 to π. So as the angle changes from -βd it corresponds to zero to π/2 when delta is zero and when delta becomes equal to +βd the maximum direction is going to π.

So essentially if this was the array as the phase difference changes from delta equal to -βd to delta is equal to +βd the beam maximum direction essentially changes from this direction this we can say the beam direction so the phase difference between the elements essentially has effect of rotating the radiation pattern for changing the beam direction of the antenna array so what we now see is each of these three parameters ratio of the currents the phase difference and the inter element spacing have a unique signature in the radiation pattern and precisely that is what we make us of when we talk about the general arrays which is not of only two elements and for N elements and we adjust these three parameters to get the desired radiation patterns.

Thank you.