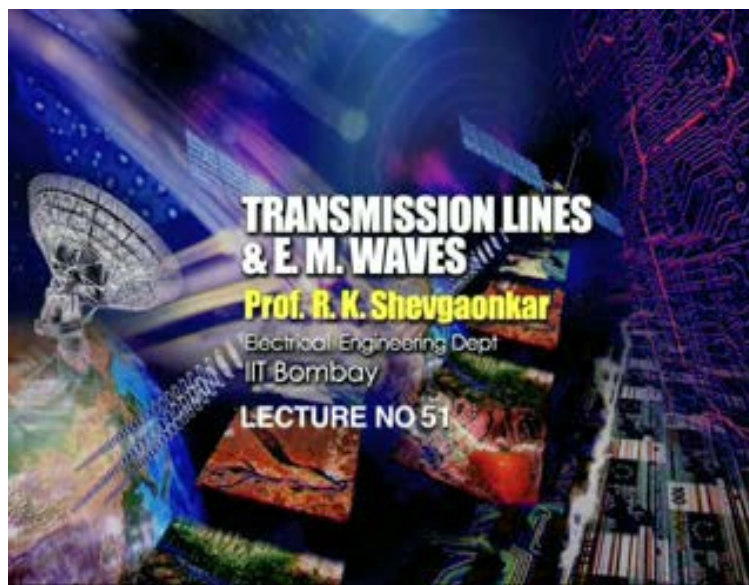


**Transmission Lines and E.M. Waves**  
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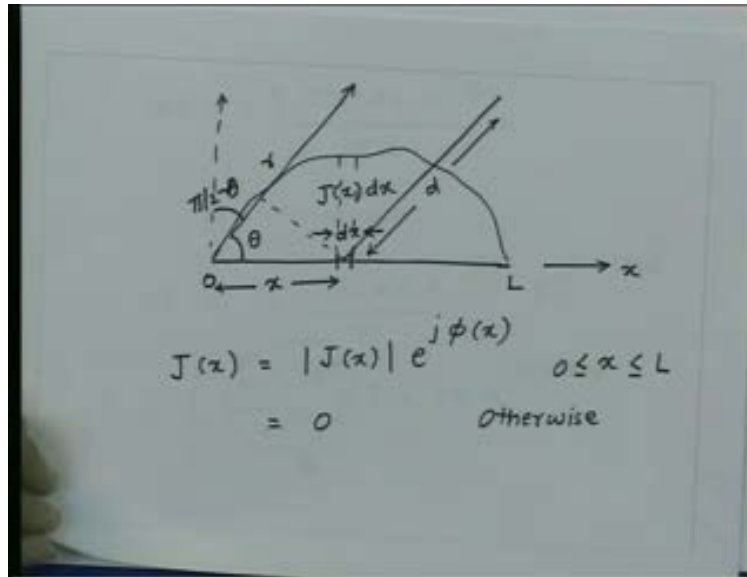
**Lecture-51**

Welcome, in the last lecture we asked the basic question that is what the relationship between the current distribution and the radiation pattern of an antenna we have analyzed the problem of the dipole where the current distribution was given and from there we found out the radiation patterns, we also saw as the length of the dipole changes the radiation pattern changes but this still did not give us a good understanding of how the current distribution affects the radiation pattern.



Therefore we ask this basic question that if the current distribution is given then what would be the radiation pattern or conversely if I wanted to have a radiation pattern what should be the current distribution. So essentially we started with some arbitrary current distribution which is complex in nature so it is having amplitude and phase which is varying in some direction and we had considered only a simple linear problem so the current is distributed along the line

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and then simplifies using the superposition of the radiation fields generated by different current elements we got a very important relationship between the radiation pattern and the current distribution and that characteristic or that relationship is the Fourier transform relationship.

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$$dE = j \eta \frac{J(x) dx}{4\pi d} e^{-j\beta d} \cdot k$$

$$d = r - x \cos \theta$$

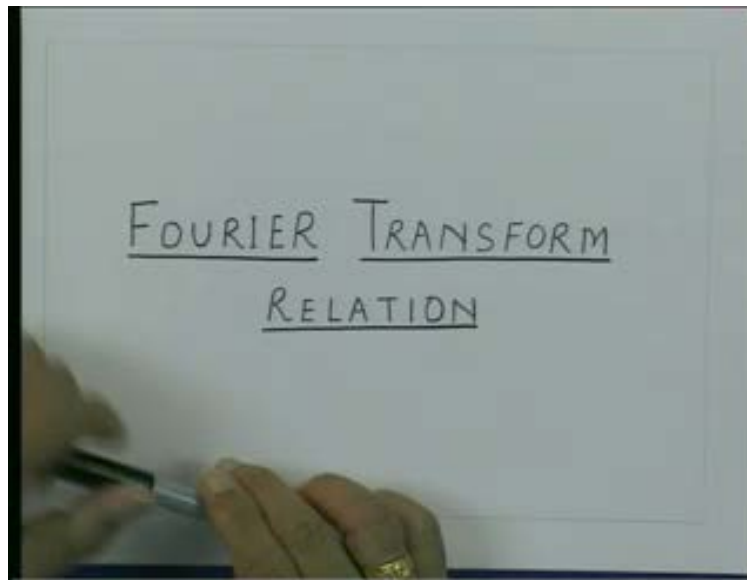
$$dE \approx k_1 \frac{J(x) dx}{r} e^{-j\beta r} e^{j\beta x \cos \theta}$$

$$E = \int_0^L \left( \frac{k_1 e^{-j\beta r}}{r} \right) J(x) dx e^{j\beta x \cos \theta}$$

$k_0$

So we have a very important conclusion that the radiation pattern of an antenna is the Fourier transform of its current distribution and this property is very useful and very important property and what that means is if the current distribution is given you have a unique radiation pattern and vice versa that means radiation pattern is given as a unique current distribution which would give me that radiation pattern.

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However, there are few things to be noted when we use this Fourier transform relationship so let me write this relationship which we got last time for the Fourier transform relationship.

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The whiteboard shows the following derivation:

$$E(\theta) = K_0 \int_0^L J(x) e^{j 2\pi \left(\frac{x}{\lambda}\right) \cos \theta} dx$$

$$= K_0 \int_{-\infty}^{\infty} J(x) e^{j 2\pi \left(\frac{x}{\lambda}\right) \cos \theta} dx$$

Variable  $l \equiv \cos \theta$ , normalized, length  $x' \equiv \frac{x}{\lambda}$

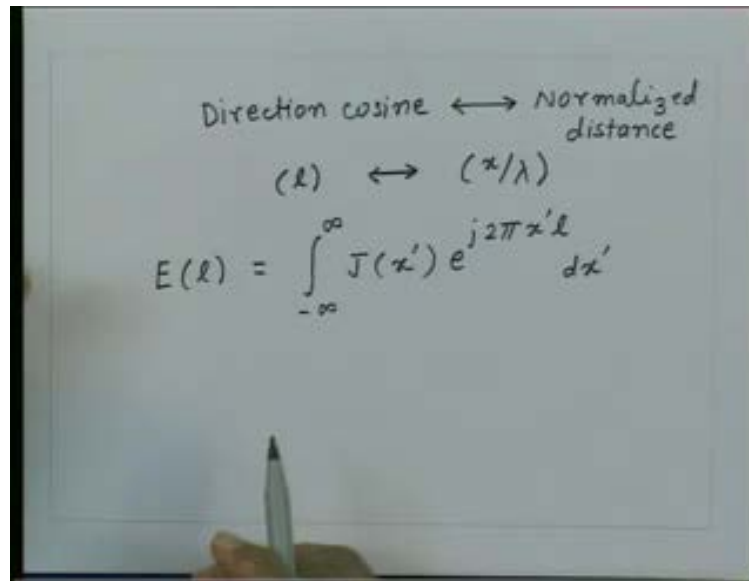
$$E(l) = K_0 \lambda \underbrace{\int_{-\infty}^{\infty} J(x') e^{j 2\pi x' l} dx'}_{\text{Fourier Transform}}$$

Note here, this quantity which was the radiation field is now a function of  $l$  where  $l$  is the cosine of the angle  $\theta$  and this  $\theta$  has nothing to do with the spherical coordinate system this is the angle which is measured from the axis or from the line for the current distribution and the distance which we got here is  $x$  prime the normalized distance with respect to the wavelength. This is a constant quantity so since the radiation pattern is a normalized pattern we do not worry about these constants which come here in the integral. So we can say in general that the radiation pattern will be given by the integral which is the Fourier integral where this is the current distribution as a function of distance and this is the Fourier term.

So we now have a Fourier relationship between these two but the parameters are not the angle in which the radiation pattern is measured  $\theta$  and also this space parameter is not the absolute distance but the Fourier pair is the cosine of the angle  $\theta$  which is nothing but the direction cosine and the normalized distance on the antenna. So the important thing here is to note is that we have a Fourier pair so we have Fourier transform relationship but the Fourier pair is direction cosine and normalized distance. So if I have a current distribution then I from the current distribution I find out the direction which is  $\theta$  cosine

of that is the parameter which is the Fourier parameter and the distance which is normalized with respect to wavelength is the special parameter for the antenna. So the direction cosine as we denoted by quantity  $l$  this is a Fourier pair with a normalized quantity which is  $x$  by  $\lambda$  and then we can write now the Fourier relationship in general without worrying about the proportionality constants that  $E(l)$  that will be equal to the integral  $-\infty$  to  $+\infty$  the current distribution which is a function of  $x$  by  $\lambda$  which we put as  $x$  prime  $e$  to the power  $j 2\pi x$  prime  $l$   $dx$  prime.

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Direction cosine  $\longleftrightarrow$  Normalized distance  
 $(l) \longleftrightarrow (x/\lambda)$   

$$E(l) = \int_{-\infty}^{\infty} J(x') e^{j2\pi x' l} dx'$$

Now if I look at this integral then I can find out the value of electric field once the  $J$  is given in terms of the normalized distance then the Fourier integral can be evaluated for all values of  $l$  from  $-\infty$  to  $+\infty$ .

However, what should be kept in mind is the  $l$  is essentially the direction cosine so if you have the current distribution the angle is going to vary for this current distribution from  $0$  to  $\pi$  and  $l$  is nothing but the cosine of that angle. So  $l$  is going to vary only from  $-1$  to  $+1$  so though this integral can be evaluated for all values of  $l$  from  $-\infty$  to  $+\infty$  only that portion of the Fourier transform is useful which lies in the range of  $l$  from  $-1$  to  $+1$ . So this range

of  $l$  then we can call as the visible range of  $l$ . So mathematically if I look at it I can get the  $e$  value for all possible  $l$ 's but those  $e$  values which will be lying for  $l$ 's corresponding to beyond  $-1$  and  $+1$  those electric fields will not be the physical electric fields.

So mathematically this relation is true for any value of  $l$  but physically if I see only that portion which lies in the range of  $-1$  to  $+1$  for  $l$  is the pattern which will represent the radiation pattern of the current distribution.

Now essentially what we are saying is we are having a range of  $l$  which will go from  $-\infty$  this is  $l$  which will go to  $+\infty$  will go to  $-\infty$ . So once the  $J$  is given to you practically the radiation pattern is written on this.

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Direction cosine  $\longleftrightarrow$  Normalized distance  
 $(l) \longleftrightarrow (x/\lambda)$   

$$E(l) = \int_{-\infty}^{\infty} J(x') e^{j 2\pi x' l} dx'$$
  

$$-\infty \longleftrightarrow \xrightarrow{l} +\infty$$

However, this is  $l$  equal to zero and this is the only range minus  $-1$  to  $+1$  which corresponds to the physical radiation pattern so we can call this range as the visible range of  $l$  which is from  $-1$  to  $+1$  so what that means is we can have the radiation pattern fixed in this which is same and we can change the pattern here without affecting the radiation pattern.

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Direction cosine  $\longleftrightarrow$  Normalized distance  
 $(l) \longleftrightarrow (x/\lambda)$   
$$E(l) = \int_{-\infty}^{\infty} J(x') e^{j2\pi x' l} dx'$$
  
Visible range  
 $-\infty \quad -1 \quad 0 \quad 1 \quad +\infty$   
 $l$

So now if I look at this problem like a mathematical problem just the  $l$  is defined which gives me the radiation pattern and then I can arbitrarily choose the function in the range from one to infinity I can arbitrarily choose the function from minus one to infinity but when I choose this arbitrary function here the Fourier transform is going to get affected what that means is keeping the radiation pattern same in the visible range I do not now have the unique current distribution so when we derive this relationship it appears that we have a unique relationship between the current distribution and the radiation pattern, however, since the  $l$  range is limited to  $-1$  to  $+1$  you may have infinitely possible current distributions which may give me the same radiation pattern in the visible range and they might be different in the range which is beyond the visible range. That means knowing the current distribution finding radiation pattern is unique but knowing radiation pattern finding current distribution is not unique.

So this relationship which was looked very attractive in the first look that is if I know this relationship now I can invert the problem and say I want to realize certain radiation pattern just by taking Fourier transform I should be able to know what the current

distribution is. There is no unique relation to that that means there are large number of current distributions which may give me the same radiation pattern in the visible range.

One may ask a question that, is it possible to arbitrarily choose the radiation pattern in this range which is beyond the visible range, the answer probably will be no, you may not arbitrarily choose this function because when you talk about this function suddenly you cannot change this function so rest will be the continuity of the function derivatives also have to be continuous and so on. So choice of current distribution which you will get will be restricting but the important thing to note here is that there is no unique current distributions which will give a given radiation pattern in the visible line though the converse is true that means the current distribution is given to you it will give you the unique radiation pattern.

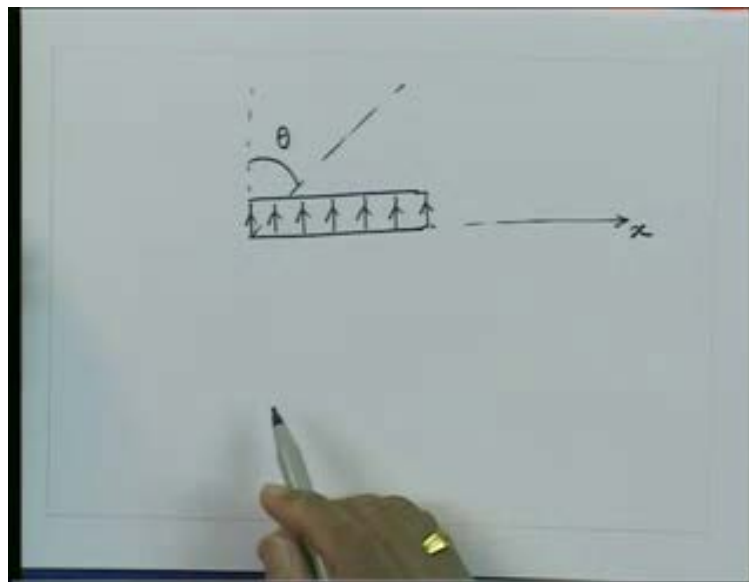
So the analysis problem that means when the current distribution is given to you and finding radiation pattern this is analysis problem is a straight forward problem but the inverse problem which is called synthesis problem that when the radiation pattern is given to you finding the current distribution that problem is not that straight forward because there is no uniqueness in that relationship. The another thing to note here from the Fourier integral is that this is simply telling you the variation of the current as a function of space it still does not tells you the basic radiation characteristics of the current what do I mean by that is let us imagine a situation let us say I am having a current sheet which is having a current which is vertically flowing like that and this is the data sheet which you are having and I want to ask what is the radiation which comes from this current sheet.

Now assuming that width of this is very very small the current distribution if I look at in this plane of paper it is uniform the current is flowing in this direction so this problem the current distribution if I look at it is a constant current distribution so this is the direction I can take as  $x$  I can measure the angle from this direction. Now since this current sheet can be visualized as combination of small small current elements which are oriented in vertical direction this current sheet can be thought of as a collection of Hertz Dipoles so



essentially I have a Hertz Dipole here so this thing can be divided into small small segments each one can be treated as a Hertz Dipole and then the total radiation field which we will get will be superposition of the fields which we will get from each of the Hertz Dipoles. Now, since we know the radiation pattern for the Hertz Dipole let us say now I measure the angle  $\theta$  from this direction that is what we do for Hertz Dipole that is the way we analyze it keeping the Hertz Dipole along the z direction. So I have angle  $\theta$  in this so this is the angle say this angle is  $\theta$  so angle with respect to this x axis is  $90 - \theta$  so in the Fourier integral we had defined the cosine of the angle which the current distribution this thing makes with the x axis is  $90 - \theta$ .

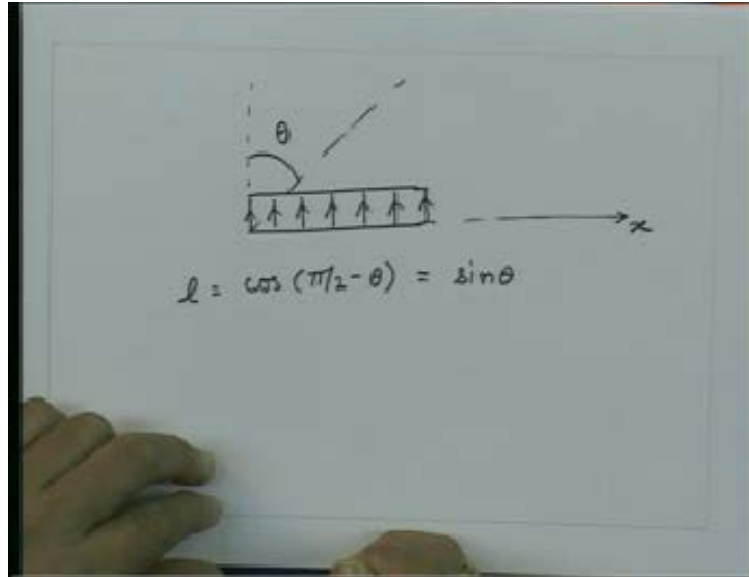
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So in this case we have  $l$  which is cosine of  $\pi/2 - \theta$  which is equal to  $\sin\theta$  and then I can write down now without worrying about whether this is a Hertz Dipole or what the uniform current distribution so I can write its Fourier transform but note here that when there is a current element which is like a Hertz Dipole this is not distributing uniformly in all directions the radiation is not going uniformly in all directions. When you apply the superposition for getting the Fourier integral we assume that each of the current elements

radiates uniformly and we simply have a superposition of the fields from different current elements.

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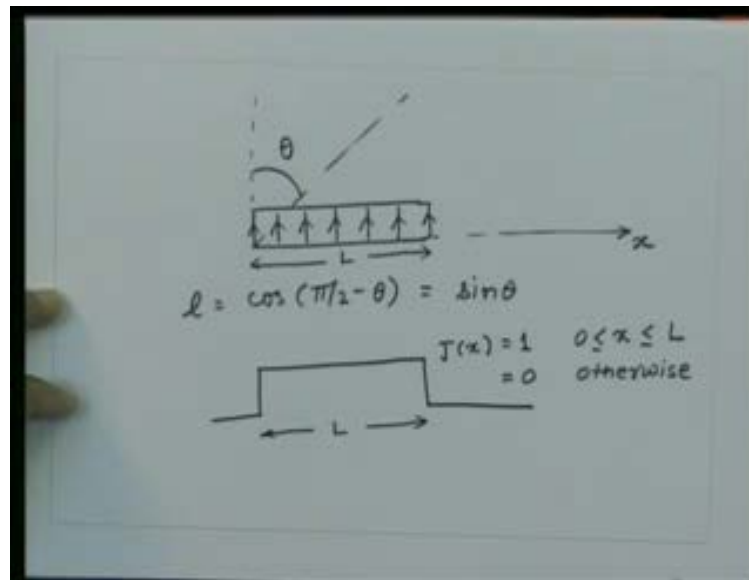


However, now this current element intrinsically has a pattern which is sine theta radiation pattern you know that when we analyze the Hertz Dipole we saw that along the axis of the dipole there is no radiation this current element is capable of radiating perpendicular to it and therefore radiation pattern for the Hertz Dipole is  $\sin\theta$  so we get the radiation pattern from Fourier transform relationship which will be corresponding to only the current distribution but the total radiation pattern has to be multiplication of the intrinsic radiation pattern of the Hertz Dipole which the radiation pattern which we get from the Fourier transform relationship.

So the original radiation pattern which the current element has that is not reflected into the Fourier transform relationship it is simply Fourier transform relationship is only between the distribution of the current and the radiation pattern, what ever are the radiation characteristics of the basic current element or basic radiating element those have to be superimposed on the radiation pattern which we get from Fourier transform

relationship. So in this case let us say this was a current sheet the current distribution will be a distribution which like that let us say the length of this current sheet is  $l$  so I have here a current distribution which is non zero over length  $l$  and is zero beyond that without loosing generality let us the current amplitude is a unity so this current  $J(x)$  is equal to unity over this range and is zero here this is 1 for  $x$  less than or equal to  $l$  and is zero otherwise.

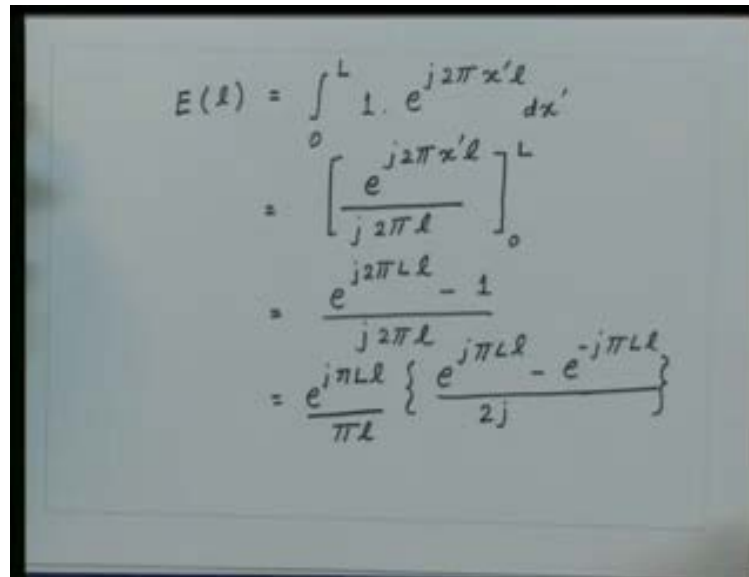
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Now we can use the Fourier transform property to find out what is the radiation pattern where now  $l$  is given by  $\sin \theta$  so the radiation pattern  $E(l)$  from Fourier transform will be equal to 0 to  $l J(x)$  which is unity over this so this is one  $e$  to the power  $j 2\pi$  and let us say the distance which I have here is normalize this  $l$  is the length in normalized quantity so let us say this is normalized length  $x$  prime this quantity will be  $x$  prime  $l$   $dx$  prime. This integral is very very straight forward so essentially from here we will get  $e$  power  $j 2\pi x$  prime  $l$  divided by  $j 2\pi$  limit 0 to  $l$ . I can substitute for these limits so this gives me  $e$  to the power  $j 2\pi L$   $1$  minus one divided by  $j 2\pi l$ . For radiation pattern since we are interested in the magnitude of the electric field we can do some simple simplification to the expression we can take  $e$  to the power  $j 2\pi L$   $l$  common from here so this will be equal

to  $e$  to the power  $j\pi Ll$   $e$  to the power  $j\pi L$  minus  $e$  to the power  $-j\pi Ll$  divided by  $2j$  and take this  $\pi l$  outside so this is  $\pi l$ .

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$$\begin{aligned}
 E(l) &= \int_0^L 1 \cdot e^{j2\pi x'l} dx' \\
 &= \left[ \frac{e^{j2\pi x'l}}{j2\pi l} \right]_0^L \\
 &= \frac{e^{j2\pi Ll} - 1}{j2\pi l} \\
 &= \frac{e^{j\pi Ll}}{\pi l} \left\{ \frac{e^{j\pi Ll} - e^{-j\pi Ll}}{2j} \right\}
 \end{aligned}$$

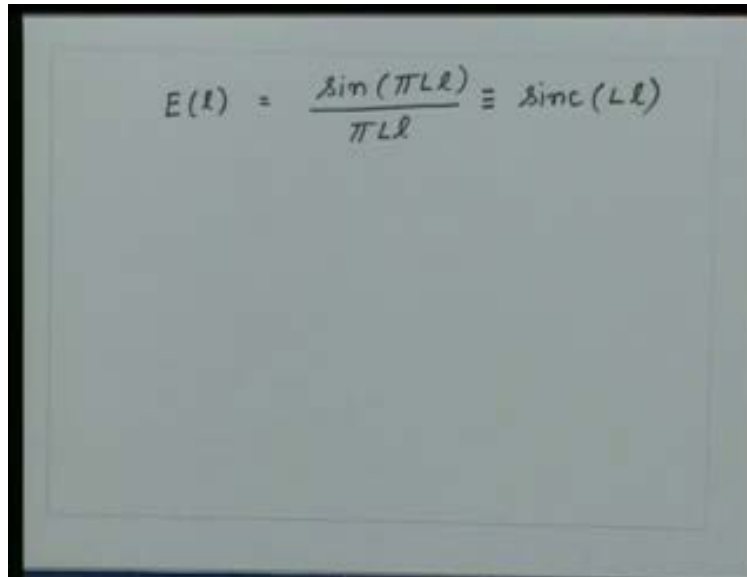
Now this quantity is nothing but the sine of  $\pi Ll$  so this we can write as sine of  $\pi Ll$  upon let us multiply numerator and denominator by capital  $l$  so this is  $\pi Ll$  into  $L$  into  $e$  to the power  $j\pi l$  into  $l$ .

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$$\begin{aligned}
 E(\alpha) &= \int_0^L 1 \cdot e^{j2\pi\alpha' l} d\alpha' \\
 &= \left[ \frac{e^{j2\pi\alpha' l}}{j2\pi l} \right]_0^L \\
 &= \frac{e^{j2\pi L l} - 1}{j2\pi l} \\
 &= \frac{e^{j\pi L l}}{\pi l} \left\{ \frac{e^{j\pi L l} - e^{-j\pi L l}}{2j} \right\} \\
 &= \frac{\sin(\pi L l)}{\pi l} \cdot L e^{j\pi L l}
 \end{aligned}$$

Where  $l$  is the constant for the given aperture size for given current distribution this quantity is constant so  $l$  is constant. This quantity gives you only the phase term so the magnitude of this is unity so the radiation pattern essentially is given by this which is the amplitude variation as a function of angle and in this case the angle is the direction cosine which is  $\sin\theta$  so we get the  $E(l)$  amplitude that will be equal to sine of  $\pi L l$  divided by  $\pi L l$  this one is nothing but what is called the sinc function of  $L l$  so this is the sinc of  $L l$ . So the radiation pattern for a uniform current distribution in this plane will be a sinc function of this product  $L$  and  $l$  which is the direction cosine.

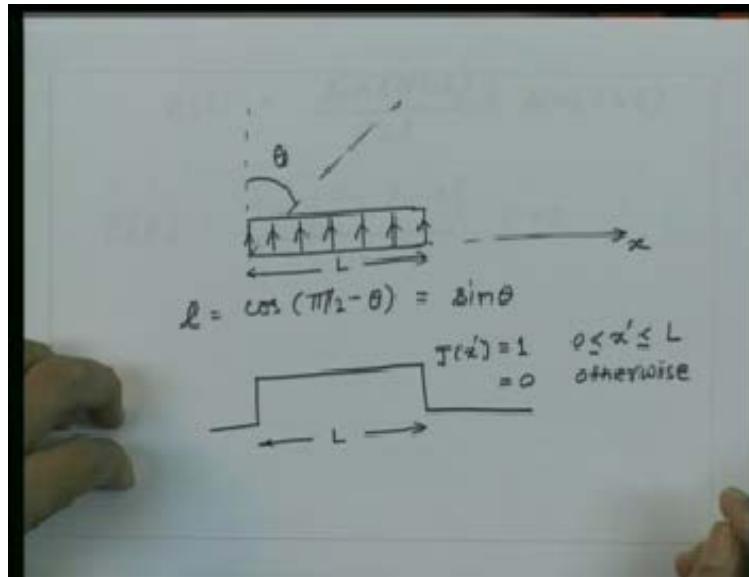
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$$E(l) = \frac{\sin(\pi L l)}{\pi L l} \equiv \text{sinc}(L l)$$

Now this is the radiation pattern we would get without considering the current elements which are forming this current sheet we simply have taken here the current distribution which is uniform and without worrying about what is the direction of the current we simply found the current the current distribution and the radiation pattern which is essentially given by this.

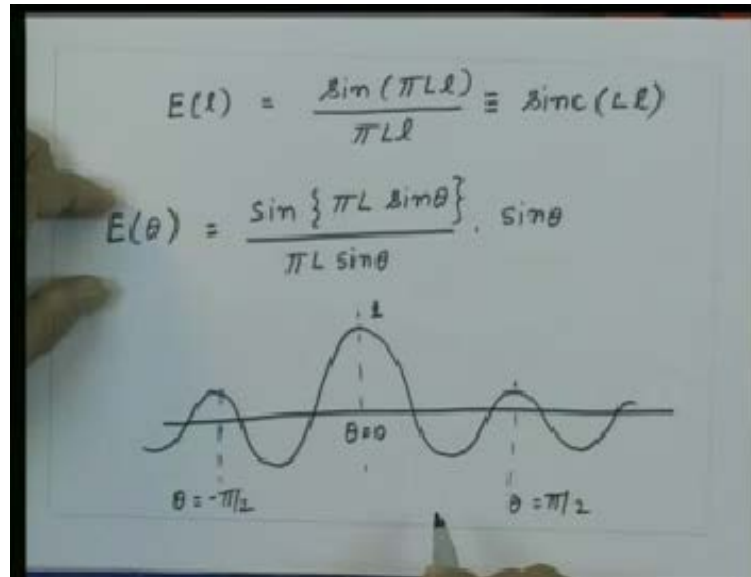
However since we know that the intrinsic Hertz Dipole has a radiation pattern sine theta the total radiation pattern must be multiplication of this radiation pattern with the radiation pattern of the Hertz Dipole because this radiation pattern will be weighted by the radiation pattern of the Hertz Dipole. So the total radiation pattern then we can write as  $E(\theta)$  that will be equal to the quantity  $\sin\{\pi L \sin\theta\}$  upon  $\pi L \sin\theta$  multiplied by  $\sin\theta$  which is the radiation pattern of the Hertz Dipole. So that is the radiation pattern you will get if you treat this like a current sheet which is in the plane of the paper and the currents are flowing upwards and if this width is much smaller compared to the wave length so that the current can be assumed constant in that direction.

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Once you understand this then this concept can be essentially expanded to any arbitrary current distribution, here if you understand the radiation characteristics of this thin current sheet may be understanding the radiation pattern for other current distribution will be quite straight forward. So knowing this current distribution and the radiation pattern now we can investigate the characteristics of this firstly as we said this quantity  $l$  has to lie between  $-1$  and  $+1$  this is the sinc function which will be varying like this so if this is taken as unity for the sinc function which will correspond to when this quantity is zero so we have here  $\theta = 0$  that is where the sinc function is maximum so this quantity  $\theta = 0$ , as I change the value of  $\theta$  from in this case now the  $\theta$  will vary from  $-\pi/2$  to  $+\pi/2$  so this angle was going from  $0$  to  $\pi$  but  $\theta$  will go from  $-\pi/2$  to  $+\pi/2$  so the range for the radiation pattern which correspond to  $\theta = -\pi/2$  to  $+\pi/2$  so this is  $\theta = -\pi/2$ , this is  $\theta = +\pi/2$ . So this is the visible range that means this is the radiation pattern which you are going to see actually in the space as the radiation pattern this portion of the radiation pattern will not be the visible portion of the radiation pattern. So that means now we will get the nulls in this radiation pattern at this location, at this location, at this location, at this location.

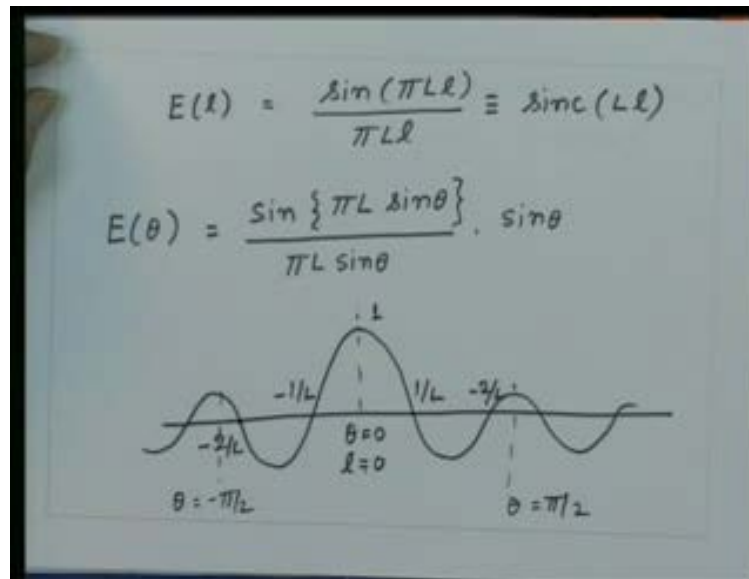
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And as we know from the property of the sinc function the nulls would correspond to when this quantity is multiples of  $\pi$  that means when this quantity  $Ll$  is an integer that is the location you will have nulls. So if I write in terms of  $l$  this is  $\theta = 0$  would correspond to  $l = 0$  also the first null will occur when this quantity is equal to one so  $l$  will be one upon  $l$  so this location is one upon  $l$  this will be  $-1$  upon  $l$  for  $l$  this will be  $2$  upon  $l$ ,  $-2$  upon  $l$  so this is  $-2$  upon  $l$ , this location will be  $-2$  upon  $l$ .

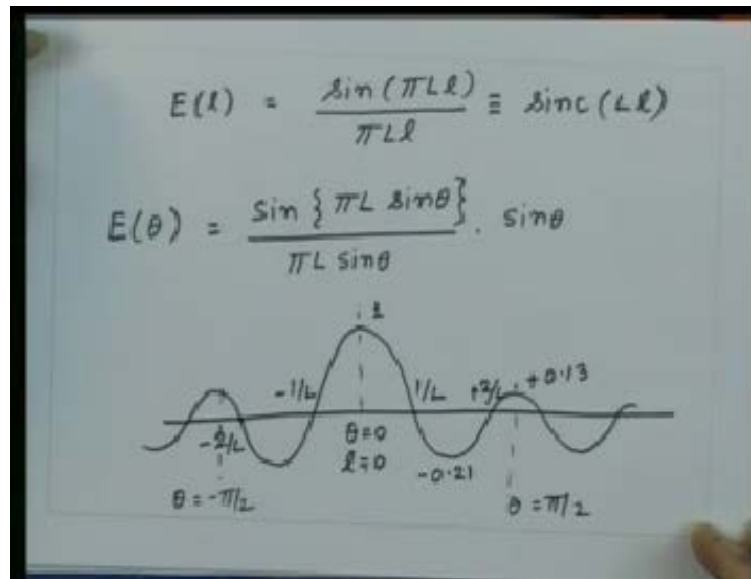


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So the nulls will be given by the directions for which the sine of the angle is one upon  $l$  or multiples of one upon  $l$ . So from here this should be  $\pm 2$  upon  $l$  so from here we can find out the directions of the radiation pattern where the radiation goes to zero called the nulls of the radiation pattern. The sinc function is a very well studied function and we know many other properties of the sinc function and that is the first negative lobe which we get here this is about twenty one percent of this peak value so if you take normalized this is unity this value will be  $-0.21$  approximately, this value the next value will be  $+0.13$  and so on. Then we can call this thing as side lobe of your radiation pattern. So for this current sheet you will have the first side lobe which will be about 21% of the main beam, the second side lobe will be about 13% of the main beam and the nulls will be equi spaced which are separated by one over  $l$ .

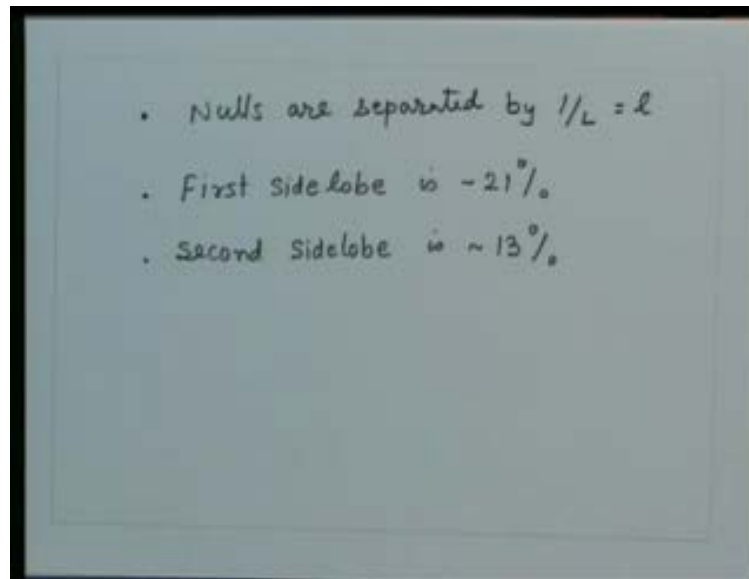
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So from here we can get some conclusions that is Nulls are separated by one over  $l$  equal to  $1$ , first side lobe is 21%, second side lobe is about 13% and so on, note here the nulls are equi spaced in the  $l$  domain they are not equi spaced in  $\theta$  domain because you are having a non linear relationship between  $l$  and  $\theta$  where  $l$  is  $\sin\theta$  so in  $l$  domain the nulls are equi spaced. So if I take this as a plot as a function of  $l$  then all this nulls are equi spaced but if i convert them to  $\theta$  then the nulls in the angular zone will not be a equi spaced with respect to the main beam.

Now we have the basic understanding developed for the radiation pattern from the current distribution.

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Also we can note one more interesting thing from this pattern and that is if we increase the length of this current sheet that means if the  $L$  becomes larger and larger this nulls will become closer and closer because when the  $L$  increases this null will come closer and closer so the angular separation between these two points what is called the beam width between the first nulls that will go on reducing that means my main beam which is given by this will become narrower and narrower, or in other words, as the size of this current distribution this aperture changes your antenna will become more and more directive because the beam width will become smaller and smaller and as we have seen earlier when the beam width becomes smaller the directivity of the antenna increases. So by increasing the length of this current sheet the directivity of this antenna can be increases.

Let us go to the next level of usage of this Fourier transform relationship and that is let us say that this current sheet gave a maximum which was in  $\theta = 0$  direction which was in this direction and there were some nulls was in some other directions. Let us now ask a question that suppose I wanted to tilt the main beam from  $\theta$  equal to zero to some other direction so in general the Fourier transform relationship what ever we have you can ask a question suppose I wanted to change the direction of the main beam so in this case I had

some current distribution for which let us say the main beam was at  $l = 0$  what should I do to the current distribution so the main beam will be shifted to some other direction let us say  $l_0$  so I wanted to shift the radiation pattern by  $l_0$ . So we can go substitute for  $l = l_0$ .

So let us say our interest is to change the beam direction by  $l_0$  so initially we had a beam in certain direction could be zero could be something else but with respect to that let us say I wanted to now change the direction of the beam by  $l_0$  and that is possible by simply shifting the radiation pattern in the  $l$  domain so the question we are asking is if I wanted to translate this electric field on the  $l$  axis by  $l_0$  what is the requirement on the current distribution what way the current distribution will get affected. So I can substitute for  $E(l - l_0)$  where the pattern is shifted by  $l_0$  that will be equal to  $-\infty$  to  $+\infty$   $J(x')$   $e$  to the power  $j 2\pi x' (l - l_0)$  into  $dx'$ . We can take this quantity out here  $e$  to the power  $j 2\pi x' l_0$  so that is  $-\infty$  to  $+\infty$   $J$  of  $x'$   $e$  to the power  $-j 2\pi x' l_0$  into  $e$  to the power  $j 2\pi x' l$ .

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• Change the Beam direction by  $l_0$

$$E(l - l_0) = \int_{-\infty}^{\infty} J(x') e^{j 2\pi x' (l - l_0)} dx'$$

$$= \int_{-\infty}^{\infty} J(x') e^{-j 2\pi x' l_0} \cdot e^{j 2\pi x' l} dx'$$


So we can say that the radiation pattern which is shifted by  $l_0$  is the Fourier transform of this quantity this term is the Fourier term. Now the new current distribution which will correspond to this radiation pattern which is shifted version of the original one is this current distribution. So what is change in this is you have the original current distribution and this current distribution is now multiplied by this quantity which is  $e$  to the power  $-j 2\pi x' l_0$  where  $l_0$  is constant. So that means this quantity is this is the phase which is varying linearly as a function of  $x'$  as a function of distance along the current distribution so any linear phase change in the current distribution will rotate the radiation pattern in  $\theta$  domain or in  $l$  domain so this is called the shift property of Fourier transform that if the function is shifted in one domain it is equivalent to having the phase gradient in the other domain and vice versa.

So when ever we want to shift the beam of the antenna essentially you have to introduce the phase gradient so in this case if initially let us say the phase distribution was constant let us say this was initial phase of the current distribution for  $jx$  now we have to introduce a phase shift of this which is negative of this so now the phase shift will be like that where the slope of this line will be equal to  $-2\pi$  into  $l_0$ . So if I take the original current distribution and if I introduce a phase gradient on the current distribution essentially the beam will be shifted in the direction of  $l_0$ . This is the principle essentially used in phased array scanning where physically the antennas are not rotated only the phase gradient is changed on the antenna structure and because of that the direction of radiation changes and this thing can be done electronically so we can change this phase gradient very rapidly and as a result the beam can really scan in the space at the rate at which the phase is changing on the current distribution.

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Change the beam direction by  $l_0$

$$E(l-l_0) = \int_{-\infty}^{\infty} J(x') e^{j2\pi x'(l-l_0)} dx'$$

$$= \int_{-\infty}^{\infty} [J(x') e^{-j2\pi x' l_0}] \cdot e^{j2\pi x' l} dx'$$


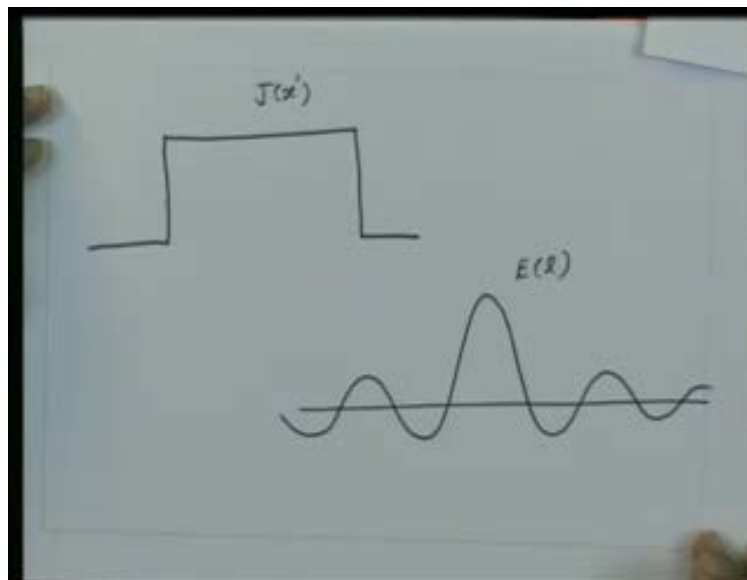
slope =  $-2\pi l_0$

So without mechanical movement now the radiation pattern or the beam of the antenna can be scanned in space by simply changing the phase gradient we can explore some other possibilities of Fourier transform. When the current distribution is uniform as we have taken here like a current sheet we get radiation pattern which is given by sinc function which is this and the important thing about the sinc function is the side first lobe level is -21% and this has nothing to do with the size of the antenna. So this quantities number is independent of  $l$  so no matter what value of  $l$  the side lobe level is always going to be 21%.

As we know from our discussion that the side lobes are essentially leakage of power in unwanted direction so ideally we would like to reduce the side lobe level to as small as a level as possible because this is the one which is a wastage of power so by reducing the side lobe level essentially we can improve the efficiency in given direction. So in practice it is always desirable to reduce the side lib level to as small as possible. As long as the current is uniform there is no possibility of now reducing the side lobes because side lobe is always going to be about 21% and that is where we can now go from the understanding of the Fourier transform and that is this side lobe levels which are the repel in the Fourier

transform are essentially because of the discontinuity in the current distribution in the space. So if I could avoid the sharpness in the current distribution then the ripple or the amplitude of the side lobe can be reduced. Precisely that is what we can do what is called the tapering of the elimination that means instead of having a current distribution which is uniform if I slowly taper it towards the end from the Fourier transform we can show that the ripple in the Fourier transform will reduce that means the side lobe level of the radiation pattern will reduce. So instead of having a current distribution if I take let us say original current distribution was like that and that was the radiation pattern so this is  $J$  of  $x$  prime and this is  $E(\theta)$ .

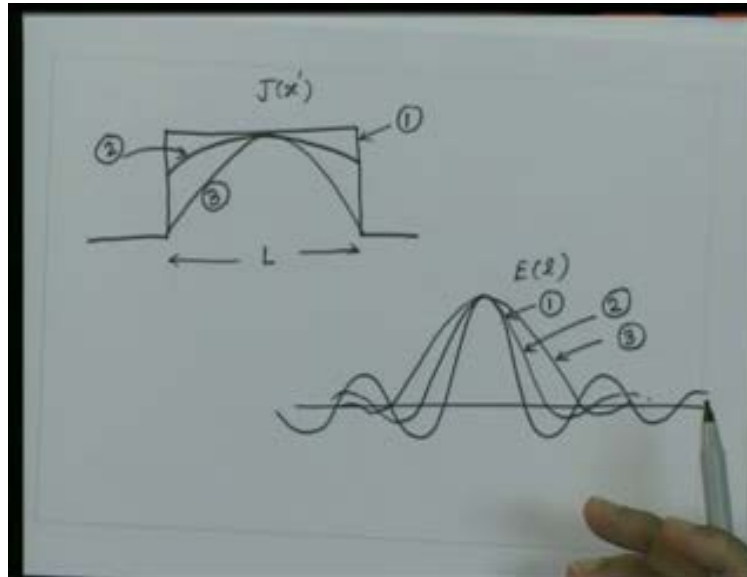
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So let us say that this current distribution is one you get a radiation pattern which will be sinc function this is one. If we now taper this current distribution to something like that then the side lobe level will come down because now the sharpness discontinuity is reduced so you get a current distribution which will look like that so side lobe level is reduced if I take the current distribution which will look like that then side lobe level will be further reduced so it will be something like this so this is the current distribution let us say number two this one is number three so this will be radiation pattern two this will be

radiation pattern number three. So by tapering the current distribution essentially we can reduce the side lobes of the radiation pattern.

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However, when we reduce the current amplitude on the edges of the current distribution when I increase the taper the effective width of the antenna radiation pattern also get altered so by putting the current taper essentially the beam width of the antenna becomes larger, or in other words, the directivity of the antenna reduces. So for the same physical antenna structure which is this current distribution if I put the current taper then the directivity of the antenna will reduce.

So we see two effects of the current taper, one is by increasing the current taper the side lobe level of the radiation pattern reduces, secondly, the beam width of the antenna increases or the directivity reduces and this we can also see from simple arguments that when ever we have a current distribution which is like his effectively we can define some effective current distribution which will be now shorter than this that we define 3dB points so you can define 3dB points for the current distribution so we will have effectively the current distribution which will be smaller than  $l$  and therefore the beam



width which is one over  $l$  will essentially increase because the effective width of the current distribution is decreases. These are certain properties of the Fourier transform which we can very easily used to investigate the radiation characteristics of different current distributions or different antennas.

The most important property which we can use from Fourier transform is the convolution property that is you know from two functions that is say two functions are  $f_1(x)$  and  $f_2(x)$  if I take these two functions and let us say they have the Fourier transform which are given by  $F$  so we have two functions which are having a Fourier pair  $f_1(x)$  which is Fourier transform  $F_1$  of  $l$  and we have another function  $f_2(x)$  the Fourier transform  $F_2$  of  $l$ . Then we know that the Fourier transform of product of these two functions is the convolution of the Fourier transform if I multiply function  $f_1(x) f_2(x)$  will be the Fourier transform which will be convolution of these two so that will be  $F_1(l)$  and convolution of  $F_2(l)$  and same is true for reverse that if I convolve these two functions that  $f_1(x)$  convolve with  $f_2(x)$  that will be the product of the Fourier transform so that is  $F_1(l) F_2(l)$ .

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$$\begin{aligned}
 &f_1(x), f_2(x) \\
 &f_1(x) \longleftrightarrow F_1(l) \\
 &f_2(x) \longleftrightarrow F_2(l) \\
 &f_1(x) f_2(x) \longleftrightarrow F_1(l) \otimes F_2(l) \\
 &f_1(x) \otimes f_2(x) \longleftrightarrow F_1(l) F_2(l)
 \end{aligned}$$

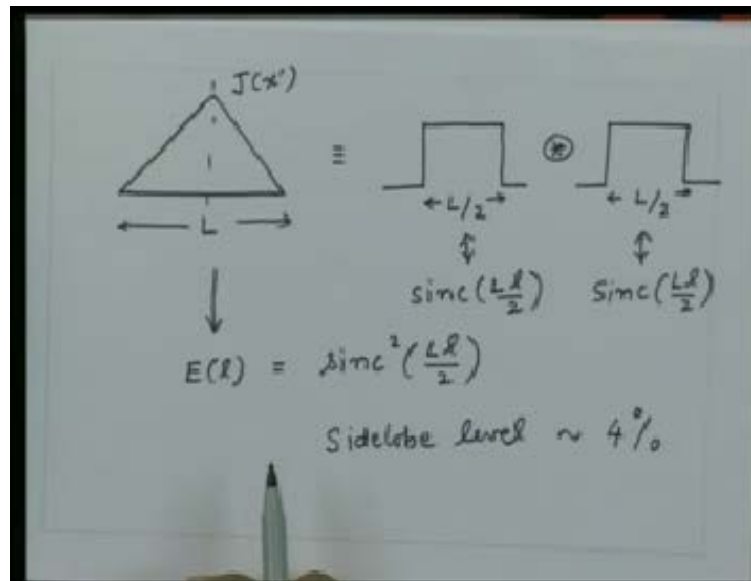
So this convolution property by using this property we can decompose a complex antenna problem into simpler problems and we can find out the radiation pattern very quickly for the complex current distributions.

Let us take a simple case let us say we want to find out the radiation pattern for triangular distribution. So let us say I want to find the current distribution is given by this this is my  $J$  of  $x$  prime of some length  $L$ . What will be the radiation pattern of this current distribution? Now this current distribution can be thought of convolution of two rectangular current distribution of length  $L/2$ . So this is equivalent to a current distribution which is convolved with a current distribution identical to this where this length is  $L/2$ . So a triangular current distribution is convolution of the two current distributions which are uniform current distributions. So this is since now the currents which are the convolution the Fourier transform the radiation pattern will be multiplication of the Fourier transform of these two.

So if I say is the rectangle we know for the Fourier transform for this which is sine sinc of  $L/2$  into  $l$  where  $L$  is now  $L/2$  so this is  $L/2$  this is now the Fourier transform of this the Fourier transform of this also will be also same which is again sinc of  $L/2$  so the radiation pattern of this current distribution will be multiplication of these two radiation patterns for Fourier transform because the current distribution is a convolution of these two current distributions so the radiation pattern would be multiplication of the two radiation patterns. So we have a situation which is like this where the individual current distributions are convolutions. So the final radiation pattern will be product of the two radiation patterns so that means the radiation pattern for this current distribution will be  $E(l)$  for this will be multiplication of these two so it will be sinc square  $L/2$ . So the pattern which you had the sinc function when you had the triangular distribution it will be the radiation pattern will be square of this function so that means this amplitude which was 0.2 that will become 0.04 approximately so the side lobe level will reduce to only 4% and this side lobe will be 1% and so on 1.6%. So the triangular current distribution will give the side lobe level as low as 4% compared to the current distribution which is uniform current distribution and that is what we argued when ever we put a current taper

that is what we have done here we have tapered the current distribution so that it goes to zero at the ends the side lobe level reduces. So in this case you will get the side lobe level approximately about 4%.

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So what we have seen now is the Fourier transform property which we have got that is a very very important property for investigating the radiation characteristics of antennas once we know the current distribution on the antenna then using the Fourier transform property we can find out the radiation pattern of the antenna, we can also ask a question if I want to modify the radiation pattern like if I wanted to tilt the radiation pattern by certain angle then what should be the changes in the current distribution and we saw if we introduce a phase gradient in the current distribution then the whole radiation pattern will be tilted by the shift property of the Fourier transform, we also saw by putting the taper on the current distribution we can reduce the side lobe level of the antenna radiation pattern.

So once you understand Fourier transform properly then many of the radiation characteristics of the antenna can be visualized in a quite straight forward manner. The

problem that knowing the radiation pattern how do we find the current distribution is still a difficult problem where as we said there is no uniqueness into that problem and there will be many solutions to the problem.

However, it looks now after the discussion that since the radiation patterns in the current distribution as Fourier transform relationship the control of the current is the sole mechanism of controlling the radiation pattern. So if you can control the current distribution on the antenna then we can control the radiation pattern also, however, as we saw in the case of let us say dipole we excite the antenna at the centre current distribution is not in our control current gets decided by its own so I do not have a good control over of the current distribution, also while changing the length of the antenna as the current distribution changes but also the terminal impedance changes. So now we have the current distribution which is related to the radiation pattern is coupled to the terminal impedances also.

So we are looking now for some mechanism by which we can control the current independently without affecting the terminal performance of the antenna so that mechanism by which the current can be independently controlled without affecting the impedance characteristics or terminal characteristics of the antenna that is what we essentially do in the arrays where different antenna elements are excited with different current so you manipulate the current distribution and therefore you manipulate the radiation pattern but the terminal characteristics of the antenna remains the same. So we will take up from here and then we will go into the investigation of the antenna arrays which gives much bigger freedom in designing the antenna radiation patterns.

Thank you.