Transmission Lines and E.M. Waves

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Lecture-50

Welcome, up till now we discussed the Hertz Dipole the extension of the Hertz Dipole to a dipole of finite size we also saw general characteristics of antennas like directivity, effective aperture, radiation pattern and so on. In this lecture we investigate some practical antennas which are used at low frequency at u h f and v h f bands, essentially these antennas lie in the category of dipole, however, what we note when we go to the low frequencies since the dipole should have length comparable to the wavelength to have a substantial radiation the size of the dipole becomes excessively large when we go to low frequencies in that situation a monopole antenna is used or essentially it finds the application in medium wave broadcasting.

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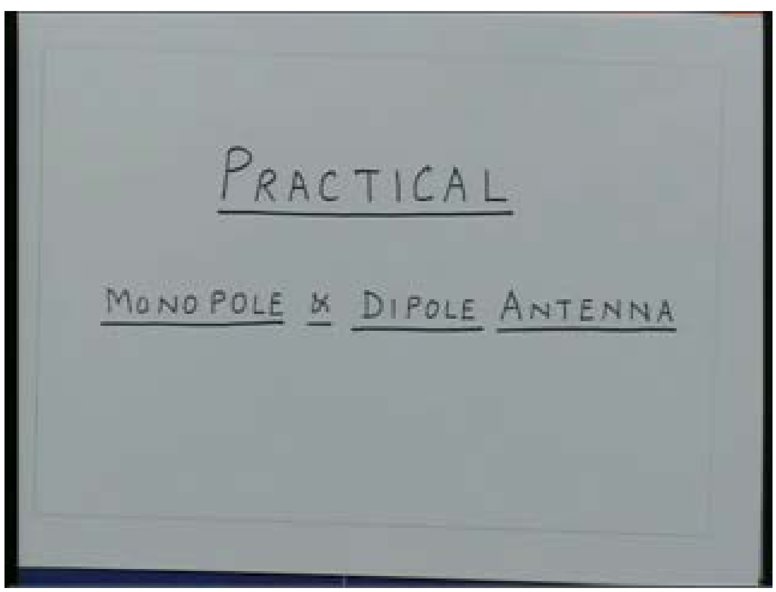

So if you go to the medium wave radio stations invariably the antenna which is used for broadcasting is the monopole antenna. In fact the radiation principle behind monopole antenna is identical to that of a dipole only thing is here we have only half of the dipole and we erect that half length of the dipole over a ideal ground plane. We have seen as we go to lower frequencies even if the conductivity of the medium is not very good the medium starts behaving more like a conductor.

Precisely that's how we make use of that when we go to low frequencies the earth which is not a very good conductor starts behaving more like a conductor and then if I have an antenna if I erect it over the ground surface it behaves as if it mounted over a ideal conducting surface and for investigating the characteristics of this monopole antenna which is erected over the conducting surface essentially we use the concept of images. So if you have a charge and if I put the charge over a conducting surface equivalently we have image of the charge so let us say if you are having a ground source like that and for understanding purpose let us say this ground is a ideal ground plane that means the conductivity of this is infinite so if I put a charge over this let us say this charge is positive then you will have the electric field which will end on the conducting surface perpendicular to it because the tangential component of the field has to go to zero so you will have the fields which will go like that so this field when they reach to the conducting surface they will be normal to the conducting surface and that is equivalent to saying that I have a negative charge which is located exactly at a same distance from here on the other side of the surface that also is going to give me the field distribution which will be identical to this field distribution.

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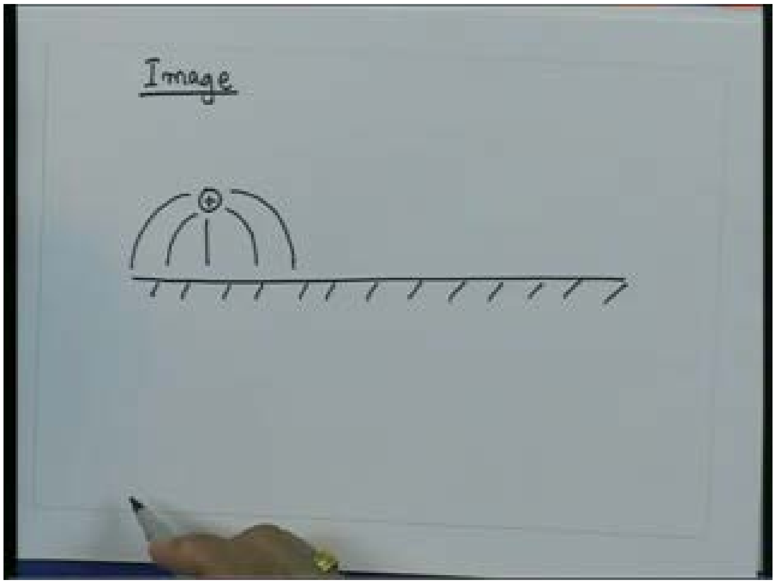

So the ground plane can be replaced by an image charge so this thing is essentially equivalent to putting a negative charge here and then replacing the ground plane, or we can say conversely if you have the positive and the negative charges we can say this is equivalent to having a charge and a ground plane, precisely that is the fact we make use of when we investigate the characteristics of a monopole antenna.

Now instead of having a charge if I have a current then let us say there is a current which is flowing like that so there is a positive charge here which is moving this way to get a negative charge we get a image of each of this charges on the other side so you get here this will be negative, this will be positive. So you will get a current which will be flowing in this direction but if I take a charge let us say this is positive here and negative here the current flows like that then the image of this positive charge will be negative and the image of this negative charge will be positive so this is positive, this is negative so the current flows in the same direction.

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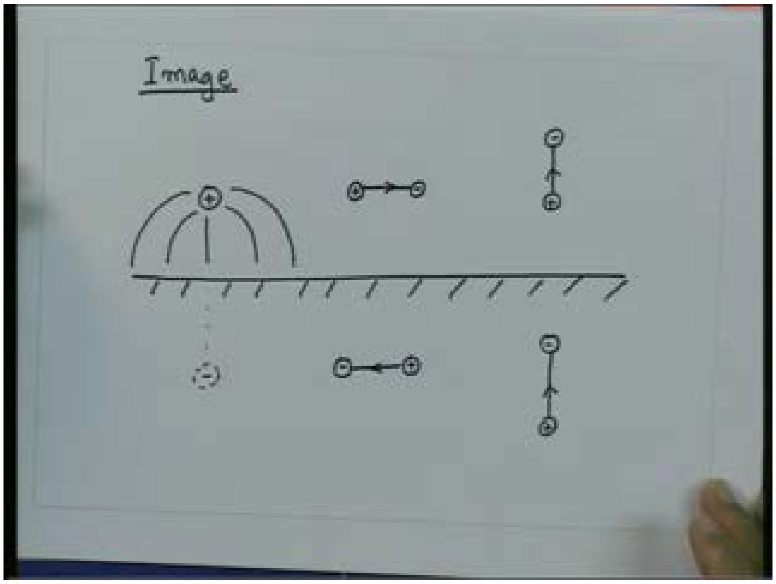

So what we see from here is if you have a charge then the image has opposite polarity if you have a current which is parallel to the conducting surface in this case horizontal then the current will have a direction which is opposite to the original element this image will be opposite compared to this, however, if the current is vertical then the image has the direction same as the original current there is no 180 degree phase shift this current and this current. So while investigating the antennas which have the current flows essentially depending upon the direction of the current of the orientation of the antenna you have to take appropriately the images and then we can replace the ground plane by its appropriate image.

So essentially the problem is to have a charge or current located over the ground plane and replace the ground plane by their corresponding images or conversely if you are having the currents which are like that and that is equivalent to saying that we have only this current and there is a ground plane we have seen the current distribution on the dipole antenna and that is the current is sinusoidal distribution over the dipole so it is something like which is symmetric these two lines are equal and we also have seen that the current which flows in the dipole is in the same direction since the current here is like that here it will be like this in this half it will belike that this will be like this.

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So we have a situation which is similar to this situation that you are having currents which are flowing in the same direction that means this dipole is now equivalent to a ground plane and a half of this dipole mounted on this if I take this and put this antenna like this, here I am exciting with a voltage between the two terminals of the antenna here the voltage will be connected between the ground and the terminal of the antenna the current distribution on this antenna will be exactly identical to this current distribution so this will be again like that this structure now we call as the monopole antenna so we have half of the dipole so we call this monopole and this monopole is mounted on a ground surface so its behavior is identical to the full wave the dipole antenna. So here we have the antenna which is monopole but its radiation characteristics are identical to the dipole antenna except few differences which will mention for any short while from now. So this antenna is dipole and equivalent of that this antenna which is the monopole antenna.

Firstly we should note the differences between these two antennas is that the radiation pattern which you are going to get for this monopole is identical to that of dipole because this is equivalent to saying that I have another current element which is located here with the same current distribution so this lower half so the radiation characteristic if I found out like radiation pattern that is exactly identical to your dipole antenna.

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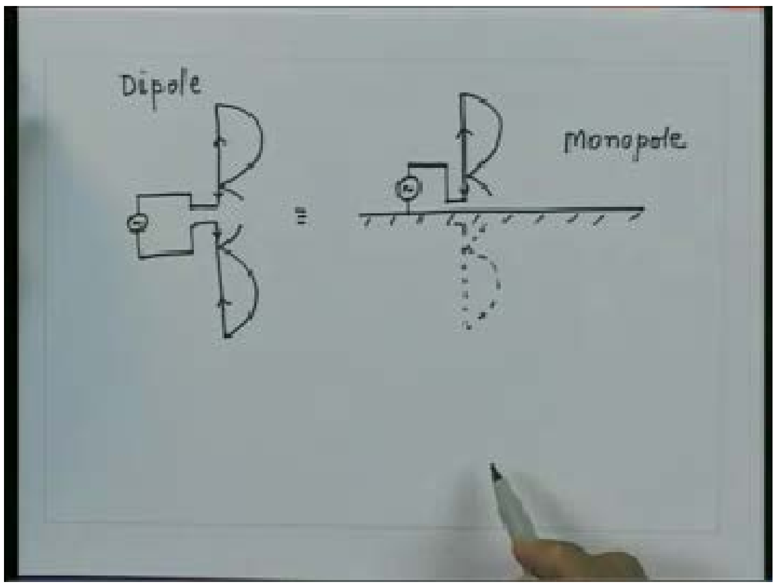

So the point which we have is it has identical radiation patterns except now that the radiation is not going below this surface because we have a conductor in this and as we saw electromagnetic wave cannot propagate inside the conductor. So in the lower half in this the energy does not propagate. So I have a radiation pattern which is identical to this only in the upper half of the hemisphere that means the dipole has the radiation which is in the infinite space where as the monopole has radiation in a semi infinite space. So the radiation patterns are identical but radiation is in semi-infinite space and since the radiation is only in the half of the space the power radiated for the same current on this antenna is half so the monopole has half the power radiated compared to dipole. So monopole has half power that of a dipole for the same current excited in the antenna.

As said differently, since the power radiated is proportional to the radiation resistance for given current the monopole has a terminal impedance or radiation resistance which is half of that of the dipole antenna.

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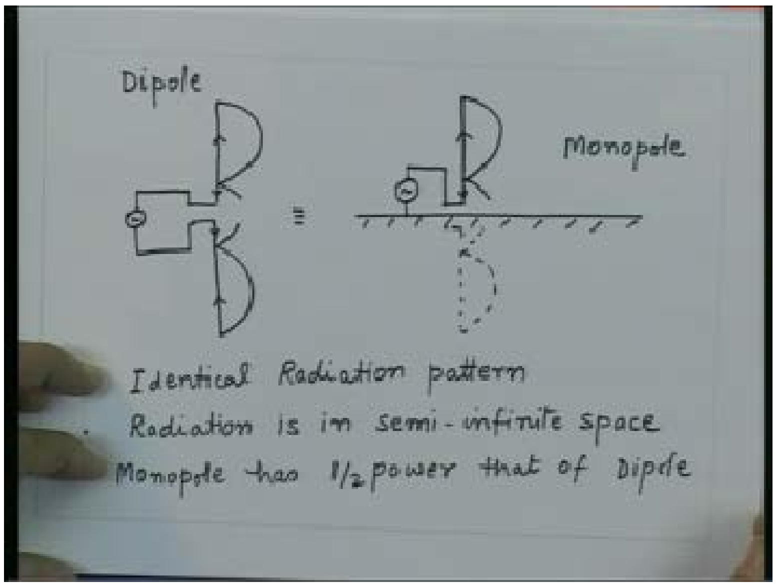

So either we can say that the monopole radiates half the power which the dipole would radiate because it is radiating only in the half space or we can say that the radiation resistance of the monopole antenna is half of that of the dipole. So what that means is, for the same current if we calculate the fields which we are going to get will be identical to that of the dipole and since the dipole has the radiation pattern which is symmetric in ø over the ground plane it will be having a symmetric distribution around this monopole antenna. Precisely that is the reason this antenna is used for low frequency radio broadcasting, one is because when we go to the medium wave the wavelength is typically the order of few hundred meters so the size of the antenna is very large, secondly if we have this antenna and want to mount like a dipole then this antenna has to be mounted at a significant height so that the ground effect does not play a significant role the radiation pattern does not get significantly affected.

On the contrary if I have a monopole then I can really mount that antenna on the ground which is vertical and then it will have a radiation pattern which is symmetric on the ground. So as we have seen for the dipole if I take the typical radiation pattern, this is my ground plane I have this monopole antenna which is equivalent to this dipole so if the length of this dipole is less than λ we have seen that there is only one maximum for the radiation pattern so the radiation pattern typically would look like that and the electric field which is generated is θ oriented for this dipole, on the ground if we see the electric field will be like that that is where the electric field is it has only the θ component and it is symmetric in ø direction and as we go to higher and higher heights the angle increases and the electric field amplitude dies down. So this antenna essentially gives you the maximum radiation anywhere on the ground surface as you go away or higher than over the ground surface then the amplitude of the electric field decreases that means this antenna is most suited to have radiation on the ground surface, precisely that is what the application in broadcasting that the purpose in broadcasting is to send the radio signals over the houses on the ground typically.

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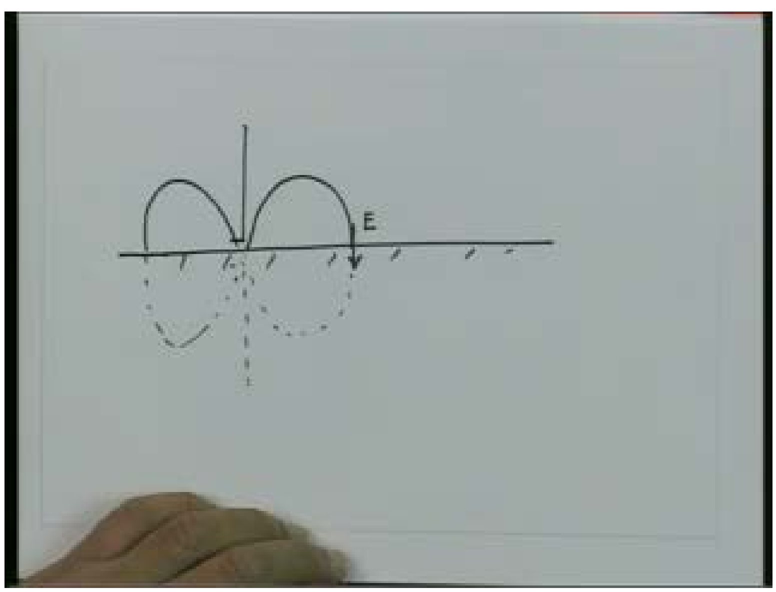

So we want that we should have a good distribution or good electric field on the ground surface also since the idea is to illuminate as large is the area possible on the ground you would like to have a uniform distribution on the ground, both these things essentially are satisfied by this monopole antenna that is the reason this antenna is a very popular antenna for medium wave broadcasting because it does not have any directional characteristic on the surface of the earth and as you go higher essentially the field dies down so as you go up above the ground surface we do not going to have any reception because most of the reception we want to have on the houses on the ground. So this radiation pattern quite suits the requirement of the medium wave broadcasting. So this monopole antenna is very much suited for medium wave broadcasting.

If you go to the radio station which have low frequency or medium wave radio station you will see a tower which is standing in front of the radio station in fact the tower itself is an antenna and this antenna is the monopole antenna. So in practice monopole antenna is a very commonly used antenna and its radiation characteristics are identical to that of a dipole principally. So if you understand the dipole then the understanding of the monopole antenna is very straight forward.

However, as you go to high frequency the antenna size will become small and then it is possible to use effectively the dipole antenna so as we go to the frequencies where the wavelength now becomes few meters like TV signals then the dipole antenna is more commonly used antenna, also the length of the dipole is typically taken as $\lambda/2$ and the things will become clear in a minute while we take that but a commonly used dipole approximately has a length of $\lambda/2$. So we have a $\lambda/2$-Dipole so as the name suggest we have the dipole whose total length is $\lambda/2$ so this is $\lambda/4$, this is $\lambda/4$ and the current is sinusoidal on that so the current is zero here so this is a quarter cycle which will get on that you will get a current distribution like that.

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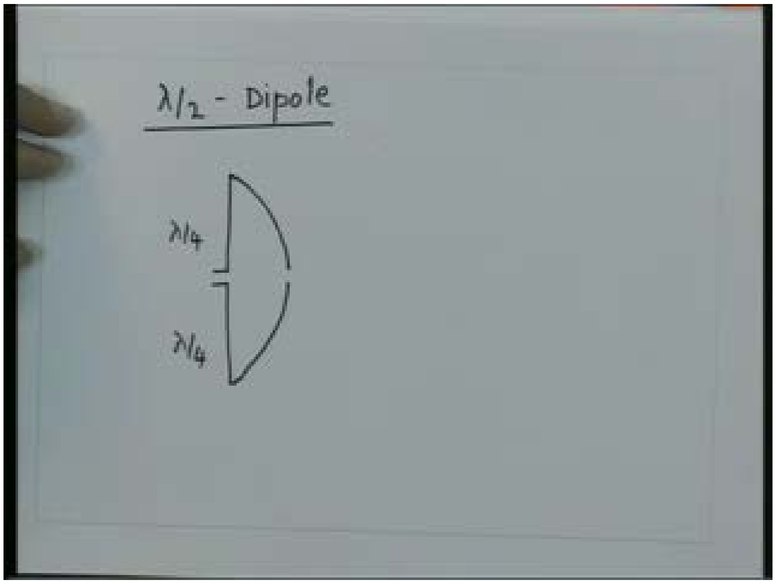

So this current will flow like, this the current will flow like this so you will get an input impedance here which would correspond to the maximum current on this dipole. We can write down the electric field for this from the general expression so as we know the radiation pattern for this dipole is given as E_θ which is j 60 I_m into e to the power -j β r upon r times the radiation pattern which is F(θ) where θ is the angle measured in this direction so this angle is θ and F(θ) which is the radiation pattern of the antenna will be equal to [cos(β H cosθ) - cos(β H)] divided by sinθ and in this case H is λ/4 so we have βH that is 2π/λ into λ/4 that is again equal to π/2. So we have this quantity βH π/2 so this cos(π/2) will be zero so the radiation pattern for this half wave dipole F(θ) will be cos(π/2) cosθ divided by sinθ. So substituting for F(θ) in this expression essentially we get the electric field for the half wave dipole.

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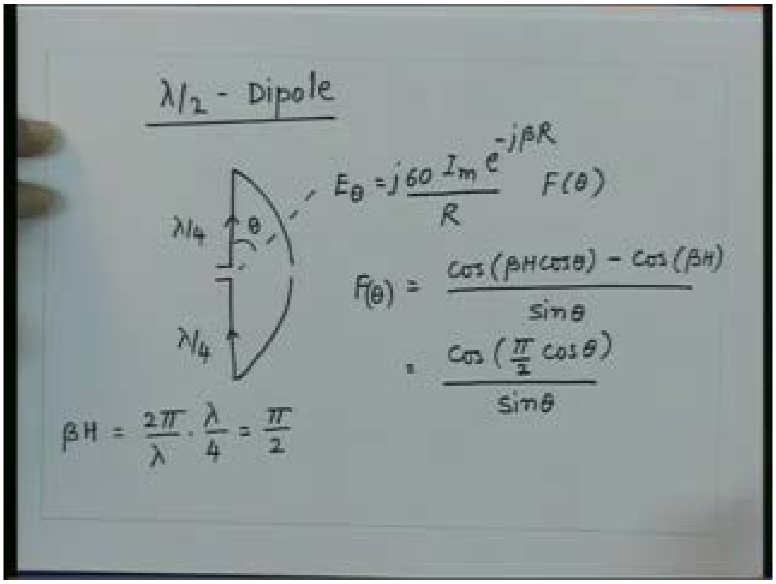

Now as we have seen it has only the nulls which are in the direction along the axis of the antenna it is also maximum in $\theta = \pi/2$ this quantity is zero so this is one so you get the maximum field which corresponds to $\theta = \pi/2$ so its radiation pattern we had seen looks like similar to the Hertz Dipole it has only maximum perpendicular to the axis of the dipole and the field is zero in the axis of the dipole.

Now knowing the electric field then we can ask how much power is going to get radiated by this half wave dipole or effectively what will be the radiation resistance of this half wave dipole. So now we can calculate the power radiated by this dipole which you can get by finding out the Poynting Vector which will be mod E square by η so I can get for here the Poynting Vector let us say P_r that is half mod E_θ square upon η where η is the intrinsic impedance of the medium which is 120π and the electric field is given by this.

From here then we can calculate the power radiated so this is W that will be integrated over θ equal to zero to π, ø is equal to zero to 2π, the power density which is this so half mod E_θ square upon η incremental surface area which is r square $\sin\theta \, d\theta \, dø$.

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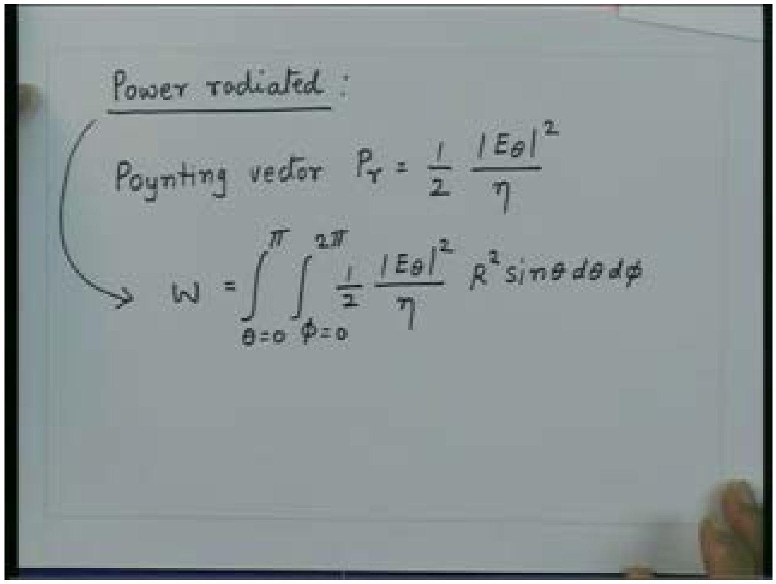

There is a ø symmetry in the problem so integral with respect to ø is simply 2π and all other constants which are here if I combine them all together essentially we get the power radiated will be thirty into I_m square zero to π, the integral for θ is cos square π /2 cosθ upon sinθ, we have here this quantity sinθ so square of that will become sine square θ, once sine square theta sine square θ so you will get sinθ here dθ. here.

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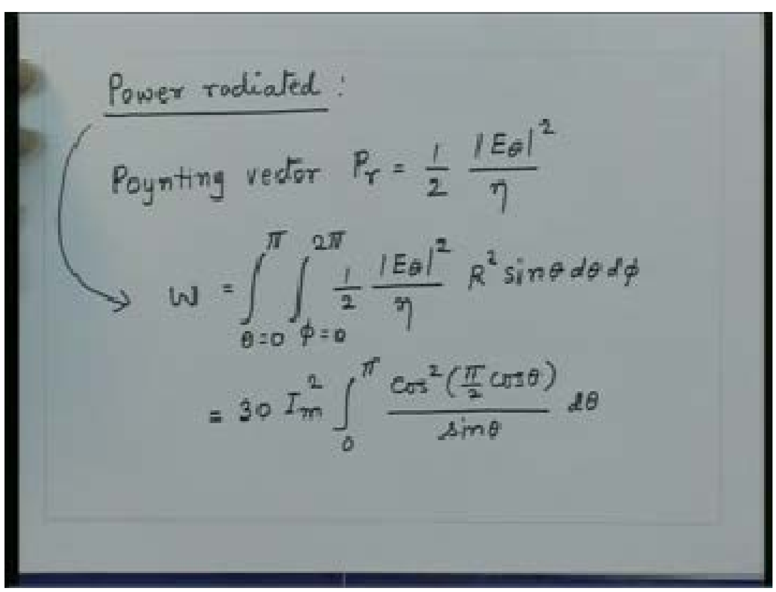

Note here the I_m is the maximum current which you see on the dipole and in this case since the length of the dipole is λ/4 this is the current which is I_m so this value of current is essentially I_m. So in general dipole the maximum current need not be at the internal terminals but if you take a dipole which is λ/2 long then the input current is the maximum current which you can see in the dipole.

Now this integral cannot be solved analytically so this integral is generally solved by numerical means and the value of this integral is 1.218 so this is equal to 30 I_m square into this integral value which is 1.218 so the power radiated by the half wave dipole is approximately 36.54 into I_m square.

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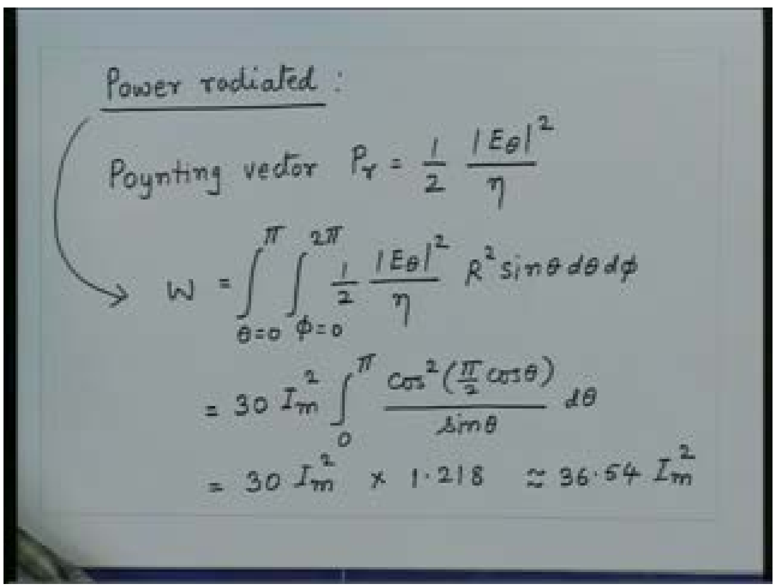

Now as we did in the case of Hertz Dipole essentially we have this power radiated we can say this is the power which is equivalently go on into a resistance called the radiation resistance so we can say that this power is equal to W which is also equal to ½ I_m square which is the input current into the radiation resistance.

So I can equate these two powers this is the power which we have got by integrating the power over the closed surface this is the total power radiated this is the power we have from the equivalent electrical circuit so this thing should be equal to 36.54 I_m square. So from here we get the radiation resistance of this antenna which is approximately 73.1ohms.

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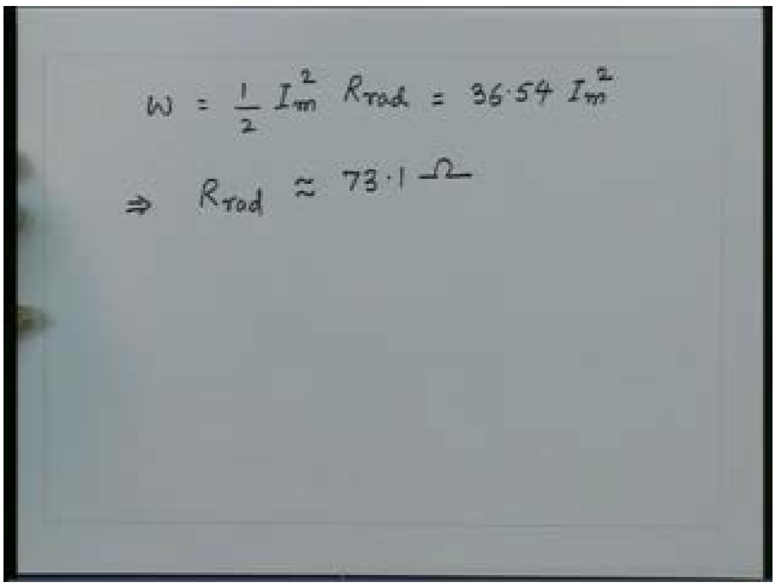

Now generally the dipole has some reactive fields and these reactive fields give the small reactance at the input terminal so the resistive part of the impedance which is measured between the terminals of the antenna is about 73 ohms, where as the reactive part has a small value and the reactive part essentially is tuned out by shortening the antenna by a small amount. So in practice we do not use the antenna which is exactly $\lambda/2$ but we use an antenna whose total length is about 0.47λ little shorter than $0.5\ \lambda$ so that the reactive part of the antenna is cancelled out and then you will see impedance at the antenna terminal which is approximately about 73 ohms.

So, a practical dipole antenna has a length of 0.47λ and this gives you the Z_{in} which is almost real and that is equal to the radiation resistance which is about 73 ohms.

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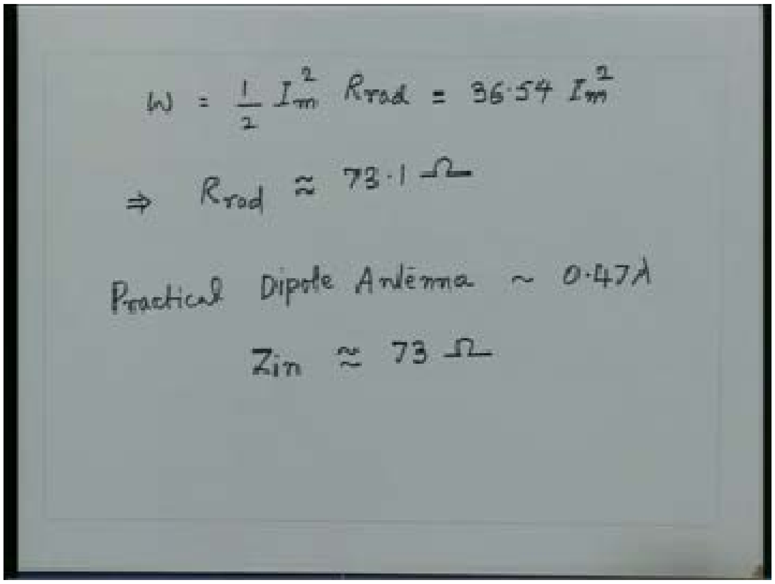

So a dipole antenna has intrinsically impedance which is not close to fifty ohms but its value is about 73 ohms, that is the reason the cable which were used initially for the dipole antennas were all standardized to 75 ohms.

So you will see in practice there are cables which are 75 ohm cables, there are cables which are 50 ohm cables. Most of the electronic equipments are standardized for the fifty ohm characteristic impedance whereas when we go to the antenna especially the dipole antenna the impedance is close to seventy five and that cable will be 75 ohm cable. So normally the antenna connections are made with seventy five ohm cables whereas the general electronic equipment will be standardized to fifty ohms.

Having what is this total radiated power then one can calculate the half power beam width of the antenna. We are interested in finding out all the parameters of this antenna which are used in practice. So now we are interested in finding out what is the half power beam width of this antenna, what will be the directivity of this antenna, what will be effective aperture of this antenna.

So the half power beam width calculation is very straight forward you take this radiation pattern and find out the points where the field would go $1/\sqrt{2}$ of its maximum value and in this case the maximum value would be 1 so you get the maximum value for this radiation pattern which is at $\theta = \pi/2$ that gives you the maximum value as 1.

So if I go to an angle where the field would reduce to $1/\sqrt{2}$ that direction will be the half power direction. So I have this half wave dipole for which the radiation pattern will look like that this is the direction which is $\theta = \pi/2$ and the electric field in this direction is unity so this point is 1, if I go to an angle where the amplitude of the electric field reduces to $1/\sqrt{2}$ and $1/\sqrt{2}$ here, this is the width which is the half power beam width of the antenna in the E plane θ half power.

So dipole antenna is symmetric radiation pattern in ø plane so there is no half power beam width defined in the ø plane, this antenna has a characteristic beam width which is only in θ direction which is this θ half power. So we can find this out essentially by equating $F(\theta)$ to $1/\sqrt{2}$. So if I take this $\cos(\pi/2 \cos\theta)$ divided by $\sin\theta$ is equal to $1/\sqrt{2}$.

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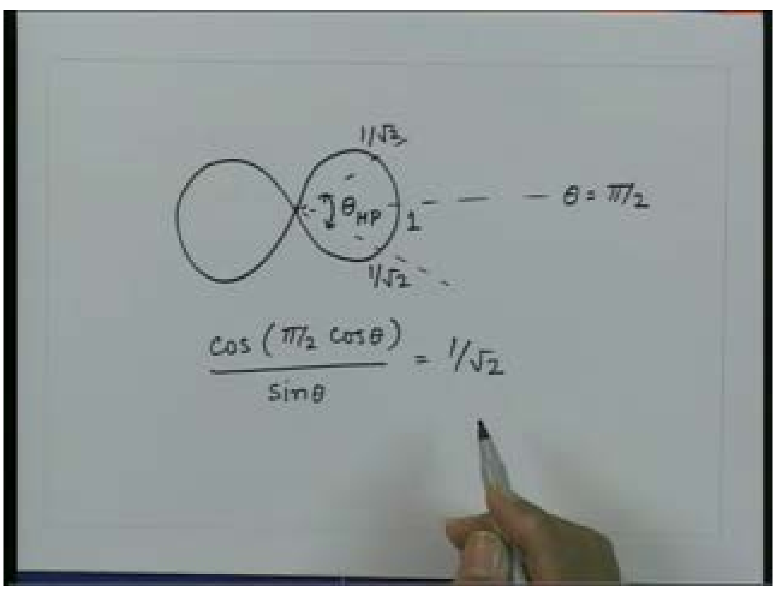

From here we can calculate these two directions where the field lies to 1/√2 value and then the difference between these two angles is the half power beam width so we can solve this to get the theta half power that is approximately 78 degrees. Remember the radiation pattern for the Hertz Dipole was only sinθ that means the half power points for simple sinθ would be 45 degrees so this angle will be forty five degrees this angle will be forty five degrees. So the half power beam width for the Hertz Dipole was ninety degrees.

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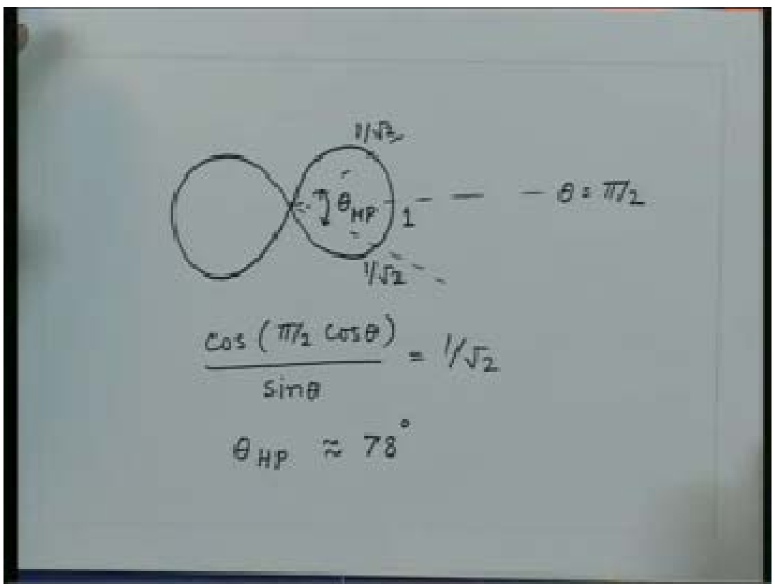

When we go from Hertz Dipole to this half wavelength dipole the angle essentially reduces from ninety degrees to seventy eight degrees that means the antenna becomes more directional so qualitatively if I look at the radiation pattern the radiation pattern of a half wave dipole and a Hertz Dipole are identical. However, if I see quantitatively the two are not same because this antenna has narrower half power beam width and consequently this antenna is more directive compared to the Hertz Dipole. And same thing essentially we will see that if I change the length from half wavelength to full wavelength as you have seen the radiation pattern remains same but the beam width becomes still smaller and antenna will become more directive.

Coming back to the half wavelength dipole, once I know this half power beam width then I can use approximate relation to find out directivity or we can go to the definition of directivity and from there you can calculate the directivity of the half wave dipole. So we will get directivity for this antenna D that is equal to 4π divided by θ equal to zero to π and ø equal to zero to 2π and square of this radiation pattern E_θ square so this integral is exactly same as what we have got here this quantity so you have here cos square (π/2 cosθ) upon sinθ dθ.

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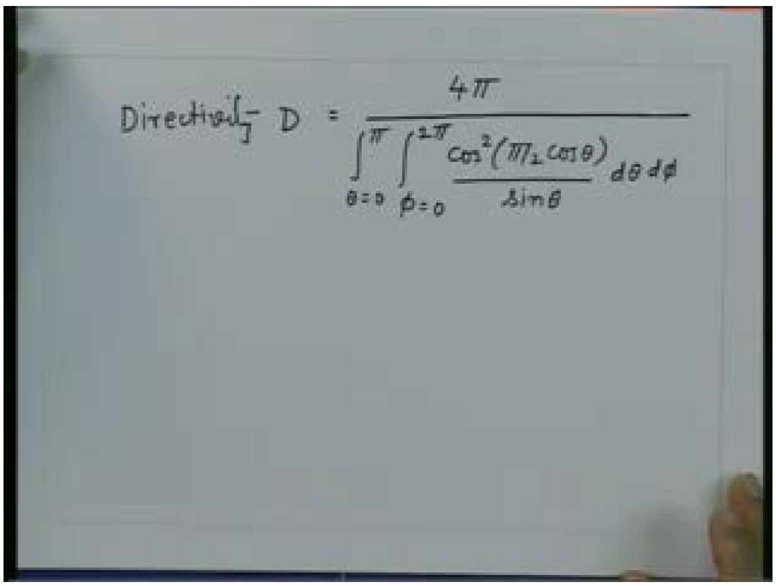

As we know the value of this integral over θ is 1.218 so this directivity is equal to 4π upon the integral over ø which is 2π and integral over θ is 1.218 so the directivity of the half wave dipole is about 1.64, as we mentioned earlier the directivity is given in terms of dB's so we can take 10 log of this quantity so it will give you the directivity in dB's so we get D in dB's that is 10 log 1.64 and that will be approximately equal to 2.15 dB.

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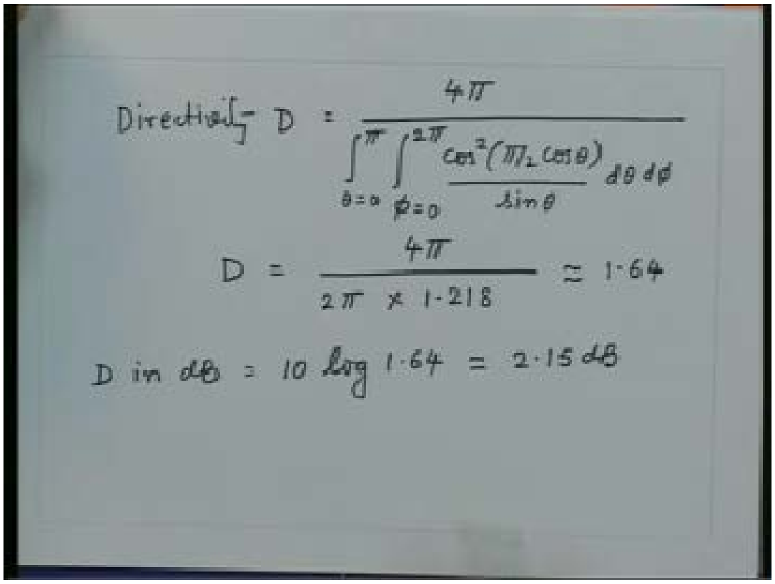

So the half wavelength dipole has a directivity of 2.15 dB that means in the direction perpendicular to the half wavelength dipole it will have an electric field or power density which will be 2.15 dB higher compared to a isotropic antenna. Once you know the directivity we can calculate the effective aperture for this antenna that means if this antenna half wave dipole is used for reception then how much power it will accept that we get from effective aperture A_e and that will be equal to λ square D upon 4π, this relation we derived in the last lecture so that is equal to 1.125 into ten to the power minus three into λ square.

So now we know the directivity we can get the effective aperture so if the antenna was used for the receiving antenna that is the effective area which the half wave dipole dispose to the incoming radiation. So now we see all the characteristic of this half wave dipole which is very commonly used it has a radiation resistance of about 73 ohms it has a directivity of about 1.6 that is in terms of dB which is about 2.15 dB and it has a effective aperture which is approximately this.

Now having understood this characteristics of this practical antennas and also in general the dipole antennas now we can pose a very general question and that is when we started investigating this half wave dipoles we said we have a current distribution which is sinusoidal which was known priory and we had said at that time finding the current distribution is the most difficult part once we know the current distribution then getting this radiation characteristic is rather straight forward problem and that time we said since the antenna can be visualized as a flared up version of Transmission Line the current distribution is sinusoidal and then we got the radiation pattern for the sinusoidally varying current distribution.

Now we can ask if the current distribution is not sinusoidal but if it is some general current distribution then what is the relationship between the radiation pattern and the current distribution, we are posing a general problem that if the current distribution in general is known which need not be sinusoidal then what is the relationship of the radiation pattern with the current distribution. So let us say we are having the current distribution which is linear like that and let us say the current is going perpendicular to the plane of the paper but its amplitude is varying and its phase is also varying along the length so let us say we have this direction as some x direction and let us say I have this direction from which I measure the angles let us say this angle is θ this θ is really not the spherical coordinate θ this is measured from a direction which is perpendicular to this line over which the current is distributed. So imagine there is a current sheet which is there whose current is varying the current flow is perpendicular to the plane of the paper but its amplitude will vary arbitrarily something like that.

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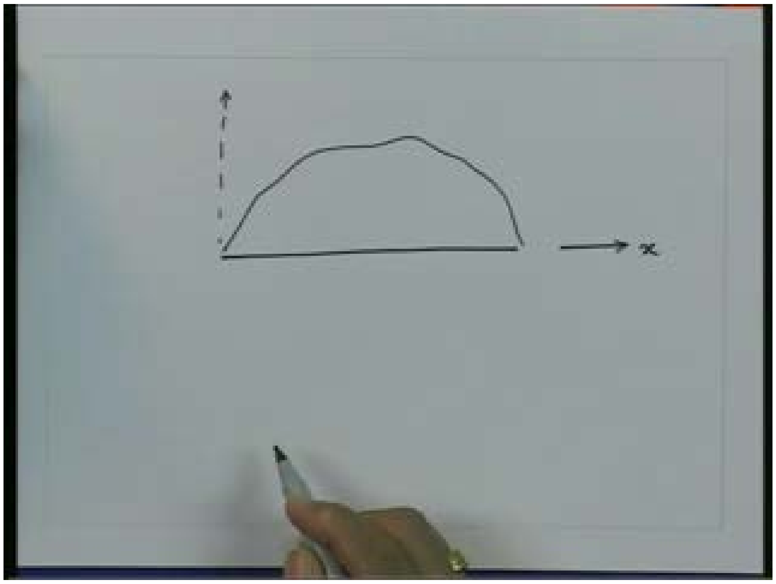

So we can now define this current by a linear current density so let us say I have linear current density and which is spread at over a length along x direction from 0 to l so length of this current sheet is l and if you are measuring the angles with respect to a direction which is perpendicular to this plane of the current sheet. So let us first say the current is flowing in this which is perpendicular to this that is given by J(x) that is some magnitude J(x) and e to the j ø(x) where this ø is the electrical angle which the current has and this is the magnitude of the current and it is zero otherwise so this is if x is between zero and l and this current is zero otherwise.

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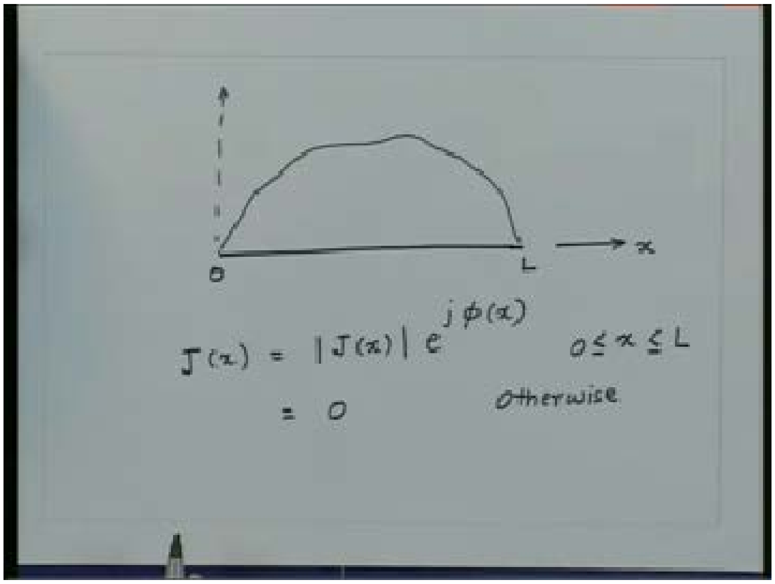

We can think of this current as small small current elements so I discretise this current distribution in the small elements and let us say I have a small current element here which is at a distance x so the current which is flowing into this is the linear current density which is the J(x) times dx if the width of this current element is dx. So the current which is flowing in this direction is J(x) dx so that is the current in this which is J(x) amplitude now J(x) into dx.

Since the currents are flowing like that the electric field which will be produced by this current in the plane of the paper will also be in the θ plane so that will be in this direction so in this plane if I go the electric field direction is like that that means if I take the field generated by this radiation field and if I take the field generated by this element somewhere here both the electric fields are going to be in same direction. So I can find out the total field by simply adding the field because of this current element this current element and so on for the entire length. So I can say that if I take the distance r which is very very far away from this current element and let us say this distance is given by d and let us say I measure all my distances from a difference point in this direction which is given by r the radiation originated by this current element will travel a distance shorter compared to this by this one which is this.

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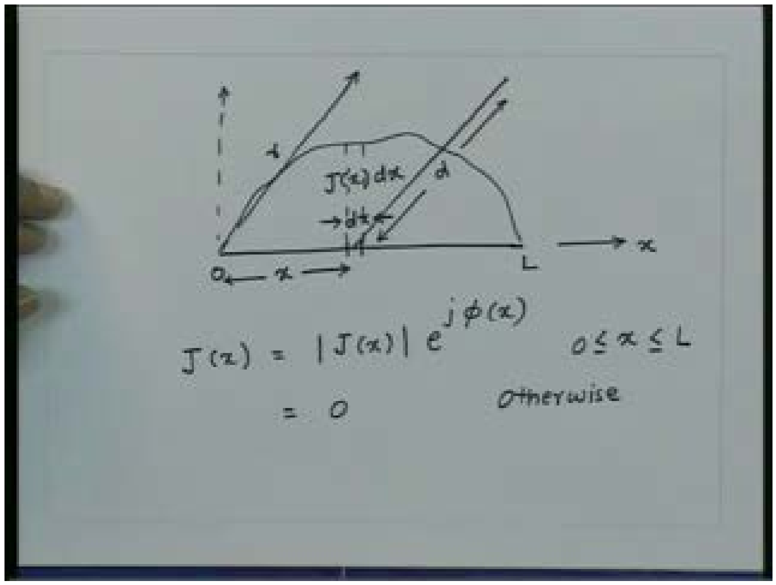

So if this angle is θ then this angle is ninety minus theta or let us say we define the angle with respect to the axis of the current element so let us say this angle is theta let us say this angle is 90 – θ. So let us define the angle not from this direction but let us say if I define the angle with respect to the axis of the current then this axis length which this radiation would be traveling compared to this one would correspond to a length of x of cosθ that means the radiation which is originating from this point will be leading with respect to the radiation originating from this point correspond to a distance of x cosθ.

Then I can write the current the electric field which is because of this small current element dE that is equal to j into η j J(x) dx that is the current e to the power -j β d divided by 4π d plus some constant which will take care of all the term which are coming like β and so on but in this constant we are not worried too much because we are interested in finding out only the radiation pattern so these all quantities η k all this will essentially get absorbed into the amplitude variation in absolute term but the relative variation of the electric field will not get affected by this constant. So in fact even this quantity 4π even all that can be kept into this constant.

Now this distance d is r - x cosθ so I can substitute for here d = r - x cosθ and as we have done approximation when we analyze the dipole this distance d can be replaced by r whereas in this case you have to actually substitute r - x cosθ so we will get the dE for a very far away point is all this quantity let me put into constant now 4π η and x and so on so let us say some constant K_1 J(x) dx e to the power -j β r into e to the power j β x cosθ divided by r, we are replacing d by r in this expression so I get electric field because of this small current element which is located here that essentially is given by this.

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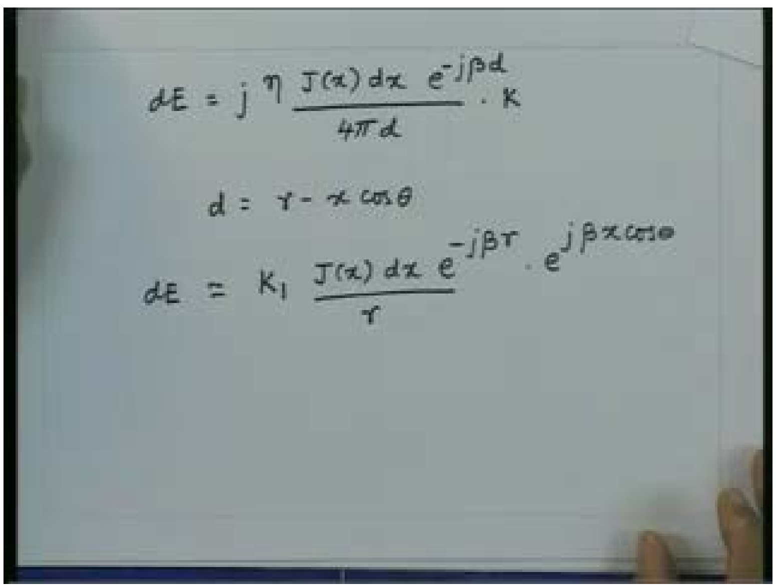

So the total electric field is nothing but integral over the length of this current distribution so I get the total electric field E that is integral zero to l this quantity K_1 e to the power -j β r upon r that is constant for given distance J(x) dx e to the power jβ x cosθ.

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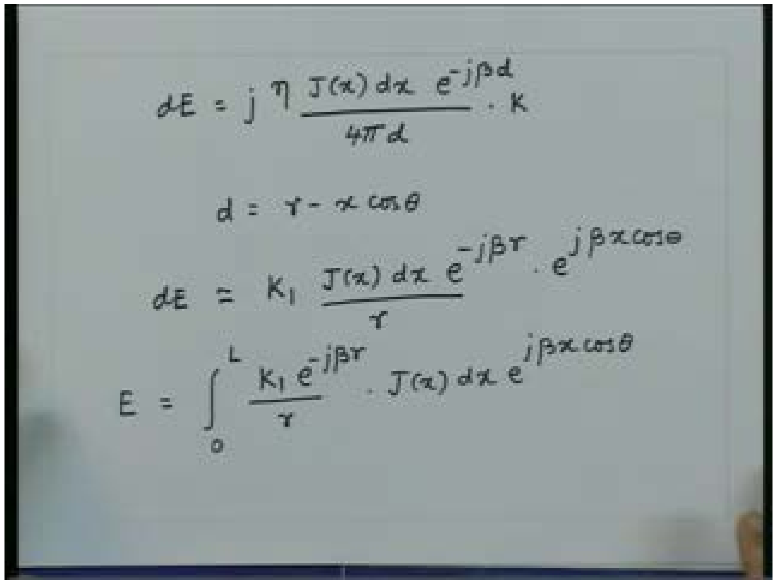

For a given distance let me put this quantity as some constant again let us call this constant some K_0 so I can write the electric field for this distribution E which will be some constant K_0 integral 0 to l J(x) into e to the power j β which is 2π upon λ I can put x upon λ into cosθ. What we can do now is we multiplied by dx. Since the current distribution is over this length from 0 to l even if I change the integration limits from -∞ to +∞ the integral contribution will only come from 0 to l so that beyond that point this j is zero so without affecting the value of the integral this thing can also be written as from from -∞ to +∞ J(x) e to the power j 2π x by λ cosθ into dx.

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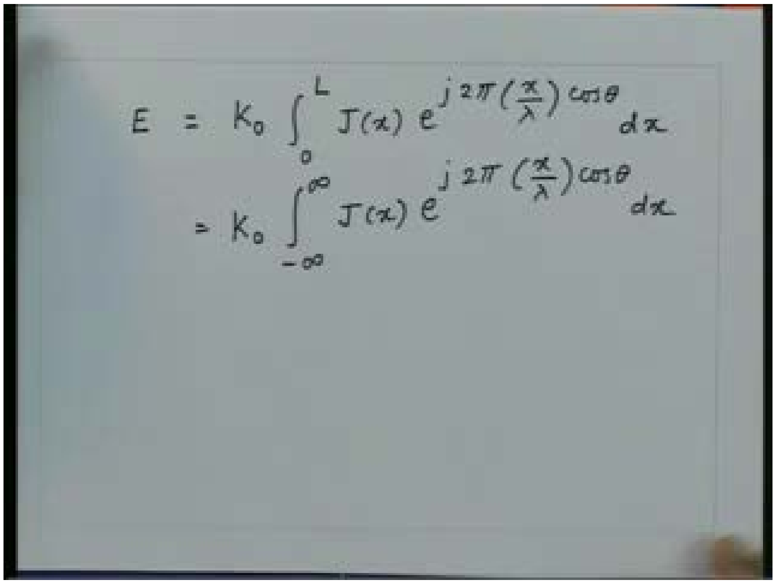

Now what I can do is I can define the length like a normalized length with respect to λ and if I do that and define the independent variable not theta but the cosθ which is the direction cosine so let us say I define variable l which is equal to cosθ and we have a normalized length x prime which is x upon λ. If I do that essentially now the E which will be a function of l direction cosine will be equal to some constant K_0 λ integral -∞ to +∞ J of x prime e to the power j 2π x prime l dx prime.

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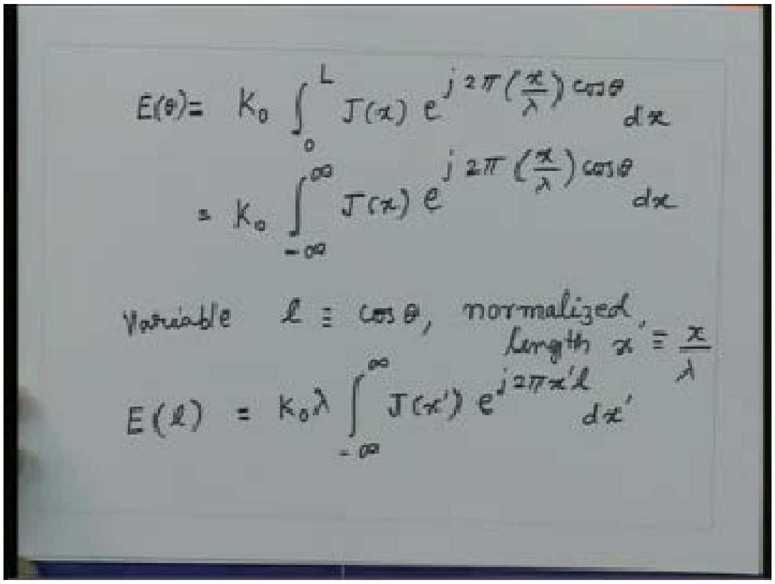

This integral if you look at is a very familiar integral and that is the Fourier integral so this integral is a Fourier transform.

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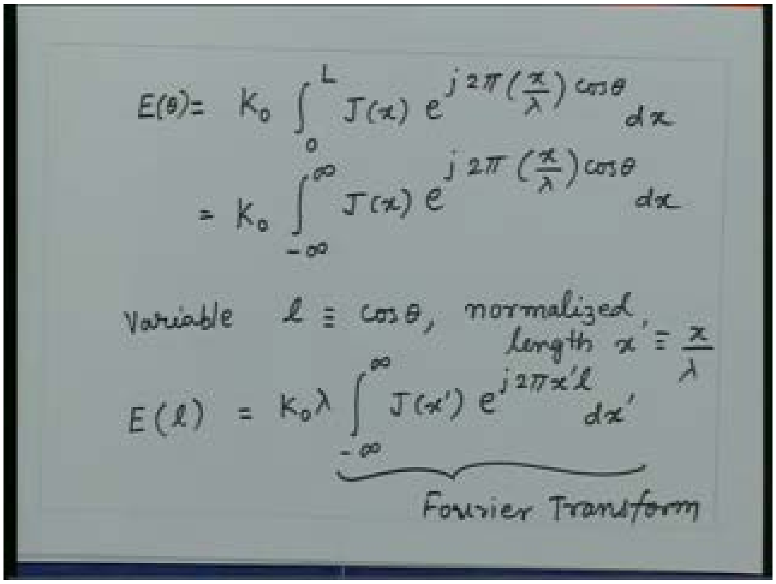

So we have a very important relationship that is the radiation pattern of a current distribution is the Fourier transform of the current on the antenna structure and this property is a very interesting property because once we establish this Fourier transform relationship then we can make use of the Fourier transform relationship which are standard to visualize the radiation characteristics of an antenna. So this relationship that the radiation pattern is a Fourier transform of the current distribution is the important relationship and having understood this relationship then we can visualize the radiation patterns of various antennas and precisely that is what we will do when we meet in the next lecture we will take some specific antenna current distributions and then by using the Fourier transform properties we will quickly try to visualize what kind of radiation pattern will be created by this antennas. So, instead of every time taking the current distribution and integrating it now we can make use of the Fourier transform properties and from there we can visualize the radiation patterns of variety of antennas.