Transmission Lines and E.M. Waves
Prof R.K. Shevgaonkar
Department of Electrical Engineering
Indian Institute of Technology Bombay

Lecture-5

Welcome, in the last lecture we introduce the concept of Loss-Less Transmission Line, we say if the resistance and the conductance per unit length of the Transmission Line is zero then there are only reactive elements in the Transmission Line so there is no loss of power because there is no ohmic element in the Transmission Line. So in an ideal situation the line is lossless if R = 0, G = 0

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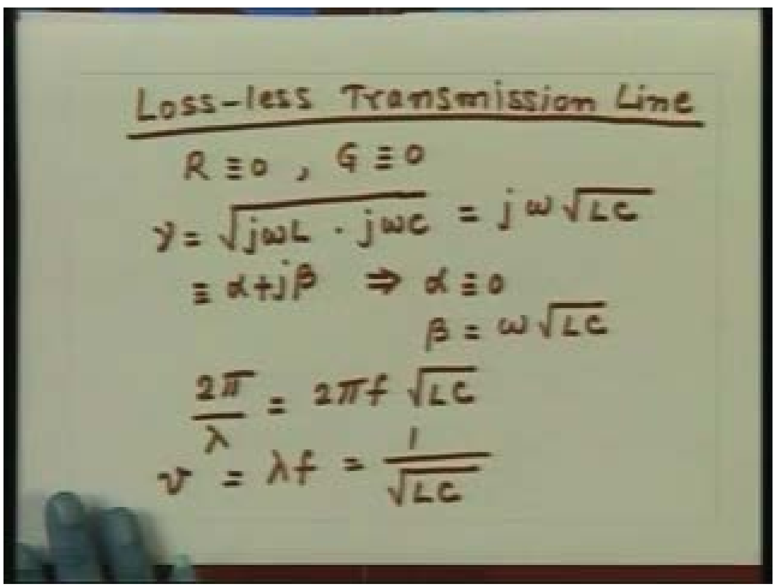

Then we introduce the concept of the Low-Loss Transmission Line which is more practical line where the resistive component R is much more less than ωL and G is much more less than ωC.

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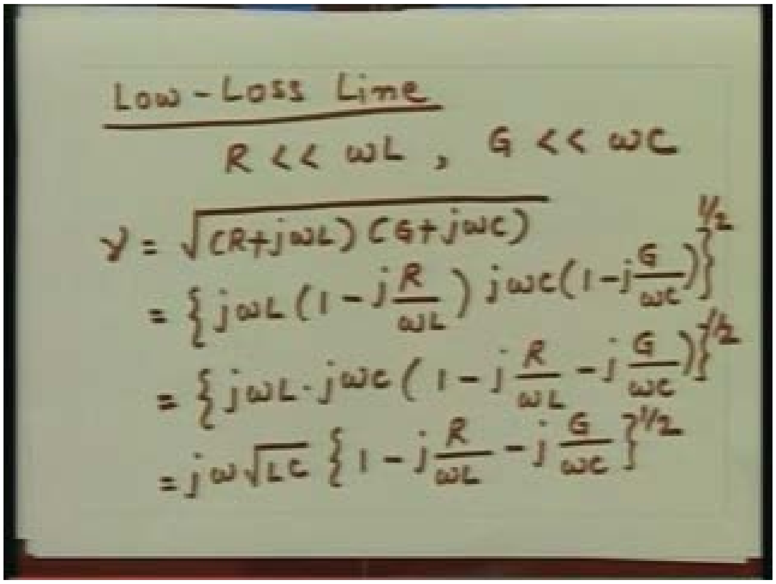

If this condition is satisfied then we said that we can create the lines still lossless. However since the loss is very small as and when we require the calculation of losses along Transmission Line we can use the value of the attenuation constant α, substituting this condition that R is much more less than ωL and G is much more less than ωC we calculated the propagation constant which can be separated into real and imaginary part and then we got the value of α and β in the approximate form.

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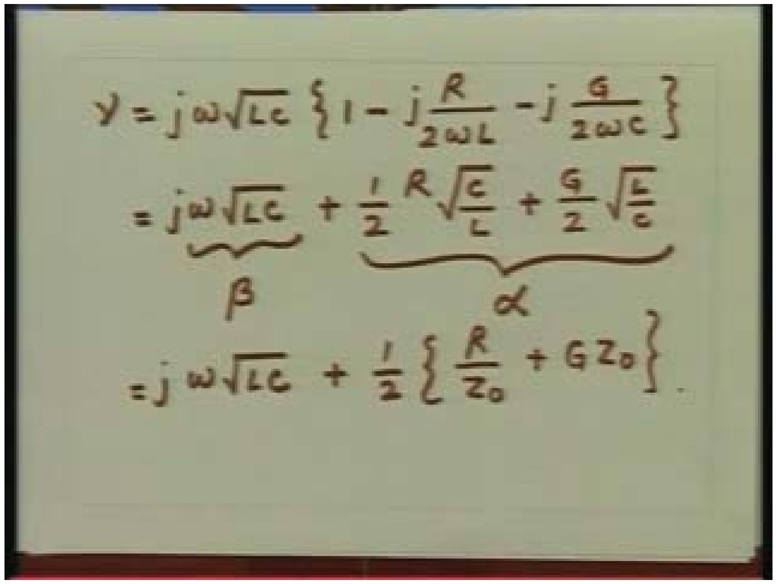

So, here this quantity was equal to β and this quantity was represented as α. However one would notice that the condition which you have defined for the low loss is now in terms of the primary constants of the line. However in the data sheet for a Transmission Line the primary constants are rarely mentioned, the parameter which I have mentioned for the Transmission Line are the effective phase constant on the line or the velocity on the cable and the attenuation constant of the line either in terms of dBs or in terms of Nepers.

Then one would like to convert this condition $R < \omega L$ and $G < \omega C$ in terms of this secondary parameters or in terms of the relationship between β and α. Since these parameters are available readily in the data sheet if I can establish a condition between these parameters for low loss nature of the line then I can find out whether a particular line is low loss at a particular frequency.

So now what we do is starting from this relationship between β and α then we can find out under what condition we can treat the line as a Low-Loss Transmission Line. Taking this value of α I can multiply this quantity here by a square of L in the numerator and square root of L in the denominator similarly I can multiply this quantity by square root of C in the numerator and square root of C in the denominator.

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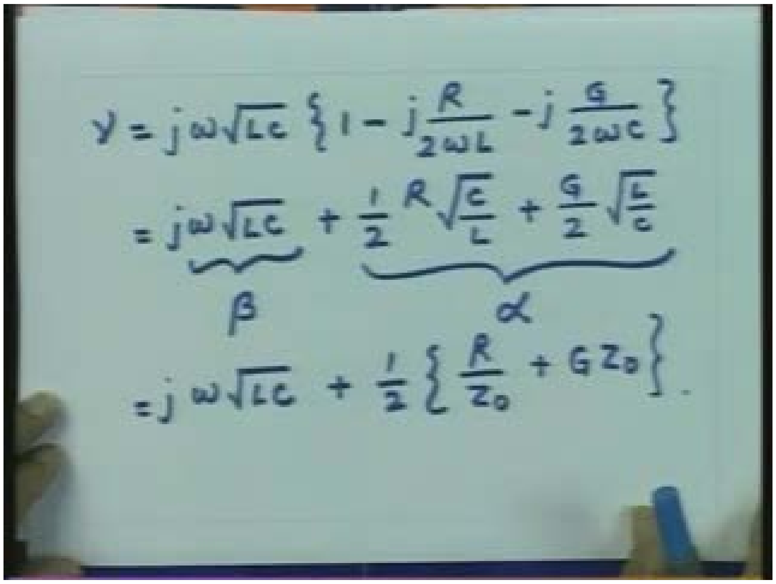

So now the α can be written in terms of this substitution so I get this value of α which is equal to $\frac{1}{2}\frac{R}{L}\sqrt{LC}$. Similarly multiplying by $\sqrt{C}$ in the numerator and the denominator in the second term we get $\frac{1}{2}\frac{G}{C}\sqrt{LC}$. Multiplying numerator and denominator by ω this can be written as $\frac{1}{2}\frac{R}{\omega L}\omega\sqrt{LC} + \frac{1}{2}\frac{G}{\omega C}\omega\sqrt{LC}$.

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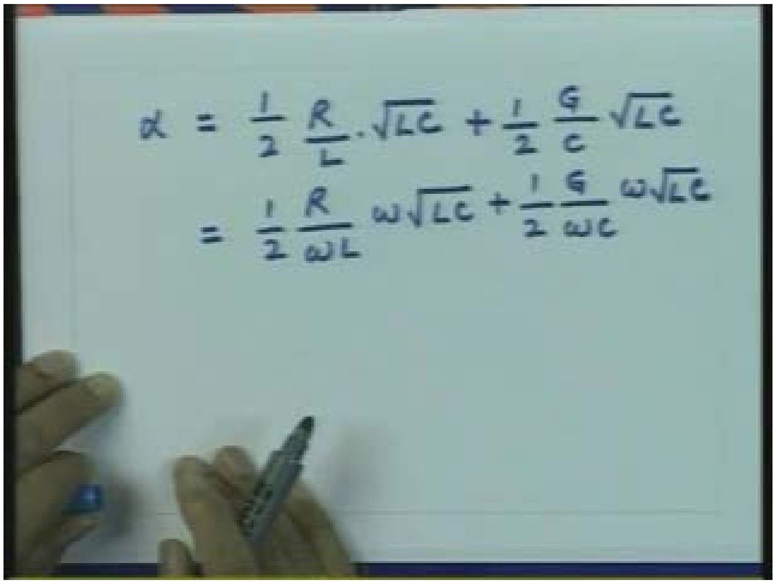

And as we have seen earlier this quantity $\omega\sqrt{LC}$ is nothing but β so this I can write as β in to $\beta\left\{\frac{1}{2}\frac{R}{\omega L}+\frac{1}{2}\frac{G}{\omega C}\right\}$.

Now from the definition of the low loss $\frac{R}{\omega L}$ is much more smaller than 1, $\frac{G}{\omega C}$ is much more smaller than 1 so this whole quantity is much more smaller than 1 so essentially what we are saying is now α is equal to β multiplied very small quantity or in other words for low loss the condition now is that α is much more less than β.

Since we know β in terms of wavelengths which is nothing but two $\frac{2\pi}{\lambda}$. Once we know the wavelength on the Transmission Line I can find out what is the value of β from the data sheet I can find out what the attenuation constant α is. If it is given in terms of dBs I will convert that in the Nepers per meter and if this condition is satisfied that the β is much larger compared to α then the line can be treated as the Low-Loss Transmission Line.

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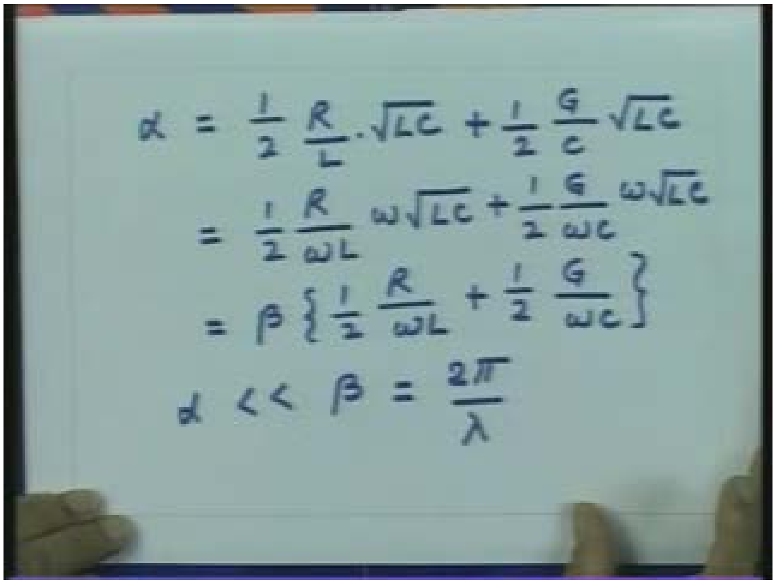

What does this physically mean we know if you travel at a distance of λ on Transmission Line the phase change is equal to 2π.

Let us say if I travel the distance for a lossy Transmission Line then the amplitude of the wave will reduce by $e^{-\alpha}$ into the distance traveled which is one wavelength. So if I consider a wave which travels a distance of one wavelength on Transmission Line then its amplitude will vary $e^{-\alpha x}$ that is equal to $e^{-\alpha\lambda}$.

If I travel a distance of one wavelength on Transmission Line substituting for λ which is $\frac{\beta}{2\pi}$ from the previous equation so λ is $\frac{2\pi}{\beta}$ I get here this is equal to $e^{\frac{-\alpha.2\pi}{\beta}}$.

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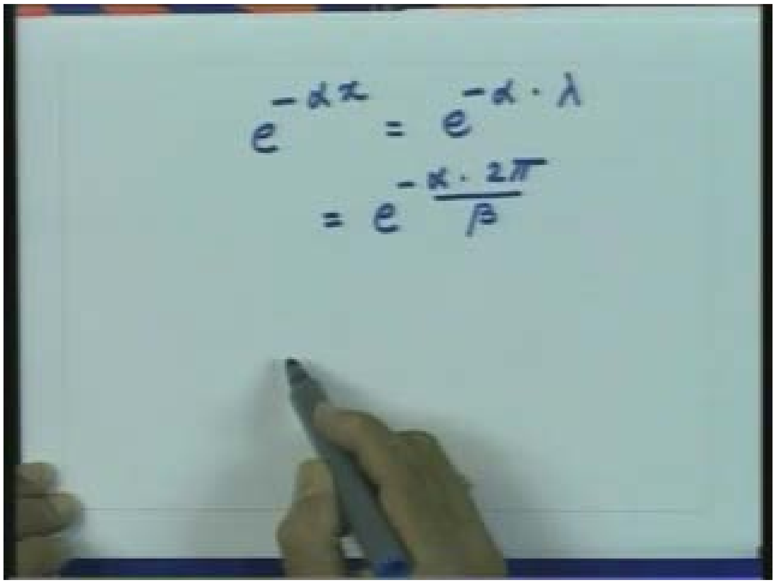

Since for low loss condition α is much smaller than β this quantity $\frac{\alpha.2\pi}{\beta}$ is much less than 1 that means the amplitude of the wave now reduces to $e^{\frac{-\alpha.2\pi}{\beta}}$ and since this quantity is very small essentially the amplitude reduction in the wave is negligibly small. So in other words what we are saying is a line can be treated a low loss transmission line provided the change in the amplitude of a traveling wave is negligibly small over one wavelength distance. Of course negligibly small is a very subjective number you can consider one percent as negligible or 0.1 percent as negligible.

Let us say as a reference value we consider one percent is a negligible quantity so when the amplitude reduces to one percent of its original value then we say that the line can be treated as a Transmission Line. Since this quantity is very small essentially when this number becomes approximately one percent that is where the amplitude will reduce by one percent so if $\frac{\alpha.2\pi}{\beta}$ is approximately $\frac{1}{100}$ the wave amplitude will reduce by one percent over a distance of one wavelength from here then I can find out what is the acceptable value of α compared to β.

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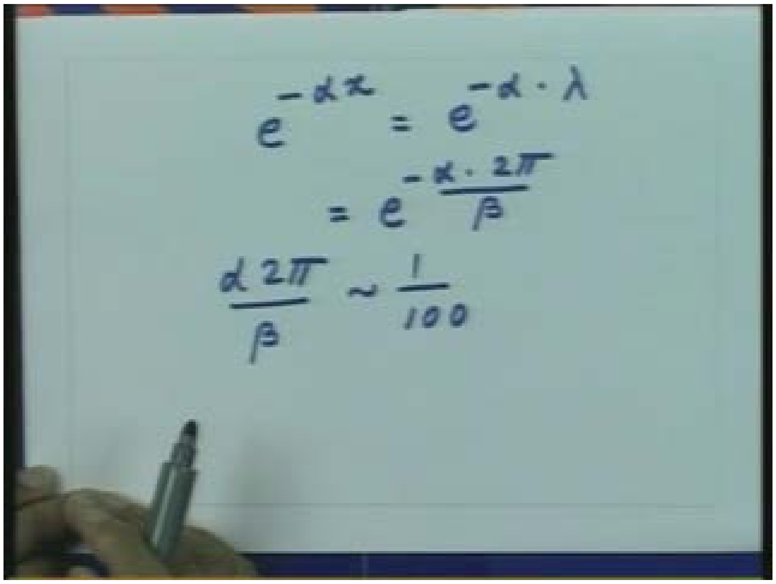

So in practice for a given line there is nothing absolute whether the line is lossy line or a low loss line for a given frequency it may be possible that α may be much smaller than β but when the frequency changes the condition may not be satisfied and the line cannot be treated as a Low Loss Transmission Line. So for a given loss on Transmission Line depending upon the frequency the line may be treated like a low loss transmission line or it may not be treated like a Low Loss Transmission Line.

Let us take a simple example to find out what are the physical parameters which we will have on Transmission Line if we just take some typical line parameters. So let us say I have a Transmission Line whose primary constants are given as L = 0.25μH/m, let us say the capacitance per unit length is 100 pico Farad per meter, let us say the conductance per unit length is zero for this Transmission Line. So G = 0 and we want to know what should be the resistance per unit length of the Transmission Line so that the line can be treated like a low loss transmission line.

Let us say the frequency of operation is equal to 100 mega hertz. From here since I know the value of L and C and the frequency I can find out what is the value of β. So β is equal to 2π into frequency which is hundred mega hertz which again is 10^8 hertz into $\sqrt{L}$ which is 0.25 x 10^{-6} or micro henry multiplied by hundred into 10^{-12} or the pico Farad. This will be equal to π radians per meter.

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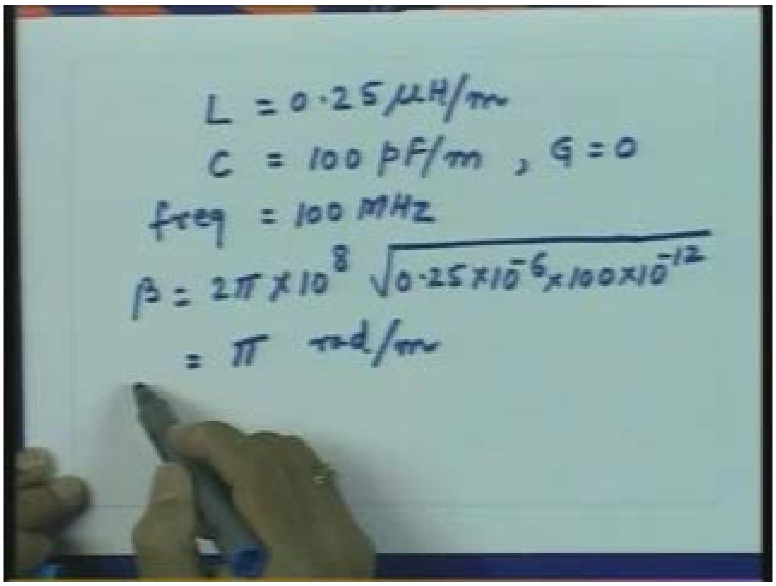

Just to take a number if I say α is less than one percent of value of β then I can treat the line as the low loss transmission line. The α should be approximately $\frac{\pi}{100}$. Now I can go back and substitute this value of α in the expression for α that is $\alpha = \frac{1}{2}R\sqrt{\frac{C}{L}}$

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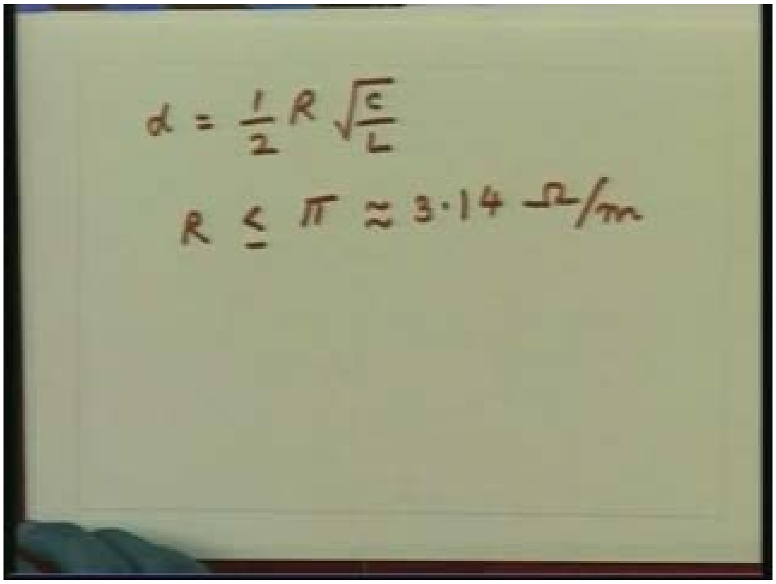

and if I substitute the value of $\sqrt{\frac{C}{L}}$ and the value of α which is $\frac{\pi}{100}$ as you have taken one percent of the value of β then I get the value of R which is less than R = π approximately 3.14 ohms per meter.

So if I accept alpha value to be one percent or less of the propagation constant β or the phase constant β then the resistance per unit length of the Transmission Line should be 3.14 ohms per meter at the frequency of hundred mega hertz. Of course as I said the frequency changes then the acceptable value of resistance will change because the line may not satisfy this condition at that frequency.

now having understood this as we mentioned earlier until and unless somebody specifically tells you that the line is a lossy line we take a liberty to create the line as a loss less line because as first order as we have seen that the phase constant for a low loss line and a lossy line is same, also the characteristic impedance of a low loss line is almost real and that is same as the characteristic impedance of loss less line.

So here onwards until and unless somebody specifically say include the losses in the calculation of Transmission Line we will treat the line to be lossless and carry out all our analysis for a Loss-Less Transmission Line. So essentially we will assume that characteristic impedance of Transmission Line is given by this so $Z_0 = \sqrt{\frac{C}{L}}$. This quantity is a real number and also the propagation constant is equal to the phase constant.

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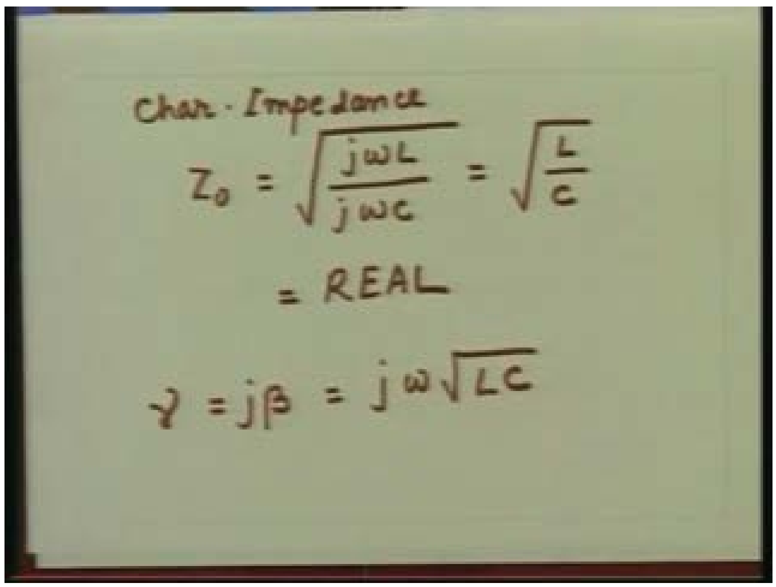

So we have $\gamma = j\beta$ that is again equal to $j\omega\sqrt{\frac{C}{L}}$. Now with these parameters we will again revisit the voltage and current expressions on Transmission Line and then we carry out the analysis of the standing waves on the Transmission Lines.

Going back to the original equation of voltage and current as we have seen for a Transmission Line whose origin has been defined at the load point as L = 0

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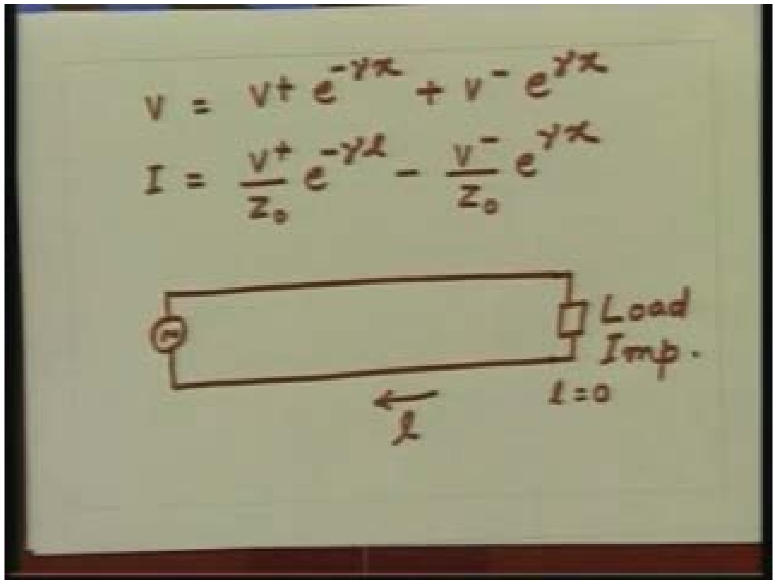

the voltage and current equation can be given by this. This represent the forward traveling wave, this represent the backward traveling wave and now we will replace this quantity γ by jβ where x will be replaced by -1.

So now we have the voltage and current as a function of distance on the Transmission Line and γ will be replaced by jβ and Z_0 is a real quantity. So we can write explicitly the voltage and current on a Loss-Less Transmission Line. So the voltage is v as a function of l that is equal to $V^+ e^{j\beta l} + V^- e^{-j\beta l}$, by taking $V^+ e^{j\beta l}$ common the same thing can be written as $V^+ e^{j\beta l} \{1 + \frac{V^-}{V^+} e^{-j2\beta l}\}$.

And this quantity as we already know is nothing but the reflection coefficient at the load end so this quantity we denote by the reflection coefficient at the load end Γ_L so $\frac{V^-}{V^+}$ as we have seen earlier is nothing but equal to Γ_L which is equal to $\frac{Z_L - Z_0}{Z_L + Z_0}$.

So now the voltage at any location on the Transmission Line can be given as $V^+ e^{j\beta l}\{1 + \Gamma_L e^{-j2\beta l}\}$.

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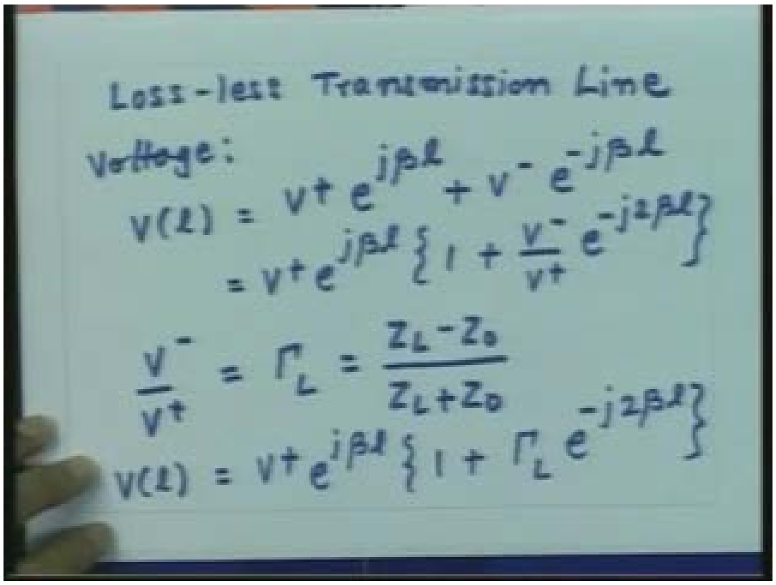

Similarly I can take the current equation I can substitute $\gamma = j\beta l$ in the current equation I can get the current that any location on the line $I(l) = \frac{V^+}{Z_0}e^{j\beta l} - \frac{V^-}{Z_0}e^{-j\beta l}$

Again taking $\frac{V^+}{Z_0}e^{j\beta l}$ common we can write down here this is $\frac{V^+}{Z_0}e^{j\beta l}\{1 - \Gamma_L e^{-j2\beta l}\}$ where $\frac{V^-}{V^+}$ will be equal to Γ_L.

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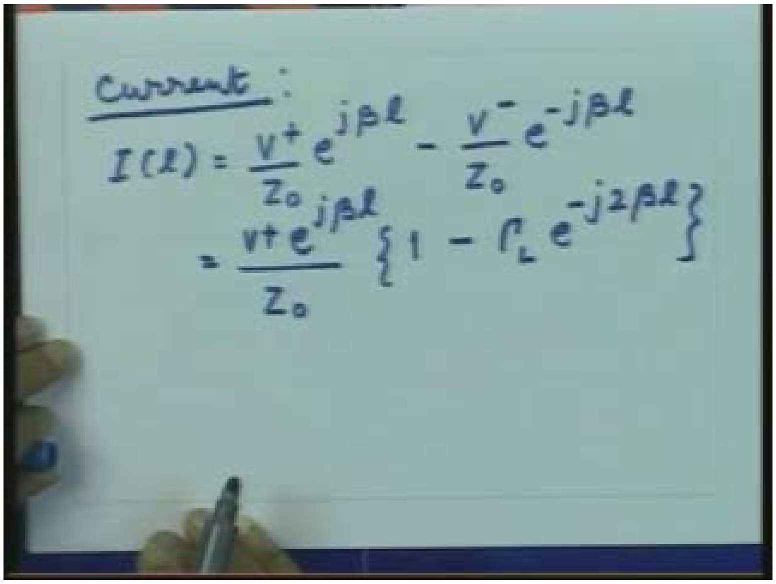

These two terms as we know essentially represent the forward and backward traveling wave so the whole expression here essentially represent super position of the forward and the backward traveling wave which is nothing but a standing wave on a Transmission Line.

So here the expression for voltage and current represent the standing voltage and standing current wave on the Transmission Line.

Now we can investigate certain features for the Transmission Line from here and first thing what we will note is that there are two terms either in voltage or current so you are having this term which is having amplitude one and then you arriving at second term whose amplitude is modulus of this quantity Γ_L plus a phase which is the phase of Γ_L plus this quantity phase which is minus j2βl. Writing very explicitly the complex reflection coefficient in terms of its magnitude and phase let us say I define $\Gamma_L = |\Gamma_L|\ e^{j\phi_L}$ where ϕ_L is the phase of the reflection coefficient at the load end.

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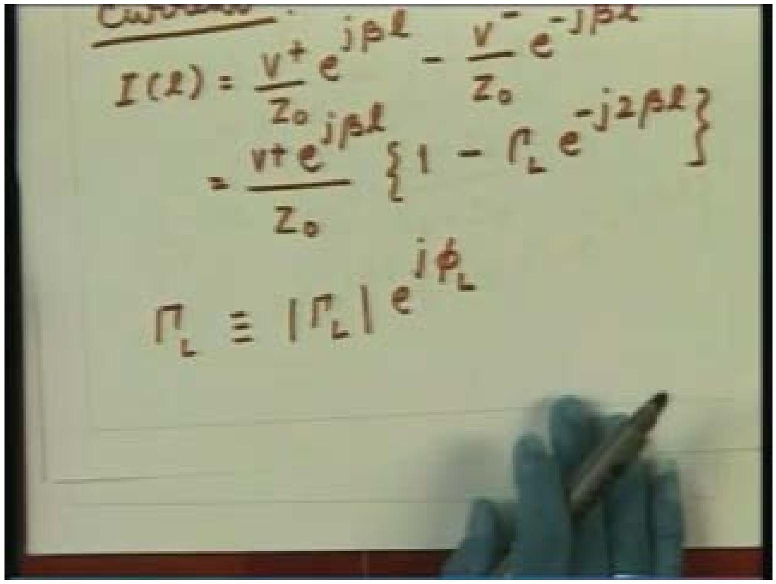

Then I can write down the current and voltage explicitly in terms of this magnitude of reflection coefficient and the phase. So finally I have two expressions here one for voltage $V(l) = V^+ e^{j\beta l} \{1 + \Gamma_L e^{j(\phi-2\beta l)}\}$ and the current $I(l)$ will be $\frac{V^+}{Z_0} e^{j\beta l} \{1 - \Gamma_L e^{j(\phi-2\beta l)}\}$.

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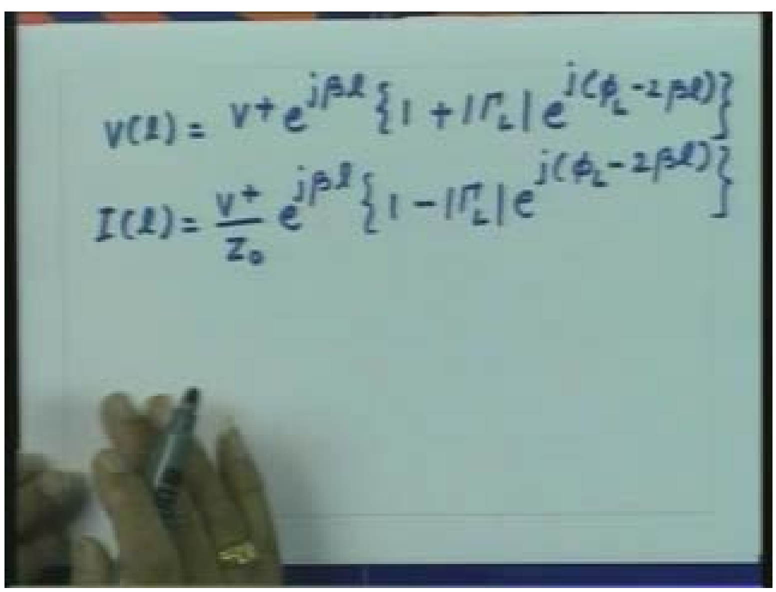

Now let us see how the voltage and current varies if I measure the magnitude of the voltage and current along the Transmission Line, what is the variation of the voltage and current along the Transmission Line.

So first thing what we will notice is as you move along the Transmission Line this quantity L is positive because we are moving towards the generator so as I move towards the generator the phase becomes more and more negative so this quantity (ϕ_L - 2βl) phase becomes more and more negative or in terms of a complex plane when the phase become more negative essentially we will move on the clockwise direction. So by moving towards the generator the phase becomes more and more negative the amplitude of this thing remains constant this term and the total voltage will be the vector sum of this term and this term is the real term, this term is the complex term whose phase is given by that and whose magnitude is given by $|\Gamma_L|$.

So essentially this is saying that this is the vector of unity one which represent the first term then I have another vector whose magnitude is $|\Gamma_L|$ and the phase of this is (ϕ_L - 2βl) and as I move towards the generator the phase becomes more negative but the magnitude of this remains constant. That means this whole quantity represent the circle where the point moves on this circle as the L changes. So this motion around this is towards generator and the magnitude of this term in the curly brackets is given by the vector which is the joining of these two points so this is nothing but $|\{1 + \Gamma_L \; e^{j(\phi - 2\beta l)}\}|$ magnitude of this quantity.

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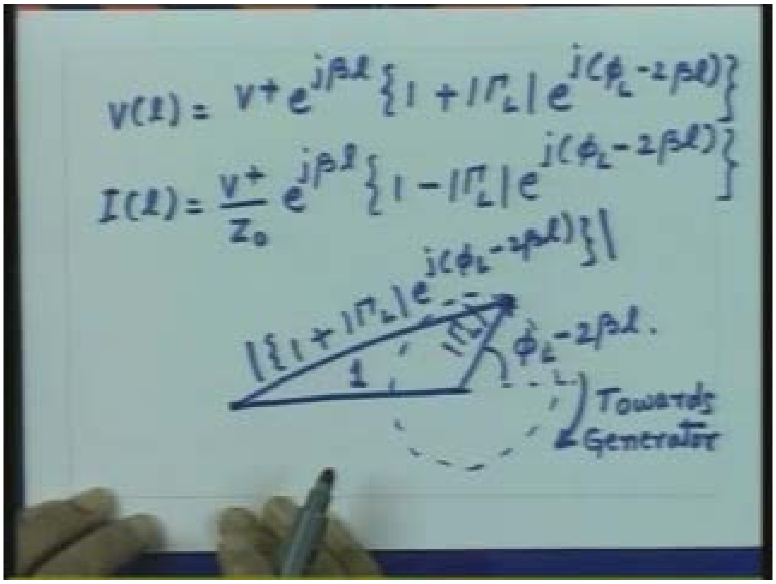

The first term is this vector which is unity the second term is this vector which is rotating as we move towards the generator on the Transmission Line and the magnitude of the total quantity here essentially varies as we move on the Transmission Line. Now the thing to note is when it moves on a circle at some phase when this quantity is zero or 2π or 4π e^{j0} or $e^{j2\pi}$ or $e^{j4\pi}$ that is equal to +1.

So I get a magnitude which is maximum which is represented by this point that is nothing but $1 + |\Gamma_L|$. Similarly if this quantity (ϕ_L - 2βl) is odd multiples of ϕ e^j odd multiples of π will be equal to -1 so I will get one minus mod $|\Gamma_L|$ that is the minimum value which I will see for this term here which is represented by this point where the two terms cancel each other.

I see that the variation of voltage and current is bound by two limits when the two terms directly add each other that time I will see the maximum voltage. If they cancel each other i will see the minimum of voltage similarly when these two terms add each other I will get the maximum current and when these two terms cancel each other I will get minimum of the current.

So the condition is when this quantity is +1 the voltage will become maximum but when this quantity is +1 and this quantity is +1 there is a minus sign here so when this quantity goes maximum at the same location l this quantity will go minimum that's a very interesting thing now. Earlier when we talked about lumped circuit wherever we have voltage higher we also have the current higher. Now what we are seeing here is that when the voltage is maximum the current is minimum and vice versa when this quantity become -1 that time the voltage will be minimum but this quantity will become plus so I will get the current maximum. Or in other words if I measure the magnitude of the current and voltages on the Transmission Line the maximum current and maximum voltages do not occur at the same location rather they are staggered in space. Wherever there is maximum voltage there is minimum current and vice versa. So the standing wave of the voltage and current are shifted with respect to each other in space on the Transmission Line.

So if I plot the magnitude with the voltage on the Transmission Line so let us say this is my Transmission Line which is terminated in some load here this is Z_L and now I plot the voltage and current on the Transmission Line let us say I plot |V| the |V| will have a variation which will go something like that this is the location where magnitude of voltage is minimum, this is the location where the magnitude of the voltage is maximum and as we saw just now that wherever the voltage is maximum the current will be minimum and vice versa so the current will go something like that so this is the plot for voltage |V| and this is plot for modulus of current. So this is this plot we are having |V| or |I| as a function of distance l = 0 and distance is measured towards the generator.

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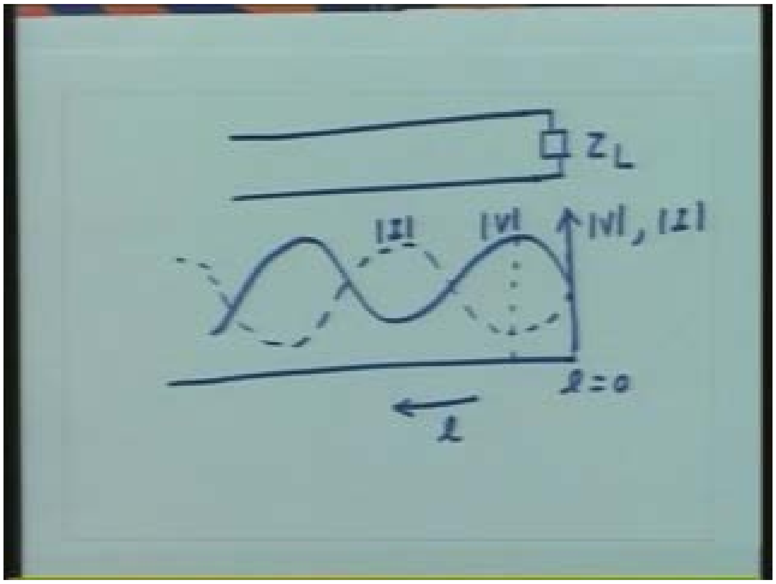

So in this location I get voltage maximum and current minimum, when I go here I get current maximum and voltage minimum. The important thing is when the standing waves are present on the Transmission Line then the voltage and current waveforms are shifted with respect to each other and maximum of voltage aligns with minimum of current and vice versa.

Now having understood this I can again go back and look at this expression of the voltage and current. If I go to the location where the voltage is maximum that means this quantity is +1 the voltage will be $|V^+|$ multiplied by $1 + \Gamma_L$ because this quantity is +1 at the same location I will have the current which will be minimum as we saw so this will be I(l) = $\frac{|V^+|}{Z_0}$ $(1 - \Gamma_L)$ where Z_0 is real since the line is loss less.

So once I know the voltage and current I can find out the impedance at this location where the voltage is maximum or the voltage is minimum or when the current is minimum or the current is maximum. The interesting thing to note here is irrespective of phase of V^+ when the voltage is maximum or minimum the ratio of V and I is a real quantity. If I take a ratio of these two $\frac{V(l)}{I(l)}$ that is equal to $Z_0\left\{\frac{1+|\Gamma_L|\ e^{j(\phi_L-2\beta l)}}{1-|\Gamma_L|\ e^{j(\phi_L-2\beta l)}}\right\}$ and when voltage is maximum or minimum this quantity is +1 or -1 so for maximum voltage (ϕ_L - 2βl) is even multiple of π that is it is 0, 2π, 4π and so on, for minimum voltage (ϕ_L - 2βl) is equal to odd multiple of π that is π, 3π, 5π and so on.

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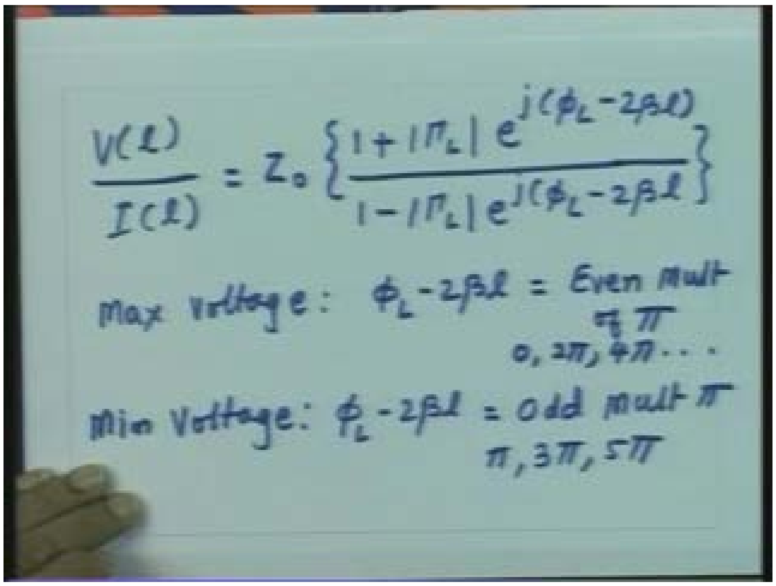

So when the voltage is maximum this quantity is +1 so Z_0 is real, now this quantity is one $|1 + \Gamma_L|$ denominator this quantity is again +1 so this quantity is $|1 - \Gamma_L|$ so the ratio of this quantity is the real quantity. When the voltage is maximum the impedance seen on the Transmission Line is real irrespective of what the line is terminated in.

Even if the line is terminated in complex impedance if I move on the Transmission Line and go to the location where the voltage magnitude is maximum, at that location the impedance measured will be always real. Similarly when I go to a location where the voltage is minimum this quantity will be equal to -1so I will get $|1 - \Gamma_L|$ you have $|1 + \Gamma_L|$ now. And again this quantity will be real so now we make a very important conclusion and that is on a Transmission Line wherever there is a voltage maximum the impedance measured is real, wherever there is a voltage minimum the impedance measured is real. So irrespective of what the impedance with which the line is terminated you will always find these points on the line where the voltage is maximum or minimum and that location the impedance measured will be purely a real quantity.

What will the value of these maximum or minimum impedances? We can substitute this so at a location where the voltage is maximum and the current is minimum that is the highest impedance you are going to measure on the Transmission Line. So this quantity when the voltage is maximum at that location we get the maximum possible impedance which we can measure on the Transmission Line. So we can get the maximum impedance which one can see on the Transmission Line and let us call that as Z_{max} is nothing but $\frac{|V_{max}|}{|I_{min}|}$.

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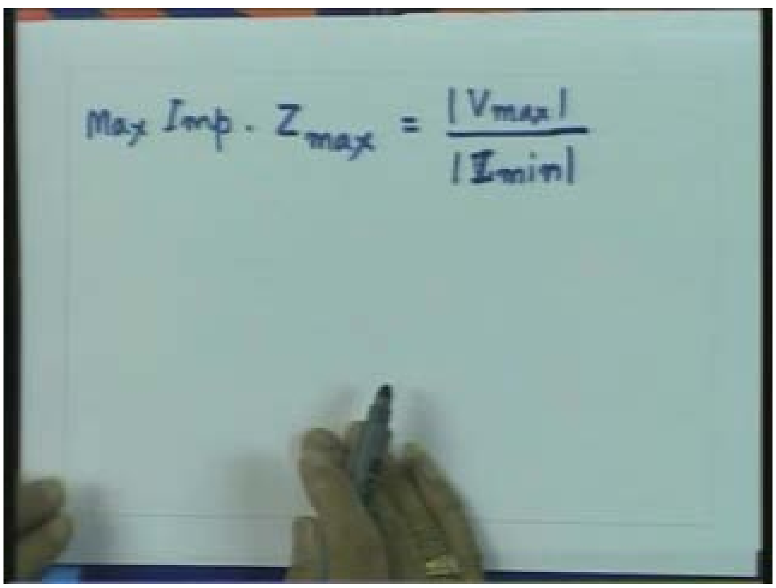

And since we have seen that this quantity when you are having the voltage maximum and current minimum that time the phase difference between them is zero so this quantity is real quantity so Z_{max} is nothing but R_{max} as resistive impedance, from here if I substitute this equal to plus one I will get $R_{max} = Z_0 \left\{ \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \right\}$.

Similarly if I go to a location where the voltage is minimum so this value will be $|1 - \Gamma_L|$ but at the same time the current will be maximum so that is the lowest value of impedance you can see on the transmission line. So you get the minimum impedance on the line which is Z_{min} which will be $\frac{|V_{min}|}{|I_{max}|}$ and that will be equal to $Z_0 \left\{ \frac{1-|\Gamma_L|}{1+|\Gamma_L|} \right\}$.

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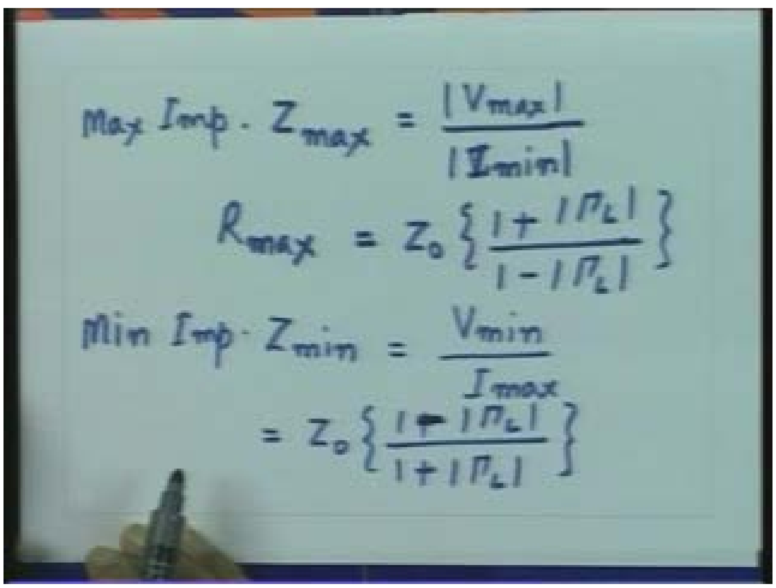

So once the load impedance on the line is known the reflection coefficient Γ_L is known its magnitude is known. I know what the maximum and minimum value of impedance is I can see on the Transmission Line. So as we move on the Transmission Line the impedance is going to vary as we saw because of impedance transformation but there is a bound on this impedance variation the lowest value of impedance which one can see on Transmission Line is Z_{min} or R_{min} is resistive and the maximum value which one can see on the Transmission Line is R_{max} which is given by that.

Now once we are having the voltage standing wave on Transmission Line at high frequencies the measurement of phase is rather complicated. You can measure the amplitude of the signal rather reliably but the measurement of phase is little uncertain. So at high frequency normally we have an effort to estimate the phase not in the direct manner but in indirect manner. By carrying out the measurements of only magnitude kind of quantities we would like to estimate the phase of the signal and as we have seen that the phase of the signal in time get translated into the phase space because the total phase which we seen on a wave is a combination of space and time or in other words the phase relationship between the two waves the forward and the backward traveling wave that is related to the time phase as well as the space phase.

And since this total phase governs the location of maxima and minima of the standing wave noting the location of maximum and minimum on Transmission Line one can estimate the phase which is there with the signal. So what we do now is we define a parameter for the standing wave which is a parameter of only amplitude variation on the Transmission Line and that quantity is called the voltage standing wave ratio. It is essentially a measure of what is the relative contribution of the reflected wave to the incident wave if the reflected wave is zero then there is no standing wave you will have only traveling wave if the reflected wave is full then you will have completely developed standing wave.

So the interference of the two waves the forward and the backward waves are going to give me this variation of the maximum to minimum. So we define this quantity the value which you get for the maximum magnitude on the standing wave and the minimum amplitude on the standing wave if I call the maximum value as V_{max} and the minimum value as V_{min} then the ratio of V_{max} to V_{min} magnitude is the voltage standing wave ratio. And this quantity is a very important quantity because without carrying out any phase measurement we can measure this quantity on the Transmission Line. Recall reflection coefficient is a complex quantity so if you want to have the complete knowledge of the reflection coefficient then we have to get its amplitude and phase. However the quantity which you are defining now is called the voltage standing wave ratio which is measured by only amplitude measurement.

So by measuring the maximum and minimum magnitude of the standing wave we get this quantity called the voltage standing wave ratio, normally it is denoted by $\rho = \frac{|V|_{max}}{|V|_{min}}$.

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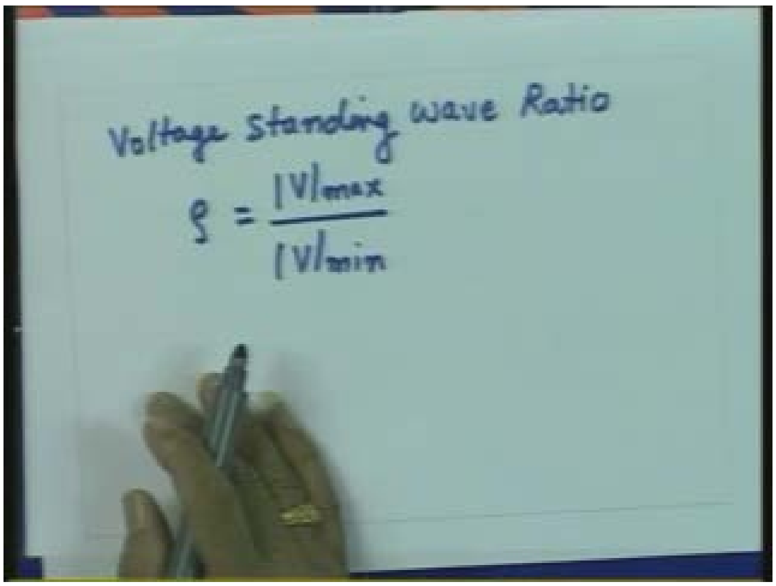

And what is the maximum value of voltage which I can see on the line is $|V^+|\{1+|\Gamma_L|\}$ and the minimum value which I can see on the Transmission Line is $|V^+|\{1-|\Gamma_L|\}$.

The $|V^+|$ cancels so the voltage standing wave ratio is $\left\{\frac{1+|\Gamma_L|}{1-|\Gamma_L|}\right\}$, in short the voltage standing wave ratio is called as VSWR. So the VSWR for a load whose reflection coefficient magnitude is $|\Gamma_L|$ is given by this.

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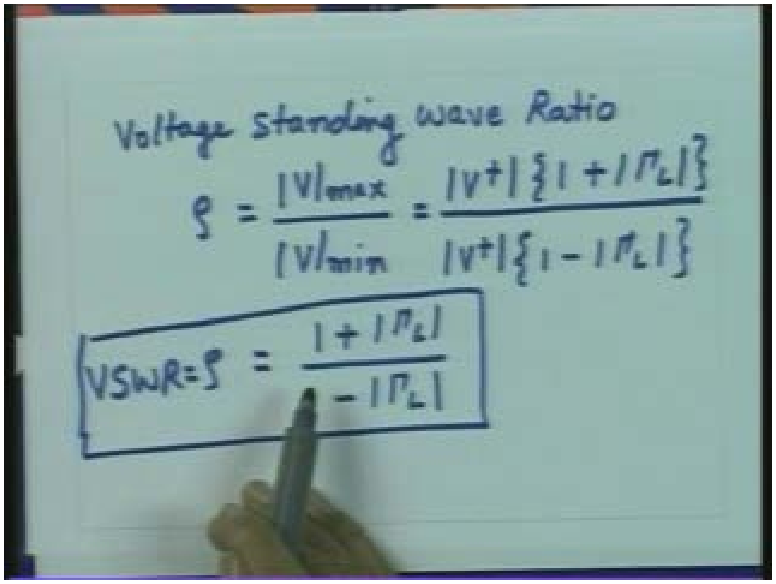

Since the line is lossless the reflection coefficient at the load is $\frac{Z_L - Z_0}{Z_L + Z_0}$ and Z_0 is the real quantity for a Loss-Less Transmission Line so Z_L can have any complex impedance terminated on the line but Z_0 is real without much effort one can see that the magnitude of this quantity is all ways less than one. So the mod of $|\Gamma_L|$ is always less than or equal to 1 and that is makes a physical sense because what the $|\Gamma_L|$ is telling you is the relative amplitude of the reflected wave compared to the incident wave.

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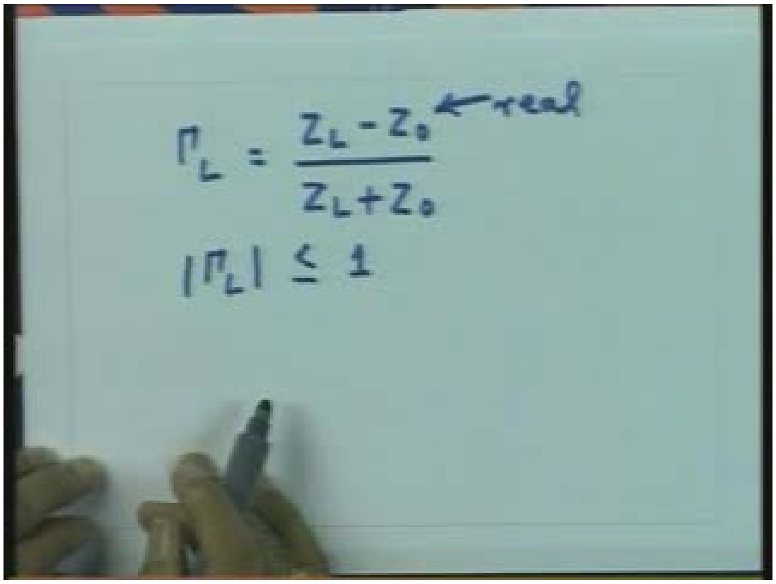

Since we do not have any energy source on the load point the part of the energy only can get reflected so the amplitude of the reflected wave has to be always less than or equal to the incident wave. So for any passive loads the $|\Gamma_L|$ is always less than or equal to 1 so in a condition when $Z_L = 0$ or $Z_L = \infty$ I will get the magnitude of $|\Gamma_L| = 1$ otherwise this quantity will be always less than or equal to one so if I want to see very specific load impedances for which the $|\Gamma_L|$ condition will be satisfied. I will see there will be three cases. Case one will be when $Z_L = 0$ that means the line is short circuited at the load end so this is a condition for a short circuited line. I substitute $Z_L = 0$ so I get $\Gamma_L = -1$ in this case I get $\Gamma_L = -1$ or $|\Gamma_L| = 1$.

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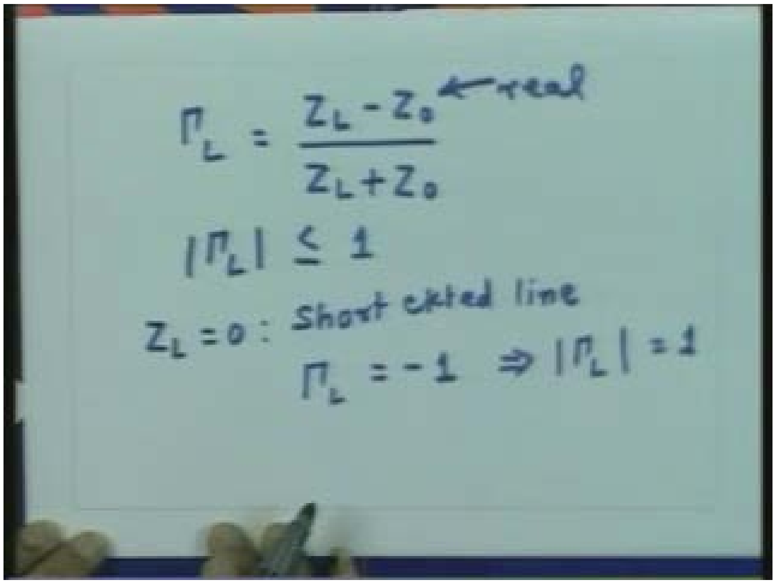

The second case if I take $Z_L = \infty$ that means the line is open circuited then again I can substitute $Z_L = \infty$, take Z_L first common so it will become $\frac{1 - Z_0}{\infty}$ so this will be equal to +1. So this will give me $\Gamma_L = +1$ giving me again $|\Gamma_L| = +1$.

The third case is that if the line is terminated in an ideal reactance so the third case is if Z_L = j times x your reactance then again so this is pure reactance then Γ_L will be equal to $\frac{jx - Z_0}{jx + Z_0}$.

This quantity in general will be complex but the $|\Gamma_L|$ will be equal to again +1.

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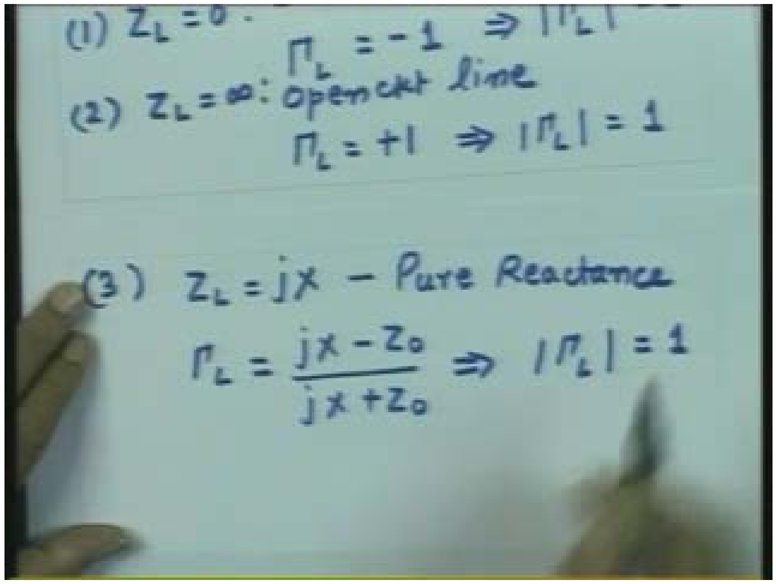

So we have these three situations when the line is short circuited, when the line is open circuited at the load end or when the line is terminated in a pure reactance the mod of reflection coefficient is one. What that means is if the line is either short circuited or open circuited or terminated in an ideal reactance there will be full reflection from the load end and that make sense that since I am having short circuit or open circuit or pure reactance there is no energy absorbing element available at the load end of the line neither short circuit can absorb power nor open circuit can absorb power nor a ideal reactance can absorb power.

So whatever power the traveling wave takes to the load end there is no option but to take the entire power back in the reflected waveform. Whatever wave reaches to the load end that completely get reflected if any of these three conditions are satisfied and that is what essentially is represented by this that the magnitude of reflection coefficient becomes equal to one. That means the entire energy with which reaches to the load it gets reflected so what we see from here is the reflection coefficient is always less than one it becomes equal to one in this special cases that is short circuit, open circuit or ideal reactance other wise the magnitude of the reflection coefficient is always less than one.

Now if I substitute this into the quantity which we have defined voltage standing wave ratio now this quantity $|\Gamma_L|$ is less than or equal to one. So in the best case when this quantity is zero we will get a VSWR equal to one otherwise this quantity will be always greater than one. So as we got the upper bound on reflection coefficient $|\Gamma_L|$. So we can also define the bounds on the voltage standing wave ratio VSWR and that is when $|\Gamma_L|$ = 0 then the VSWR = 1. However when $|\Gamma_L|$ goes to 1 that time this quantity is infinity so we have VSWR ρ its bounds are one equal to or less than equal to infinity.

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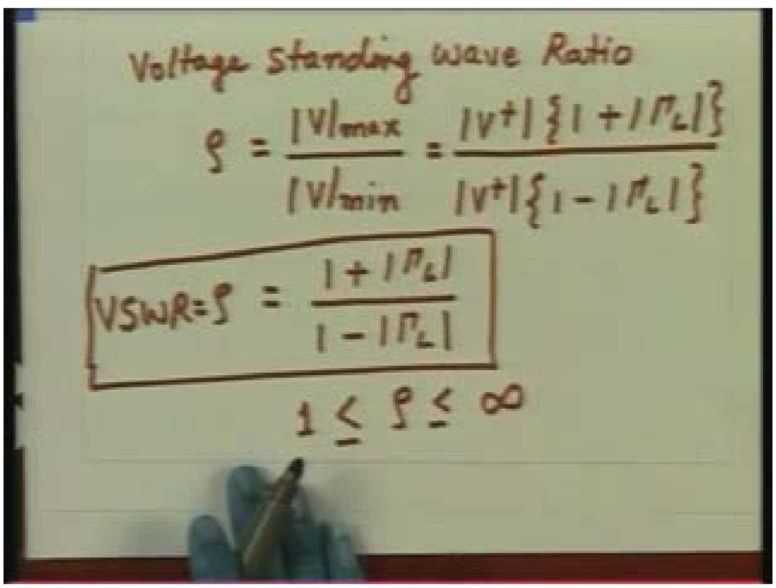

And which is the better case? The $|\Gamma_L|$ = 0 represents no reflected wave on Transmission Line that means we have full power transfer to the load. So $|\Gamma_L|$ going to zero is the best case as far the reflection is concerned or in other words when VSWR = 1 that is the best case for the power transfer on the line. As the VSWR increases it indicates higher and higher value of $|\Gamma_L|$ that is higher and higher reflection on the Transmission Line or lesser and lesser efficiency of power transfer to the load.

So in every circuit design our effort is to make the VSWR as small as possible or as close as to 1 as possible. Higher the value of VSWR indicates more mismatch on the Transmission Line or higher value of the reflected wave on the Transmission Line.

So this quantity VSWR is one of the very important quantity in the measurement at high frequencies. Whenever we design a circuit we essentially try to measure the VSWR on the Transmission Line and we try to make the VSWR as close to one as possible, making sure that the circuit is efficiently transferring power to the load end of the line.

Now once you have defined this parameter ρ then one can relate this ρ to the maximum and minimum impedance which one can see on Transmission Line. We have already seen that the maximum value of the impedance which we can see on the line is R_{max} = Z_0 $\left\{\frac{1+|\Gamma_L|}{1-|\Gamma_L|}\right\}$ but now you know this quantity is a measurable quantity and that is nothing but the voltage standing wave ratio ρ.

So the maximum value of the resistance or the impedance which we can again see on the line is nothing but Z_0 into wave standing wave ratio ρ.

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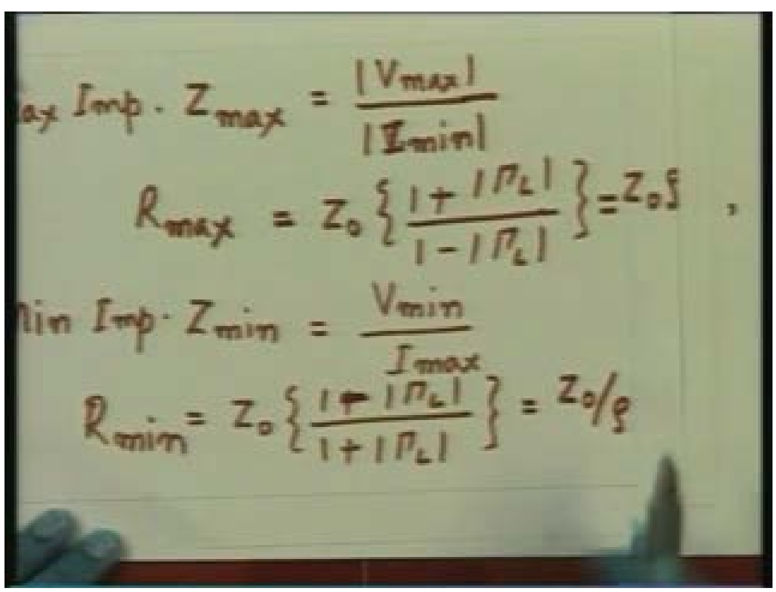

Similarly the minimum resistance which I will see on the line is this is $\frac{1}{\rho}$ so this is equal to $\frac{Z_0}{\rho}$.

So I get from here $R_{max} = Z_0\ \rho$ and R_{min} on the line is equal to $\frac{Z_0}{\rho}$. So for any line which is terminated in arbitrary impedance if I move to a location where the voltage is maximum then I know the value of the impedance at that location because measuring the voltage maxima and minima I can find out the value of this voltage standing wave ratio ρ, I know the characteristic impedance align apriori so I know the maximum value of the impedance which line will show.

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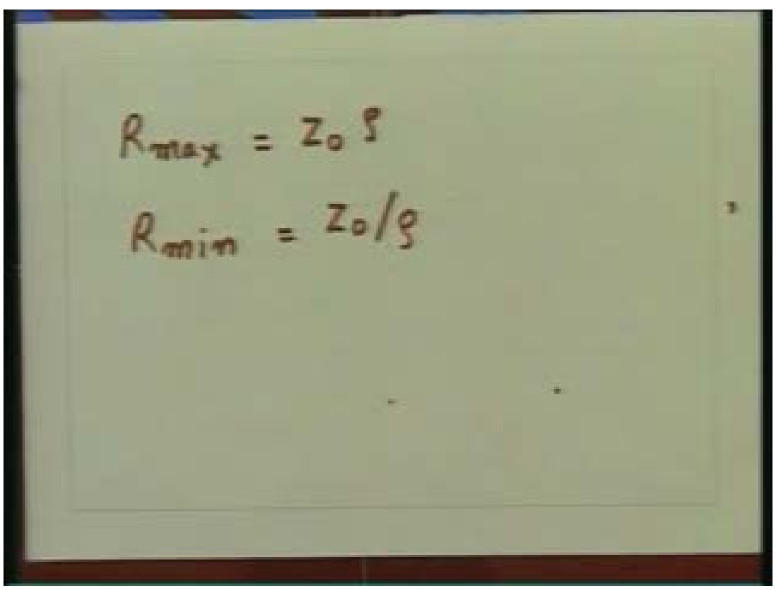

Similarly at the location where the voltage is minimum the impedance measured will be R_{min} and that will be the characteristic impedance divide by the voltage standing wave ratio. Now if you recall we had mentioned that if the impedance is known on any point on Transmission Line then you can always transform that impedance to any other location on the Transmission Line. Then immediately it strikes to us that knowing the location at the voltage maximum or minimum the impedance value is known there. so now I can transform either side of Transmission Line towards the load so that I can get the load impedance.

Now this essentially opens a measurement technique for the unknown impedance at high frequency if you are having a complex impedance its measurement is quite tedious because we cannot re-measure the phase very accurately. But I have a mechanism of measuring the phase indirectly as we already mention the phase get reflected into the standing wave pattern or the location of the voltage maxima and minima. So if I measure the voltage standing wave ratio and the location of the voltage maxima and minima I can always transform this impedance R_{max} or R_{min} to the location of the load which is nothing but the load impedance.

So if I transform from the load impedance to voltage maxima I will get this impedances but I can work backwards and say if I know the location of the voltage maximum from the load, I know the value of impedance at that location, I know the distance of load from that point so transformation of this impedance to the load end should give me the load impedance. When we go to the application of Transmission Lines we will see that this technique is used for measuring the complex impedances and at that time we will explicitly derive the expression for the unknown impedance which is terminated to a Transmission Line.

Thank you.