Transmission Lines and E.M. Waves
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Lecture-49

Welcome, up till now we looked at the antenna as the source of electromagnetic wave that is we excited the antenna with the current and the effect of that was the radiation. The antenna is used for transmitting electromagnetic energy but as we said at the beginning antenna is a transducer which converts the electrical quantities like voltage and current into the electromagnetic fields and vice versa that is electromagnetic quantities like electric and magnetic fields to the voltage and current.

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Today we will investigate the characteristics of an antenna as a receiving transducer that means when the electromagnetic wave is incident on an antenna what kind of currents and voltages are induced in antenna, how much power is received by the antenna, what kind of response the antenna has to incoming electromagnetic wave as a function of direction as a function of polarization like these are the questions essentially we will investigate in this topic called the receiving antenna.

And then we will also define some relationship between the parameters of the transmitting and the receiving antenna, that means if you take a antenna and if you use the antenna for transmitting purpose and if you use the same antenna for the reception purpose how the parameters of the transmitting and receiving antennas are related that some derivations will be carried out in this lecture. The properties of transmitting and receiving antenna are related to the Reciprocity theorem. The theorem as such is beyond the scope of this course however just to get a gist of what the theorem says the theorem says is that what ever properties the antenna has while it was in transmitting mode the same properties it would have in the receiving mode also, what that means is if the antenna had certain directions for maximum power that means antenna was putting more power in certain directions when the same antenna is used for reception purpose the antenna will be capable of receiving more power from that direction compared to other direction when it was transmitting less power while transmitting more. Also, if I look at the electric field and if I change the direction of the electric field keeping the direction of the wave same then as the electric field changes essentially the polarization is changing and because of that the voltage induced into the antenna terminals will vary and the power delivered to the load connected to the antenna terminals will vary so we will have a response of the antenna to the incoming polarization also.

So again from the Reciprocity theorem the antenna will respond maximally to that polarization which it is capable of transmitting that means the antenna has a state of polarization and that is what we had said would be the state of polarization of the incoming wave to which the antenna respond maximally when we discuss the topic of polarization. So the polarization characteristics are also same for transmitting and receiving antenna.

So as we said the Reciprocity theorem is beyond the level of this course, however, what we will do is we will try to validate these arguments which Reciprocity theorem puts forward that the antenna would have the same behavior while receiving as it was in transmitting and that we can see for a simple dipole which we have investigated the Hertz's Dipole.

Now, we will see the characteristics of the receiving antenna and let us say if I have a Hertz's Dipole of some small length dl some voltage is going to get generated between the terminals of this antenna let us say this voltage is called the v_{oc} open circuit voltage, now if the wave which is uniform plane wave which is incident on this antenna and let us say this wave comes from this direction so this is the direction the wave and this is the transverse electromagnetic wave so you have a electric field which will be just this is E let us say the dipole is oriented in the z direction as we have taken earlier so the wave is incident on this from an angle θ with respect to z axis.

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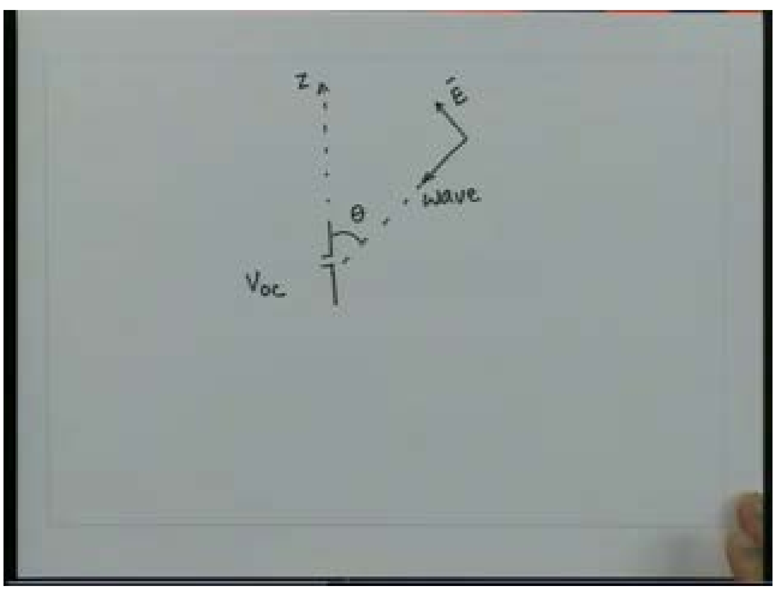

Now due to this electric field we have E dot dl so there is a voltage induced between this wire this is the Hertz Dipole and that is a dot product of dl which is the length of the dipole and the electric field. So as we can see from here the E dot dl the voltage v_{oc} will be equal to E dot dl where dl is the length of the antenna so here the length is dl, if this angle is θ then the angle which it makes with the z axis is 90 – θ so the dot product of these two that is open circuit voltage will be the electric field E dl into cos(π/2 – θ) that is equal to E dl into sinθ. That means when the wave is incident from this direction when θ is equal to ninety degrees so this quantity is one so you will have a maximum voltage induced between the terminals of the antenna where as when the wave is incident from this direction when θ = 0 or θ = π that time the voltage induced between the terminals of the antenna would be zero. Then I can say that the reception pattern for this Hertz Dipole will be having a directional dependence which will be sinθ. So it will get a maximum induction from the wave which is coming from the direction θ equal to ninety degrees that is perpendicular to it and when the wave comes along the axis of the antenna then the induced voltage will be zero because dot product will be zero.

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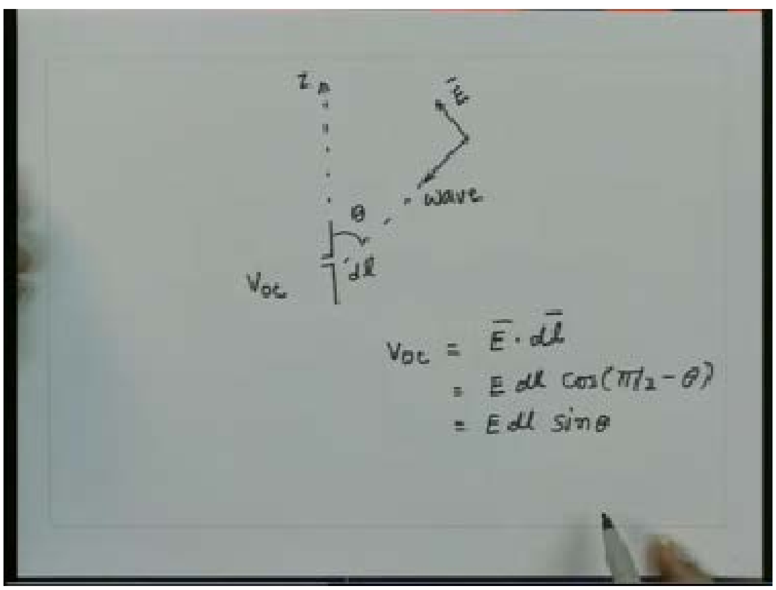

So if you recall precisely the same variation the antenna had while transmitting that is the radiation pattern of this antenna was sinθ for the Hertz Dipole so this quantity sinθ which tells me the variation of the open circuit voltage for this antenna as a function of the angle of the incoming uniform plane wave incident on this antenna that variation is sinθ. So we have a validation that is the radiation characteristic or what we call the radiation pattern for a transmitting antenna is identical to the directional dependence of the receiving antenna. So this antenna is capable of giving you maximum field θ equal to ninety degrees direction so it also receives maximally from that direction it does not put any radiation field along the axis of the dipole so it does not also receive any field from the axis of the dipole.

Now one can say what the effect of polarization on this is, so again let us say suppose the electric field was perpendicular to the plane of the paper this electric field when it comes here this is perpendicular to the Hertz Dipole so E dot dl is identically zero so when the E is perpendicular to this you have the dot product equal to zero so no voltage is induced you will have a voltage only induced by that component of the electric field which lies in the plane of the paper because that is the one which is going to give me dot product with this the component of the electric field which is perpendicular to the plane of the paper will give me the dot product zero because the angle between this electric field and dl will always be zero.

So what we see is as I go on a given distance from the dipole if I draw a circle around this and if I imagine a situation that the wave is coming from different directions so at a distance r from the Hertz Dipole if I take various waves which are coming and impinging on this this field will give me the voltage this field here will be like the phi incoming wave so it will give me the voltage which is maximum and there is no voltage induced because of the electric field which has the component which is perpendicular to this. So this is precisely the behavior that now your antenna is responding to only θ component of the electric field which is tangential to the circle. If you take electric field which is perpendicular to this then the induced voltage is equal to zero. Precisely that is the behavior we see for the transmitting antenna that the Hertz Dipole used to radiate an electric field which was linearly polarized and the linear polarization was in the direction θ.

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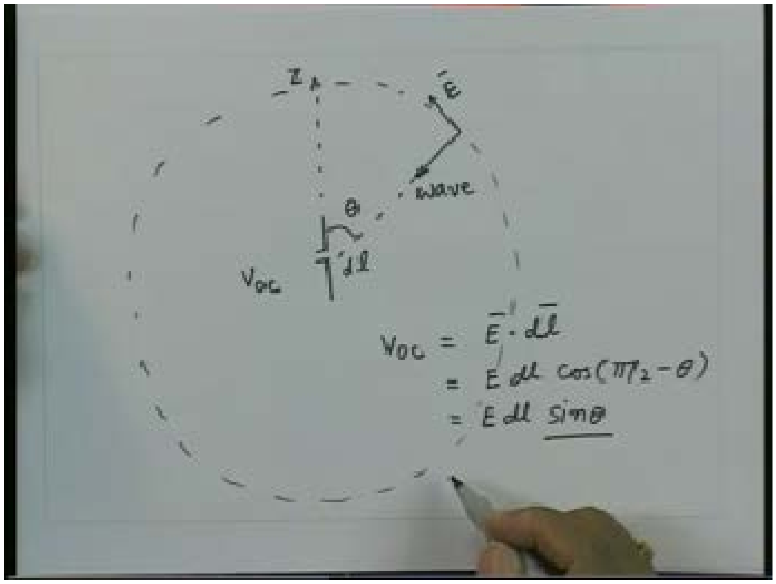

So here also we see the Hertz Dipole response to the incoming electric field which is oriented along θ so the polarization characteristic also identical for the transmitting and receiving antenna. In fact this property is true for any general antenna though we saw this validity for Hertz Dipole in fact when we go in practice we take any antenna and its radiation pattern will be identical while receiving and transmitting mode its polarization behavior will also be identical while transmitting and receiving mode that means the antenna will respond maximally to that polarization to which it is capable of transmitting.

Having understood this behavior then there is one parameter which is very special to the receiving antenna which cannot be defined for transmitting antenna and that is called the effective aperture of the antenna. Now the idea is as follows, let us say I have an antenna when it was used in a transmitting mode so let us say I have a general antenna which it was transmitting mode so when I saw from the circuit point of view between the terminals of the antenna we see an impedance which is the input impedance of the antenna which is z. So from the circuit point of view this is equivalent to an impedance which is some resistance value which is equal to the radiation resistance of the antenna so it is R + jx. So an antenna which is seen from the circuit point of view between the terminals appear like a impedance where R is the measure of how much power is radiated by the antenna and x is essentially related to the reactive field the inductance and the electrostatic field which are around the antenna. So the power radiated by the antenna is essentially related to this quantity R.

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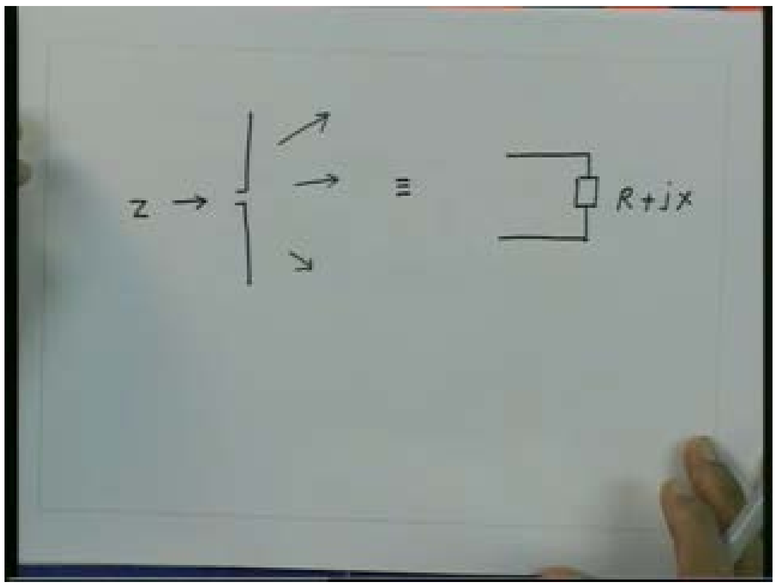

Now again by the reciprocity theorem essentially the impedance between the terminals of the antenna is identical when antenna was transmitting and receiving that means while receiving if I look between the terminals of the antenna this will appear like a voltage source because voltage is induced between the terminals that is the v open circuit voltage with an internal impedance which is the same as the antenna impedance which is R + jx. So this was the case which was transmitting antenna if I take an antenna which is receiving antenna then we have a situation this is the antenna on which the wave is incident and this antenna is now connected to a load impedance to which the power is to be delivered and let us say this load is given by Z_L so this antenna is now equivalent to the open circuit voltage which is induced between the terminals of the antenna so I have here a voltage source which is v_{oc} then I have a internal impedance of this antenna which is R + jx and this is connected to load impedance to which the signal is received so this impedance is equal to Z_L.

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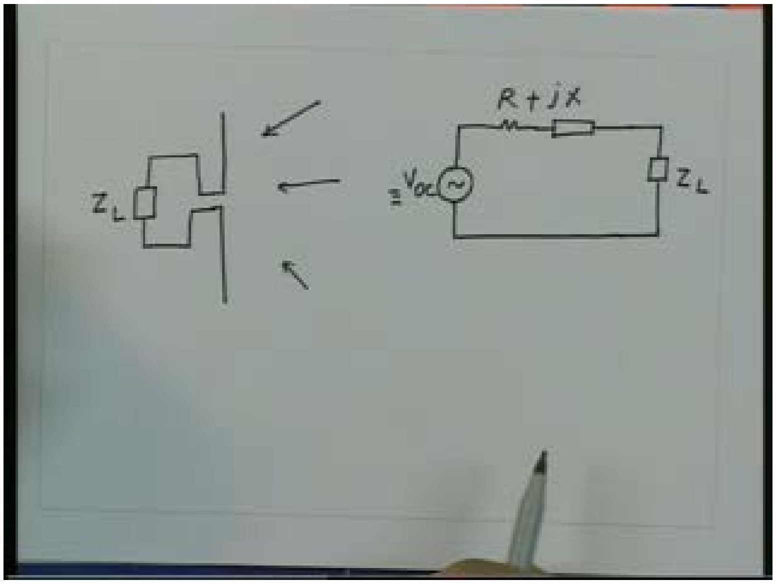

So if I see from circuit point of view, a receiving antenna is equivalent to an open circuit voltage that is the voltage which will measure between the open circuited terminals of the antenna the internal impedance of the antenna is the same impedance which you would measure between the terminals of the antenna when antenna is in transmitting mode and that is the load impedance to which finally the signal is delivered in the receiving mode.

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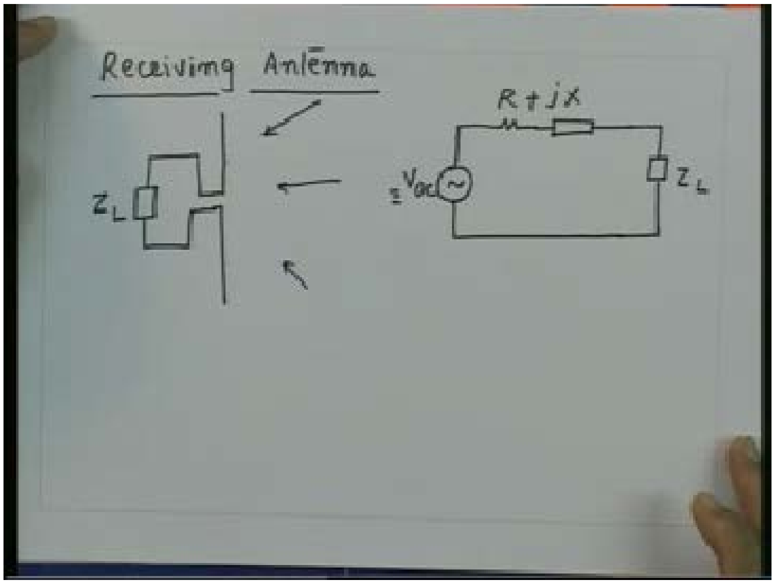

So the receiving antenna can be equivalently represented by this circuit. Then the power which is delivered to this antenna is when ever there is a complex conjugate of this one that is where the power is maximally delivered to this so now let us assume that everything is perfectly matched that is the wave which is incident on the antenna is coming from that direction for which the antenna has maximum response the polarization of the wave is adjusted in such a way that I get a maximum response the load impedance is chosen in such a way that I have a complex conjugate match that means that is the maximum power I can extract from this wave to this load. So from this antenna, under every match condition polarization direction of maximum radiation and the complex conjugate load I will get the maximum power. So I can say the maximum power delivered to the load under fully matched conditions and fully matched conditions means direction of maximum radiation the polarization and the complex conjugate match then the maximum power which you can extract will be equal to the $|v_{oc}|$ square divided by 4R. I can just find out the current from here and then multiply by the voltage across this by conjugate of the current I will get the power delivered to the load.

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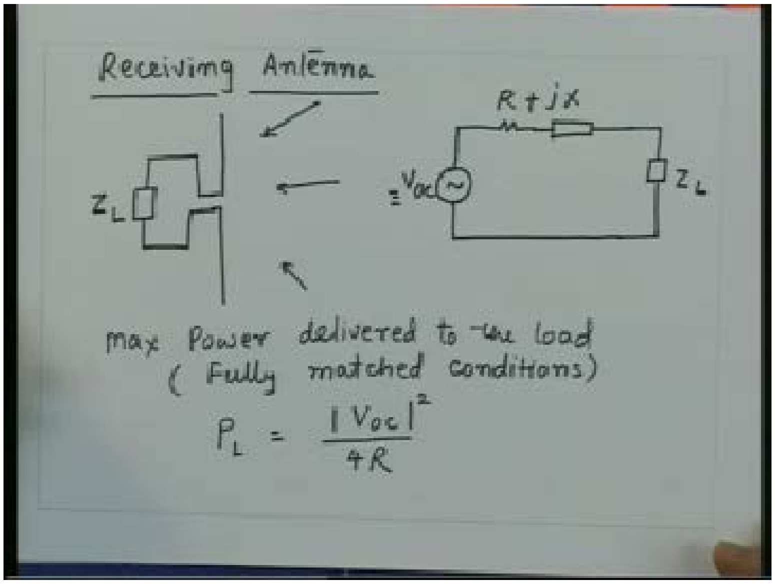

Now I can ask a question that the wave which is incident on this antenna has a power density called the Poynting Vector and this antenna has extracted a power which is P_L that is the power which is made available to the load which is connected to the antenna. So if I take the ratio of this power which is delivered to the load by the Poynting Vector of this load which is incident on the antenna which has the units of watts per meter square I get a quantity which has a dimension of area that quantity then I call is the effective aperture of my antenna receiving antenna.

So we say that if S is the power density of the wave then the effective aperture A_e will be the power delivered to the load under fully matched condition divided by the power density of the wave.

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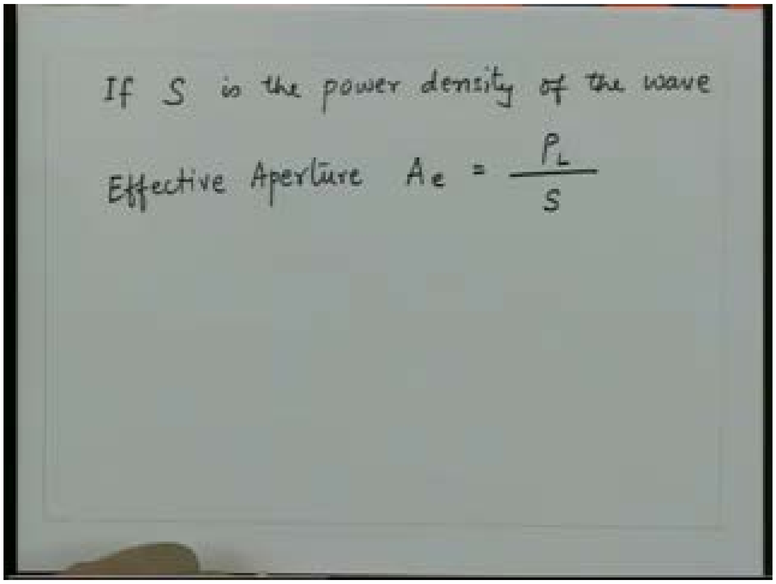

So this aperture is telling as when the wave was coming which I have certain power density and let us say I had a piece so I can cut a piece from this wave so I got some power from that area so the antenna essentially effectively put the piece and cuts a piece from this wave front which is coming in and that is the one which sort of delivered the power to the load. So this effective aperture in some cases is related to the physical area of the antenna ((20:40)) we cannot have any direct relationship between the effective aperture and the physical area of the antenna.

For example, when I take a thin antenna like a dipole the physical area is extremely small but I will still have the effective aperture which will be substantial because this can still deliver the power to the load. So this is a parameter which is a unique parameter to the receiving antenna because this quantity is not there for transmitting antenna and then it will be interesting to find out what is the relationship of this effective aperture which the parameters of the transmitting antenna and I knew for transmitting antenna other than direction of maximum radiation pattern there was a quantity called directivity which you should tell us the focusing power of a antenna in a given direction it would be interesting to ask a question does it have any relation to the aperture and vice versa that means are the directivity and the effective aperture related in some sense.

If I make a physical antenna like a parabolic dish the larger the antenna I make the antenna beam becomes smaller so directivity increases and since parabolic dish antenna has a large physical area it appears as if it would have given you a larger effective area so increasing the size of a parabolic dish antenna would give me high directivity it would give me higher effective area also.

So it looks intuitively possible that the effective area and directivity are in direct proportion in some sense because if one increases the other quantity also is increasing especially that is what it looks like for the parabolic dish.

So what we do now is essentially derive a general relation of the effective aperture and the directivity of the antenna when the antenna was receiving the transmitting mode. For that, let us again go to the circuit model of the antenna so let us say I have a transmitting antenna to which some voltage v_1 is given because of that some currents flow so let us say this antenna is given by one then at some distance I have receiving antenna at the moment I must specify what type of antenna it is let us take general case so I have some antenna here which is radiating I have another antenna which is receiving and let us say this antenna is antenna two so this is the receiving antenna and through radiation and other fields you have here coupling here between these two antennas let us say there is some distance r between these two antennas and you are having a coupling through radiation

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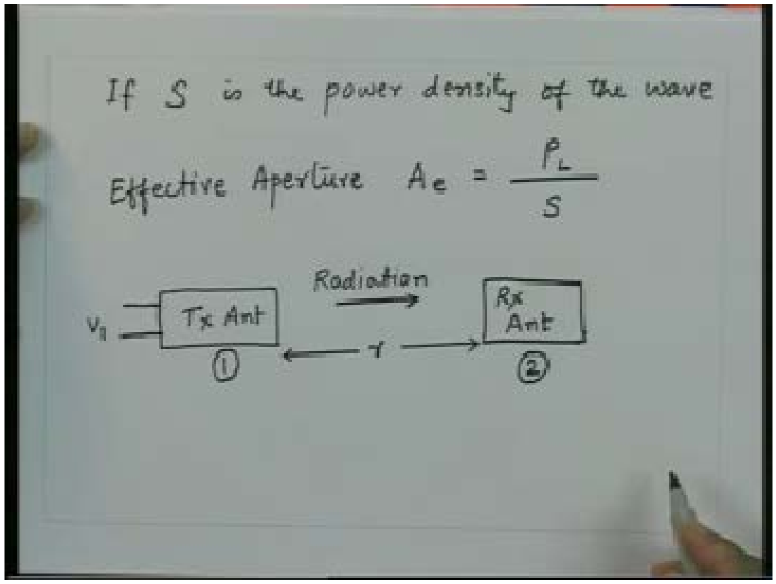

So the receiving antenna will essentially develop some voltage across the terminals which will be connected to the load and the power delivered with the load. We can write down the equivalent circuit of this so when I say V_1 voltage is applied to this some I_1 current is going to flow and the internal impedance of the antenna is Z_1 so this is equivalent to an impedance which is Z_1 so I have voltage which is V_1 because there is some current flow between the antenna terminals which is I_1 and this impedance is Z_1. And the effect of the coupling of the field between these two can be accounted for by electrical parameter called the mutual impedance between these two so I have between these two a quantity called mutual impedance, let us say this is Z mutual and on the receiving side we have the voltage which is v_{oc} then it will have a internal impedance of the antenna which is say Z_2 and it is connected to a load which is the complex conjugate of the Z_2 so this is Z_2 conjugate.

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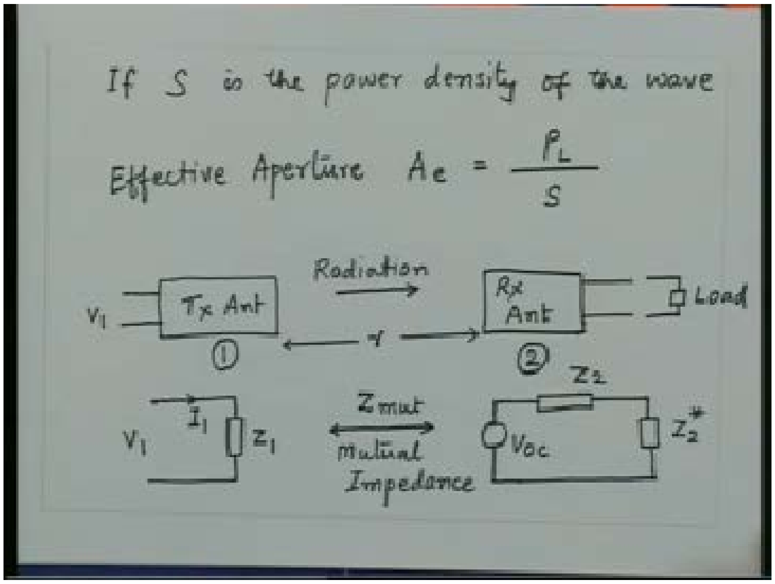

So a two antenna system in which one is receiving and other one is transmitting essentially can be written equivalently like this and this quantity which we have here Z mutual is reciprocal quantity that means what ever is the mutual coupling between this antenna and this antenna and if you transmit this and receive this I have this mutual impedance the same thing is true when you transmit this and receive from this antenna. So Z mutual is a parameter which is independent of direction that means from which antenna it is transmitting or receiving this is Z mutual quantity is same from the Reciprocity theorem. Then beyond this, we can essentially view the circuit analysis to find out how much power is transmitted and so on.

So firstly if the current which is flowing into this circuit which is I_1 and this is Z mutual that means this tells me what is the voltage induced into this because of this current that is what the thing which is given by Z mutual is so z_{oc} is z mutual into I_1.

So by definition I will get the open circuit voltage v_{oc} is equal to z mutual into I_1. And then for complex conjugate match load which is $Z_2 = R_2 + jx_2$ the maximum power transferred will be |V| square upon $4R_2$ so we have the power received P_L will be equal to

$|v_{oc}|$ square upon four $4R_2$ which is equal to mod Z mutual square mod I_1 square upon $4R_2$.

How much is the power transmitted by the antenna is corresponds to the radiation resistance of this antenna so if I say Z_1 is $R_1 + j\ x_1$ then the power transferred will be mod I_1 square into R_1 so we have power transmitted by the antenna P_t will be equal to mod I_1 square into R_1.

Now this power goes into the space induces the voltage here that we had traveled a distance of r and as we have seen from the antenna there is a spherical wave which is created by the antenna so you are having a power density which varies as total power divided by four pi r square if the antenna is isotropic if the antenna has a directivity then the power density will be enhanced by directivity in the maximum direction so if I say that the two antenna systems are perfectly aligned to look into each other for maximum radiation that means they are completely matched again in terms of the radiation patterns the power density which you get on location on the receiving antenna will total power divided by 4π r square into directivity of this antenna.

So let us say the directivity of antenna one is given by v_1 so I will get the power density at the receiving antenna that is S will be equal to the power radiated multiplied by directivity D_1 divided by 4π r square. Then I can substitute for P_t so this will be equal to mod I_1 square R_1 into D_1 upon 4π r square.

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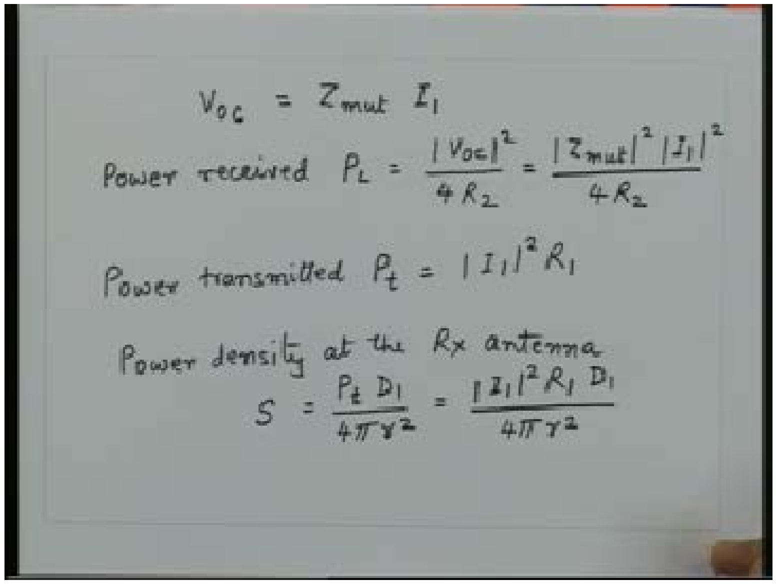

Now as you have designed the P_L by S that is the effective aperture so I can write the power received from this will be the effective aperture into the power density. So essentially what I can get is the effective aperture multiplied by this power density that should give me the power received and that power received should be the same as this power which is delivered to here. From here I can get power received by receiving antenna will be equal to the power density multiplied by the effective aperture of antenna two.

So note here in this case the receiving antenna is two transmitting antenna is one so one has the directivity parameter the receiving antenna has effective parameter which is effective aperture and that we are calling as A_{e2}.

So this antenna is having effective area which is A_{e2}. So I can substitute for S so now this quantity would be mod I_1 square R_1 into D_1 upon 4π r square into effective area for aperture two for antenna two.

Now as we have said this received power must be the same as the power which is delivered to the load under fully matched condition so we can equate this two to get Z mutual square mod I_1 square divided by $4R_2$ that is equal to this quantity which is mod I_1 square R_1D_1 A_{e2} divided by 4π r square.

From here, I can calculate this quantity Z mutual so I get from here Z mutual square here I_1 square and 4 will get cancels so you will get so you will get R_1 R_2 D_1 A_{e2} divided by πr square.

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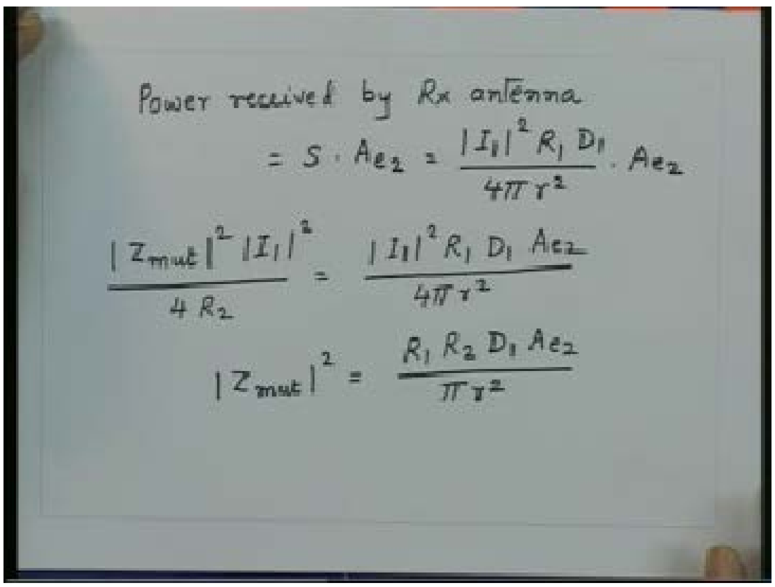

Now let us interchange the role of receiving and transmitting antenna that means this is the one which is transmitting so it has a directivity let us say is D_2 and this antenna is the receiving antenna so its aperture is A_{e1} and here Z mutual parameter which we have defined which is the mutual inductance between the two antenna is same whether this is transmitting or this is transmitting so I can calculate this quantity z mutual from taking this antenna as transmitting g and this antenna is receiving antenna. If I do that then this R_1 and R_2 will remain same and D_1 will be replaced by D_2 because now that is the transmitting antenna A_{e2} will be replaced by A_{e1} because that is the receiving antenna.

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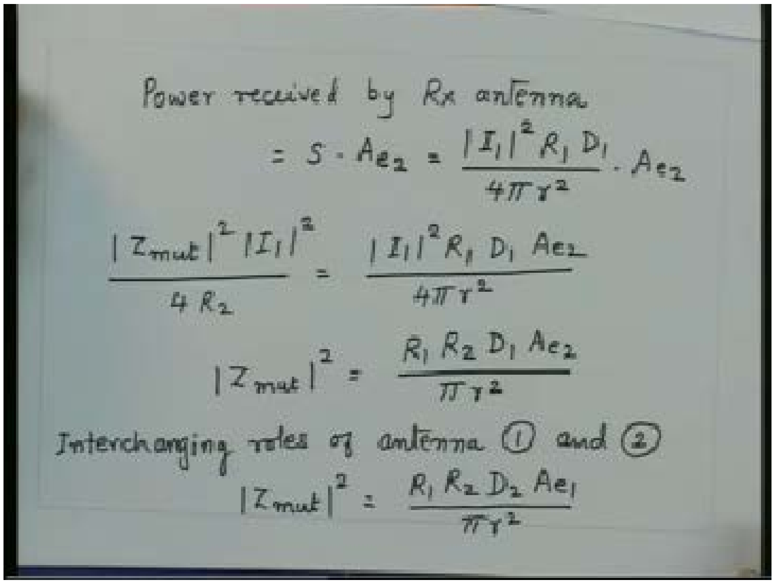

So interchanging the role of transmitting and receiving antenna let us say interchanging roles of antenna one and two that means now two is transmitting antenna and one is receiving antenna we can get Z mutual square that is R_1 R_2 D_2 A_{e1} divided by π r square. That means this quantity D_1 A_{e2} is equal to D_2 Ae_1, from here we essentially get D_1 A_{e2} = D_2 Ae_1 that is $D_1/A_{e1} = D_2/A_{e2}$.

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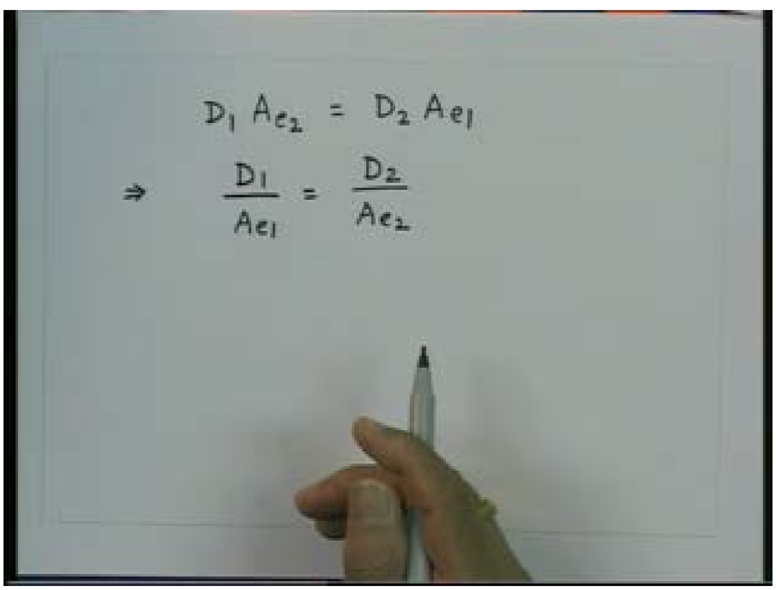

Now since we have not considered here a specific antenna this relation that the directivity of a antenna and the effective aperture this the ratio of this two must be independent of which antenna pair we choose, that means this quantity D or directivity of a antenna and this effective aperture this quantity must be a constant quantity because now this ratio now is independent of which antenna pair you take. So this quantity must be some constant let us say k whose value we evaluate for one antenna then essentially we got a relationship between the directivity and the effective aperture in general for any antenna. So first thing what we note from this derivation is that for an antenna directivity and the effective aperture are directly proportional to each other which is constant k and that constant k can be evaluated by finding out the directivity on the effective aperture for any one antenna and since we have investigated the simplest antenna which is the Hertz Dipole may be we can calculate the value of D and A_{e1} for the Hertz Dipole and from there we can calculate the value of k.

So let us say for Hertz Dipole we have a radiation pattern E which will be given by sinθ, we have a directivity for the Hertz Dipole D which is 4π divided by integral θ from 0 to π, ø from 0 to 2π mod E square And this pattern is normalized pattern its maximum value is one so this is mod E square which is sin square θ and the solid angle sinθ dθ dø.

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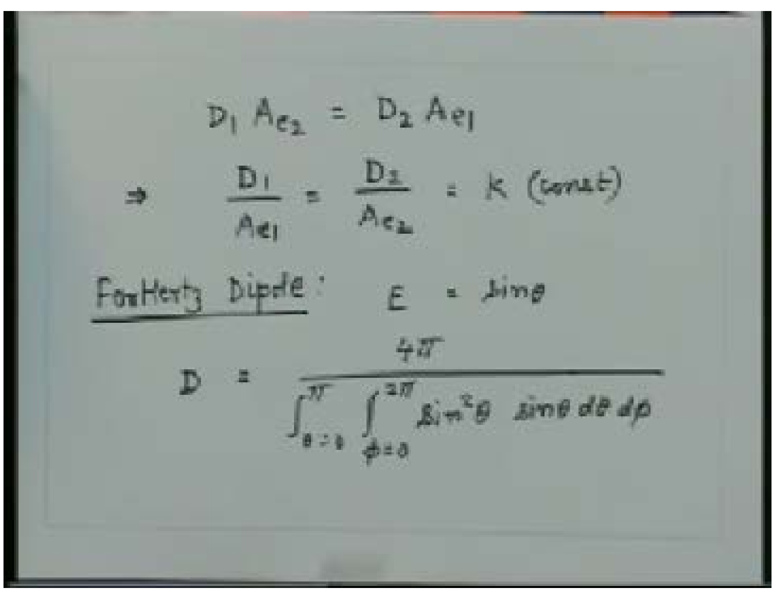

And this integral we had calculated earlier so this will be 2π over ø so this is 4π upon 2π integral 0 to π sin cube θ dθ and this integral we had evaluated which was 4/3 so if I substitute that this is 2 upon 4/3 that is equal to 3/2. So the directivity of the Hertz Dipole is 3/2.

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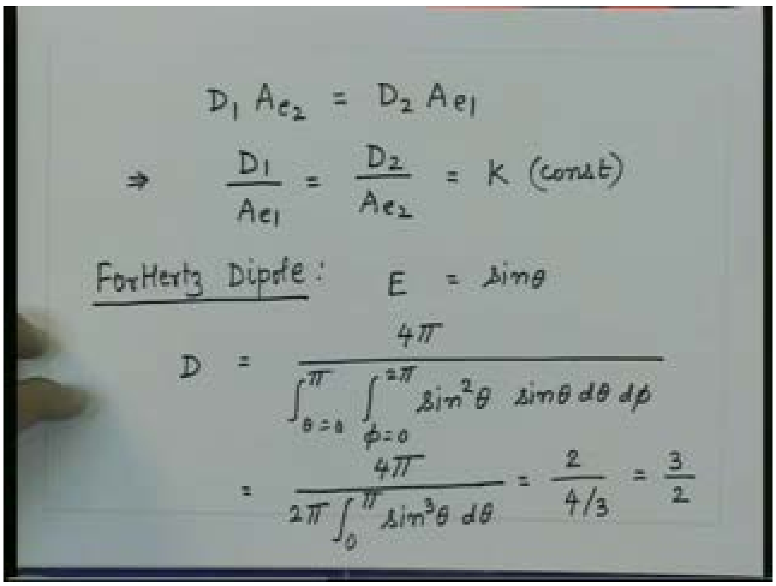

And also we can calculate the effective aperture for the Hertz Dipole when the wave is incident so that the maximum voltage is induced. So let us say we have a Hertz Dipole and the wave which is incident from the maximum direction which is this direction has an electric field which is E so the Poynting Vector S for this will be mod E square upon η where η is the intrinsic impedance of the medium so which is equal to mod E square divided by 120π where 120π is the intrinsic impedance of the medium.

The voltage induced because of this electric field in this which is E dot dl so if the length of this Hertz Dipole is let us say l then the v_{oc} the open circuit voltage from here will be equal to E into l, everything is matched polarization is also matched so dot product is the maximum dot product which is E into l

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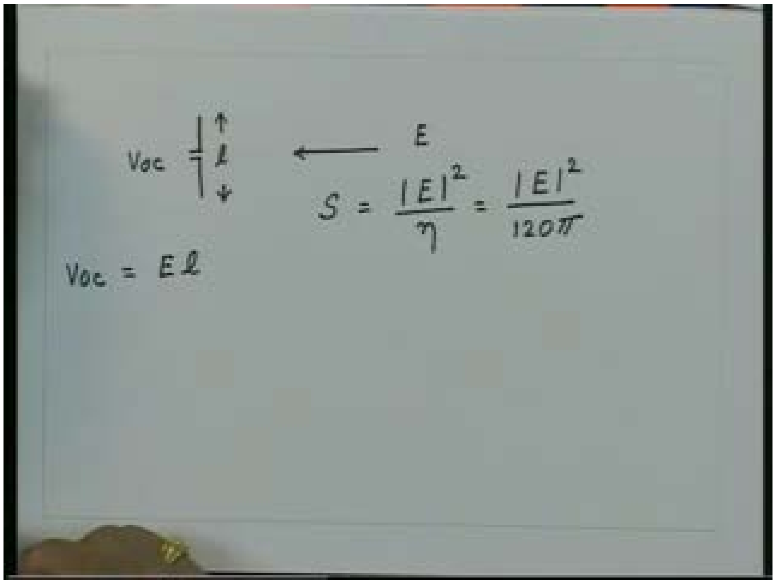

Now if I look into the terminal of the Hertz dipole we see a resistance which is nothing but the radiation resistance which you already derived so this is essentially equivalent to a voltage source and a resistance which is the radiation resistance so the maximum power which I can extract from here that is P_L from this antenna will be mod v_{oc} square divided by four into the radiation resistance of the Hertz Dipole. Now I can substitute for v_{oc} which is mod E square l square upon four and radiation resistance is 80 π square into λ upon l whole square.

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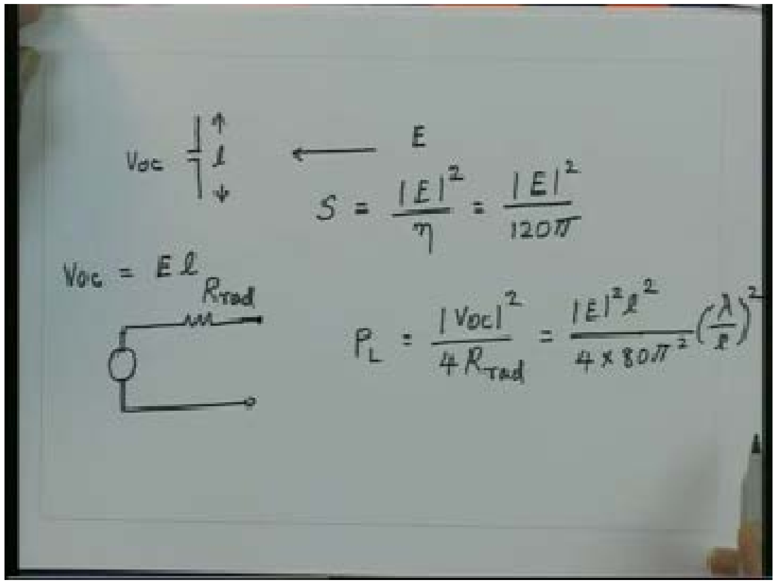

Now I know the Poynting Vector of this wave which is coming and incident on the antenna, also I know the power which can be maximally extracted from the Hertz Dipole which is given by this so for this antenna the effective aperture A effective will be P_L divided by the Poynting Vector so the E square will cancel out, the l square has cancelled out this so I get from here λ square 3/8 into λ square upon π.

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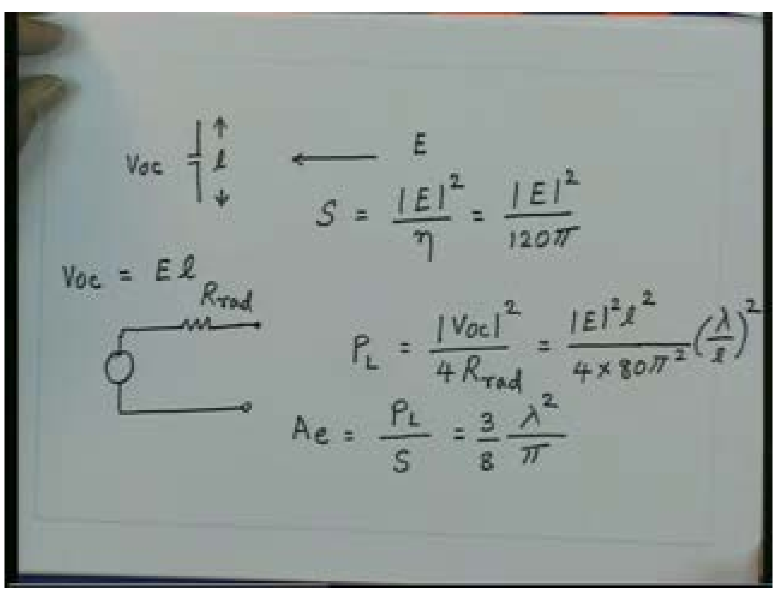

Firstly, the interesting thing to note here is for the Hertz Dipole the effective aperture is independent of the length of the dipole you see no where the dipole is coming into picture that means as long as the current is uniform across the dipole the assumption of Hertz Dipole is valid the effective aperture has nothing to do with the physical size of the antenna that is a very interesting property of the Hertz Dipole and same is true also for the directivity of the Hertz Dipole the directivity also is not a function of the length of the antenna. So both these quantities the directivity and the effective aperture for the Hertz Dipole are not functions of the physical size of the antenna.

The second interesting thing to note here is the effective aperture essentially scales as λ square so for a given antenna since the length is not coming into picture as I decrease the frequency of the antenna that means as the wavelength increases the effective aperture of the antenna goes on increasing. So if I take a small piece of wire and as the frequency decreases essentially λ is the one which is going to increase and the effective aperture which is offered by the antenna will increase and that is a very interesting characteristic of the Hertz Dipole.

However, now solving this two that getting this quantity the directivity of the Hertz Dipole which is 3/2 and the effective aperture which is this then I can substitute in this relation and find out this constant k the proportionality constant so now we get the proportionality constant k that is the directivity which is 3/2 divided by the effective aperture which is 3/8 into λ square upon π, here 3 will gets cancel and 2 will gets cancel so I will get 4π. So this will be equal to 4π upon λ square.

By substituting these things here then I can get a general relation between the directivity and the effective aperture for any antenna and that will be the directivity upon effective aperture for an antenna will be equal to 4π upon λ square or the directivity is equal to 4π into effective aperture upon λ square.

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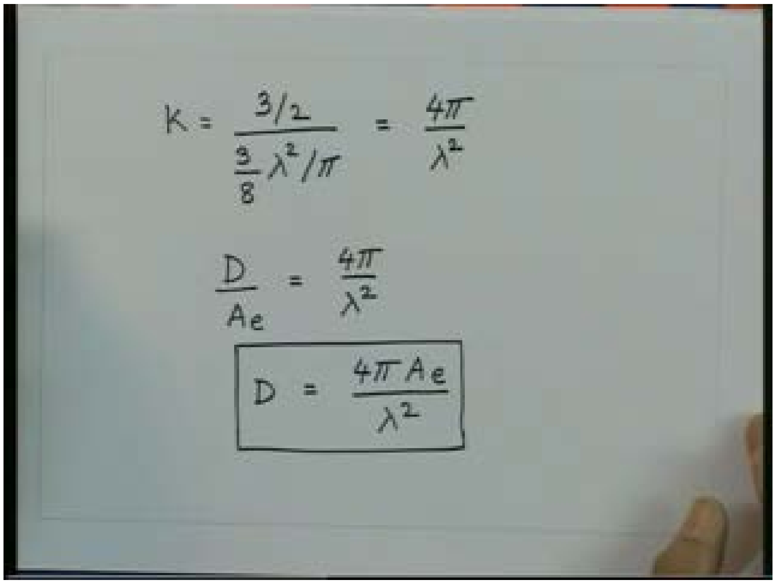

This is the relation which essentially relates the effective aperture of an antenna and the directivity of the antenna and as we have seen that there is a proportionality relationship that means larger the directivity of the antenna higher will be the effective aperture of the antenna and directivity is related to the beam width of the antenna that means narrower the beam of the antenna higher will be the effective aperture of the antenna.

Now although we have derived this relationship for the maximum direction which was typically maximum radiation and that is how directivity was coming into picture, if you define a parameter called a directive gain which is the ratio of the radiation intensity in the z direction divided by the average radiation intensity then also this relation would be true only thing this quantity will not be directivity this will be directive gain in that particular antenna. So now we can say that one antenna has given of course when everything is matched I get effective aperture I get directivity of the antenna but then I can say that if I do not have a radiation coming from the direction of maximum reception but suppose it was coming from some arbitrary direction antenna is not going to respond maximally and so was the case when the antenna was transmitting it would not have transmitted maximum field in that direction but it would have given me some enhancement or decrease in that direction which I call the directive gain. So if the antenna is receiving a wave from some angle θ and ø so let us say I have some antenna here and the wave is coming from some angle which is θ and ø. Then I can define the effective aperture with the antenna offers to this wave coming from θ, ø direction that effective aperture is A_e which is a function of theta θ and ø, and I have a directive gain for this antenna also where the antenna is transmitting which has a variation G(θ, ø).

This relation is true for any direction and for any antenna. So we can say that the directive gain of any antenna is equal to 4π into effective aperture for that direction divided by λ square. When we take a direction which is for the maximum reception or maximum radiation the G will become equal to directivity, this will become the effective aperture of the antenna for maximum radiation that will give me essentially this relation which is this.

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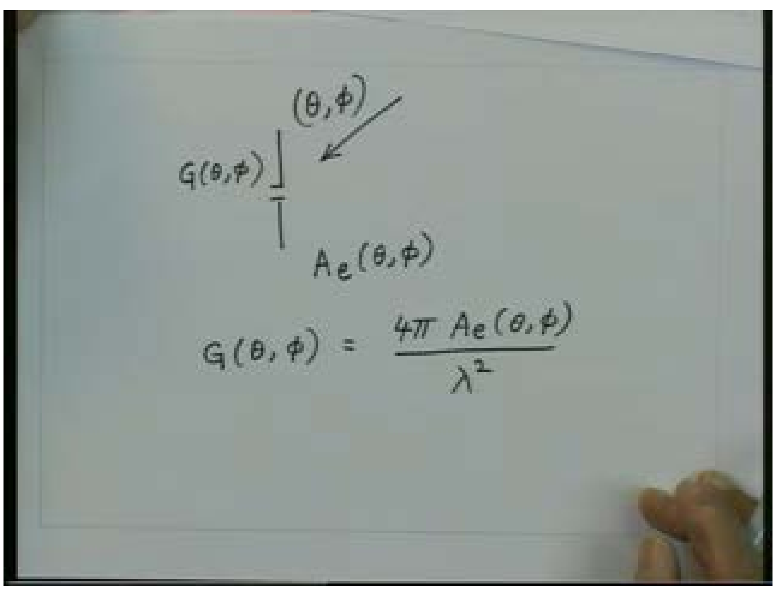

So the relationship between the directive gain we can call this quantity as the directive gain maximum value of G(θ, ø) is nothing but Directivity by definition. So this relation is a very useful relation in the antenna analysis because it relates essentially the parameters of an antenna when in transmitting mode and its parameter when the antenna is in receiving mode. So this relationship comes very handy when essentially we do the calculation for the antennas in transmitting and receiving modes.

Just to take an example here, if I take a parabolic dish and find out what would be the effective aperture and the directivity of the antenna. Let us say if I take a simple parabolic dish which is circular and let us say a diameter D the beam which is produced by this antenna is approximately in radians it is λ/D so you get θ produced by this will be approximately the beam width or the half power beam width will be λ/D radians.

We have circular beam so I can get the area of this beam essentially π into θ upon 2 whole square so now I can get the directivity for this antenna which will be equal to 4π upon the solid angle of this beam which is approximately π into λ/2D whole square. So from here the π will gets cancel so this you will get 16 D square upon λ square.

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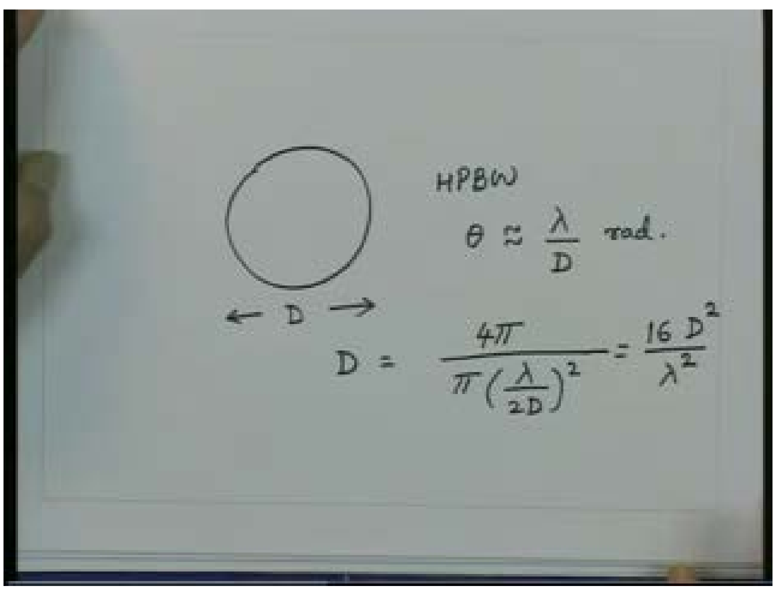

So what should be the effective aperture of this antenna A effective is λ square upon 4π into D so that is equal to λ square upon 4π into the directivity of the antenna which is

16D square upon λ square where λ square will gets cancel so this will be equal to 16 upon 4π into D square.

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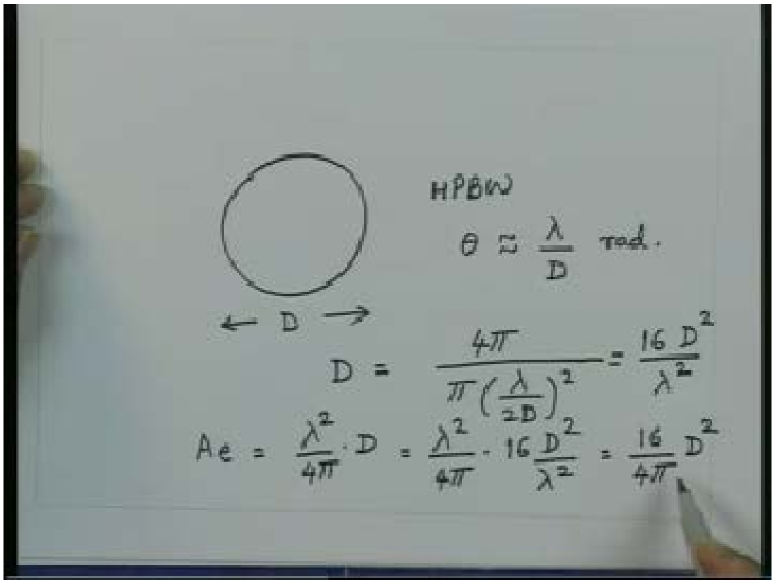

So if you see this quantity is close to about this square and so is the physical area if I calculate it is π upon 4 D square so this quantity is now very close or if I cancel this, this will become four so this will be equal to 4 D square upon π. So the physical area of this parabolic dish is very close to the effective aperture for this antenna.

However, as I mentioned this relationship is only true for the antennas which are having aperture kind of thing in general when I take a dipole which are very thin the physical area is very small but its effective area could be large depending upon its directivity or the length of the dipole and so on. So this relation which we derive today between the directivity of the antenna and the effective aperture is an extremely important relation because using this relation then I can go from properties of transmitting antenna to the receiving antenna and vice versa.

Thank you.