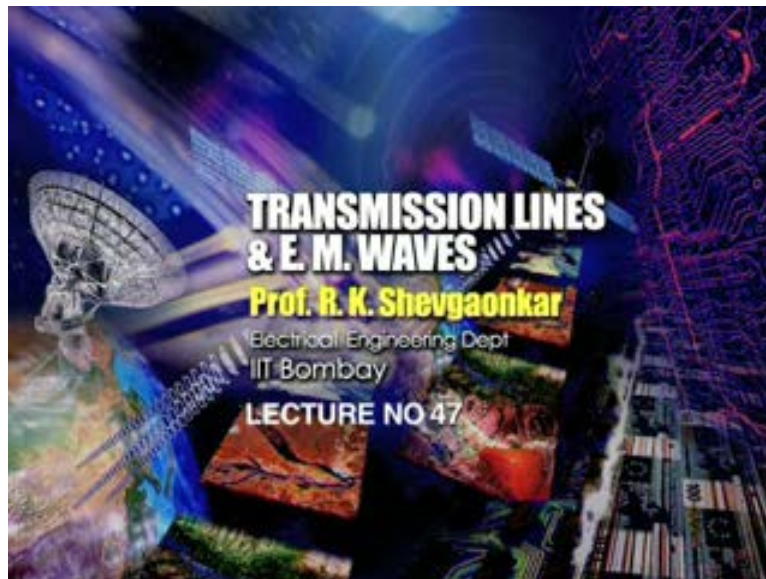


**Transmission Lines and E.M. Waves**  
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**Lecture-47**

Welcome, in the previous lecture we investigated the characteristics of the small current element called the Hertz Dipole we saw that Hertz Dipole is not a very efficient radiator because the Radiation Resistance is very small. So if you have a dipole of length  $0.1\lambda$  it gives you a Radiation Resistance of only about eight ohms. So in practice the Hertz Dipole does not have much utility.

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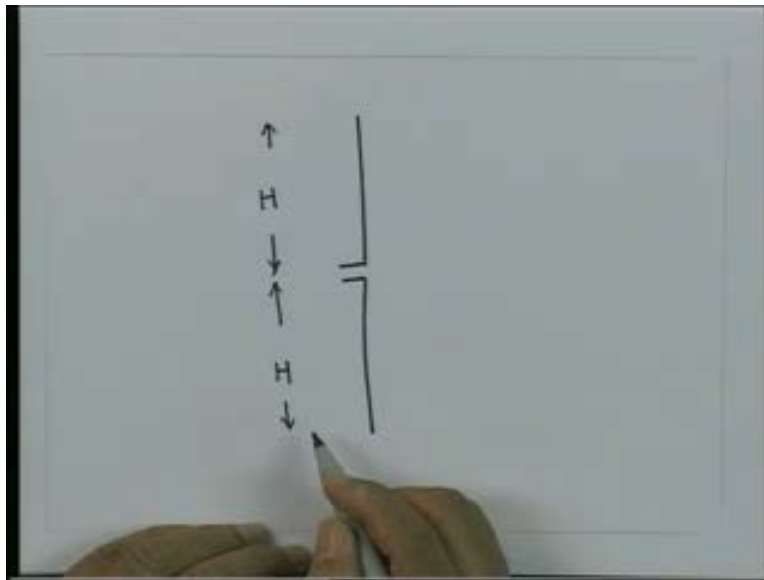


However, the Hertz Dipole has now created a foundation for investigating more complex antenna structure. So the next logical step from the Hertz Dipole is to investigate the radiation characteristic of a linear thin dipole that means if I consider a piece of wire which is having a length comparable to the wavelength then what would be its radiation characteristic that means radiation pattern, what would be the input impedance of this

antenna, what kind of polarization will be created by the antenna, are the issues if we investigate then this antenna can be used in practice.

So dipole antenna is one of the antennas which find a wide application in the field, in fact sometime back most of the TV antennas were essentially the dipole antennas. So as the name suggests this is the dipole that means there are two halves of this antenna and this antenna is excited at the centre so essentially the structure is a linear wire which is excited at the centre either by a voltage source or a current source and let us say the length of this wire is  $2H$  that means half of this dipole is  $H$  so we have this is  $H$  half length and this is  $H$ .

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And if you can excite this at the centre, because of the excitation at this point the current is going to spread on this wires and as you have seen the extreme case of Transmission Line the dipole antenna can be visualized so without going into much detail of how the current distribution is obtained on the dipole we will essentially investigate the radiation characteristics assuming that the current distribution on the dipole is known.

As I mentioned earlier the current distribution calculation is rather complex issue in fact you require methods like method of moments or other numerical techniques to find out the current distribution on this wire in a self consistent manner. However, once the current distribution is given to you then finding the radiation pattern and input impedance is very straight forward thing.

So in this course we do not investigate how the current is calculated on the dipole we assume by some crude arguments the variation on the dipole and once the variation is known then we can visualize this dipole as collection of small small Hertz Dipoles and then by simply applying superposition we can find out the total electric field due to this dipole.

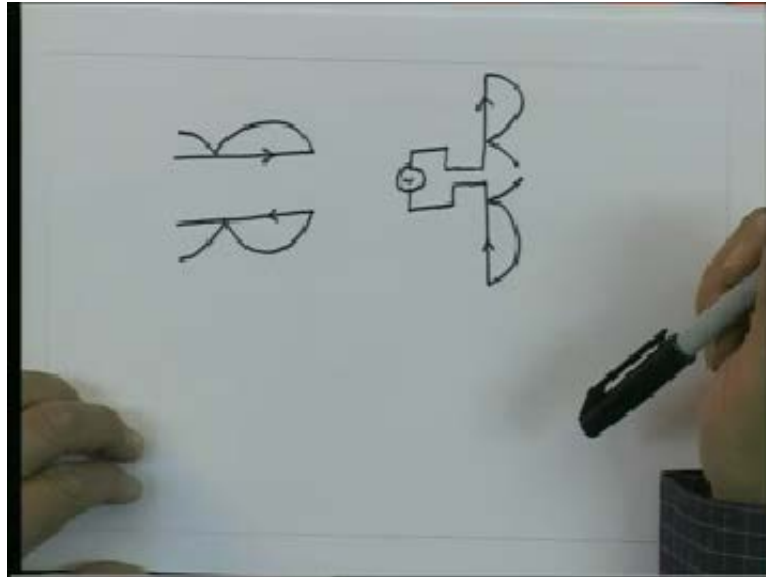
So as we had put forward the argument that if you consider a Transmission Line which is open circuited at this end then you have a reflection at this end of Transmission Line so the current is zero at this point and you have a standing wave which will be created on the Transmission Line. Similarly this will happen on the other wire of Transmission Line so the current is flowing here this way, the current will be flowing in the opposite direction in the second wire.

Now if you slowly flare up so that ultimately it becomes completely opened up version which is the dipole then this thing can be visualized as a flaring up version of Transmission Line and the current distribution still is this where this current will be flowing upwards so this current also flows upwards. So essentially the current which is going to be here they will be in the same direction so here the current flows like that here the current flows like that so the current will be coming out of this the current will be going inside this so you can think of this as a voltage or current source to the input terminals of the antenna.

Now the rigorous calculation also show as I mentioned the rigorous methods like method of moments or some other methods that if the dipole is very thin then the current essentially flows along the length of the dipole and then in an ideal case when the

diameter of the dipole is practically zero the current distribution on this dipole is almost sinusoidal.

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So we know the distribution that we can see from the flaring of photo transmission line also that means this is the standing wave when we have a sinusoidal distribution on this which is a standing wave pattern this pattern remains even when the antenna is completely flared up to become a dipole.

Now at this point it is not very convincing that when the Transmission Line is completely flared up still the arguments like standing wave patterns are valid but it so happens that even the rigorous solution gives you the same kind of current distribution as you would have got from the standing wave pattern on a Transmission Line.

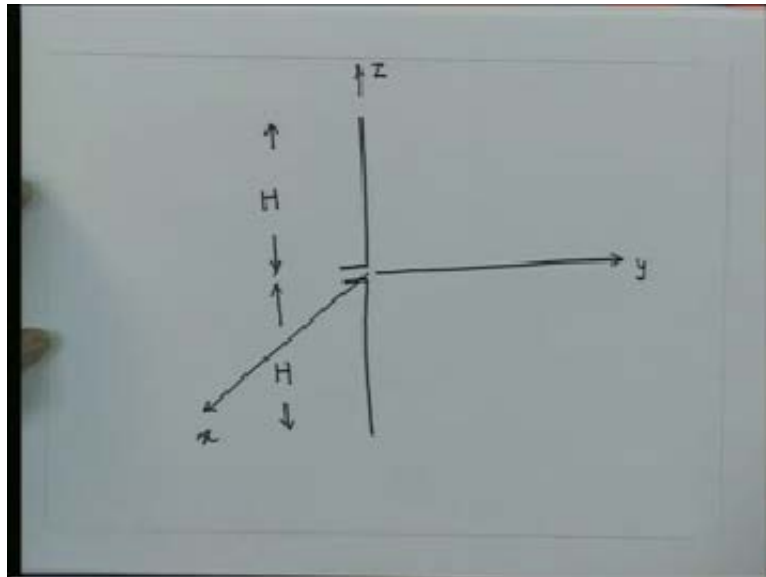
So, now we know the current distribution and few logical tests we can conduct on the current distribution and that is when we go to the tip of the antenna the current must go to zero because there is no path for the antenna and that is also true in the case of transmission line that if you have a open circuited Transmission Line the current at the

open circuit end must go to zero. Also we know that on the Transmission Line the current is symmetric so two conductors will have equal in opposite direction for the current, however, when you have a flared up version of this the currents travel in the same direction so you have a symmetry of the current distribution on the dipole and you have the current zero at the tip of the dipole.

So essentially these are two conditions under which one can investigate the problem that means the input current which flows on the antenna is not in the control of the designer what you can do is you can simply design the length of this antenna since the current has to go to zero at the end of the antenna and this function is a sinusoidal function depending upon the length it might have certain value of the current at the input terminal of the antenna. So input terminal current is decided by the length of the antenna there is no independent control of controlling the current for given voltage excitation.

Once you have this current distribution then we can visualize the dipole antenna at the collection of the Hertz Dipoles. So let us say first I define the coordinate system, as we have done earlier this is let us say y direction, this is x direction and this is z direction so I am assuming that the dipole antenna is oriented along the z direction and it is having a current which is  $I(z)$  which will be given by the standing wave pattern we have got on the Transmission Line.

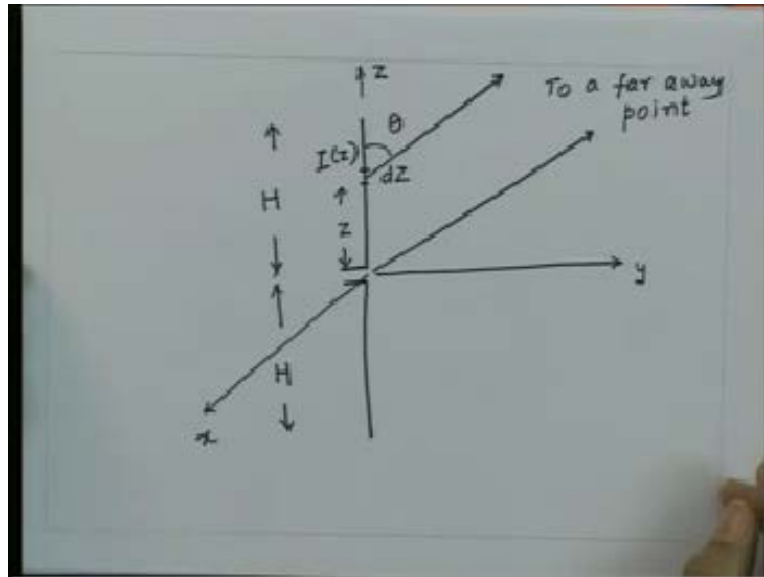
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Now as we said this dipole can be visualized as a collection of small Hertz Dipoles so let us say we divide this into small pieces and added height of  $z$  from the centre of the dipole we have infinitesimally small current let us say that is  $dz$  and the current which flows in this dipole is given by  $Iz$  so here we have current which is a function of  $z$  and then there is a small current element we are considering at a distance  $z$  from the origin along the dipole  $dz$ . So the field at a very far away point because we are considering only the radiation fields so essentially this should go to a distance substantially far away from the dipole so that the electrostatic and the induction fields have reduced to very small value so essentially we have got only radiation field that means we go very far away from the antenna theoretically we should going to infinite distance from this so that the two other fields go to zero and then we apply the superposition of the fields due to each of this small current elements.

So let us say I measure all the distances from the centre of this dipole let us say I want to measure a field which has a very far away point in the direction which makes an angle  $\theta$  with respect to the  $z$  axis so these two are to a far away point.

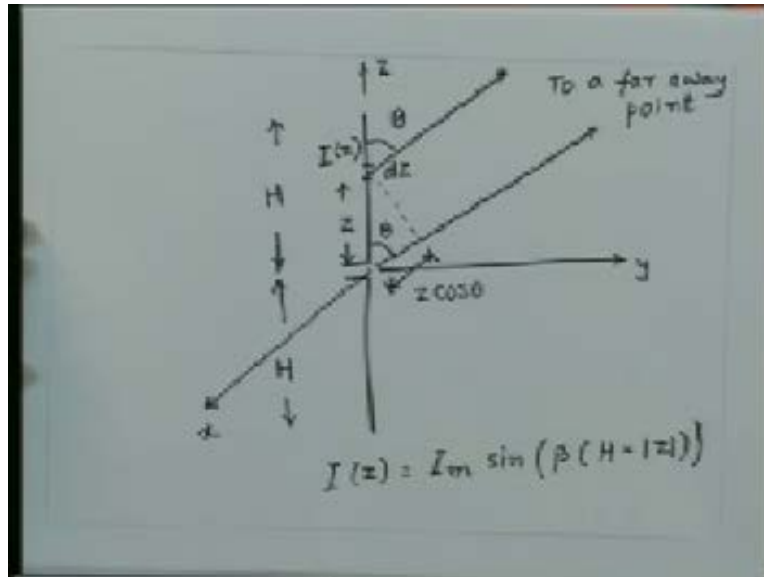
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Since the point is very far away these two lines are almost parallel so if I drop a perpendicular from this point on this line then the field which is generated from this current element will be traveling a distance shorter compared to reference point which is the centre of the antenna or in other words the field which are induced because of this the radiation fields at far away point will be leading in phase corresponding to a distance which is this distance. So if this angle is  $\theta$ , this angle is also  $\theta$  so this distance which you have here is equal to  $z \cos\theta$  so we have got a distance which is  $z \cos\theta$  because this angle is also as  $\theta$ . So what that means is the radiation coming from this Hertz Dipole with respect to the centre of the antenna leads in phase corresponding to the distance which is  $z \cos\theta$  so while superposing the field we have to put an additional phase which is corresponding to this distance.

Now the current distribution which you have on the antenna is from the standing wave pattern so this thing can be written as  $I(z)$  is equal to some maximum value  $I_m \sin[\beta(H - |z|)]$ . You can verify this is mod of  $z$  so when  $z = 0$  you have a current which is  $I_m \sin\beta H$ , however, when  $z$  is equal to plus or minus  $H$  then the current will go to zero so we will have a current distribution which will be something like that.

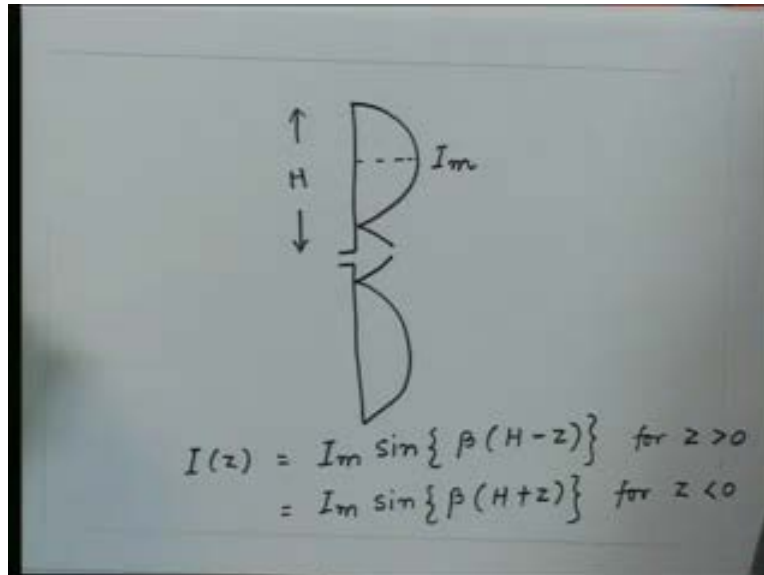
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So the current distribution for the dipole where this is the value which is  $I_m$  and the height of this antenna is  $2H$  that means half of the height  $H$  this is the current distribution which you have that is essentially given by this so I know now the current distribution on the dipole for which I write down explicitly for  $z < 0$  which is lower half of the dipole and  $z > 0$  which is for the upper half of the dipole I can explicitly write the current which is  $I(z) = I_m \sin\{\beta(H - z)\}$  for  $z > 0$  and  $I(z) = I_m \sin\{\beta(H + z)\}$  for  $z < 0$ .



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So this thing which you have written the current distribution now we can explicitly write this for the lower half of the dipole which is for  $z < 0$  and for the upper half of the dipole which is for  $z > 0$ .

Once we know this current distribution as we said we simply apply and find out the field because of the infinitesimally small current element find out the extra phase which is coming because of the distance and simply superimpose the field over all antenna elements so we will get total field at a far away point which will be the electric field due to the entire dipole.

So let us say the Hertz Dipole which you have got that is this Hertz dipole and now we are treating this like an incremental element so the electric field which is generated by this current element  $E_\theta$  we know that the Hertz Dipole which is oriented in  $z$  direction generates an electric field which has only  $\theta$  component so since all these elements are oriented in  $z$  direction all the elements are going to generate only  $\theta$  oriented components so if I go to every far away point the electric field due to all of these elements are going to be in the same direction. So essentially we require only an algebraic addition of the

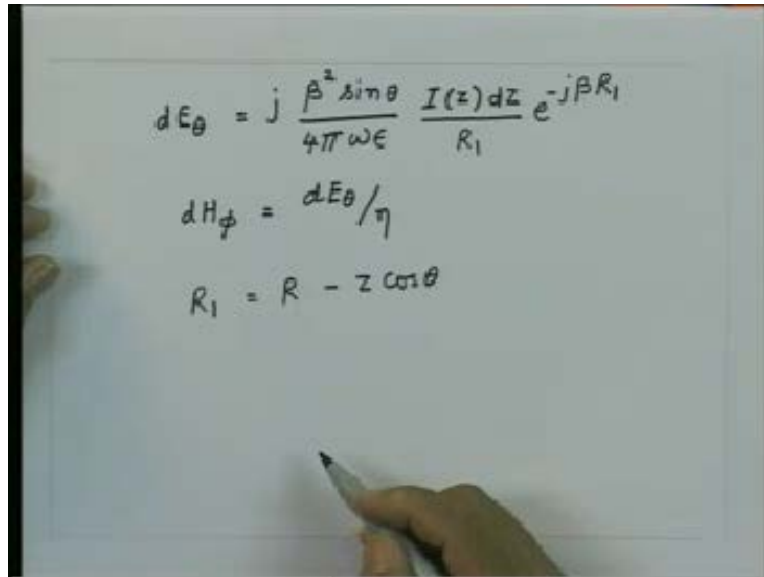
electric fields because they all are oriented in same direction the  $\theta$  direction and apply with the proper application of the phase which is coming because of the extra length introduced by the path.

So the infinitesimally small current elements the Hertz dipole fields as we derived let us call that as  $dE_\theta$  will be equal to  $j\beta \sin\theta$  divided by  $4\pi\omega\epsilon$  into  $I(z)$  into  $dz$  that is now the current element, at location  $z$  the current flowing is  $I_z$  this is the length of the current element so this is the current moment for that infinitesimally small element divided by  $R_1$  that is the distance of the point from this and this is the distance from the centre is given by  $R$  so here this will be  $R_1$  e to the power minus  $j\beta R_1$  and as we know the wave radiation field essentially generates a transverse electromagnetic wave so the magnetic field is related to the electric field through the intrinsic impedance of the medium so the magnetic field  $dH_\phi$  will be nothing but  $dE_\theta$  divided by the intrinsic impedance of the medium.

So we do not explicitly calculate the magnetic field for the dipole we simply calculate only the electric field and as we know this relation as and when the magnetic field has to be calculated we can divide the electric field by the intrinsic impedance of the medium and we can get the magnetic field. So essentially we do most of the discussions only for the electric field when ever we discuss the radiation characteristics of antenna

Now as I mentioned the  $R_1$  is the distance which is shorter compared to  $R$  by this distance. So I have  $R_1 = R - z \cos\theta$ , now I can substitute for  $R_1$  because I am measuring all the distances from the centre of the dipole. So this quantity will be e to the power  $-j\beta R - z \cos\theta$  and this will be again  $R - z \cos\theta$ .

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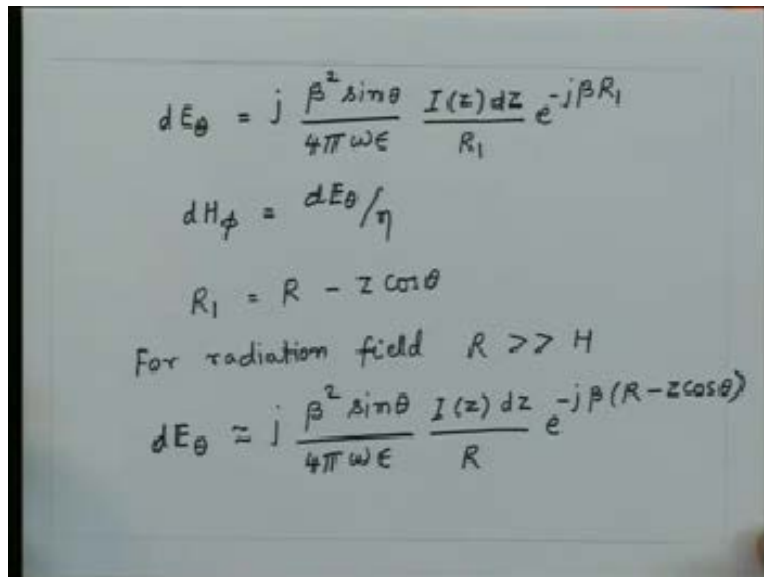
The image shows a whiteboard with three handwritten equations. The first equation is  $dE_{\theta} = j \frac{\beta^2 \sin \theta}{4\pi \omega \epsilon} \frac{I(z) dz}{R_1} e^{-j\beta R_1}$ . The second equation is  $dH_{\phi} = dE_{\theta} / \eta$ . The third equation is  $R_1 = R - z \cos \theta$ . A hand holding a pen is visible at the bottom of the frame.

$$dE_{\theta} = j \frac{\beta^2 \sin \theta}{4\pi \omega \epsilon} \frac{I(z) dz}{R_1} e^{-j\beta R_1}$$
$$dH_{\phi} = dE_{\theta} / \eta$$
$$R_1 = R - z \cos \theta$$

Now what we do is if you look at these two quantities, one is this one which is coming in the denominator here the  $R$  is coming in absolute term what it means is that the electric field is going to change equivalent to replacing this  $R_1$  by  $R$  then there will be correction which will be coming due to this quantity. Now since we are considering a distance which is very far from the antenna the maximum  $z$  we can have on this antenna is equal to  $H$ , this  $H$  is much smaller compared to  $R$  that is what we have in practice for radiation. So we have  $R$  is much more greater than  $H$  for radiation field and since  $z$  is always going to be greater than or equal to  $H$  this  $R$  is much more greater than  $H$  typical example is if I take dipole antenna which is having a size of say  $\lambda$  or  $\lambda/2$  let us say if I am operating at a frequency of hundred mega hertz the wavelength will be three meters the dipole size will be the order of 1.5 meters or two meters or three meters or something like that, however, when I take the dipole antenna for sending radiation I will be sending radiation over distances of few tens of kilometers so  $R$  will be of the order of kilometers whereas  $H$  will be order of only few meters. So this condition  $R$  is much greater than  $H$  is very well justified when we go in the calculation of the radiation field.

So this quantity  $R_1$  can be replaced by  $R$  because this quantity is much smaller than this quantity in this term. Can we do the same thing here can we replace  $R_1$  by  $R$  in this expression also, the answer is no, we cannot do that the reason is we are talking about the change in distance but this is related to  $\lambda$  because when you multiply this quantity by  $\beta$  this is going to be  $\beta z \cos\theta$  which is a phase and this absolute phase does not have any meaning when you talk about a phase change comparable to  $\pi$  you have got essentially the cancellation of the two fields. So here when we talk about the approximation here the distances absolutely do not come into picture they are relative distances because this is the phase quantity and when the phase becomes comparable to  $\pi$  you cannot really neglect the effect of the phase change, however, in this case the amplitude does not get significantly affected because this  $R$  is much greater than  $H$ . So with this understanding we can now make a approximation to the field  $dE_\theta$  which is approximately  $j \beta^2 \sin^2\theta$  upon  $4\pi\omega\epsilon$  into  $I(z) dz$  upon  $R$  where I am replacing  $R_1$  by  $R$  in this and this will be e to the power  $-j\beta(R - z \cos\theta)$ .

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The image shows a whiteboard with handwritten mathematical derivations. The equations are as follows:

$$dE_\theta = j \frac{\beta^2 \sin\theta}{4\pi\omega\epsilon} \frac{I(z) dz}{R_1} e^{-j\beta R_1}$$

$$dH_\phi = dE_\theta / \eta$$

$$R_1 = R - z \cos\theta$$

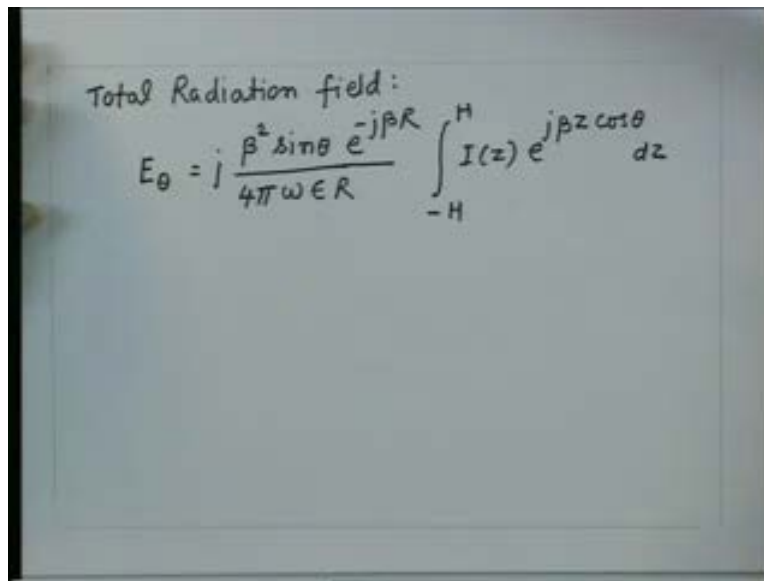
For radiation field  $R \gg H$

$$dE_\theta \approx j \frac{\beta^2 \sin\theta}{4\pi\omega\epsilon} \frac{I(z) dz}{R} e^{-j\beta(R - z \cos\theta)}$$

Now all these quantities are not functions of  $z$  except this phase term and this current distribution. Then as we said the total field which was going to get here is superposition

of this field's phase term is now accounted for this  $z \cos\theta$  so the total field which you are going to get due to this dipole will be integral of this  $d\theta$  over the total length. So we get the total radiation field  $E_\theta$  is equal to  $j \beta^2 \sin\theta$  upon  $4\pi\omega\epsilon R$  into  $e$  to the power  $-j\beta R$  into integral  $-H$  to  $+H$   $I(z) e^{j\beta z \cos\theta} dz$ .

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Handwritten equation for the total radiation field  $E_\theta$ :

$$E_\theta = j \frac{\beta^2 \sin\theta e^{-j\beta R}}{4\pi\omega\epsilon R} \int_{-H}^H I(z) e^{j\beta z \cos\theta} dz$$

It has to be substituted for this  $d\theta$  and integrated over the total length of the dipole which is from  $-H$  to  $H$  so from here I just find out the integral from  $-H$  to  $+H$  and I get the total electric field from the dipole.

Now we can substitute for the current of the two half as we have got here for  $z < 0$  and  $z > 0$  separately and can find out the total integral. If we do that then this quantity can be written as  $j 60 I_m$  where all constants which are there they have gone into that like  $\beta$  will be related to the frequency and the permittivity and permeability of the medium so this will be  $I_m e$  to the power  $-j\beta R$  divided by  $R$  and will be some function of  $\theta$  which we can write explicitly. The reason for doing this is this is the amplitude term which you are going to get at a distance of  $R$  but as we saw we are interested in essentially the relative distribution of the field at the function of the angle what we call as the radiation pattern

so this function of  $\theta$  will be essentially will be the radiation pattern for this dipole that is what we are interested.

How do you get this parameters here is we know this quantity  $\beta$  is  $\omega$  square root of  $\epsilon$  so we can take this term  $\beta$  square upon  $4\pi\omega\epsilon$  so this term will be  $\beta$  square upon  $4\pi\omega\epsilon$  will be equal to  $\omega$  square into  $\mu\epsilon$  divided by  $4\pi\omega\epsilon$  which is equal to  $\omega\mu$  upon  $4\pi$  or we can write this as  $\beta\eta$  for the intrinsic impedance of the medium also upon  $4\pi$  and when we integrate this we are going to get one  $\beta$  which will cancel with this  $\beta$  so essentially we will get just quantity  $\eta$  divided by this which is hundred and twenty five for the free space impedance so this will be equal to  $30\beta$ .

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Handwritten mathematical derivation of the total radiation field for a dipole antenna. The text is written on a whiteboard or paper. The derivation starts with the title "Total Radiation field:" followed by the equation for the electric field  $E_\theta$ . The equation is written in two forms: first as an integral, and then simplified using the intrinsic impedance  $\eta$ . The final result shows that the term  $\frac{\beta^2}{4\pi\omega\epsilon}$  simplifies to  $30\beta$ .

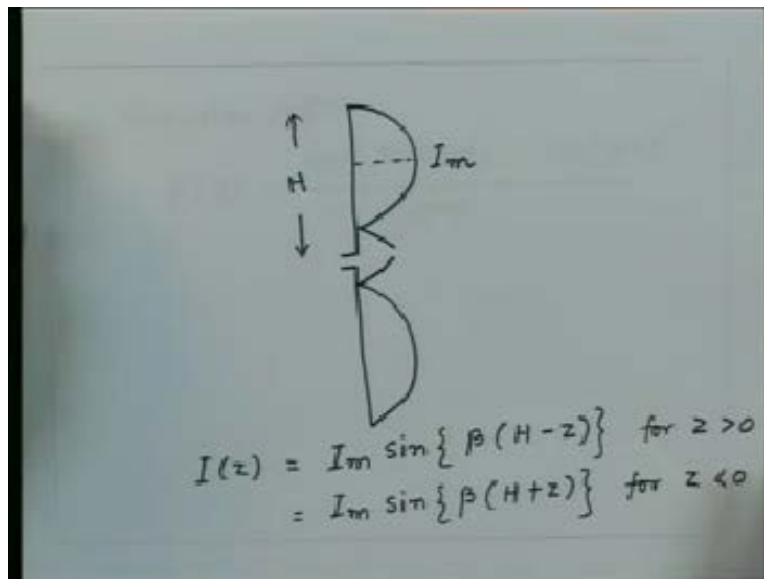
$$\begin{aligned} \text{Total Radiation field:} \\ E_\theta &= j \frac{\beta^2 \sin\theta}{4\pi\omega\epsilon R} e^{-j\beta R} \int_{-H}^H I(z) e^{j\beta z \cos\theta} dz \\ &= j 60 I_m \frac{e^{-j\beta R}}{R} F(\theta) \\ \beta &= \omega \sqrt{\mu\epsilon} \\ \frac{\beta^2}{4\pi\omega\epsilon} &= \frac{\omega^2 \mu\epsilon}{4\pi\omega\epsilon} = \frac{\omega\mu}{4\pi} = \frac{\beta\eta}{4\pi} = 30\beta \end{aligned}$$

So having this quantity  $\beta$  square upon  $4\pi\omega\epsilon$  equal to  $30\beta$  if I substitute here then this integral essentially will get simplified to this. So as I mentioned this term essentially gives you now a variation of the amplitude of the electric field as a function of  $R$  and that we have seen for the Hertz Dipole which varies as one over the distance, in this case also the same thing is true that the electric field amplitude will be inversely proportional to a distance from the centre of the dipole. We also have a phase term which is  $e$  to the power

$-j \beta R$  which is same as what we get for a spherical wave which has traveled a distance  $R$  from the source.

So we are not worried about this term if we are interested in finding out the absolute amplitude for the electric field then we can make use of this. However we are more interested in this quantity called the radiation pattern and this one when we solve this integral we get called radiation pattern as we have mentioned the radiation pattern is a normalized quantity so this total constant does not really matter we want only relative variation of the field amplitude as a function of angle so radiation pattern would be  $F(\theta)$  equal to  $\cos(\beta H \cos\theta) - \cos\beta H$  divided by  $\sin\theta$ . If we just solve this integral that is the thing essentially we are going to get as a function of  $\theta$ .

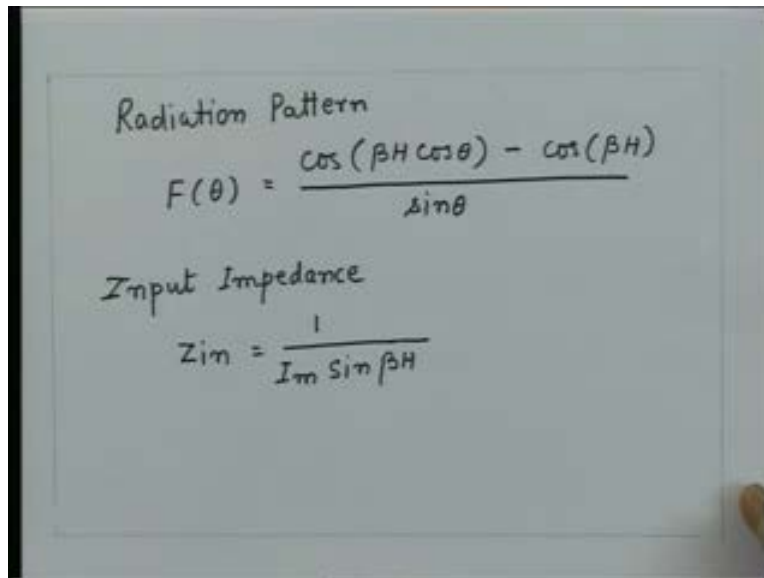
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So now we get a radiation pattern for the Hertz Dipole and then we can also calculate what the input current is when the length of the Hertz dipole is given to you so the two quantities in which you are interested in, one is what is the relative distribution of the electric field as a function of angle other one was for a given length of the dipole how much is the input current for a given excitation of the voltage. So without losing

generality if I say that excitation is one volt the terminal of the dipole then the current what ever is there in the terminal one upon current will give me the input impedance of the dipole. So we get input impedance of the dipole which is  $z_{in}$  which will be one upon the current at the input of the dipole that is at  $z = 0$  so if I substitute  $z = 0$  in this  $I_m \sin \beta H$  will be the current.

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Handwritten notes on a whiteboard:

Radiation Pattern

$$F(\theta) = \frac{\cos(\beta H \cos \theta) - \cos(\beta H)}{\sin \theta}$$

Input Impedance

$$Z_{in} = \frac{1}{I_m \sin \beta H}$$

So, few things can be noted now immediately that the input impedance which in some sense has been a measure of power radiated by the antenna is no more a monotonic function of the length of the antenna. We saw that in the Hertz Dipole the radiation resistance was related to the length of the dipole in the square fashion that is the radiation resistance was proportional to the square of the length of the dipole. However, in this case the input impedance is having rather complex relation so it depends upon the value of  $H$  so you have a sinusoidal distribution on the dipole and then depending upon the value of  $H$  you may get some value of the input current.

So the input impedance may vary over a very wide range in fact when this quantity is  $\pi/2$  then that time the input impedance will be one upon  $I_m$  whereas when this quantity is zero



or  $\pi$  that time this will be zero and the input impedance will go to infinity because the current will go to zero so I may get the highest value of the input impedance which will be as high as infinity and we get the reverse value when this is maximum that is one so that will be equal to one upon  $I_m$  so essentially the input impedance may vary from one upon  $I_m$  and when this quantity is zero or  $\pi$  when  $\beta H = \pi$  but  $\beta H$  is equal to  $2\pi/\lambda$  into  $H$  so when  $\beta H$  becomes  $\pi$  will be when  $H$  is equal to  $\lambda/2$ . So this  $\lambda$  will cancel two will cancel will get pi so the total length of the antenna will be  $2H$  which will be equal to  $\lambda$  so if I consider a dipole of one wavelength long then its input impedance will be infinity because in that case the current will go to zero at the terminal.

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Handwritten notes on a whiteboard:

**Radiation Pattern**  

$$F(\theta) = \frac{\cos(\beta H \cos \theta) - \cos(\beta H)}{\sin \theta}$$

**Input Impedance**  

$$Z_{in} = \frac{1}{I_m \sin \beta H}$$

$$\beta H = \frac{2\pi}{\lambda} \cdot H$$

Diagram showing the relationship between  $Z_{in}$  and  $H$ :

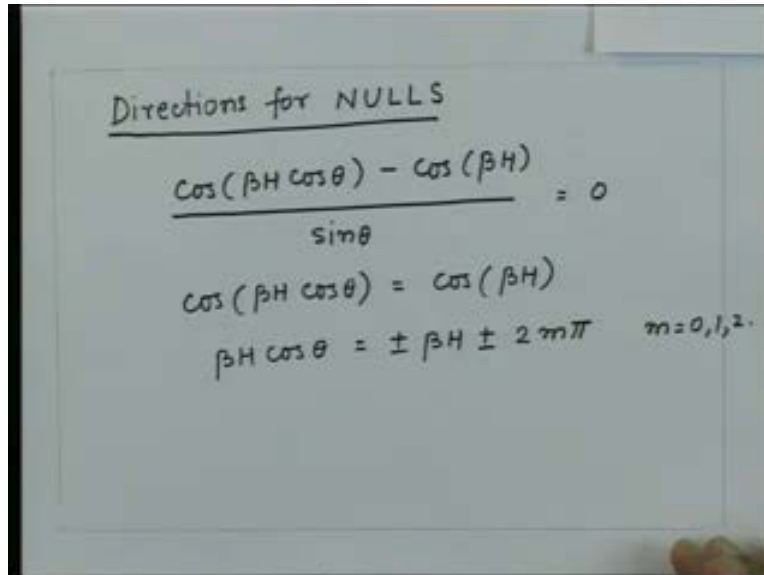
$Z_{in} \rightarrow \frac{1}{I_m}$	$\rightarrow \infty$
$\downarrow$	$\downarrow$
$H = \lambda/4$	$H = \lambda/2$

Now if I consider  $H = \lambda/4$  then this  $\beta H$  will be  $\pi/2$  and I will get the minimum input impedance of the antenna. So this value essentially corresponds to  $H = \lambda/4$  and this value corresponds to  $H = \lambda/2$ , of course these are not the unique values this quantity will go to zero again for  $2\pi$  and multiples of that so as the length of the antenna increases we will see that the impedance will become one upon  $I_m$  again it will go to infinity again it will go to one upon  $I_m$  and so on.

So there is no monotonic trend for the variation of the input impedance as the length of the dipole changes. What happens to the radiation pattern when we change the length of the antenna? That is also very complicated again immediately it is clear that as the length of the antenna increases this function might have multiple maxima and this function might go to zero many times at the function of angle  $\theta$ . So normally instead of investigating the radiation pattern in great detail what we normally do very roughly is we try to see what are the directions in which the radiation pattern becomes zero that means what are the directions in which there is no electric field if you take two conjugative directions where the electric field has gone to zero then in between these two directions the electric field must have gone to maximum somewhere so the calculation of the maximum electric field or the direction of electric field is rather tedious task but finding the direction where the electric field goes to zero is rather simpler because we can equate this radiation pattern to zero and we can find out directions for which electric field would go to zero then we say roughly between two conjugative directions of zero field there must be a direction where the electric field goes to maximum.

These directions where electric field goes to zero is called the directions of **nulls** so null in radiation pattern is defined as that direction in which the electric field is zero. So we can get the directions for nulls by equating the radiation pattern of this dipole to zero. So I can get this by  $\cos(\beta H \cos\theta) - \cos\beta H$  upon  $\sin\theta$  is equal to zero or from here I have  $\cos(\beta H \cos\theta)$  will be equal to  $\cos\beta H$  or  $\beta H \cos\theta$  will be equal to plus or minus  $\beta H$  and plus or minus  $2m\pi$  where  $m$  is an integer which is equal to 0, 1, 2 and so on.

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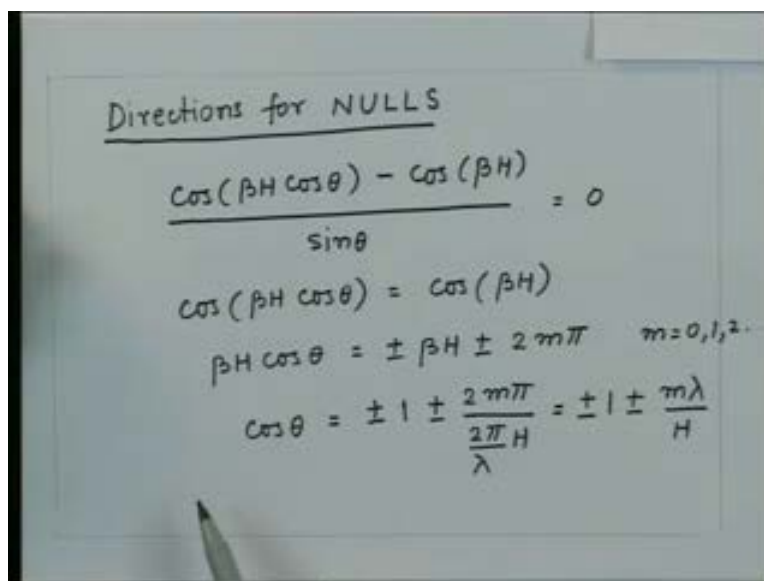
A photograph of a whiteboard with handwritten mathematical derivations. The title 'Directions for NULLS' is underlined. The first equation is  $\frac{\cos(\beta H \cos \theta) - \cos(\beta H)}{\sin \theta} = 0$ . The second equation is  $\cos(\beta H \cos \theta) = \cos(\beta H)$ . The third equation is  $\beta H \cos \theta = \pm \beta H \pm 2m\pi$  with  $m=0,1,2$ .

Directions for NULLS

$$\frac{\cos(\beta H \cos \theta) - \cos(\beta H)}{\sin \theta} = 0$$
$$\cos(\beta H \cos \theta) = \cos(\beta H)$$
$$\beta H \cos \theta = \pm \beta H \pm 2m\pi \quad m=0,1,2$$

From here, then I can find out the directions of the nulls cos of this angle null that will be equal to  $\pm 1 \pm 2m\pi$  or  $-2m\pi$  upon  $\beta H$  where  $\beta$  is  $2\pi/\lambda$  into  $H$  so that  $2\pi$  cancels this will be  $\pm 1 \pm \lambda m/H$ .

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A photograph of a whiteboard with handwritten mathematical derivations. The title 'Directions for NULLS' is underlined. The first equation is  $\frac{\cos(\beta H \cos \theta) - \cos(\beta H)}{\sin \theta} = 0$ . The second equation is  $\cos(\beta H \cos \theta) = \cos(\beta H)$ . The third equation is  $\beta H \cos \theta = \pm \beta H \pm 2m\pi$  with  $m=0,1,2$ . The fourth equation is  $\cos \theta = \pm 1 \pm \frac{2m\pi}{\frac{2\pi H}{\lambda}} = \pm 1 \pm \frac{m\lambda}{H}$ .

Directions for NULLS

$$\frac{\cos(\beta H \cos \theta) - \cos(\beta H)}{\sin \theta} = 0$$
$$\cos(\beta H \cos \theta) = \cos(\beta H)$$
$$\beta H \cos \theta = \pm \beta H \pm 2m\pi \quad m=0,1,2$$
$$\cos \theta = \pm 1 \pm \frac{2m\pi}{\frac{2\pi H}{\lambda}} = \pm 1 \pm \frac{m\lambda}{H}$$

So we will have nulls whenever this condition is satisfied so knowing the value of  $H I$  can try for all possible values of  $m$  for which the  $\cos\theta$  the magnitude is less than or equal to one and that will give me directions for the nulls.

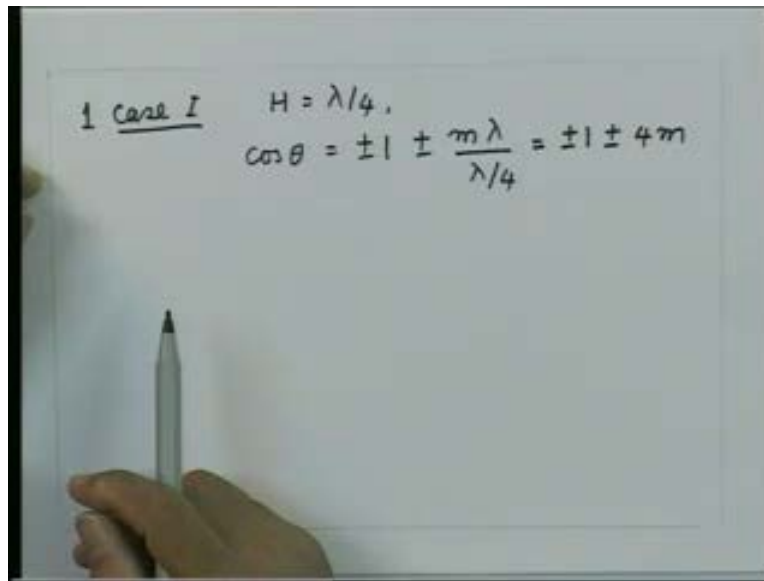
First thing we notice is as the  $H$  increases there will be more and more values of  $m$  for which you will have this quantity less than or equal to one that means there will be multiple directions in which the electric field will be zero so the number of nulls will increase as the length of the dipole increases. One thing, however, can be verified and that is when  $m = 0$ ,  $\cos\theta = \pm 1$  that means  $\theta$  corresponds to the direction  $\theta = 0$  or  $\theta = \pi$ .

So there is a null which is in the direction  $\theta = 0$  or  $\theta = \pi$ , however, one should notice that when  $\theta = 0$  or  $\pi$  even this quantity goes to zero the denominator goes to zero so just blindly we cannot say that there is a null for the  $\theta = 0$  because we have to actually find out what is the limit of this expression when  $\theta = 0$  or  $\pi$  and one can verify that when  $\theta = 0$  in fact the limit of this expression goes to zero. So there is indeed a null in the direction  $\theta = 0$  and  $\theta = \pi$  and that also makes physical sense that since the dipole is a collection of the Hertz Dipole and the Hertz Dipole has a null along its axis that means at  $\theta = 0$  and  $\theta = \pi$  the superposition of the field should be zero in that direction since the individual element which is the Hertz Dipole in which we divided this dipole into has null in the direction of  $\theta = 0$  and  $\theta = \pi$  every linear dipole also has null along its axis so if I go along this axis either  $\theta = 0$  or  $\theta = \pi$  a dipole will have null in that direction that means the dipole does not radiate along its length along its axis it always radiates in the direction which is not along the axis.

Does that it radiates always perpendicular to it as was in the case of Hertz Dipole? That is not straight forward, in fact as we find out the directions of the nulls and approximately we say between two nulls there will be one maximum so that we have to see for different lengths of the dipole what will be the radiation patterns and from there we can broadly try to visualize the radiation patterns.

So we can take some specific cases and find out the radiation patterns of the dipoles. Let us say if I take the length of the antenna which is  $\lambda/4$  this quantity  $H$  is now  $\lambda/4$  so dipole length is  $\lambda/2$ . So let us consider some length take case-I where  $H = \lambda/4$  that means the dipole is  $\lambda/2$  long so  $\cos\theta$  for the nulls will be equal to  $\pm 1 \pm m\lambda$  upon  $\lambda/4$  so that will be equal to  $\pm 1 \pm 4m$  where any value of  $m$  which is not equal to zero will make this quantity greater than one so that means there are nulls only corresponding to  $m = 0$  and these nulls are the same that  $\theta = 0$  or  $\theta = \pi$ .

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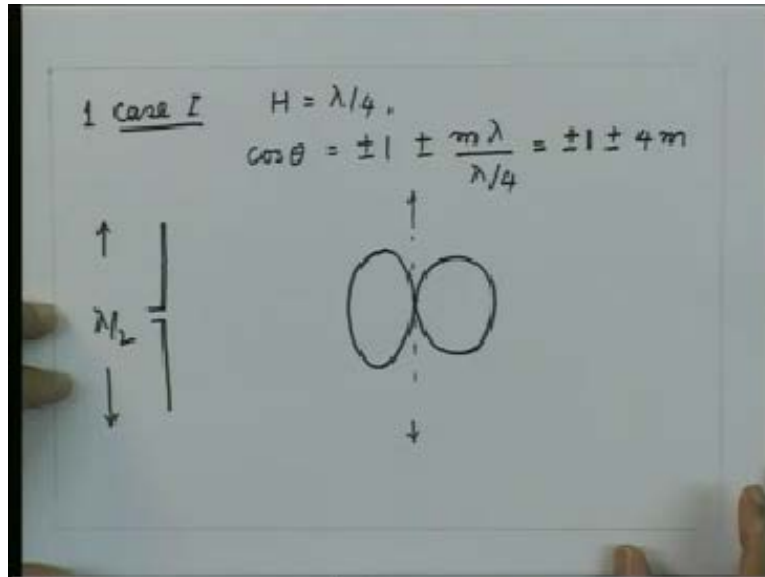
1 Case I  $H = \lambda/4$ ,  
 $\cos\theta = \pm 1 \pm \frac{m\lambda}{\lambda/4} = \pm 1 \pm 4m$

So if you are having a dipole of length  $\lambda/4$  then you get only the same nulls which are  $\theta = 0$  and  $\theta = \pi$ . So the dipole is of  $\lambda/2$  length so this length is  $\lambda/2$  and there are only two nulls one is in this direction other one is in this direction.

The pattern is symmetric in the  $\phi$  direction because the Hertz Dipole is also the symmetric pattern in  $\phi$  direction so superposition will again give me the pattern which is in the  $\phi$  direction. So these two nulls are there and since the problem is symmetric there has to be maximum between these two nulls so the maximum is in this direction so we

get a radiation pattern which is same as the Hertz Dipole radiation pattern for an antenna which is this.

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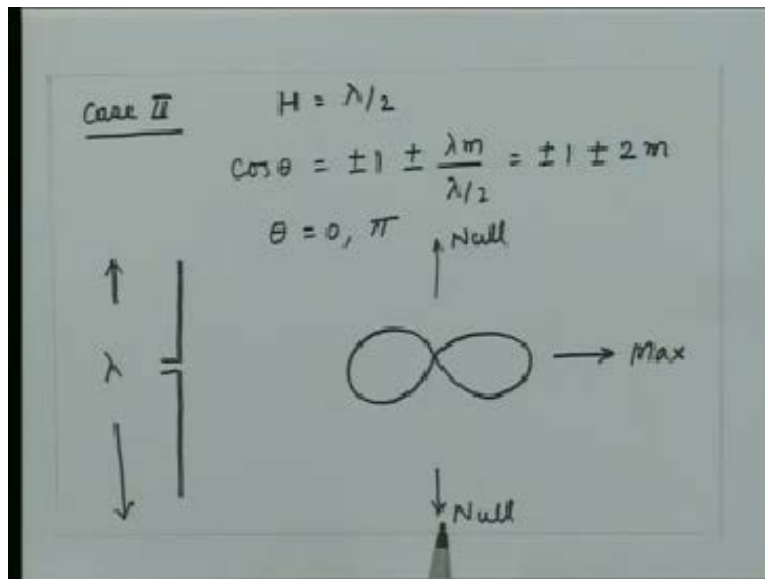


So this radiation pattern is identical at least shape wise to the Hertz Dipole so by changing the length from the antenna to the Hertz Dipole to a physically realizable length which is  $\lambda/2$  the radiation pattern does not get significantly altered the nulls remain same the direction of maximum remains same so this is the direction in which we have maximum radiation. Now note here there is no one direction here the entire plane is the direction in which the radiation is going maximum because we have seen yesterday this is like a apple so if you take the cut of the apple vertically that is this radiation pattern.

We can increase the length of the antenna and go to case-II that is I increase the length of the dipole  $H = \lambda/2$  that means I have made dipole which is one wavelength long so in this case the  $\cos \theta$  will be  $\pm 1 \pm \lambda m / \lambda/2$  so it will be  $\pm 1 \pm 2m$ , again we have  $\pm 1$  nulls for  $m = 0$  but also those nulls for which this magnitude will be less than one again would correspond to  $\pm 1$  for  $m = 1$ . So the directions of nulls are still  $\theta = 0$  or  $\theta = \pi$ .

So even for a dipole which is one wavelength long which is this  $\lambda$  the direction of nulls are still the same which is  $\theta = 0$  and  $\theta = \pi$  and the direction of maximum again symmetry will be half way so direction still remains same which is maximum like this. So the radiation pattern becomes something like that, again in the direction of the maximum so till we increase the length of the dipole up to  $\lambda$  the shape of the radiation pattern does not change significantly of course something must be happening so if I say this apple gets more and more compressed like this so this thing gets little elongated it does not look like a simple  $\sin\theta$  function so this radiation variation as a function of  $\theta$  will become more sharper compared to this but qualitatively the radiation pattern still has two nulls and it has direction of maximum which is like this shape but still resembles very much like a apple though is a compressed apple.

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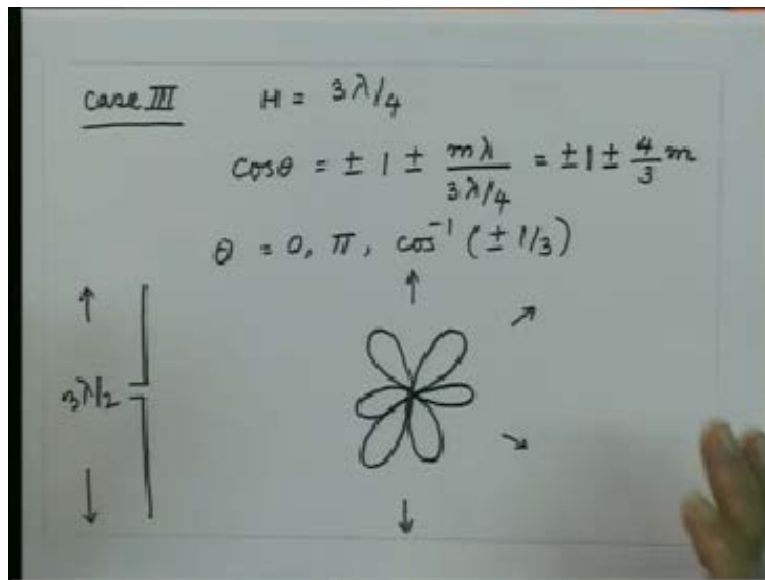


However, once I go beyond this take one more  $\lambda/2$  if I take the case- III where the  $H = 3\lambda/4$  and in this case we will get  $\cos\theta = \pm 1 \pm m\lambda$  divided by  $3\lambda/4$  that is equal to  $\pm 1 \pm 4/3$  into  $m$ . And now one can see that you can get the nulls for  $m = 1$  also by appropriate sign so by taking appropriate sign for  $m = 1$  we can actually find out the nulls corresponding to one for which this quantity will be less than one the magnitude of this

will be less than one. So those angle would exist so I can get a radiation pattern for this dipole so there will be nulls corresponding to  $m = 0$  which will be theta equal to zero and  $\theta = 0$  and  $\theta = \pi$  and there will be nulls corresponding to  $m = 1$  also. So now as we got nulls  $\theta$  which will be equal to  $0, \pi$  and if I take  $m = 1$ , I would take the sign appropriately so this will be if I take plus one sign here I have to take  $-4/3$  so this becomes  $-1/3$  so that will be corresponding to  $\cos$  inverse of  $\pm 1/3$ .

Now the length of the dipole has become  $3\lambda/2$  and the directions of nulls are one is this  $\theta = 0$  and  $\theta = \pi$  but I have got two nulls which are symmetric one is this direction other is this direction and as we said between every two nulls there has to be maximum radiation so now the radiation pattern would look something like that and this is the figure of revolution essentially you will get a pattern it will look something like that.

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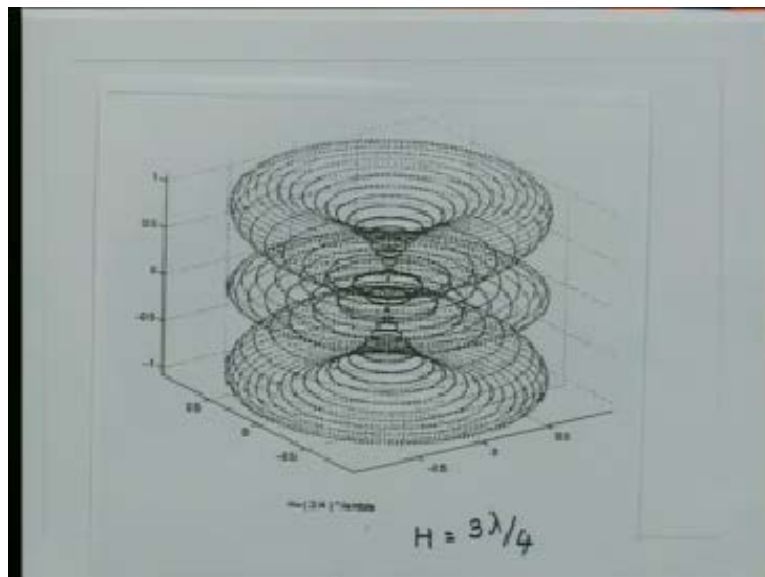
So we got directions for nulls earlier there were only two directions now in fact you got a cone like this over which there will be nulls and there will be maxima which are multiples because there is a radiation which is maximum here this, there will be maximum here and so on and as we saw earlier the radiation pattern is a three



dimensional figure so essentially what we are seeing this as a section of three dimensional figure so if I imagine three dimensional figure by revolving along the axis of the dipole the radiation pattern would essentially look like that.

So this corresponds to  $H = 3\lambda/4$  so you see here this is the direction of null which we have seen this is another direction of null so all this directions there is no radiation in this there are direction here which is maximum there is a direction null here again which is null and there is a null which is going downward.

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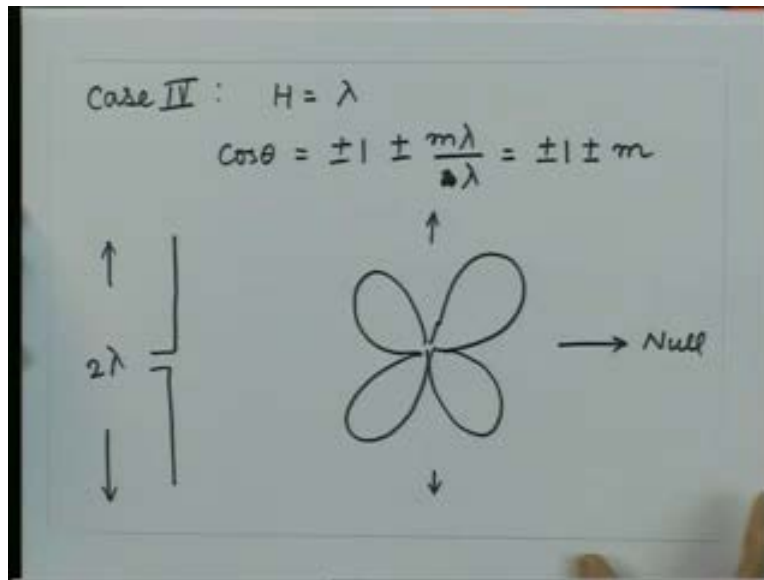


So this looks more like some kind of a flower vase which is created so the radiation in three dimensions essentially would look like a flower vase for  $3\lambda/4$ . So our understanding that as  $H$  increases there will be more nulls radiation pattern will become more complicated that is what essentially appears from here.

We take one more case and that is let us say case-IV where  $H = \lambda$  and then the  $\cos\theta$  will be equal to  $\pm 1 \pm m\lambda/\lambda$  so that will be equal to  $\pm 1 \pm m$ , again what we will notice is  $m = 0$  will give me the two nulls but  $m = 1$  will also give me null which will again be same as

$\pm 1$  with appropriate sign. So now when the dipole length has become  $2\lambda$  again I get only two nulls which are in this direction but now there is one null which is coming from here corresponding to when this quantity goes to zero that means theta becomes equal to  $\pi/2$ . So I got one more null in this direction which is null so I have got three nulls and the radiation pattern would become something like this.

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So if I revolve this around the figure around the axis of the dipole you will get a radiation pattern which is like this so this is for  $H = \lambda$ . Now we draw a very important conclusion that as the length of the dipole is increased the radiation pattern gets modified more nulls come but there is no systematic evolution of the radiation pattern and the impedance which you see for different lengths of dipole also does not change systematically as the length of the dipole. You will investigate more characteristics of the dipole and in general other antennas from the radiation patterns.

Thank you.