Transmission Lines and E. M. Waves
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Lecture – 45
Radiation for the Hertz Dipole

In the last lecture we derived the magnetic vector potential for the hertz dipole. We saw hertz dipole is the basic unit on which we can develop and we can find out the fields for more complex current distribution. So the analysis of hertz dipole is very important because once we understand the fields developed by the hertz dipole we can always find the electric and magnetic fields generated by any other current distribution.

So we had got this hertz dipole which is nothing but a small current element and from there we got the magnetic vector potential. we considered a coordinate system where the hertz dipole is oriented in the z direction and from there we found that the magnetic vector potential at any point in the space will be oriented in the same direction as the current element that everywhere in the space we will have the magnetic vector potential which will be oriented in z direction.

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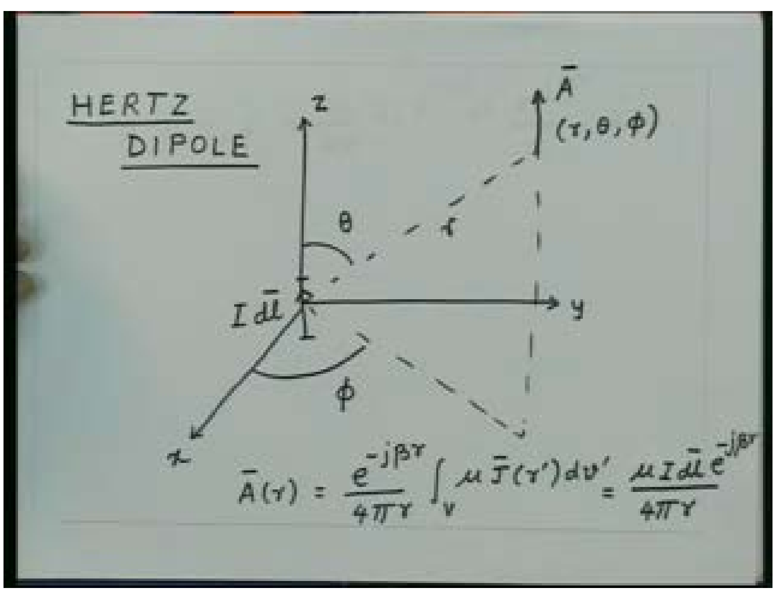

So the magnetic vector potential at a distance r from the origin essentially is given by this which we wrote explicitly as the z component which was essentially given by that.

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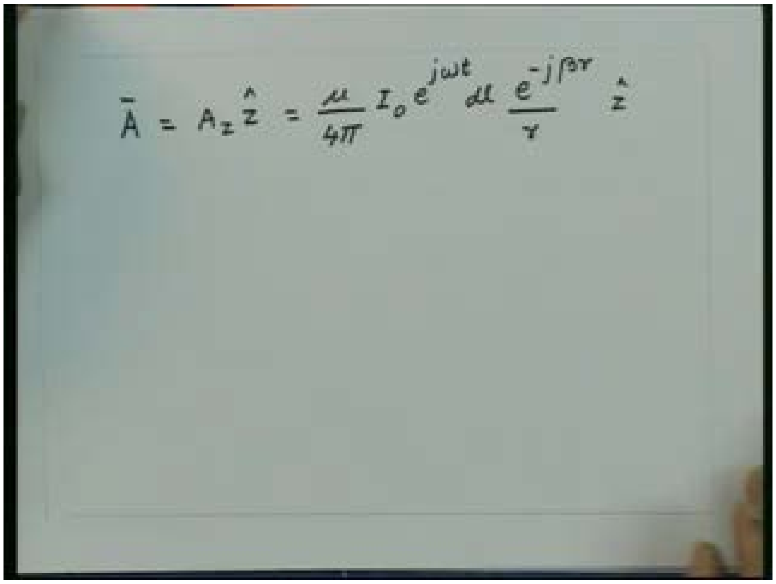

Now, since we are talking about the spherical coordinate system (Refer Slide Time: 3:14) and we are going to investigate now the fields in the spherical coordinate system, essentially we have to convert this A z component in the appropriate spherical components that is the r component and the theta component. So note here this is the radius vector, this is the direction which is theta direction; the radially outward direction this direction is the r direction (Refer Slide Time: 3:42) and phi direction is perpendicular to the plane of the paper which is going this way. So this vector potential does not have any component in this direction perpendicular to the plane of the paper that is 90 degrees with respect to this so the vector potential in the spherical coordinate system would have two components: the r component and the theta component. So since this angle is theta this angle is also theta so this component will be A z cos of theta, this component will be negative of A z sine of theta so you will have theta component which is minus A z sine theta and r component which is A z cos theta. So from here we get now the component from the spherical coordinate system.

So I have from here A r is equal to A z cos of theta; A theta which will be minus A z sine of theta and A phi in this case is equal to 0 where A z is essentially this quantity; so if you put the whole thing together that is this quantity A z.

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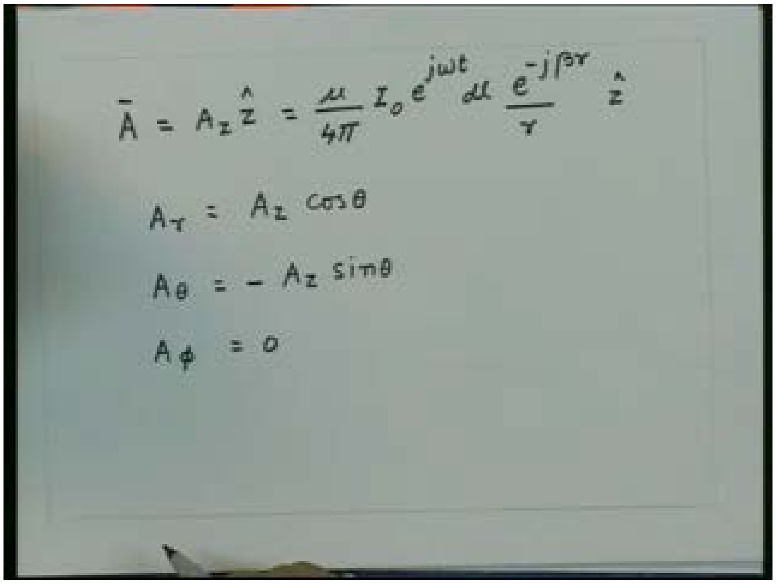

Now, once we have the knowledge of the vector potential then we can go now to the definition of the vector potential and from there we can find out the magnetic field. We can use mu times h which is b is equal to del cross of A or the magnetic field h is 1 upon mu del cross A.

So once I know these components of the vector potential A r A theta A phi then I can substitute into this and I can get the magnetic field.

Write in this curl explicitly. This is equal to 1 upon mu and I can write the curl in the spherical coordinate system which is 1 upon r square sine theta determinant r r theta r sine theta unit vector phi d by dr d by d theta d by d phi A r r A theta r sine theta A phi.

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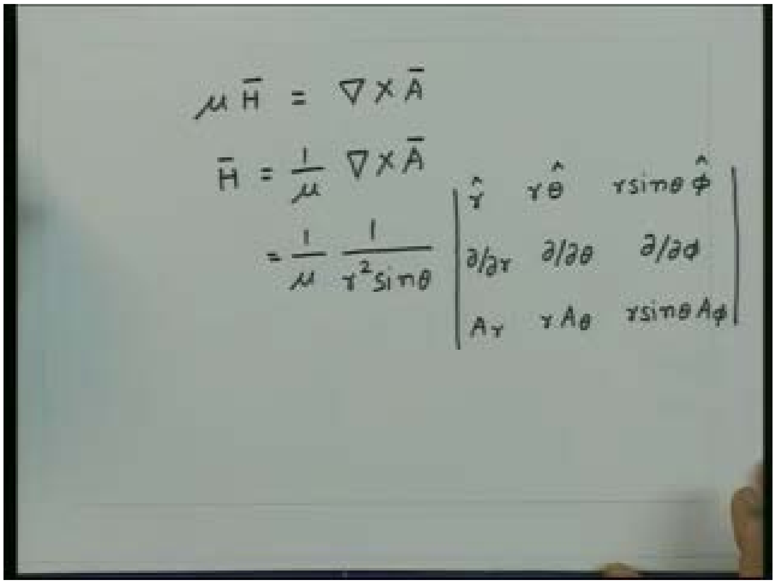

Now, since the current element which we have got here (Refer Slide Time: 7:12) is symmetric in the phi direction because it appears same no matter from which direction we see; only when we see in different direction in theta the element will appear differently. See if i see from this direction in theta it will look like a line, then I see from the top it will look like a point and so on. So we have a theta dependence for these currents but there is no phi dependence because it appears symmetric from all directions. As a result this quantity d by d phi is identically equal to 0 in this case and we also have seen that the phi component of the magnetic field vector potential is also 0.

So substituting now into this we get the magnetic field h that is equal to 1 upon mu r square sine theta determinant r r theta r sine theta phi d by dr d by d theta d by d phi is 0 then the r component A r which is A z cos theta so this is A z cos theta, the theta component is minus A z sine theta so this is minus A z sine theta and the phi component is 0 you get this is 0.

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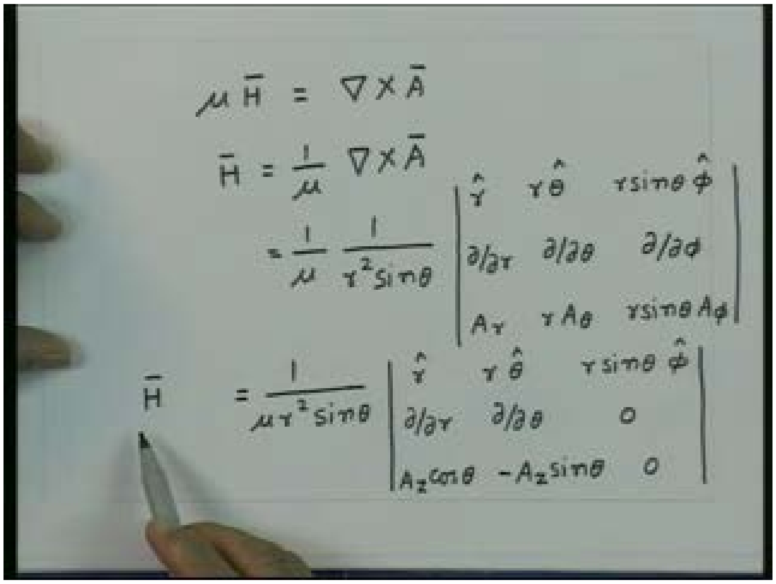

So the vector magnetic field essentially is given by the determinant which is this. Now it is immediately clear that since these two elements are 0 here the r component and the theta component are identically 0. The only component which you will get will be corresponding to the phi component which will be d by dr of this quantity minus d by d theta of this quantity (Refer Slide Time: 9:33).

So essentially what we get from here we get H r is equal to 0 H theta is also equal to 0 and H phi equal to 1 upon mu r d by dr of minus A z sine theta minus d by d theta of A z cos theta; from here (Refer Slide Time: 10:27) d by dr of minus A z sine theta minus d by d theta of A z cos theta; so this is the phi component of the magnetic field.

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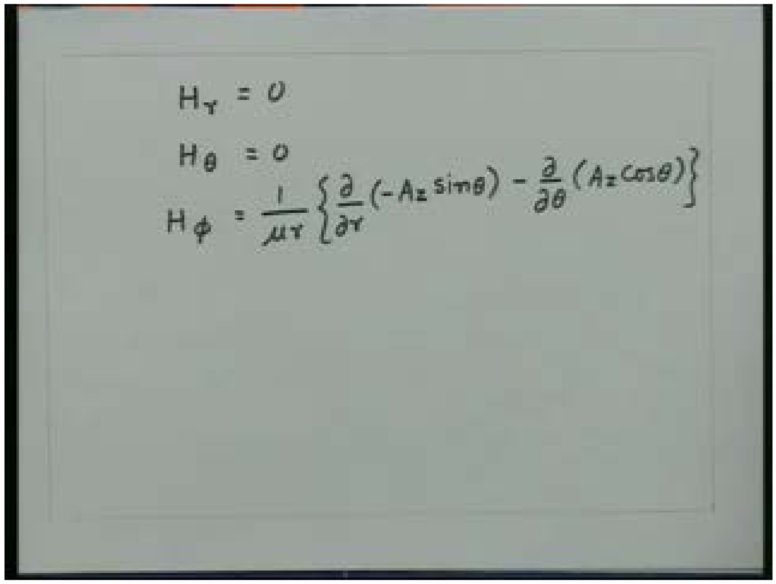

Substituting now for A z from here A z is this quantity and taking derivatives with respect to r and theta we can get the magnetic field phi component which is equal to I 0 dℓ e to the power j omega t minus j beta r sine theta upon 4pi r or I can take r inside the bracket this will become j beta upon r plus 1 upon r square.

So the hertz dipole essentially generates the magnetic field which are phi oriented. That means the magnetic field essentially are going round this current element. So no matter where you go in the space essentially they will be essentially spiraling around this z axis because everywhere they are going to have the phi component.

So magnetic fields as you would have got, suppose this current element wall of infinite length we would get the magnetic field from the Ampere’s law which will be like the loop around this the same kind of magnetic field is generated by this current element also.

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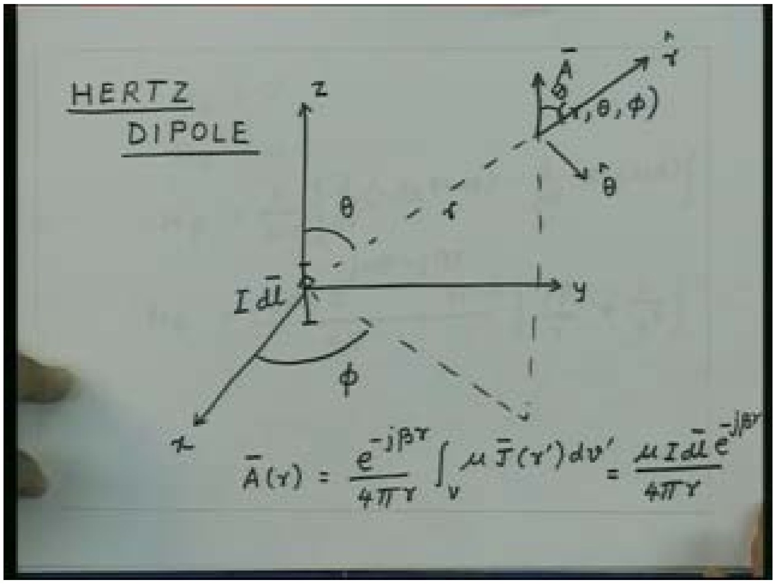

Once we get the magnetic field then we can go to the Maxwell's equation and now we want to find out the electric field not at a location of the source but somewhere in the space where there are no charges or no sources so essentially we can substitute the magnetic field in the source free Maxwell's equation and then find out what is the electric field.

So we can take this magnetic field now and substitute into a source free Maxwell's equation which is del cross H that is equal to j omega epsilon times e. And since I know h i can find out E so the electric field now at the observation point that will be equal to 1 upon j omega epsilon del cross H.

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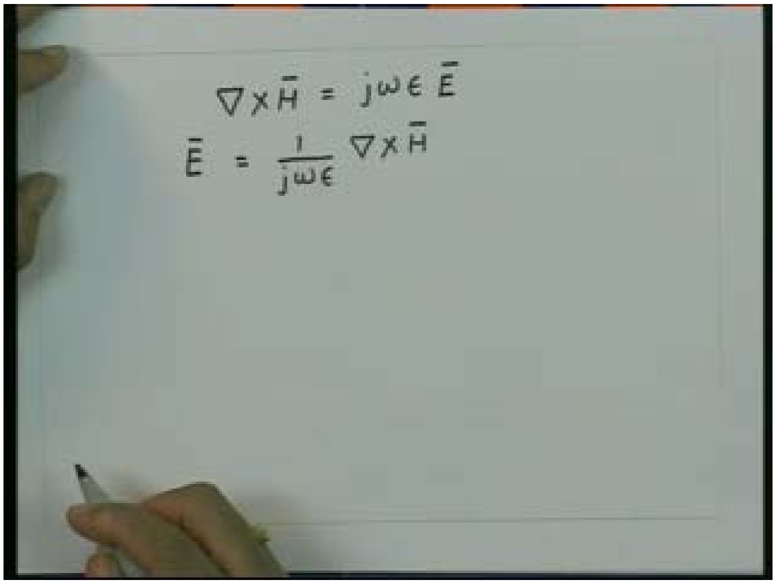

Again I can use the determinant in spherical coordinates. So this quantity will be 1 upon j omega epsilon, 1 upon r square sine theta r r theta r sine theta phi d by dr d by d theta d by d phi which is 0 because we have seen earlier that there is a symmetry in phi so this quantity will be 0 and now we do not have a component which is H r component for the magnetic field, there is no theta component for the magnetic field, the component which we have is only phi so H r is 0 H theta is 0 so I can substitute this quantity is 0 first (Refer Slide Time: 14:33) this is 0 this is 0 and this is phi.

Now again simplifying and taking out the components so the r component essentially will be d by d theta of H phi and this quantity theta component will correspond to the derivative of this quantity d by dr of H phi. So now just taking this derivative and separate out you get the component E r that will be equal to I 0 d ℓ cos of theta e to the power j omega t minus j beta r upon 4pi omega epsilon beta upon r square minus j upon r cube; and theta component E theta will be equal to I 0 dℓ sine theta e to the power j omega t minus j beta r upon 4pi epsilon j beta square upon omega r plus beta upon omega r square minus j upon omega r cube and the phi component of the electric field is identically zero in this case.

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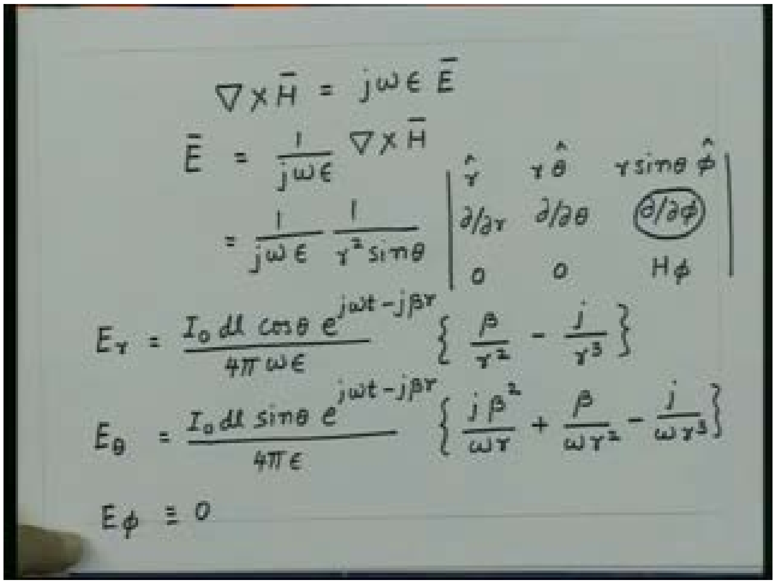

So now note here (Refer Slide Time: 17:20) the magnetic field or the hertz dipole will be going around z axis which is like that but for the electric field we have two components: one is the r component which is like that and other one is the component which is in theta direction which is like this. So depending upon the location this electric field direction will be changing, the magnetic field direction will change only in this plane which is parallel to this xy plane because we have only the phi component.

Now up till now the derivation is very straightforward. Essentially what we did we took the magnetic vector potential substituted into the Maxwell's equation, found the magnetic field, once we have a knowledge of the magnetic field we again substituted it into the Maxwell's equation and we got the electric field.

Now we try to look at these terms which we have got various terms into the electric and magnetic fields.

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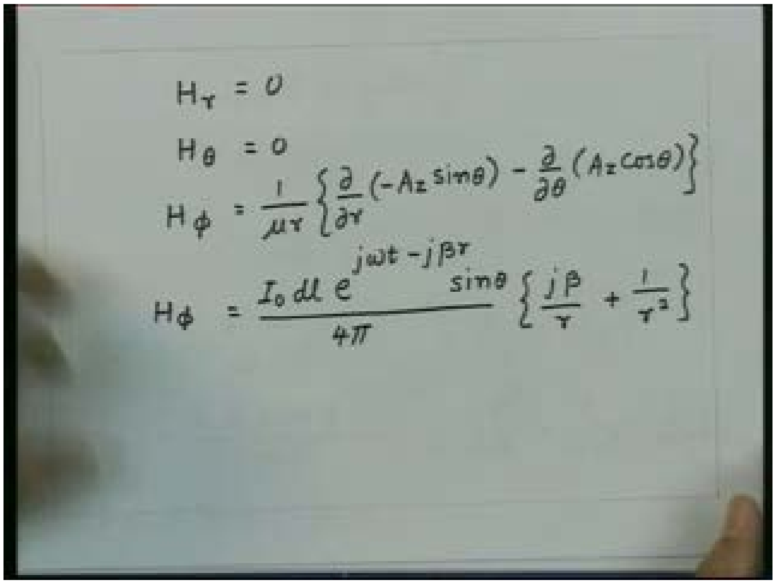

So if I look in general the field expression the H phi and these two components E r and E theta I noticed there are three types of field which are generated by the hertz dipole depending upon the variation of the field as a function of distance. One variation is 1 over r and the other variation is 1 over r square and the third variation is 1 over r cube; same is for magnetic field, we do not have 1 over r cube term but I have 1 over r square and I have a term which is 1 over r (Refer Slide Time: 20:04).

We know that from the Ampere's law that if I have a current then it produces the magnetic field and the magnetic field has a variation from from a current that varies as 1 over r square. So I know this term which is varying at 1 over r square is same as what we used to get from the simple Ampere's law. This phenomenon was the induction phenomena. So if we had a current it would produce a magnetic field surround it and that magnetic field had a variation which was 1 over r square so this term which varies as 1 over r square we call as the induction current. So we have these fields, we denote them as induction term because it was the magnetic induction I had when the current was flowing in the wire (Refer Slide Time: 20:02).

However, if I look at the electric field here I have a term which varies as 1 over r and I also find a term which is 1 over r cube. Now this term (Refer Slide Time: 20:21) which is 1 over r is a term which we have not seen before, it is a term which is a new term this term beta upon r or same time here which is varying as 1 over r and that is the term, essentially we will try to interpret in detail because that is the one which is contributing to the radiation.

So firstly we have three types of fields 1 over r variation 1 over r square variation and 1 over r cube variation. This term as we already saw we call as the induction term or induction fields. The 1 over r term which is a new term and that is the term which we call as the radiation field and the third term which varies as 1 over r cube we call as the electrostatic field and we will try to understand why the terms are called that way. The induction term is very clear because that is the way the magnetic induction was due to a current. So 1 over r square field is a very familiar field, we have seen it earlier and using the same understanding we call that field as the induction field.

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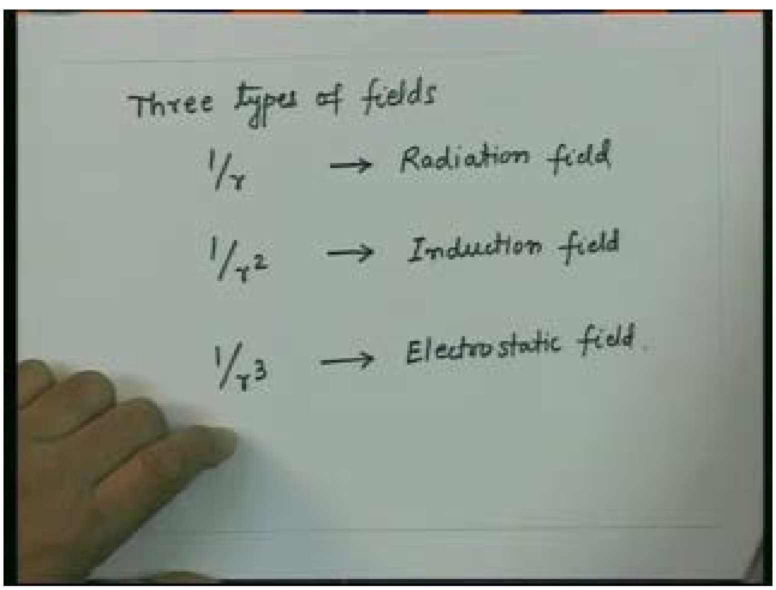

If I look at this field which is varying as 1 over r, if I look at the expression here (Refer Slide Time: 22:18) this is the quantity which is beta square upon omega and as we know beta square is nothing but omega square mu epsilon that is what we have seen earlier. So this quantity whatever we have that varies as omega square mu epsilon upon omega r so this field 1 over r that varies as omega mu epsilon upon r. That means this term which is varying as 1 over r it is proportional to frequency. So this quantity 1 over r term this is proportional to omega.

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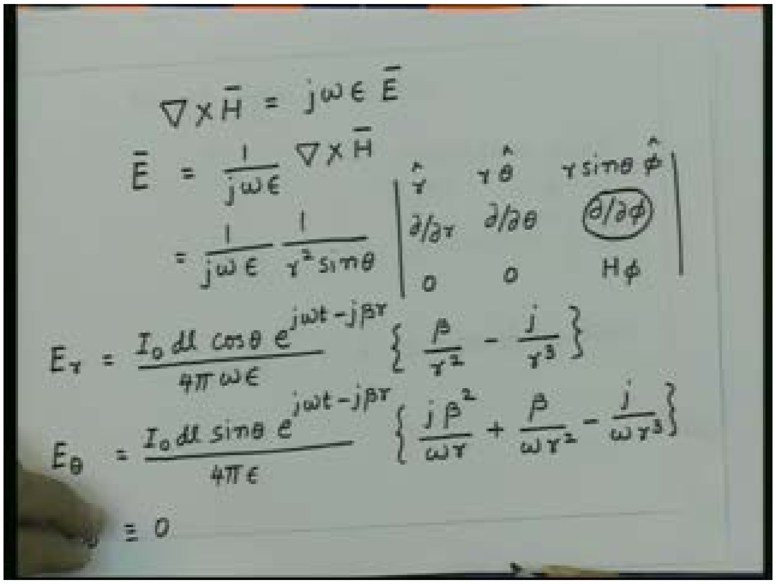

If I look at this term here which is beta upon omega r square, since beta is proportional to omega this term is independent of omega and this term is inversely proportional to omega. So this field induction field is independent of omega and this field electrostatic field whatever we are calling which is varying as 1 over r cube this is inversely proportional to omega 1 upon omega.

So, as we go to the lower frequencies this is the term which is going to dominate. This term is independent of frequency so it is always present and this phenomena is essentially a higher frequency phenomena. So what that means is that this field whatever is the radiation field (Refer Slide Time: 24:12) is a higher frequency phenomena essentially and that is what is the radiation that when we go to the extremely low frequency there is no radiation but as I go to higher and higher frequency for the same current the I 0 into dℓ we will have now this term dominating more and more as the frequency increases that means for the same current amplitude if the frequency is increased the dominance of this field increases. So radiation becomes stronger and stronger for the same current as the frequency increases and that we understand from our basic arguments which we have put for the radiation that: as the frequency increases the rate of change of current increases that means the charges get accelerated decelerated in more way and because of that we have more radiation field so this relation that the radiation field is proportional to omega agrees with what basic understanding we had put forward for getting a radiation from the current. This variation also is okay because this is the field which is even generated by the dc current so its independency for omega is also understandable.

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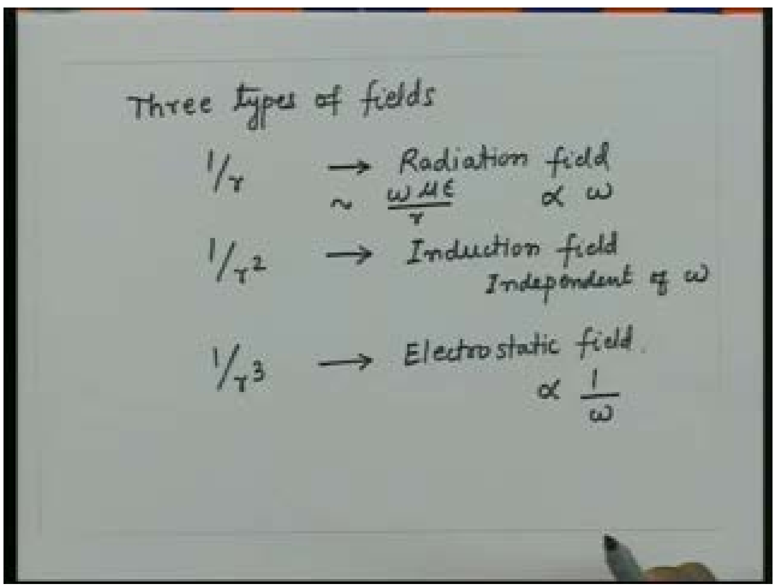

What is the origin of this field which is varying as 1 upon omega; that means this field is going to be more dominant as we go to the lower and lower frequency. As the term we are calling it here the electrostatic field that means this must be related to some kind of charges which are there. But we have not put any charges anywhere in our analysis, we simply assume there is a current I which is flowing into this current element and then we investigated the fields which are induced because of this small current element.

However, if you look at very carefully now one can ask a question this is my current so element, let me enlarge it little bit, current is flowing into this current element but where does the current go; the current flows into this so one half cycle the current will be flowing upwards, in the other half cycle the current will be flowing downwards and there is no closed path now for this because these things are closed by the fields. So essentially when the current is flowing in this direction (Refer Slide Time: 26:44) the electrons are moving downwards, positive charges effectively are going upwards. So when the current goes upwards there is accumulation of charge on these two points on the two ends of the current element. So when the current flows this way the positive charges are moving here and the negative charges will be moving in this direction. So in one half cycle the positive charges will get accumulated at this end and the negative charges will get accumulated in this end. In the next half cycle when the current reverses direction the negative charges will get accumulated here and the positive charge will get accumulated here (Refer Slide Time: 27:30).

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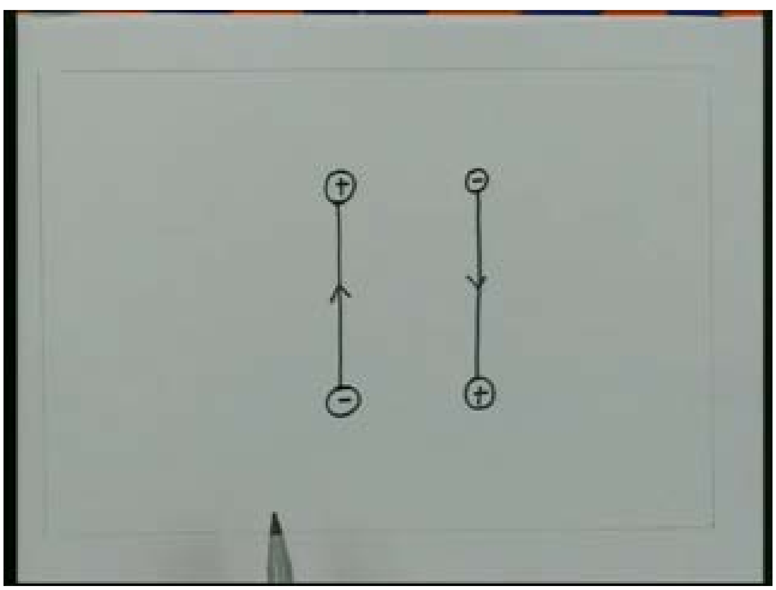

So essentially this current flow which we are talking about which is time varying is equivalent to having time varying charges which are accumulated at the two ends of this this current element which is saying that I am having an electric dipole whose charges are changing as a function of time and they essentially give you this current in this element so essentially you are having a dumbbell of charges equal and opposite charges which are located at the two ends of this current dipole. So as the current is oscillating positive and negative this dumbbell essentially is oscillating so it is positive, next half cycle will be negative and so on. So essentially we are having an oscillating dumbbell which is equivalent to the current flow which we have here.

So though we did not say explicitly when we started with this current in this small current element that there are charges in this system, we simply started with a current flow in the small current element these charges will get accumulated at the ends of the hertz dipole and then they will produce their field which will be electrostatic field. So if I consider this electric dumbbell and now find out what is the the field produced because of these charges I can find out some observation point is very close so let us say I take some point here, some point here, I put some plus positive charge so I will get field which is like this the field will be like this, so resultant field will be always the vector some of these two fields which is produced because of these two charges.

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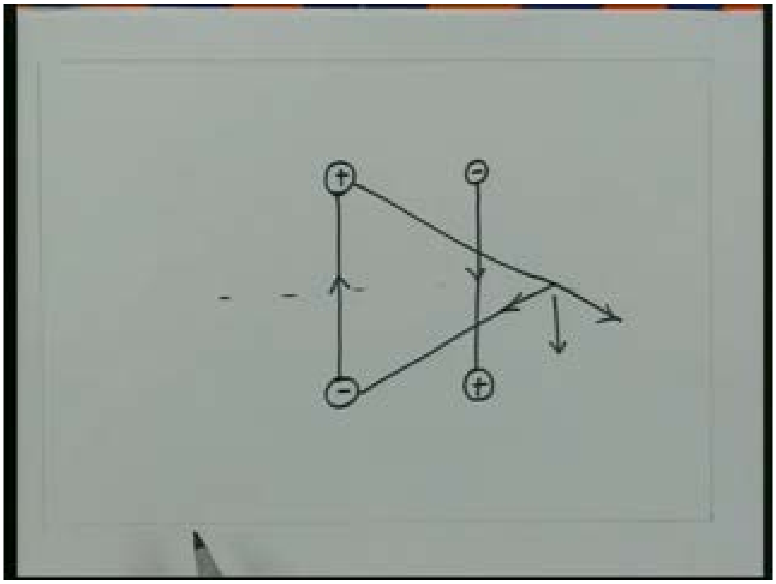

One can show that this field now varies as 1 over r cube where r is the distance from the center of this dipole. So this term which we are getting here 1 over r cube term is essentially electrostatic because this field is produced by the dumbbell of this electric charges which are separated by a distance dℓ. So if I just without getting into the time varying thing if I consider two equal and opposite charges separated by distance dℓ and find the field produced by these charges then I will get the resultant field which will vary as 1 over r cube and that is what is the field which we have got here.

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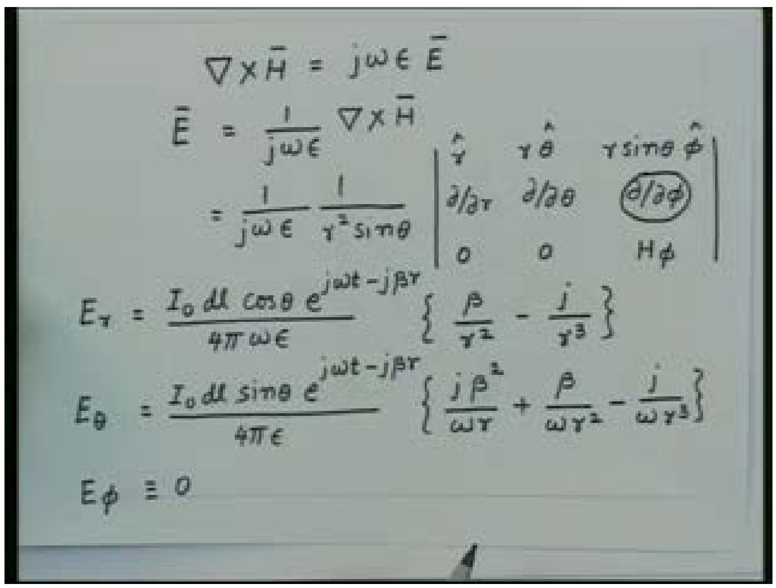

Why this field (Refer Slide Time: 30:30) then varies as 1 over frequency because this frequency becomes smaller and smaller, for the same current more and more charges get accumulated here because the charge essentially is integral of the current over time. So if I have a time period which is very large for the same current amplitude I will be accumulating charges over the time period and as the time period increases I will get more accumulation of the charge. So, as the frequency becomes smaller and smaller the amplitude or the the amount of charge which will get accumulated on the ends of the hertz dipole will go on increasing and one can see that when if we go to frequency which is dc the current will be always flowing upwards and then if I take as a steady-state in finite time these charges will go to go to infinity.

So, as the frequency goes to zero this field essentially will go to infinity because in a steady-state it will be the infinite charges which will be resting on the two ends of this dipole. So this nature that the field which we have got here is because of a charge which is now the collection of the charges which are due to the current should vary inversely as a function of frequency because lower the frequency that means more accumulation of the charges and higher will be the field because of these electrostatic charges. So this field therefore then we call electrostatic field, (Refer Slide Time: 32:14) with this field we call the induction field and why there is no 1 over r cube variation for the induction field because there is no magnetic charge accumulation. So we get a variation which is 1 over r square which is same as the induction field and we get a field which is 1 over r which is the one which is radiation which should contribute to the power flow; that we will see a little later.

So this field (Refer Slide Time: 32:39) is the new field which the magnetic field has, there is no equivalent of 1 over r cube term here because there is no accumulation of the magnetic charges. For the electric field we have accumulation of electric charges and because of that we get a field which will be electrostatic field and that will vary as 1 over omega. So these there fields now: the radiation field, the induction field and the electrostatic field will be generated by the simple dipole hertz dipole. Our interest, however, is only in this field (Refer Slide Time: 33:17) because we are investigating now only the radiation field. So we will try to only take that component of that field which will be varying as a function of omega.

So what we conclude from this analysis is that you take any time varying current the time varying current is going to generate three types of fields; one which will vary as 1 over r and this field will be linearly proportional to the frequency, the other field which will be varying as 1 over r square this field is independent of frequency and third one is the electrostatic field which will vary as 1 over r cube and this field will vary as inversely proportional to the frequency.

Interestingly one can see from the expression for these fields that all these three terms become equal so I can take the magnitude of this term and make it equal to this; for the same distance even this term will become equal to this so if I take a distance from the current element where these terms will become equal so I get a distance beta square upon omega r equal to beta upon omega r square one beta will cancel, omega will cancel, r will cancel so from here I get r is equal to 1 upon beta which is nothing but 1 upon 2pi upon lambda so this is lambda upon 2pi.

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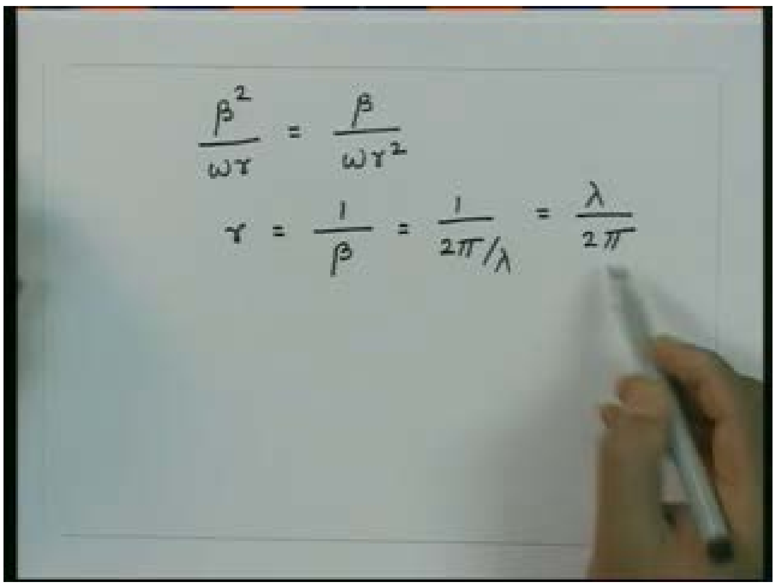

Taking approximately pi 3.14 so this I am taking approximately this quantity as 6 we say that approximately at r equal to lambda by 6 these three terms will become equal in magnitude wise. So we have... if I go from the hertz dipole a distance lambda by 6 approximately then the radiation field, the induction field and the electrostatic field would be equal and as I go less than lambda by 6 this term will increase, this will increase further and as I go beyond lambda by 6 this term would have died down rapidly and the only term which will survive will be this term (Refer Slide Time: 36:06).

So if I plot this now as a function of distance to different fields so this is e magnitude and plotting as a function of distance I will have three types of fields: one is the electrostatic field which will be like this, so other will be the induction field will be like that and the third will be the radiation field which will be like this and this is the loop point which is lambda by 6 approximately or lambda by 2pi to be precise where these three fields are going to be equal so this is 1 over r, this is 1 over r square and this is 1 over r cube.

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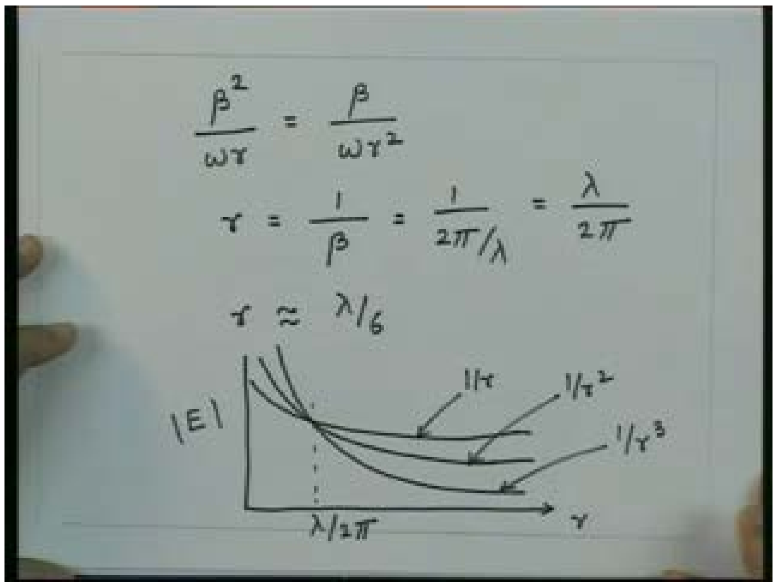

As we see as we go to lower and lower frequencies this is the term which will dominate. So if I go very close to the hertz dipole essentially the influence will be only the electrostatic field. If I am somewhere in between then I will get the induction field and if I go very far away from the dipole very far away means much much larger compared to lambda by 6 then practically the radiation fields will be present in the in the medium.

So now this distance lambda by 6 is now the reference point and on the basis of this distance with respect to this I can now divide the field into two categories. So if I go to a zone which is very short compared to lambda by 6 I call those fields as the near fields and those fields will be essentially dominated by the induction and the electrostatic field whereas if I go to a distance which is much larger compared to lambda by 6 then the field which will be dominated will be only radiation field and I call that field as the far field.

So now depending upon the distance I now classify the fields. So we say near field, this for r much much less than lambda by 6 and then we have fields which are called the far fields which will correspond to r r much much greater than lambda by 6.

Ideally if you want only 1 over r term which is the radiation field then we should go to a distance almost tending to infinity where 1 over r square and 1 over r cube field would have died down with a substantially low value and I will get a field which is only 1 over r which is the radiation field.

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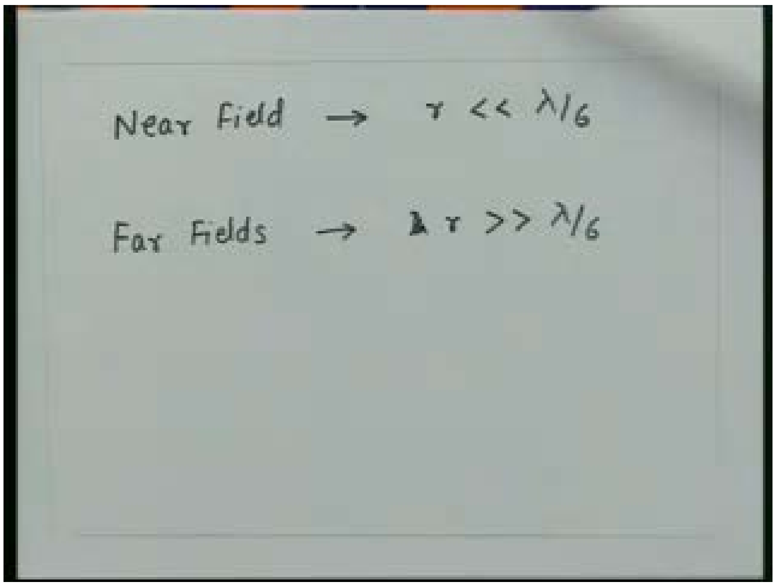

So basically we see now something interesting and that is if I go now very close towards to the hertz dipole we have a term which is 1 over r square and 1 over r cube so I will have a component which is this component e theta (Refer Slide Time: 39:51), I will have component which is E r because it has the r square 1 over r square and 1 over r cube, the magnetic field will have a term which is 1 over r square but if I go to the zone which is far field zone where r is much much greater than lambda by 6 then these fields are negligibly small so I will get only radiation field. There is no radiation field component corresponding to E r so this will be negligibly small so this will be taken as zero and we will have a theta component which will be given by that.

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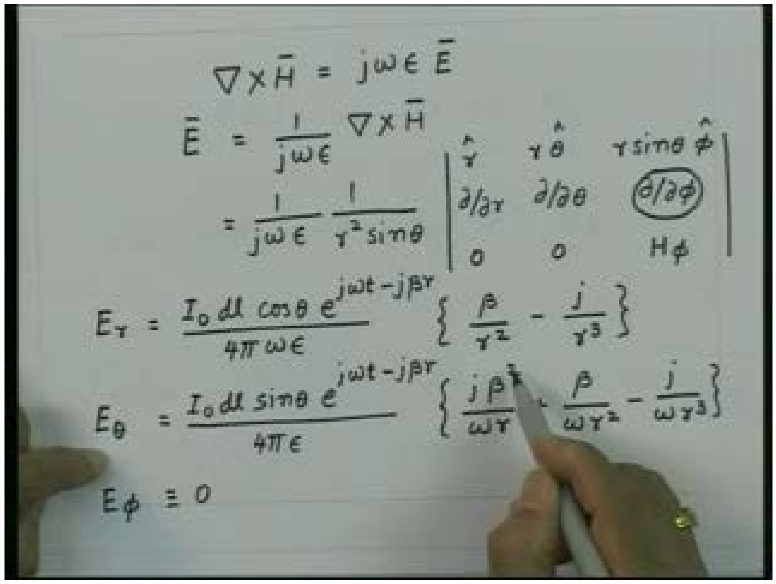

So what we see in near field you will have a component E r, E theta and H phi all three components will be present whereas when I go for the far field the E r component is negligibly small and we will have only two components which is E theta and H phi. Since we are interested here only in the radiation field we can take appropriately only those terms which are having 1 over r variation. So I can take this quantity is zero now, this is the term (Refer Slide Time: 41:17) which gives me radiation field so I can write down only for far field that is what our interest is; we get the component E theta which is j I 0 dℓ beta square sine theta e to the power j omega t minus j beta r upon 4pi omega epsilon r and we get H phi that will be equal to j I 0 dℓ sine theta beta e to the power j omega t minus j beta r upon 4pi r.

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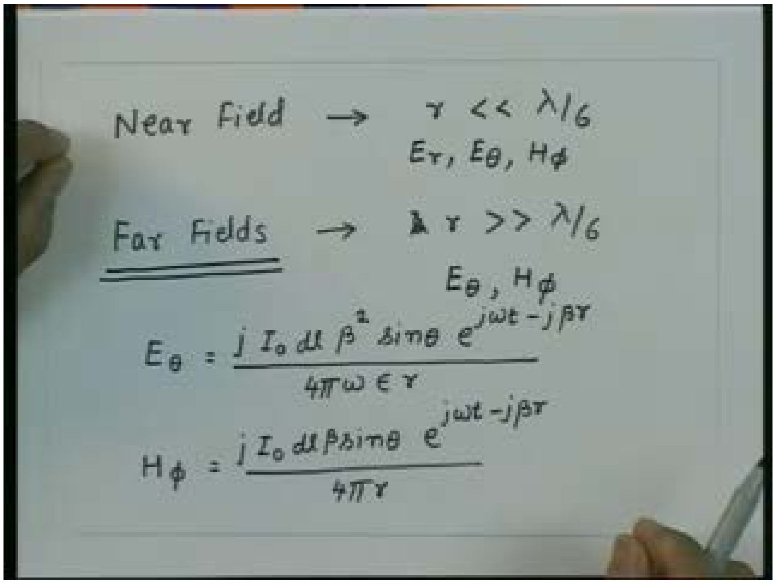

So, for the far field we have only these two components. Again few things can be noted here. Firstly, both the fields have a variation as a function of angle theta the sine theta. That means the fields are zero at theta equal to 0 and fields are maximum at theta equal to 90 degrees. So that means both the fields are maximum in this direction when theta equal to 90 degrees and along the length of this hertz dipole that means along the axis of the dipole these two fields are identically zero; one of the ratio.

That means the fields are not uniformly created in the space, they have a directional dependent. You have maximum field when we go perpendicular to the hertz dipole and along the hertz dipole the fields this far fields are identically zero. Second thing what we note from here is if I take a ratio of these two quantities or before that the electric and magnetic fields have this term j with respect to this current I 0 e to the power j omega t that means the field which are now generated what we call as the far field or radiation field they are 90 degrees out of phase with respect to the current; and what is the reason for that?

The reason is simple that right from the beginning we said the radiation is a phenomena which is related to the rate of change of current. So if we are having a time varying current which is e to the power j omega t the rate of change of current will be equivalent to multiplying the quantity by j omega so you have the fields which are 90 degrees out of face with respect to the current so this j which you get here in this field essentially supports our basic argument which we had for the radiation that if we accelerate the charges that means if we have a rate of change of current we will get the radiation field that is what we see from here.

The third thing which we note from here is if I take a ratio of this quantity with theta and H phi this quantity will cancel, I 0 dℓ will cancel, sine theta will cancel, r will cancel, 4pi will cancel but what we will get we will get beta square here and here we will get beta so the ratio of these two quantities if I take E theta and H phi that will be equal to beta upon omega epsilon. And I know this beta is nothing but omega square root mu epsilon divided by omega epsilon. So this is equal to square root of mu upon epsilon which is nothing but the intrinsic impedance of the medium eta.

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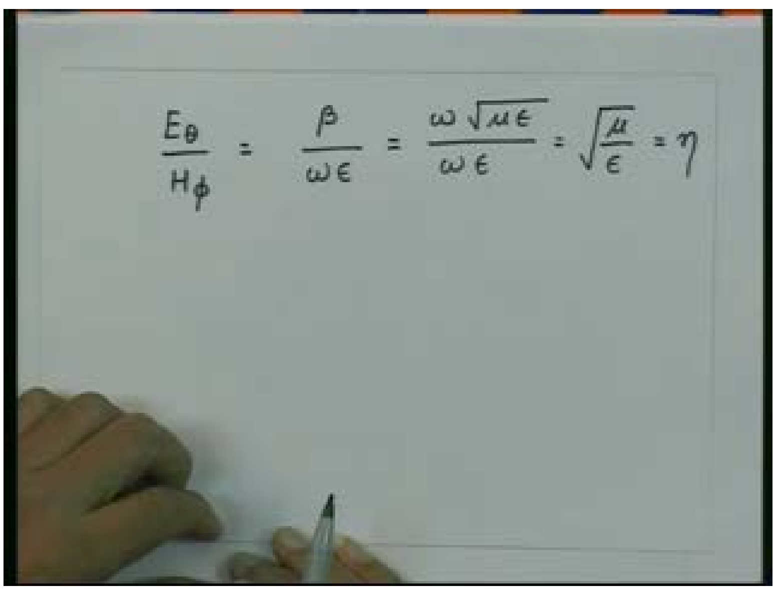

So note what we have got here. The wave which we have got now or the field which we have got is... this represents this spherical wave that means the wave is traveling in the r direction that is what we... right from the beginning we got from the solution.

We got a component E theta for the electric field and component H phi for the magnetic field which is perpendicular to the plane of the paper; and ratio of the electric and magnetic field is equal to the intrinsic impedance of the medium. That means these fields are having exactly identical properties as we used to have for uniform plane waves that the electric field, the magnetic field and the direction of the wave propagation are perpendicular to each other that is what we have here the wave propagates in all direction, the field is in theta direction electric field and the magnetic field is in phi direction so these three are perpendicular to each other and the ratio of the electric and magnetic field is equal to the intrinsic impedance of the medium (Refer Slide Time: 47:03).

So this wave is spherical wave and it has a transverse nature that the electric and magnetic field both are transverse to the direction of the wave propagation. So we have got here what is called a transverse spherical electromagnetic wave and it has properties all properties of transverse electromagnetic wave like the ratio of the electric and magnetic field should be decided by the medium properties which is the intrinsic impedance of the medium and the electric field, the magnetic field and the direction of the wave propagation should be perpendicular to each other.

So, though the wave which is created by this is spherical, every point in space if you go around this hertz dipole you will see a transverse electromagnetic wave. Though the direction of the transverse electromagnetic wave will be varying at different points in space but any location if you ask what is that nature of the electromagnetic wave it is a transverse electromagnetic wave. So essentially if I take the observation point, draw a sphere passing through that point the radially outward direction direction gives me the direction of the wave propagation and the tangents which are drawn to the surface at that location perpendicular to each other they give you a direction of the electric and the magnetic fields.

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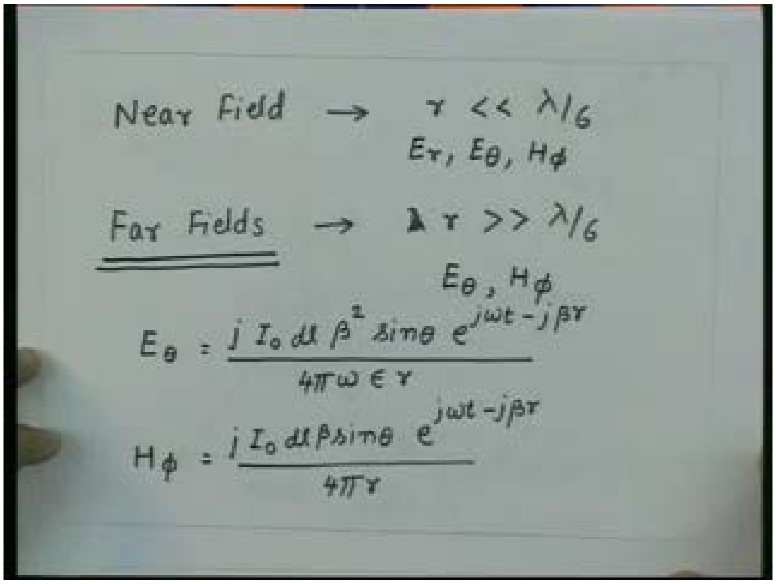

We can also see some interesting things from here and that is the electric and the transverse nature of the electromagnetic wave is there at every point in space and its direction will be changing when you go to different locations but at a given point how the electric field is varying as a function of time.

Let us consider this point here the electric field is having a theta component which will be positive or negative. That means at this location the electric field is always oriented in this direction as a function of time. That means this wave is a linearly polarized wave. The direction of the electric field will change from location to location; here theta direction is this, if I go to a point here theta direction will be vertical, if I go here theta direction will be like that. So from one location to another location direction of the electric field will change but if I ask what is the polarization which is the direction of the electric field at a particular location that wave is linearly polarized.

So now we have got very interesting conclusions and that is if we consider the small current element what is called the hertz dipole then it generates variety of fields, it generates induction field, it generates electrostatic field, it generates radiation field; however, if we take only the radiation fields then these radiation fields satisfy all the properties which the transverse electromagnetic wave has. So it generates the spherical wave but the electric and magnetic fields are perpendicular to each other and they are also perpendicular to the direction of the wave propagation. Also, the ratio of the electric field and magnetic field is equal to the intrinsic impedance of the medium. And the polarization which the wave had at any point for the hertz dipole will be always linear polarization.

So the hertz dipole essentially generates an electromagnetic wave which is spherical but it is linearly polarized. So on this concept then we will develop more and then we will try to investigate more practical antenna systems. We will see how much power is radiated by the current element and so on and so on.