Transmission Lines & E. M. Waves
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Lecture – 43

Up till now, we discussed propagation of electromagnetic wave along a guiding structure. That means we had a structure which can guide electromagnetic waves along it. For example, we had a coaxial line where energy was confined between the two conductors and the energy was moving along the length of that coaxial cable. We had a parallel wire line where there were two conductors which were parallel to each other and electromagnetic energy was guided along the length of these conductors. Then we discussed propagation of electromagnetic waves along the media interfaces. Again the media interface was the guiding mechanism. So, electromagnetic wave used to be guided by this interface.

Then we went to structures like waveguides where again the conducting boundaries were guiding the electromagnetic energy along with them. If you recall, we also have said when we started discussing the topic of electromagnetic waves that let us not worry at that moment how the electromagnetic waves are generated. Somehow, we ask the question, if the time varying electric and magnetic fields existed, in what forms they would exist and we concluded that they would exist in the form of electromagnetic waves.

So, without asking the question how these fields were generated, we investigated the characteristics of the electromagnetic waves. Now we ask the question however, how the electromagnetic waves are generated and that is the topic which is covered under the title what is called radiation. A device which generates electromagnetic radiation is called antenna.

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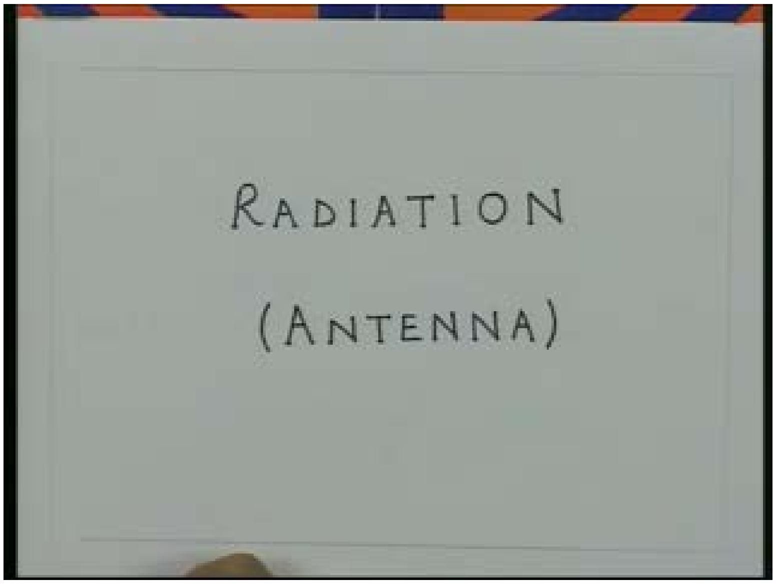

So, this topic essentially deals first with the principles of generation of electromagnetic waves and then we will talk about the practical devices which can generate electromagnetic waves from currents and voltages and which can convert the electromagnetic energy electromagnetic waves to currents and voltages when these waves impinge on the structure, what is called antenna.

So in this topic, essentially we investigate the characteristics of this device, what is called antenna, which essentially is the transducer which converts the electrical quantity, like current and voltage, into the electromagnetic quantities like electric and magnetic fields and vice versa. So, when we use this device like a transmitting device, we excite this with the voltage and current, and antenna can generate electromagnetic wave, the same structure is capable of receiving electromagnetic wave. That means when the electromagnetic wave is incident on the structure, you get currents and voltage at the terminals of the structure, what is called antenna.

So, first we will develop the foundation of radiation and then we will go to the practical structures like antennas. Now, the basic for radiation is the accelerated charges. If you recall, in the electrostatic case, we had a charge; and then the charge essentially produced what is called electric field. The effect of this charge could be felt by electric field. If the charges are kept in motion, uniform motion, then they constitute current and then we know there is a constant uniform current; gives you the magnetic field.

So, if you have a charge which gives you electric field, if the charges are kept in motion, then they give you the magnetic field. Question one can ask is, if the charges are accelerated, and that is what will happen if you are having time varying current; so, if you take a current which is varying as a function of time, the charges are no more moving with the uniform field. So, they get accelerated and decelerated. Then, what kind of field should be generated?

Thus the question essentially is asked under this topic - what is called radiation. So, as we know from our basic physics laws, that when the charges are accelerated, there is a possibility of radiation; they throw energy in their surroundings. So behind radiation, there has to be acceleration of charges because accelerated charges are capable of throwing the energy in their surrounding environment.

However, every accelerated charge may not give you the radiation. So, when we say that you require acceleration of charges or the time varying current for generation of radiation, we say that if we have a time varying current, there is a possibility of radiation and the reason for this is very simple.

When we talked about the coaxial cable or parallel wire transmission line, there also we are having time varying currents. Electrons are moving on the conductors and we have seen that the electrons get accumulated; again they move; they get accumulated somewhere else. So, we have a movement of electrons back and forth along these conductors. So again, there was acceleration and deceleration of charges. But this was not giving you radiation. The energy was guiding along these conductors and there was no energy loss from this structure.

What do I mean by that is that if you are having a guiding structure, if the energy starts moving perpendicular to that, that is a loss of energy because that energy now cannot be collected at the other end of this guiding structure. So, this structure which is guiding structure, can guide the electromagnetic energy; but it will not give you an energy which will be going transverse through the length of this structure and in that case we say there is no radiation from the structure. So, we have acceleration, deceleration of charges. But we do not have radiation from this structure.

So, though we have acceleration and deceleration of charges, since we do not have radiation, I cannot make a general statement that every accelerated and decelerated charges would give you radiation. But we can say that there is a possibility because accelerated and decelerated charges are capable of giving radiation.

So then, why these things do not give radiation? Because, you have two currents which are flowing equal and opposite in the two conductors of a transmission line.

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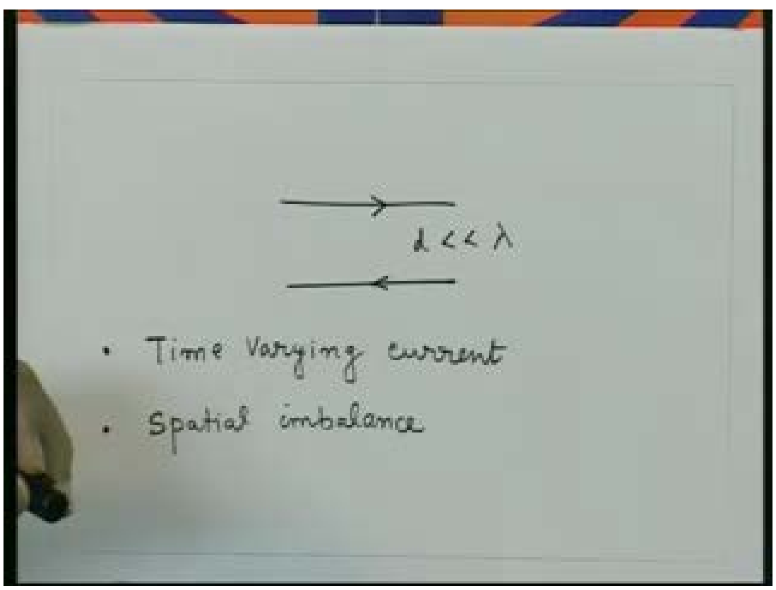

So, if I consider a structure which is a structure like this, I have a current which is flowing this way in this conductor, and the current equal and opposite current flows in the conductor next to it. So, if I take any small section of transmission line, I have charges getting accelerated. They will generate the fields because of the current which is in this direction and this conductor will generate the field which is...which will be due to the current in the opposite direction.

Since the spacing between these two is much much smaller compared to lambda, so d is much smaller compared to lambda. Therefore, whatever are the fields which are generated which could have given you the power leaving the structure, they almost cancel each other and because of that you do not have any power radiated by this structure. However, if I separate the distance between these two, then the d will become comparable to lambda and then the same cancellation - equal and opposite current cancellation - will not take place in all directions.

It is possible that when I move in these directions perpendicular to the plane of the paper, they might cancel each other but when I go in some other direction, it is possible that now this phase which is coming because of space, that might compensate for the phase difference between these two currents and then I might get the radiation from the structure.

So, then it appears that to have a radiation, there are two things...must be there: one is, we must have time varying currents because time varying current will give me the acceleration of charges. Secondly, I must have a special imbalance of the current. If I have two equal and opposite currents which are located close to each other - and close to each other means within much shorter than the wavelength - then the possibility of radiation is very weak. So, we should have what is called the spatial imbalance.

We have spatially separated this current which are even equal and opposite, then there is a possibility of radiation. So then, when we talk about the structure like antennas however, now our effort is to essentially identify and investigate those structures where these conditions are satisfied.

Firstly, that means you take a structure and excite this structure with time varying current; and secondly, you make sure that there are no equal and opposite current very closely spaced to each other in space; and then there will be a possibility of radiation from this structure. So essentially, the radiation phenomena is related to the time varying current that is the crux of the matter.

That means, any time varying current you take and if this condition let us say is satisfied, then you will have a radiation from the structure. What that means is, any low frequency, no matter how small the frequency is, any frequency which is not D C is capable of giving you radiation, because no matter how small the frequency is, there will be always time varying current; there will be acceleration and deceleration of charges and because of that, there would be radiation.

So radiation phenomena essentially is attached to any time varying current. Also, we will note here that as the frequency increases, the acceleration and deceleration of charges will be more now because essentially the rate at which the current is changing is higher. So, we expect that as the frequency increases for the same amplitude of the current, same peak current or RMS current, we should get more radiation because now there is more acceleration and deceleration of charges. So this phenomena, the radiation phenomena, essentially is a high frequency phenomena.

As you go to higher and higher frequency, this phenomena should become more and more dominant and as you go to low frequency, this phenomena should become weaker and weaker and weaker. And when you go to D C, this phenomena ceases. So that means, whatever understanding or analysis we have developed for the circuits, electronic circuits, at low frequencies where the currents were flowing into their components as we said, they were like guiding, guided structures.

So, there was no interaction between different electronic components but at very far away distance because the fields were, more or less, localized fields; like the capacitive fields, electrostatic fields or the induction fields. So, if I separated the distance between the two electronic components, they would not really show an interaction.

However, if you go to radiation, the radiation is the phenomena which is now going to take the electric and magnetic fields over much far away distances, where they are taking the power away from the structure. So now there is a possibility of interaction between different electronic components because of this phenomena, what is the radiation phenomena.

So radiation phenomena is essentially the higher frequency phenomena and it...not only it takes away the power from the structure, but it also starts showing the interaction between different electronic components which were separated by substantial difference compared to wavelength and earlier were not showing any interaction.

So now, when we talk about the radiation or the antenna, our essential job is to design those structures which will give this radiation efficiently. What that means is, if I excite this structure with voltage and current, it should put sufficient amount of power in the form of electromagnetic waves which will take the power away from this structure which is antenna.

Just to give a little feel, let us say conceptually, suppose I had a transmission line and as we said, if the transmission line's separation is much smaller compared to lambda, then there is no possibility of radiation because there are two currents which are equal and opposite. We also know that the characteristic impedance of the transmission line depends upon l and c, the inductance per unit length and the capacitance per unit length of the line. And l and c depends upon the transmission line parameters; that is, the conductor size and the spacing between the conductors. That means the characteristic impedance is a function of the spacing between the two conductors. So, if I vary the spacing between the two conductors, the characteristic impedance of the line changes.

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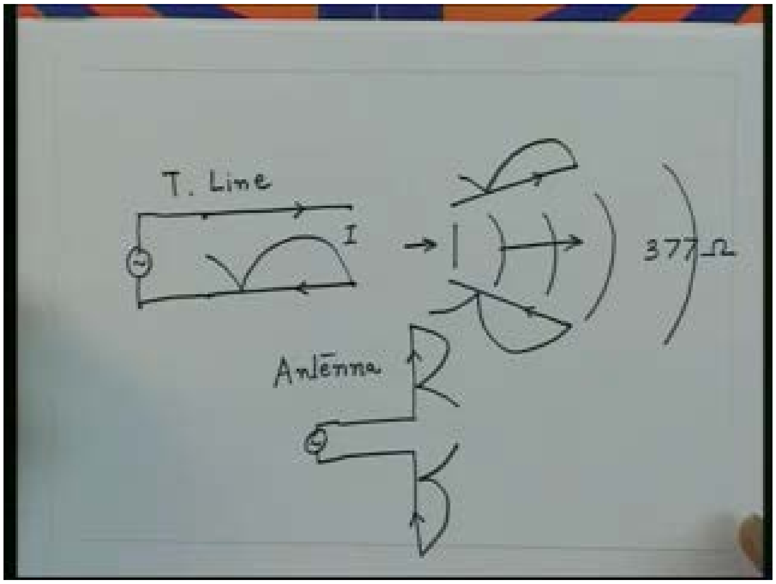

Let us very crudely, very qualitatively, say that if I take a line and slowly start flaring this line. That means this was the line on which the electromagnetic wave was traveling or the voltage or current wave was traveling; and this end, if it is open circuited, then we have a current minimum at this location and voltage is maximum and current will be zero at every lambda by 2 distance on transmission line.

So if I plot the current, it will be zero here. Again it will be zero here and it will be zero and so on. So here, this is the current distribution on the transmission line I. So, we have a standing wave because the transmission line is open circuited at this end. This is connected to some voltage source at this location, other end of the transmission line. So, we have a standing wave structure on this transmission line and here we are now interested in current. So, this is the current standing wave on the transmission line.

Now, what we do is we slowly start flaring this transmission line. So let us say, I take this structure and the other two points. So from here, I slowly flare the structure like that. Now, if this flare is gradual, I will get now a transmission line whose characteristic impedance is changing along the length because the spacing between the two conductors is changing. So, the inductance and capacitance between the...these two conductors are changing. So as a result, the characteristic impedance change of this line. So, the impedance which I see at this point, though this end is open circuited, the transform impedance goes on changing as I flare this angle wider and wider.

But if this flaring is slowly...just smooth, then I still have a standing wave current distribution on this. That means the current is zero here. So, it will be zero here; it will be like that, that and same will happen here. Now note there are two conductors here and if I assume that the current in this flows this way, the current...if this is going to flow in the opposite direction and that is what we said; these two currents are so close to each other that they will cancel; the fields will get cancelled and there will not be any net radiation from this structure.

Now, as you flare further and further, this is the direction of the current, and this is the direction of the current here, and again the distribution will be like that. So, on this end which is same as this end, you have a current zero and then the current have a standing wave pattern which will look like that. Now, if I just go by the circuit point of view, we know that the free space has an impedance. It has the intrinsic impedance which is 377 ohms. So, if I look like at a medium or transmission line, this transmission line essentially is terminated into 377 ohms because it sees an impedance ahead of it when the waves come; wave comes from here and tries to reach at this point.

This wave, electric and magnetic fields, they see a medium ahead of it which is infinite and it has a impedance of 377 ohms. This line has a certain characteristic impedance which is varying as a function of this flaring. So, if I flare in such a way that the impedance now seen at this point of the transmission line is equal to 377 ohms, essentially the structure sees the match load. So, whatever waves come from here, it sees the impedance which matches with 377 ohms.

So, the wave which is coming from this side comes here and it gets launched as if you are having a perfect continuity of the transmission line; and the waves get launched into the medium. So, you had electric fields which are like that. It will become like this, will become like this, like this, and slowly the wave will get launched into the free space. This current distribution which you have got here, standing wave, essentially it was obtained with the understanding that this is the guiding wave structure. We can take extreme case and still say that well, this nature of standing wave still is retained. We can open up this structure completely and I get a structure say like that; and here again, the current distribution is going to be like that. So, these two conductors which I had, they are opened up. So, this is the current like this; but now, this current is like that.

So, I have now got the conductor structure which is completely opened up and these two currents are now flowing in the same direction. So in this case, the two currents, they were generating fields which would cancel each other when I go far away from the structure. But in this case, now the currents are flowing in the same direction and now there is no cancellation of fields. So, you have now the fields which can carry the power away from the structure.

So, if I excite this structure with some voltage or current, then the field which will be induced by this structure would give me a power flow into the space. So, this is a very crude way of saying that if I take a transmission line which was a guided structure and slowly flare it out, I will get a structure which will start radiating energy into the space. So, I will get a structure which will be an antenna structure. So, I have here a line, transmission line, and the open version of transmission line will become a structure which will be antenna.

Of course at this stage, one may say that if we are taking the transmission line and opening it like this, are this assumption that your standing wave and all that will remain. There is no simpler way of convincing that yes, this current distribution will remain still like that; but rigorous analysis shows that if you take a very thin wire and if you open it up like this, the current distribution still remains a standing wave type of distribution as you would get on the transmission line.

So at this point, without getting into the details of this qualitatively, one can say that if you have time varying currents and if you...opposite currents separated by distance in terms of lambda, or if I have current which are flowing in same direction, then there is a possibility of radiation from the structure. So, I can get a structure unlike this.

Then the questions which are of interest for antenna are firstly, if I excite this structure with certain voltage or current, how much power will be radiated? That is the question one can ask. One can ask also a question, would the power be radiated equally in all directions or would there be specific directions in which the power would go? Knowing the antenna structure, can I say in which direction the power will go or which direction the power will not go? These are the questions one normally asks for the antennas.

So, when we discuss the topic of antenna, essentially the problem is to find how much power will be radiated by the structure, which direction the power will be radiated and which direction the power will not be radiated and so on. What kind of polarization will be developed by the structure? So, when the electromagnetic wave is launched in the three dimensional space, what will be the nature of electric and magnetic fields in the three dimensional space; and these are the questions essentially we will answer in this topic what is called radiation.

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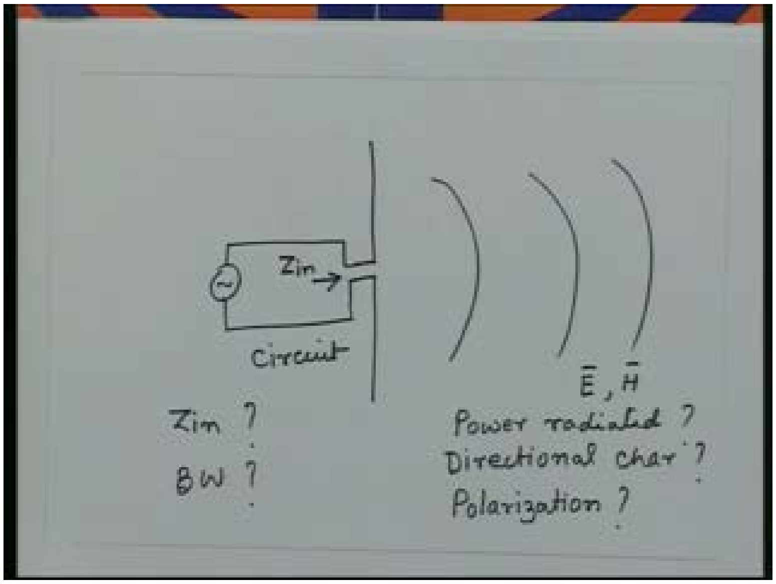

You will also see that if you look at the antenna structure, the antenna structure is actually having a dual nature; and that is on one side of the antenna, you see an electrical characteristic. If I consider, let us say, an antenna structure like this, on this side you have generated the fields. So, you are having electric and magnetic fields. But on the other side you are going to connect the structure to the electrical circuit. So, either...even if you are exciting it or receiving the wave, you will be connecting this to a circuit. You are having a, let us say, a voltage source; to it this will be connected.

So, on one side of the antenna you have the electromagnetic waves. So, it has the wave characteristics. The other side of the antenna which is at the terminal, you are having a circuit kind of behavior for the antenna. So, antenna actually is...has a dual characteristic. On one side it has a circuit behavior; on the other side it has electromagnetic wave behavior. And when we have circuit behavior, then the parameter which will be of interest would be, what is the impedance which we will see between the terminals of the antenna?

So, what is the reactance and resistance which will be measured between the terminals of the antenna? What will be the bandwidth this element would have? So, if I treat this antenna like a two port unit, how much bandwidth this electrical unit has?

So the question one would be interested in is what is the input impedance of this antenna? What is the bandwidth of the antenna? Whereas, if I go on the other side which is the electromagnetic wave side, then we will ask how much is the power radiated? What is the directional characteristic of the antenna? What is the polarization of the antenna and so on.

So antenna, when we investigate this topic on antennas, essentially we have to see from both the sides. We have to see the antenna as a circuit element when we see from the input side of the antenna; whereas, we have to look at the antenna like a source of electromagnetic waves and then we have to investigate the characteristics in terms of the electromagnetic waves.

Having said these general requirements then, now we require the mathematical foundation for analyzing this problem of radiation. We have seen that whenever we start a problem in electromagnetics, a new problem, we essentially fall back on Maxwell’s equations and ask for the solution which is consistent with the Maxwell's equations.

Up till now, we have solved the Maxwell’s equations, what we called as a source free Maxwell's equations. That means there were no free charges and there were no free currents in the medium and we found the solution for the electric and magnetic fields. Later, we had relaxed the condition that the medium has a finite conductivity and because of that we had the conduction current density because of the fields because we had the finite conductivity.

When we discuss the problem of radiation however, we have to firstly separate out the two regions; one is what is called a source region where you have the currents and charges and the effect region where essentially you are going to get the electric and magnetic fields. So, we have essentially some region here which we call as source which consists of the currents J and also it will be having the charges rho.

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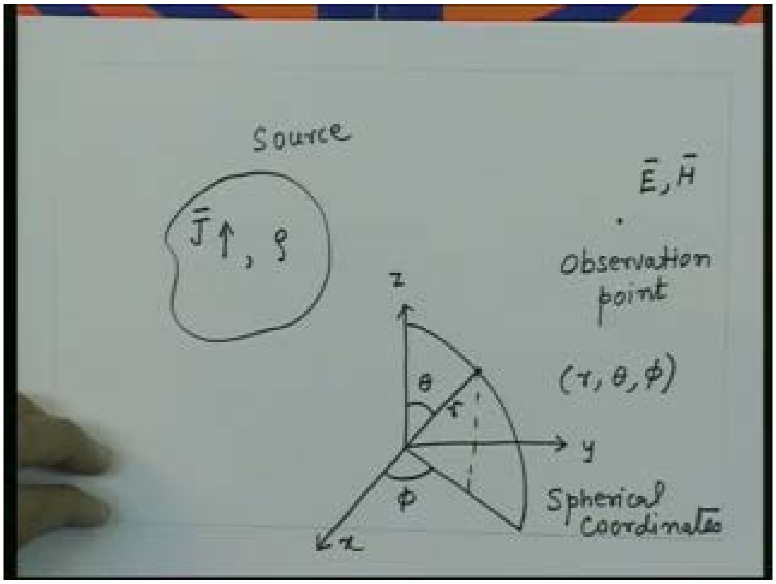

So, we have the conduction, the current density; we have the charges for the volume charge density rho; and because of these currents and charges, we have to find out the electric and magnetic field at some point here, which is the observation point. So, this point we want to find out what is the electric and magnetic field.

So firstly we will note here that when we go here in the source, then the conduction current density is sigma times the electric field which is at that location here. But if I go here, then the conductivity is zero because this space we are talking about is a unbound space, which is like free space. So the conductivity for this medium is zero.

So if I go here, I have sigma times here the conduction current. But if I go here which is the observation point, then the conductivity is zero and there is no conduction current. So when I solve now the Maxwell's equations in this region, the Maxwell's equations are source free; whereas if I go here, the Maxwell's equations are driven by the sources which are the conduction current and the charges. And essentially we have to establish the relationship between the electric and magnetic fields with the sources that is objective.

So we have here what is called an observation point. Now, since we are investigating the power generated by the sources, the currents in charges, essentially this source is located at a finite distance. So, the source now is at observer distance from the origin of any coordinate system. Earlier when we investigated the uniform plane wave, essentially if you have to generate a phase front which is of infinite size, ideally you will require a source which will be of infinite size and that problem essentially we had circumvented by saying that, let us assume that the wave is coming from a very large distance. So, the source was essentially pushed to infinity. So, we never defined, we never worried about what is the coordinate origin when we talked about the uniform plane wave; and that was possible in Cartesian coordinate system because all the coordinates x, y, z, there were translation. You could translate it without changing the direction of the unit vectors on x, y, z. So, the origin can be shifted anywhere.

In this case, since now the source has to be located and the things are going to be with respect to the source, the appropriate coordinate system now is not the Cartesian coordinate system but that will be a spherical coordinate system. So we say with respect to the origin in the spherical coordinate system, we have the source location and then we generate the fields in the three dimensional space and since we are investigating the power which is going away from the source, essentially from this point which we can call origin for this coordinate system, the fields will be taking power away from the origin.

So, the coordinate system which we use normally for antennas is the spherical coordinate system. So, if we have this is x, y and z, the spherical coordinate system, some point it will be...this is phi. So, let us say some point here p from z axis, this radius vector x and angle theta. This distance is r; and if I take a projection of this and this, that is the angle which will make phi.

So essentially what we get, we get a coordinate now – r, theta and phi. So we essentially get a distance of the point, observation point from the source which we can keep at the origin and then the directions in which the power will be flowing. So we have two angles now. The theta which is measured with respect to z direction and then phi is the angle which is of the projection taken on the x y plane measured from the x direction.

So in this topic the antenna, essentially we will be handling the coordinate system which is the spherical coordinate system. So, spherical coordinates. With this, let us now go to the general Maxwell's equations and then see how do we formulate the problem for radiation. That means the formulation of the Maxwell's equation in terms of the sources which are the currents and the charges.

Now, if I try to see the relationship between electric and magnetic fields with the sources directly, then the problem rather is very complicated. So as we have done when we talked about electrostatic case, that the electric field was related to what is called a scalar potential, we can define another potential what is called the magnetic vector potential; and then the problem is simplified to some extent because the magnetic vector potential as we will see will be related to the currents; electric scalar potential will be related to the charges. So, we can find essentially the solutions for the potentials and once you get the solution for potentials, then we can find out the electric and magnetic fields.

So, problem essentially is divided into two steps: first, define the potentials which are consistent with the Maxwell's equations, find the solutions for the potentials and then from this potential, find the electric and magnetic fields and from there you find out the power or the radiation from the sources. So, starting with the Maxwell's equation, that del dot B is equal to zero, this identity will be satisfied, if I define this B vector by a curl of some other vector.

So, if I define this quantity B is curl of some vector, then the del dot del cross A will be identically equal to zero. So, this Maxwell's equation will always be satisfied. So, note the physical quantity which we are having is the magnetic flux density, which is B. Only for our mathematical convenience we are defining this quantity and here this operation is similar to what we have got for the electrostatic case, which was, the gradient of the scalar potential was equal to electric field.

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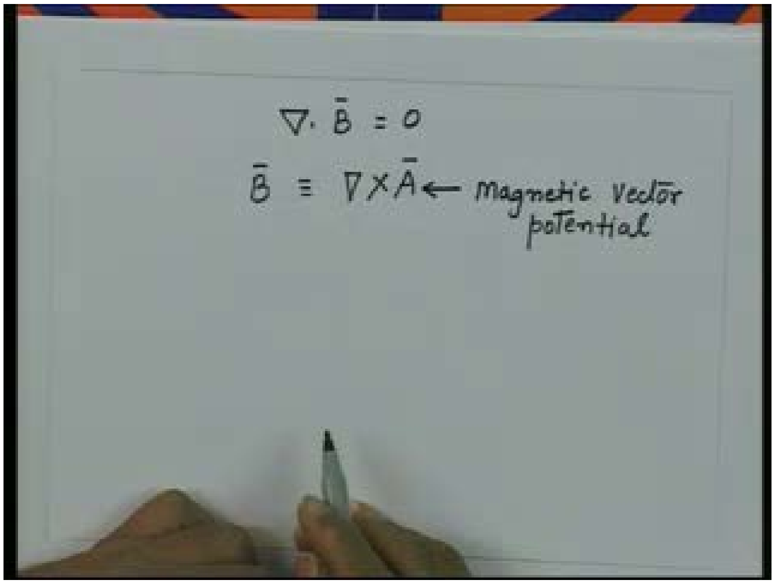

This quantity, we call as the magnetic potential and since we are having an operation, curl operation, this is the vector quantity. So, this we call as the magnetic vector potential. So, first we define this quantity for mathematical convenience what is called magnetic vector potential; and now we formulate the problem in terms of the magnetic vector potential. Taking the other equation...so this was one equation, if I take another Maxwell's equation which is del cross E is equal to minus d B by d t and let me use the dot notation; that is, d by d t is represented by dot. So, this thing can be minus B dot. The dot is same as d by d t.

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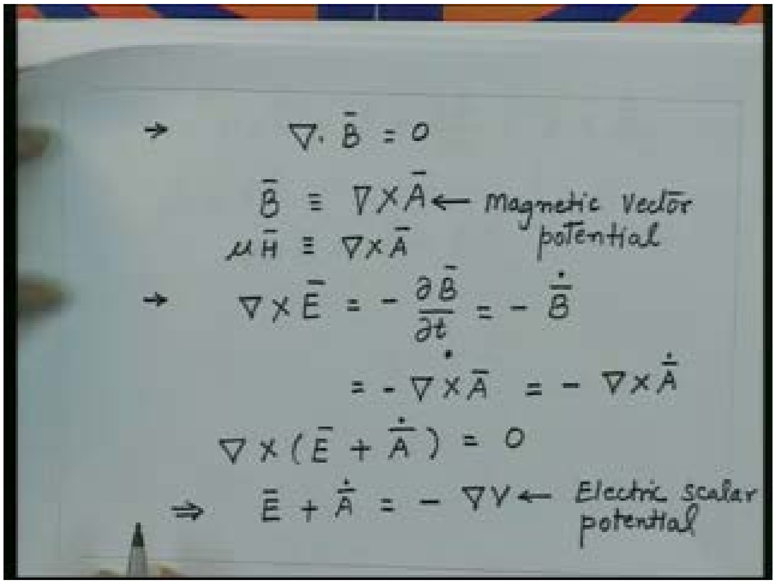

So, I can substitute now for B from here. So, this will be equal to del cross A dot, whole thing, the minus sign. Now again using the arguments that thus we are not worried about the variation of space as a function of time as we did earlier when we talked about Maxwell's equations, we can now interchange. So, it is d by d t of this quantity. We can interchange the space and time derivatives. So, this quantity can be written as minus del cross A dot. I can bring this term on this side. So, this gives me del cross E plus A dot. That is equal to zero.

Now again, this is an identity which we have got. So, if I define this quantity as gradient of some scalar, this will be del cross del of scalar quantity; will be identically equal to zero. So from the vector identity, we can say that this quantity now is equal to the gradient of some scalar function. So, this essentially gives me that E plus A dot; that is equal to gradient of some scalar function. And let us call that as minus del of V. So, V the scalar function is the gradient of the scalar function and I am using this minus sign to be consistent with the electrostatic case.

That means if I put time derivative equal to zero, we get this electrostatic case and in this case I know the electric field is equal to minus times the gradient of the scalar electric potential. So, V is the electric scalar potential; same as what we have seen earlier so that we define this quantity equal to minus del of V, so that this relation is consistent when time derivatives are set to zero. We have put d by d t equal to zero. This will be zero and I will get the electric field which is minus times the del of V and V is, as we know, is called the electric scalar potential. So, this is one relation we now have. That electric field is now represented in terms of the derivative of the magnetic vector potential and the gradient of the electric scalar potential.

We can take another Maxwell's equation. So, we can take del cross H. That is equal to J plus d D by d t which is equal to J plus D dot; and now knowing the relation that D is equal to epsilon times E, we can get del cross H which is equal to J plus epsilon E dot. Again while doing this we are assuming that epsilon is not changing as the function of time. So medium is stationary and then we can write down essentially this equation.

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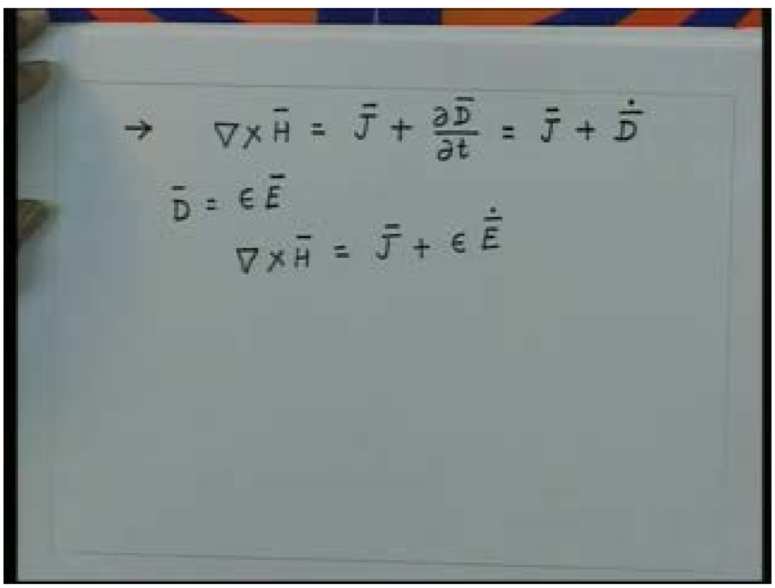

Then knowing from here, this we can write as mu times H equal to del cross A. So, H will be 1 upon mu into del cross A.

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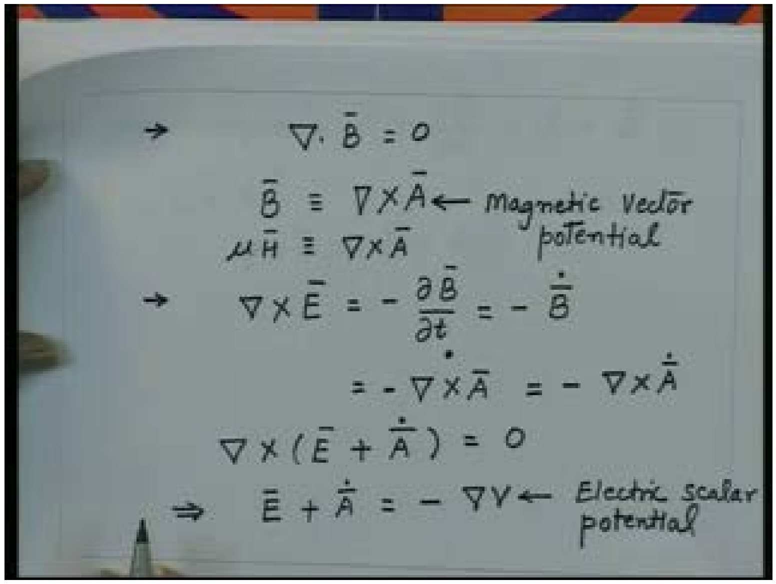

I can substitute in this. So I got here, del cross 1 upon mu del cross A; that is equal to J plus epsilon E dot. Again, saying that the medium permeability is not changing, so I can take it out. You have here del cross del cross A. That is equal to mu times J plus mu epsilon times E dot; and we can use now the vector identity to expand this triple product.

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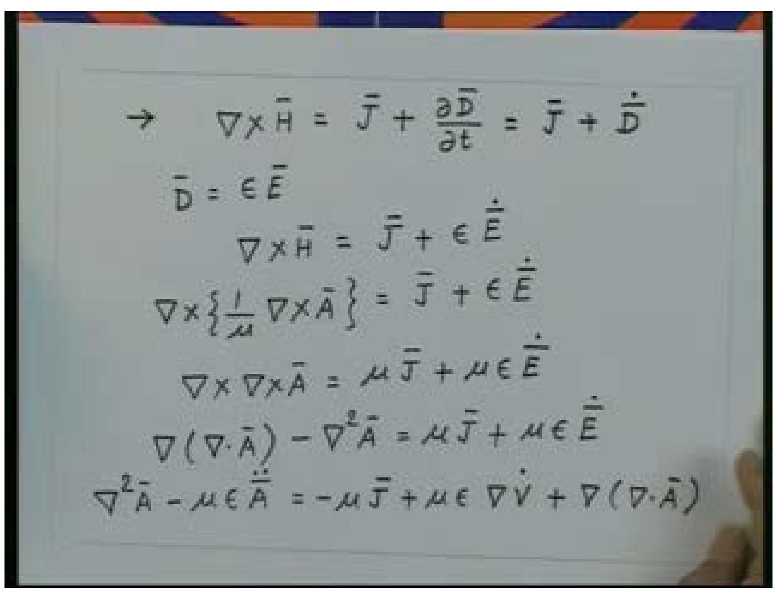

So, the vector identity is del of del dot A minus del square A. That is equal to mu J plus mu epsilon E dot. We can rearrange the terms for this and also substitute for the electric field from here. That is E equal to minus A dot minus del V into this.

I can substitute for electric field and then arrange the terms to get del square A minus mu epsilon A double dot. This double dot is coming because of this E dot and E is having this quantity A dot. So, this epsilon that is equal to minus mu J plus mu epsilon del of V dot plus del of del dot A. So from this curl equation, essentially we get now a relation between the vector potential and this quantity what is the source; that is the conduction current density into the medium.

We can use now the third relation and that is del dot E. That is equal to rho of one epsilon. But before we do that, let us look at this equation and let us try to see if the sources were not there. Then what equation essentially we would get from the Maxwell's equation? We already solved this problem earlier. As we have said, when the medium was source free, that means when there was no conduction current, when there was no charges, we had got a solution that was the wave kind of solution and the equation was the wave equation.

So that means whatever case we are considering now with charges, if we put charge or the currents equal to zero, then we must get the equations which are consistent with the source free solution or source free equation. So firstly, what we will note here, this quantity here, the vector potential if we are writing, if I say the time derivatives or the time variation for all the electric magnetic fields, vector potential, all these quantities is harmonic with angular frequency omega, then the second derivative of this quantity will be equal to multiplying by minus omega square.

So, if I put current equal to zero and if I had considered this quantity as time varying quantity which are e to the power j omega t, then I know I should get an equation, del square something plus omega square mu epsilon. That quantity should be equal to zero. That is the wave equation.

However, what we see now, there is a new term will be here which was not there when we investigated the wave propagation problem in the unbound medium. So, the first thing to note here is that we have an additional term here now over and above what we would get from a simple wave equation, when I put the source term zero. Is this equation or does this equation give you unique solution? The answer is No.

This equation cannot give you unique solution and the reason for this is, this quantity vector potential which you have to define here as we...as you know is, we defined for our mathematical convenience. So, we defined some vector field which we called as negative vector potential for which the curl is defined which is the magnetic flux density; this is the physical quantity.

So, just by defining the curl of a vector field, the vector field is not uniquely defined. The vector field is uniquely defined, if we define this curl and divergence in the space. Then and then only you can define uniquely the vector field. So up till now what we have done, we have simply defined the curl of this vector field which is magnetic vector potential; but the divergence of this vector potential is completely undefined and because of that, there is no uniqueness in the magnetic vector potential. I cannot tell the unique solution in this problem.

So, we still have a freedom of choosing the magnetic vector potential's divergences and we should choose the divergence in such a way that this equation, whatever we have should reduce to the wave equation when we put the source free condition. That means if I say that these two quantities are equal to zero because some of these two equal to zero, then this will be equal to zero. If I put now the source which is J equal to zero, I will get a equation which will be this equation which will be a simple wave equation.

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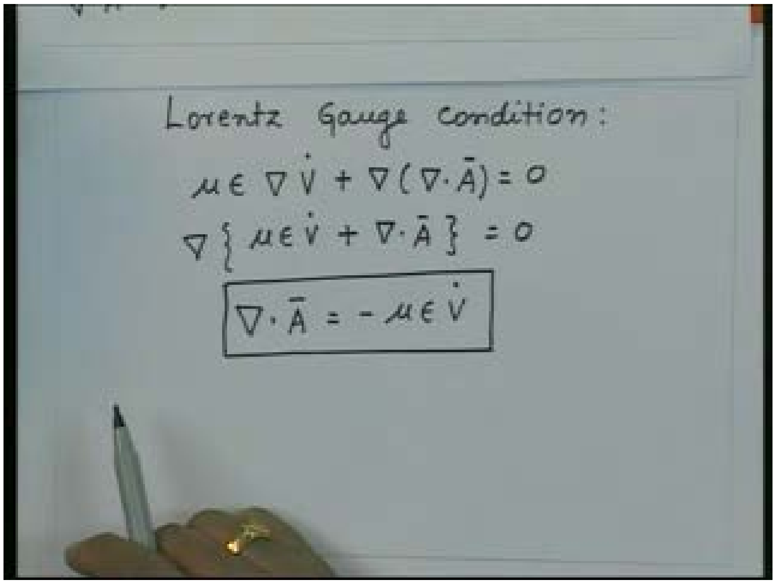

So, to make this equation consistent with the wave equation and the source free condition, we defined the divergence del of A such that this quantity goes to zero. That condition then is called the Lorenz Gauge condition. So, Lorenz Gauge condition makes this quantity equal to zero. So essentially this condition is mu epsilon del of V dot plus del of del dot A; that should be equal to zero. I can take this as divergence. So, I can write this as del of mu epsilon V dot plus del dot A; that is equal to zero. And since this has to be true for any space, essentially we get from here the condition that del dot A should be equal to minus mu epsilon V dot.

This is the condition which is the Lorenz Gauge condition which is defining the divergence of the magnetic vector potential. So now, we have defined the curl of the magnetic vector potential which is related to the magnetic flux density. We have defined the divergence of the magnetic vector potential by what is called Lorenz Gauge condition.

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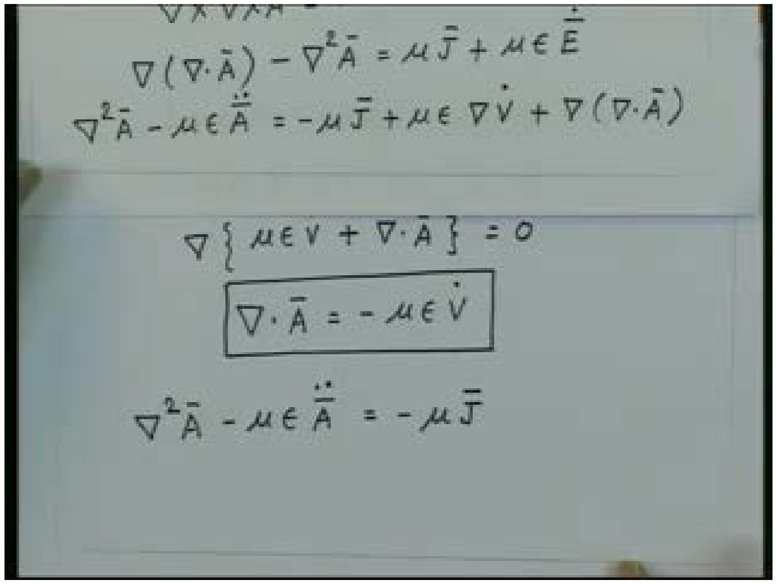

Then, we can get a solution to the magnetic vector potential uniquely and by doing this, essentially, this equation now reduces to del square A minus mu epsilon A double dot. That is equal to minus mu into J. What we can do? We can just take this condition which we had got for electric field from here and we can substitute for A dot from this. So, I can take a divergence of this equation.

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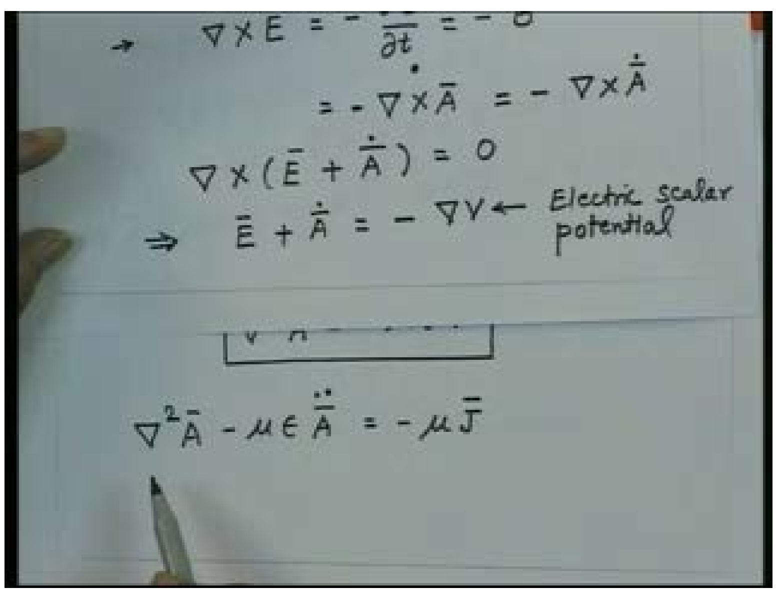

So, if I just use this relation which we got from first equation...say, if I get del dot E plus del dot A dot is equal to minus del square V and I know del dot E is rho upon epsilon from the Maxwell's equation ; so this quantity can be written as rho upon epsilon plus del dot A. If I get from Lorenz Gauge condition, the del dot A will be minus mu epsilon V dot and we want del of A dot here.

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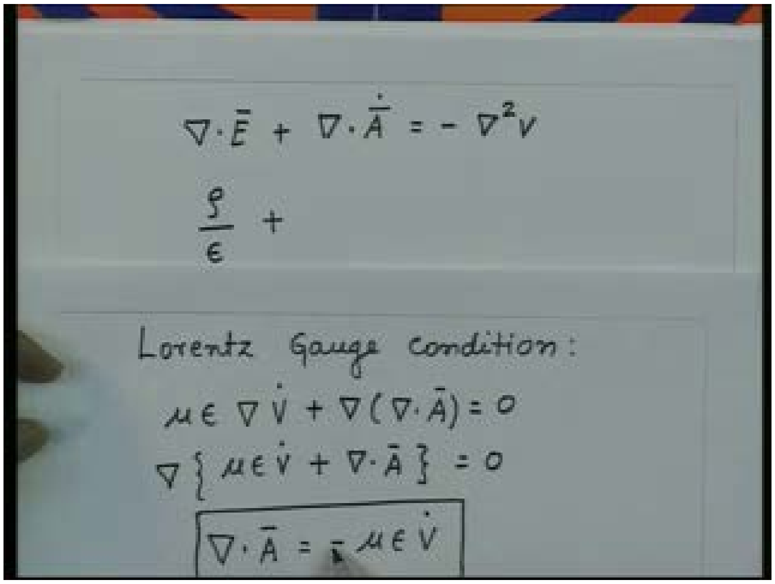

So, this will be minus mu epsilon V double dot. So, this will be minus mu epsilon V double dot. That is equal to minus del square V. So, if I rearrange the terms, we get the equation from here which will be del square V minus mu epsilon V double dot. That will be equal to minus rho upon epsilon.

So you see, these are interesting things which has happened by defining what is called the Lorenz Gauge condition and that is...we now got this equation; one for the magnetic vector potential which is this and one for the electric scalar potential which is this; and these two equations are identical equations: del square operator with the scalar operator.

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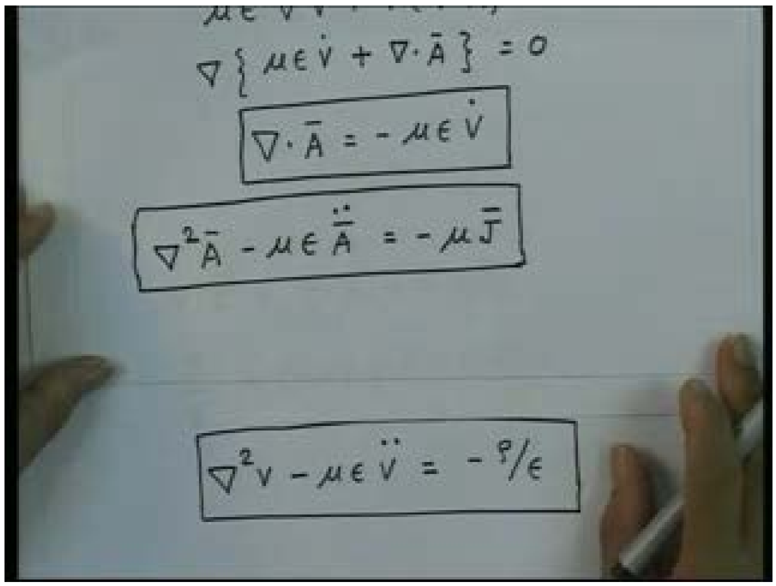

So here, the vector is because of the conduction current density J and an A; and this equation is the scalar equation. But if I see functionally, these two equations, these two equations are identical equations. So essentially, by defining the Lorenz Gauge condition, we not only decouple the two potentials the magnetic vector potential and the electric scalar potential, but we get the equations which are identical to the wave equation which in case of source free, if I put J equal to zero and rho equal to zero, I will get the wave equation.

So, what we conclude first before we go to the radiation problem that when we have a time varying sources, then we have the electric and magnetic vector potential and both of these are essentially governed by the wave equation. So, the magnetic vector potential and electric scalar potential, both have a wave behavior in the three dimensional space.