Transmission Lines & E M. Waves
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Lecture – 42

We are discussing an important topic in waveguides and that is calculation of attenuation constant. We have seen that in practice, the material with which the waveguide is made is not ideal conductor. So, we have ohmic losses in the walls of the waveguide and also the dielectric which is filling the waveguide may not be an ideal dielectric and because of that we may have dielectric losses in the dielectric.

Then we saw last time that this attenuation constant due to these two components, one is because of the finite conductivity of the walls and the finite conductivity of the dielectric material filling the waveguide are tided independent. So, we calculate the attenuation constant due to each of these components and then, say that the total attenuation constant is sum of the two attenuation constants, under the assumption that losses are very small in a good waveguide. And last time we calculated the attenuation constant due to dielectric material introducing the concept of the complex permittivity.

So in the dispersion relation, if you replace the dielectric constant by the complex dielectric constant for the lossy medium then separating the real and imaginary parts we can get the attenuation constant due to the dielectric losses then we develop the frame work for calculating the attenuation constant due to the conducting losses in the wall and we saw that the calculation of attenuation constant in general can be visualize as the propagation of the fields inside the waveguide and in presence of losses say electric and magnetic fields both will have a variation e to the power minus alpha z, if this mode is propagating in z direction and alpha is attenuation constant.

Then, in general the power will be proportional to e to the power minus 2 alpha z because the power is proportional to the square of the electric field or the magnetic field. So if we replace this power by some constant initial power at z equal to 0 times this variation e to the power minus 2 alpha z and differentiating this with respect to z, we get dw by dz minus 2 alpha e 2 the power minus 2 alpha z and then, we can write the attenuation constant alpha from here which is minus dw by dz upon 2 w.

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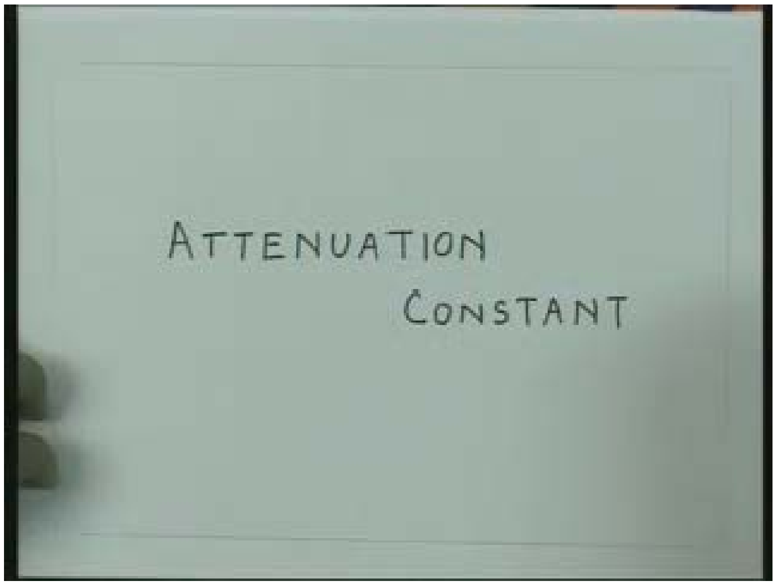

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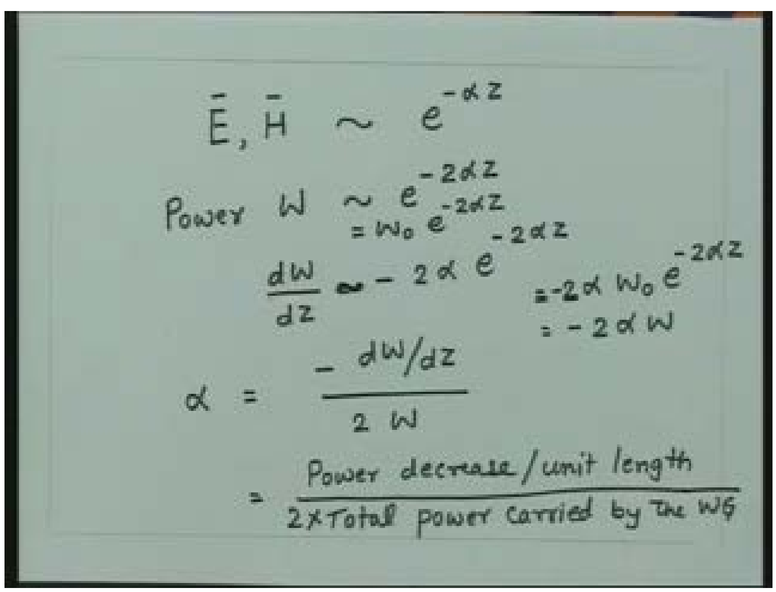

Physically, we can interpret this quantity is the power decrease per unit length and divided by two times the power carried by the waveguide. So you have to put factor 2 here, so essentially in general if we have to calculate the attenuation constant of a wave guide then we require 2 quantities, one is what is the power loss per unit length of the waveguide and what is the total power carried by the waveguide, if these we have know these 2 quantities then I can calculate what is the value of alpha. We also mentioned very clearly that in general the fields which are there in a waveguide are function of the losses and the losses depend upon the field. So in general actually the problem is very complex because the losses and fields depends upon each other.

However, if you consider a waveguide which has low losses then we can assume that even in the presence of losses the fields are not disturbed significantly that means the fields of a lossy waveguide are almost same as the fields of a losses waveguide and once we know the fields then one can use this relation to find out the attenuation constant of the waveguide that is what preciously, we are going to do now.

So we will take today 2 cases, one is a waveguide which is a parallel plane waveguide which is simpler case and we will consider there are more which is the transverse electromagnetic mode. Once we develop the understanding how do we calculate the losses in the waveguide then we will go to the more useful mode in rectangular waveguide that is the TE 1 0 mode.

So let us calculate first the attenuation constant for the TEM mode in a parallel plane waveguide. So we talk about calculation of alpha and let me put a suffix c here to denote the conducting losses in for TEM in a parallel plane wave guide. So today we do this calculation of the attenuation constant and that will give us some feeling when we use the structures like coaxial cables or the parallel wire line, how the losses are actually calculated.

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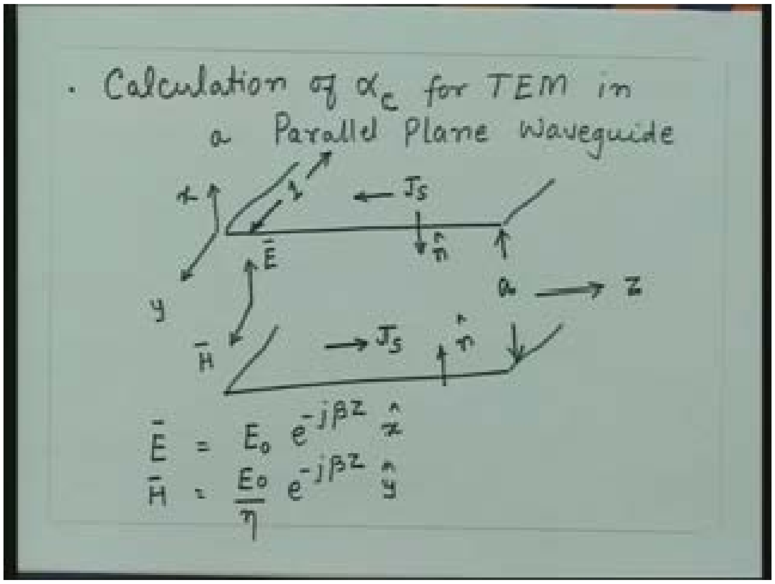

We have seen the calculation of these losses from the knowledge of resistance and the conductance in the transmission line. However, one may ask a question how do we know this resistance and conductance in the transmission line. So essentially starting from the fields we can calculate how the losses are going to take place in this conducting surfaces.

Now let us consider now a parallel plane waveguide, here like that and the wave is propagating in this direction which is the z direction. This direction we have taken as the x direction which is perpendicular to the planes of the waveguide and the direction which is coming outwards this that is the y direction. Then, we have seen for the TEM wave we have the electric field which is like that and the magnetic field which is oriented in the y direction.

So in this case, this is the direction of the magnetic field so we have here e, we have here h and the wave is propagating in this direction and we have also seen for this mode, the ratio of the electric and magnetic fields is equal to the intrinsic impedance of the medium filling the waveguide. So we have a relation between e and h that is equal to eta. So if I express the electric field e which is some amplitude e 0, e to the power minus j beta z then h will be equal to e 0 divided by eta, where eta is the intrinsic impedance of this medium e to the power minus j beta z. Only, if I put this as a vector this will be in x direction, h will be in y direction. Now since the waveguide is infinite in the y direction if we go by this relation and find out what is the power, total power carried by the waveguide then this power will be infinite because we have the cross section of this waveguide if I consider the total waveguide in y direction, the total power carried will be infinite, the total loss will be infinite, you will not get any meaningful answer. So normally what we do we can defined the attenuation constant per unit width of this waveguide. So I can say that if I take a piece of this waveguide which is of unit width in y direction.

If I take this is 1 in y direction, what is the power loss in the unit width of the waveguide? What is the power carried by unit width of the waveguide and then, from their I can calculate the attenuation constant of the waveguide. Now note here this h which is not varying as the function of y this constant whose amplitude is e 0 upon eta. On this wall the magnetic field will be in y direction here also the magnetic field will be in y direction and as we saw last time that the normal to the surface will be upwards, here unit normal which is like that and we will have unit normal for the upper surface which will be like that.

So if I go by n cross h, so this is n, h is coming out, so if I use n cross h right hand then the j will be flowing in this direction. So I have that the surface current which flows here which is js and similarly, on this wall surface current will be flowing this way which will be js. Again n cross h, say n is downwards and h is coming out if I go by that then from here and to h will give you the direction of js and the amplitude of js is exactly same as this amplitude which is e 0 upon eta. Now once we are visualize this fields and current inside the waveguide then these fields are going to now move with a phase velocity inside this waveguide in the z direction.

So all this quantities are going to have a sinusoidal variation as a function of time. So if I consider any location, the fields and the current, surface current are going to vary sinusoidally as a function of time. So if I take the peak amplitude of this and then find out simply the RMS value of the electric and magnetic fields and the surface currents then I can find out the RMS power loss and the RMS power carried by the waveguide.

So for unit width of this waveguide, the power carried by the waveguide per unit width will be simply half e cross h conjugate, integrated over the cross section which is having a height which is a or d and width of the wave guide which is equal to unity. So let us say I have the height of this wave guide which is given as a, then the power carried by the wave guide per unit width.

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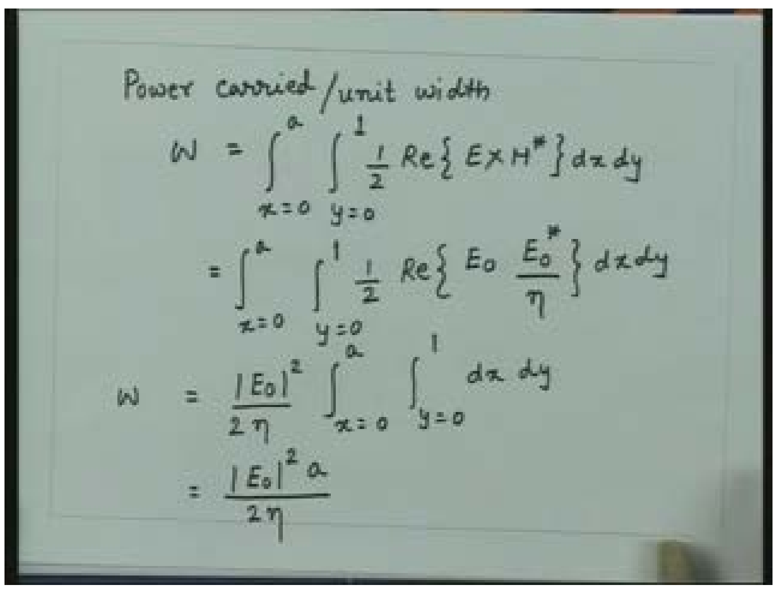

So we can get power carried per unit width that is w that is equal to integral over the cross section. So in the x direction the limits are from 0 to a from y direction the limits are from 0 to 1. So we have integral x is equal to 0 to a y is equal to 0 to 1 half real part of e cross h conjugate integrated over area of cross section dx dy.

Now the electric field is x oriented and the magnetic field is y oriented, so e cross h gives me the z direction. So this power is going to flow in the z direction and e cross h conjugate if I take this will be e 0 multiplied by e 0 conjugate upon eta conjugate and if we consider that the dielectric which is filling this waveguide now is ideal dielectric, eta is a real quantity. So essentially this will reduce to the integral x equal to 0 to a y equal to 0 to 1 half real part of e 0, e 0 conjugate upon eta conjugate but for dielectric eta is real that is same as eta dy. So the total power per unit width of the waveguide that is this quantity is constant.

So you get mod e not square upon eta 2 times integral x is equal to 0 to a y is equal to 0 to 1 dx dy. So essentially with respect to y the integral is 1 with respect to x, the integral is a so this is equal to simply mod e 0 square a upon 2 times eta. So we got now the total power carried by the parallel plane waveguide for a transverse electromagnetic mode per unit width of the waveguide. The second quantity which we need is the power loss per unit length of the waveguide. As we have seen earlier, when we introduce the concept of what is called the surface resistance.

If you know the conductivity of this medium we can calculate the surface resistance and once you know the surface resistance the power loss per unit area is half rs into mod j square. So the power loss in this wall, so we get here power loss per unit length which is equal to minus dw by dz that is integrated over the area of the surface per unit length that means in z direction, we have unit length and y direction also we have taken width equal to unity, so in this y also we have unit length.

So the integration for limits for the area are from 0 to 1 in z and from 0 to 1 in y and js will be same as the magnetic field, so this peak amplitude will be e 0 upon eta and will be having a sinusoidal variation in time. So per unit length if I take a RMS value and integrate over this that will give me the total loss in this wall and since, there are two walls the total loss will be double of what we calculate from the each wall.

So that gives me 2 times factor of 2 is for 2 walls and the integral limit for y is equal to 0 to 1, for z 0 to 1 per unit length of the waveguide half rs which is a surface resistance mod of js square dy dz and if you recall, the rs was the real part of the intrinsic impedance of this conducting wall. So just remind you that the intrinsic impedance eta c for the conductor is square root of j omega mu upon sigma which we can separate out into square root of omega mu upon 2 sigma plus j square root omega mu upon 2 sigma. This quantity was the surface resistance, this quantity was surface reactance. We are interested now for the loss in this quantity with a surface resistance. So I can substitute in this expression and for js is same as n cross h which is which is h which is 0 upon eta.

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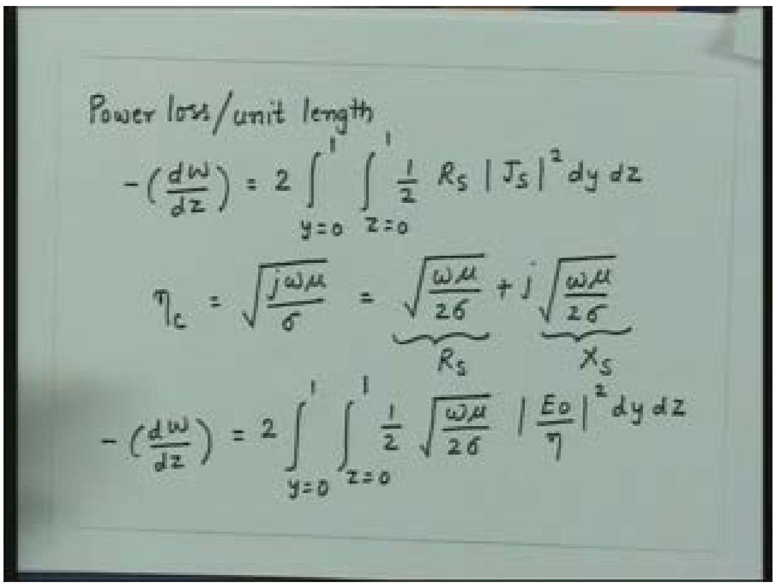

So I get the power loss per unit width minus dw upon dz equal to 2 time y is equal to 0 to 1, z equal to 0 to 1 half rs which is this quantity omega mu upon 2 sigma js square which is e 0 upon eta square dy dz. Again all this quantities are constant as if function of y and z we can take this thing out and we can get the power loss minus dw by dz that is equal to this factor of 2 will cancel with that.

So we will get square root of omega mu upon 2 sigma mod e naught square upon eta, an integral this now will be for y from 0 to 1 dy that will be 1 for z, z equal to 0 to 1 dz that will be 1, so the integral value is equal to 1. So, I now get the quantities now which are needed for calculation of the attenuation constant one is the total power guided by the waveguide and second one is the power loss per unit length of the waveguide.

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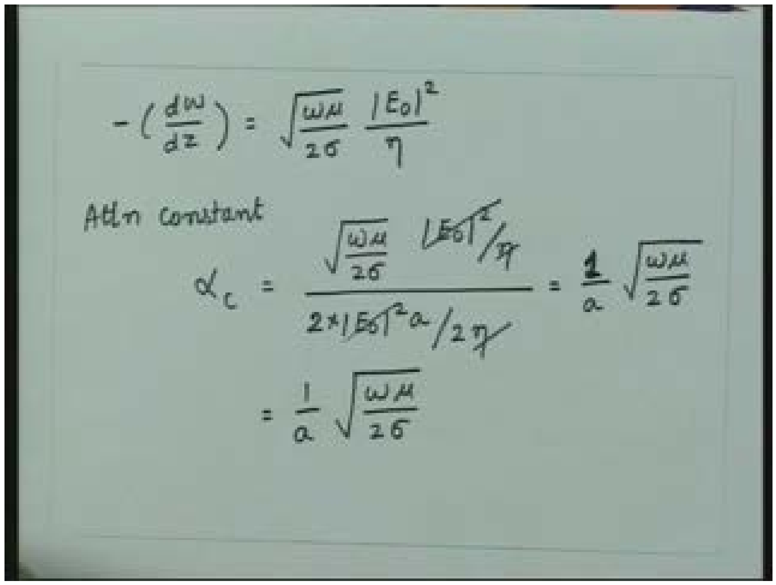

So the attenuation constant for this case alpha c will be minus dw by dz which is this so square root of omega mu upon 2 sigma mod e not square upon eta divided by the total power carried which is mod e naught square a mod e naught square a upon 2 eta. The e naught square will cancel, eta will cancel will get this quantity cancels with this, this cancels with that you will get this is equal to 2 upon a square root of omega mu upon 2 sigma.

So we get the attenuation constant for the parallel plane waveguide which is given by that no sorry, there is a factor 2 here, so I have to multiply 2 times this, so there is no 2 here this should be equal to 1. For the attenuation constant e is the power loss per unit length divided by 2 times the power carried. So, alpha c is 1 upon a square root of omega mu upon 2 sigma. Few things can be observed here and that is the attenuation constant is inversely proportional to the height of the waveguide and that make sense here because if you consider the parallel plane waveguide.

Since magnetic field is not varying as a function of height, function of x, no matter what is that separation between these two planes the power loss per unit length is same, it does not change. However, the power carried by the waveguide increases as the height of the waveguide increases.

So the attenuation constant, so the power carried by the waveguide is more but the power loss per unit length is small and as a result the attenuation constant is small. Second important thing which we notice here is the attenuation constant is inversely proportional to the square root of the sigma and that we also understand that if we have a good conductor sigma becomes large and the attenuation constant will be small and in the limit when the sigma tends to infinity the attenuation constant will be 0.

So if we consider a waveguide which is made of ideal conductor then we will have the attenuation constant 0 but the important thing to note from this expression is that the attenuation constant is the function of frequency. So as the frequency increases as omega increases, the attenuation constant increases that is the reason if you take any guiding structure like a coaxial cable or parallel plane line or anything. As you go to higher and higher frequency the signals have more attenuation because the attenuation constant is a at a function of frequency.

So at low frequencies the attenuation constant is manageable but when you use the same structure for carrying the signal at high frequencies, the loss becomes excessive and the signal cannot be transmitted efficiently from one point to another. You can also see this from another angle just suppose, we had to send a broadband signal using this structures that means if the signal occupies a large bandwidth, the lower end of that frequency spectrum will go efficiently but the higher end of the spectrum will not go efficiently, this amplitude will be reduced that means the waveguide in structure has a effect like a low pass filtering.

So your signal get distorted as if the signal has passed through a low pass filter. So whenever we have this guiding structures invariably, you will see the effect will be like a low pass filtering and the signal will get distorted that means there will be blurring of the sharp edges in the signal in time signal gets smoothed out because you get a low pass filtering action because of the loss behavior of this waveguide.

So in conductor loss in general one can say that varies as the function of frequency and as we increase the frequency, we get the higher loss right for the same waveguide in structure. With this understanding for the simpler problem, now we can go to the mode in which we are more interested in and that is the transverse electric mode 1 0 mode, TE 1 0 mode because that is the mode which is going to propagate inside a rectangular wave guide and if you recall we have already derived the fields for the transverse electric 1 0 mode, it had 3 components E y, H x and H z and E x, E z and H y are 0 for this mode last time, we also saw the current distribution because of these fields.

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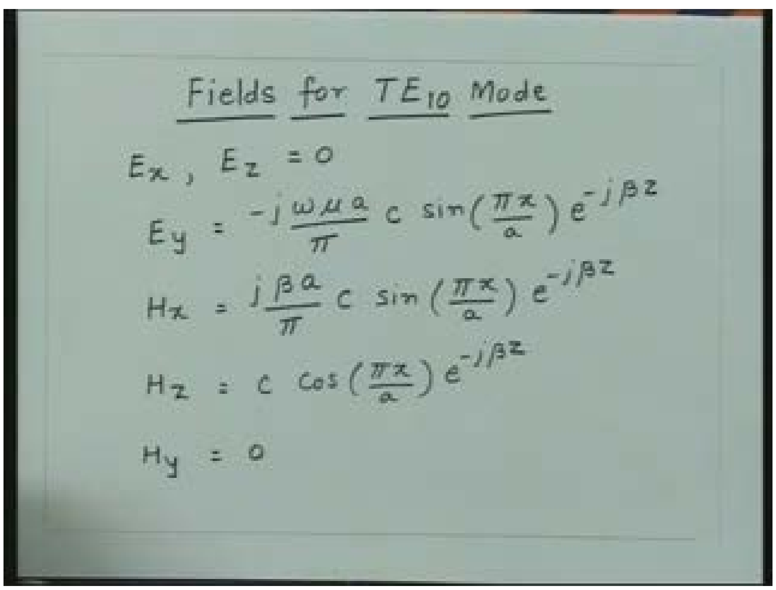

So now what we can do we can using this fields now, we can calculate what is the power carried by the waveguide, what is the loss in 4 walls of the waveguide and then from their we can calculate the attenuation constant. So let us first take the waveguide, this is the waveguide. In the x direction I have a width of the waveguide which is a, in the y direction I have height of the waveguide which is b and the wave is going to propagate in z direction. This is z, so what we do first, first we calculate total power because I know now the cross section I take per unit length of this waveguide and calculate the loss per unit length.

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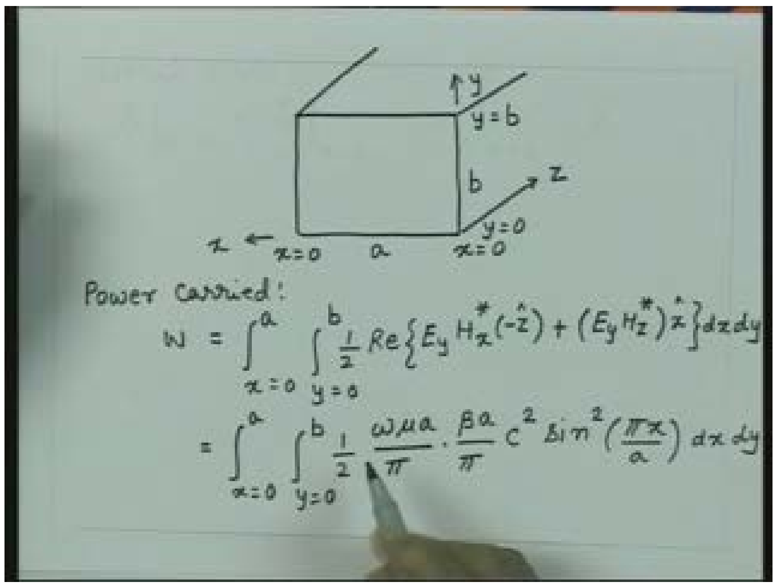

Again we use the same argument since at every location the field is going to vary as a function of times sinusoidally. We take the peak value of the magnetic field and just take the RMS value from there and then, you calculate what is the power loss by integrating over the length of the waveguide.

So first we take total power, power carried that is w that will be equal to x going from 0 to a, y going from 0 to b, half real part of e cross h and the real part since these 2 quantities, this is minus j, this is j. So if I calculate from here e cross h conjugate, only this product will give me real part, product of these 2 will give me imaginary.

So E y times H z which gives me the power flow in the x direction that is imaginary only this power which is E y, H x that is the one which will give me the real power flow in the z direction. So from here real part e, E y, H x conjugate and E y, H x will give me the power flow in the minus z direction plus I have E y, H z conjugate that gives me the power flow in the x direction, integrate dx dy.

Now as I mention this quantity is purely imaginary from here. So the product which we require is this and this minus z, if I take H x conjugate, this will become minus j if I take a product of these that will become plus from j square will become minus and then which is minus sign will compensate for that. So you will get x equal to 0 to a, y equal to 0 to b half multiplication of these 2. So that gives me, omega mu a upon phi beta a upon phi, c square sin square phi x by a dx dy. Again, so there is no variation in terms of terms of y this function this is a function of x only.

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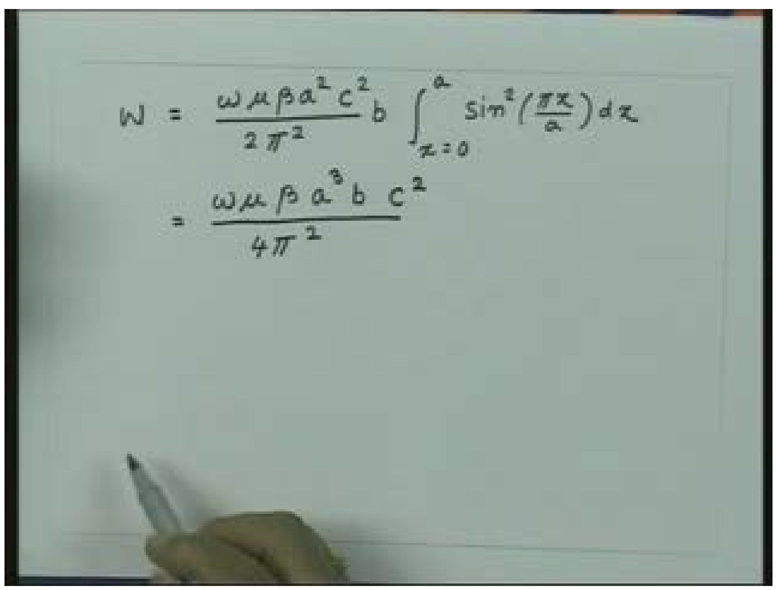

So I can combine these terms and get the total power inside the waveguide w that is omega mu beta a square c square upon 2 phi square, the integral with respect to y will be b. So multiplied by b integral x equal to 0 to a sin square phi x by a into dx and this integral will be only having a value of half. So you can get finally the expression which is omega mu beta a cube b, c square upon 4 phi square.

So if I know now the fields for the TE 1 0 mode then we can calculate the total power carried by the wave guide. So power calculation for the loss in the waveguide is rather tedious because in case of parallel plane waveguide, we have we had only 2 planes and the current distribution also very simple that the current was moving only in z direction, so we got integrated very easily. If you recall the current which is flowing on the surface of this waveguide is quiet complicated last time we had shown that it has more like some kind of a flower, kind of pattern, so on this wall the current is constant because z the component of the magnetic field which is H z that does not vary as a function of y.

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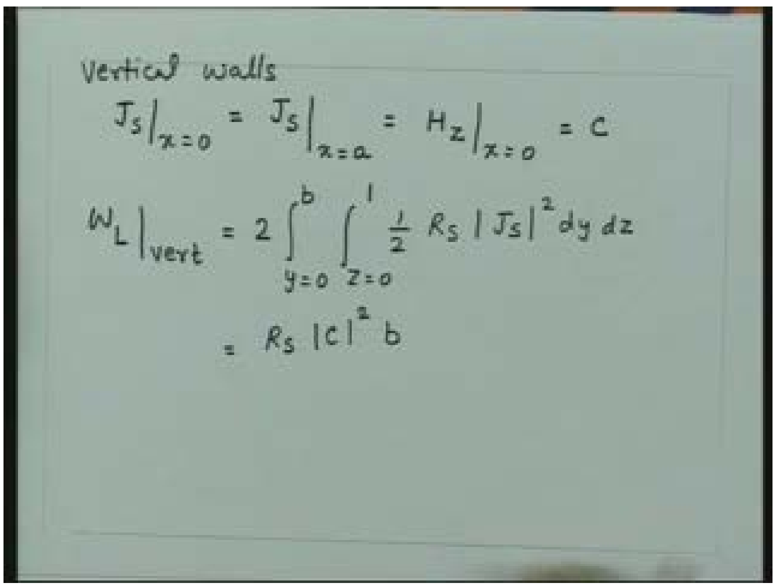

So we on this wall essentially we have the current which is oriented in this direction and it does not vary as a function of y. However, if I go to this surface then we have a 2 component of the magnetic field that we have H x also which is non-zero at y equal to 0 and y equal to b and we also have H z component which is given by that. So if I look at the waveguide I will have on this surface both the components of the current which is x and z. However, if I go to this surface I will have the current which will be only y oriented.

So we can get now the current on this wall as the surface current js at x equal to 0 wall through at x equal to 0 that is same as the surface current at x equal to a wall and that is equal to the H z at x equal to 0. So if I take this component and if I put x equal to 0 or x equal to a essentially this quantity will be equal to c. So that is equal to the amplitude c from here then we can calculate the loss of the waveguide but we have now the 2 components of the currents, one in this wall which is z oriented and other one which will be x oriented.

So in fact we have to find the total current in this wall which will be the vector sum of the 2 currents on the top surface and the bottom surface of the waveguide and from there essentially I can calculate the total loss inside the waveguide. So first let us calculate this loss for the side way walls which is this wall for which we will have that these 2 walls and for which the magnetic field is given by this.

So x equal to a gives me the current in this which is not varying as function of y on this wall also I can calculate from here the current which is not varying as the function of y. So these are the currents which are on the vertical walls. Recall in this waveguide here this is x equal to 0, this is x equal to a, this is y equal to 0, this is y equal to b. So x equal to 0 and x equal to a represent these walls the vertical walls and in this the magnetic field is only the z component of the magnetic field which is having an amplitude constant and is not varying as a function of y.

so if I calculate now the loss in these walls these 2 walls per unit length of this waveguide it is the integral of half rs into js square on these walls and since, there are 2 walls it will be double of that. So we can get the W L in vertical walls that will be 2 times y equal to 0 to b z per unit length, so z equal to 0 to 1 half rs mod js square dy dz and js is given by this which is c, so from here we get rs mod c square integral y to y equal to 0 to b will give you b and 0 to 1, for z will give you one. So loss in the vertical wall is this multiplied by b, so calculation of loss in vertical wall is quiet simple because we have only one component of the surface current which is the y oriented surface current.

However, when I go to the top surface as I mentioned we have 2 component of the current because we have 2 magnetic field component H x and H z. So we have to find the total current on this surface and then calculate the loss on the top and bottom surface so we have the surface current.

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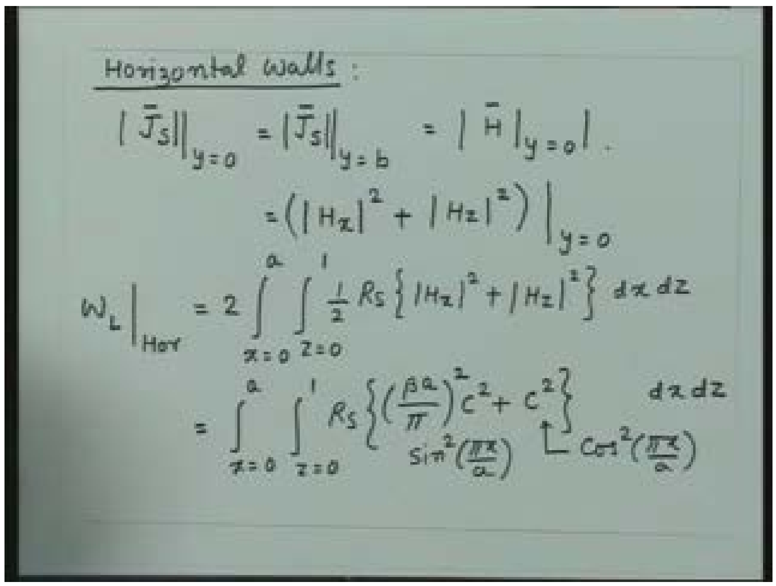

So let us consider now the horizontal wall, we get the surface current js at y equal to 0 that will be same as surface current js at y equal to b and if we are interested in finding out the mod of this quantity that is what we will be needed in our loss calculation. So, that will be equal to mod of the magnetic field js is the vector quantity now because it has 2 components x and z at y equal to 0.

So first you find out the total component which is the magnitude of the 2 components H x and H z. So this quantity will be equal to mod of H x square at y equal to 0 plus mod of H z square at y equal to 0. So from here we have the fields H x and H z, since it is not varying as a function of y fields are same that y equal to 0 or y equal to b or for that matter any value of y in this cross section. So substituting this now for 2 components H x and H z, I can calculate the total surface current magnitude and then you can calculate the losses in the vertical walls. So you get the loss W L in the horizontal wall that will be equal to 2 times integral and now these are 2 walls. So you are x limits are from 0 to a and z will be from 0 to 1.

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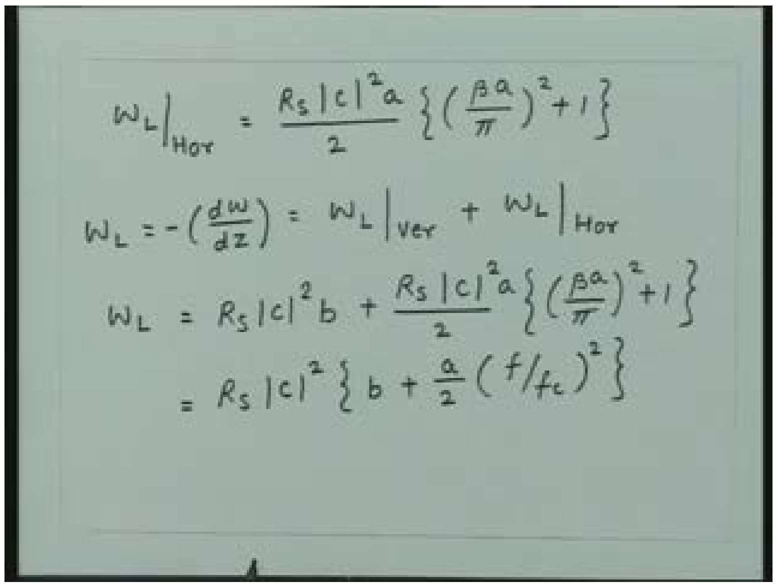

So this is x equal to 0 to a, z equal to 0 to 1 half rs into mod H x square mod H x square plus mod H z square at y equal to 0 dx dz. Substituting from this expressions for H x and H z, you can calculate this half will cancel with 2. So this will be x equal to 0 a, z equal to 0 to 1 R s beta a upon phi whole square c square plus c square. This term will be having a variation which is sin of phi x. So this will be multiplied by sin square phi x by a and this will be multiplied by cos square phi x by a integrated over dx dz. For solving, this integral essentially we get the w loss for the horizontal walls that will be R s mod c square a upon 2 into beta a upon phi whole square plus 1. So the total loss is the loss in the horizontal wall and the vertical wall. So we get the total loss W L which is minus dw by dz that will be equal to W L vertical plus W L horizontal.

So that will be equal to if I had the two terms one which I got for the vertical wall and the horizontal wall I will get that as W L equal to R s mod c square b plus R s mod c square a upon 2 beta a upon phi whole square plus one which I can write in terms of the frequency and the cutoff frequency of the mode because I know the cutoff frequency is related to the dimension of the waveguide. So if I do some rearrangement and write this expression this plus is R s mod c square to b plus a upon 2 into f upon f c whole square.

So I know the 2 things which are needed one was the total power carried by the waveguide which we got and second is the power loss per unit length of the waveguide and as we saw once we know these 2 quantities then I can get the attenuation constant for the waveguide and that will be so this quantity W L which is minus dw by dz that divided by 2 times the power gives me the attenuation constant.

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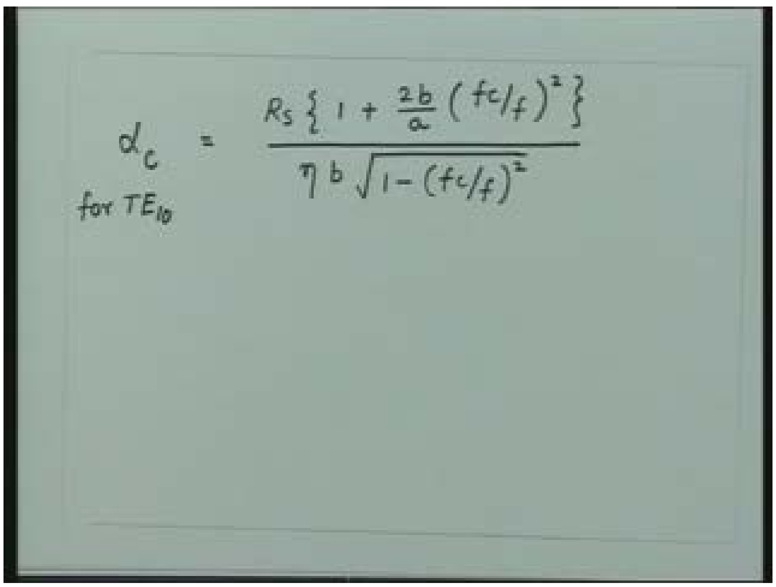

So finally we get the attenuation constant, so we get alpha c for TE 1 0 and that is equal to R s 1 plus 2 b upon a fc upon f whole square divided by eta b square root of 1 minus fc upon f whole square is the matter of only algebraic manipulation that from here W L and w which we have derive we can write this quantity in terms of a cutoff frequencies and the frequency and I doing little algebraic manipulation, we can get the expression for the attenuation constant again since the R s is square root of omega mu divided by 2 sigma the attenuation constant again increases with frequency.

So s equal to higher frequencies we get the more losses inside the waveguide but we also note that the attenuation constant now is also a function of frequency related to the cutoff frequency and as the frequency approaches the cutoff frequency, again the attenuation constant becomes very large and as we know when frequency approaches the cutoff frequency, there is no propagation of the mode the propagation ceases.

So the power essentially bounces back and forth between the 2 surfaces and the power essentially is absorb in the ohmic losses in this walls and the attenuation constant becomes very large. So having understood now these 2 cases, one was the rather simpler case which was tem mode and other one is this case which is the more useful mode but that is the mode which is most of the time going to propagate inside a rectangular waveguide.

We now can calculate the attenuation constant for any mode so we have now explain the philosophy of how the attenuation constant is calculated inside a waveguide and using this philosophy, the attenuation constant can be calculated for any arbitrary mode inside a waveguide. So let me summarize what we did the quantity attenuation constant is very important when we use a waveguide in structure in practice, we assume that the waveguide in structures are efficient that means the losses are very small on this waveguide structures.

However, whatever they are small losses we would like to get a measure of that loss, so we require this quantity attenuation constant in a practical waveguide and then we saw that the attenuation constant of a waveguide can consist 2 components, one coming because of the dielectric loss due to the finite conductivity of the dielectric filling the waveguide, other one will be the loss due to the finite conductivity of the waveguide walls, then assuming that the losses are small for both the dielectric as well as conductor, we separately calculate the attenuation constant due to dielectric losses and due to the conducting losses and then the total attenuation constant is some of these two attenuation constants.

For calculation of the attenuation constant due to dielectric, we use the concept of complex permittivity. So we replace the dielectric constant by complex dielectric constant in the dispersion relation and by separating real and imaginary parts, we get the attenuation constant due to dielectric losses. For conducting losses, we calculate the total power flow inside the waveguide and the losses per unit length on the walls of the waveguide and then from there we calculate the attenuation constant of the waveguide, this essentially completes our discussion on the normal propagating waveguide.

So what we have seen in this that we started with a wave which was in unbound medium and slowly we try to trap the wave inside a more and more bound structure. So if you look at the our journey from the beginning of the propagation of plain wave, we started for a wave in unbound medium then we put an interference so essentially try to restrict the propagation of wave in the semi-infinite space then we put two planes. So we trap the wave between 2 planes we got a structure what is called a parallel plane waveguide then we restricted the wave even from other two direction so that wave is trapped now in a closed pipe which is what is called rectangular waveguide. So we realize a practically useable structure what is called rectangular waveguide by trapping this wave in this 4 walls of this rectangular waveguide.

Of course we can also close from other two sides that means we can cut this pipe and sort of close this pipe from the both sides and then the wave can be trapped completely inside the structure that structure essentially is called resonator which is beyond the scope of this course, so we do not discuss resonators.

But if you look at the journey of our discussion essentially we started from an unbound medium is slowly captured the wave into a more bound structure and that is the way the electromagnetic waves are guided in a practical system. So, waveguide is an extremely useful device for guiding the electromagnetic waves efficiently from one point to another.