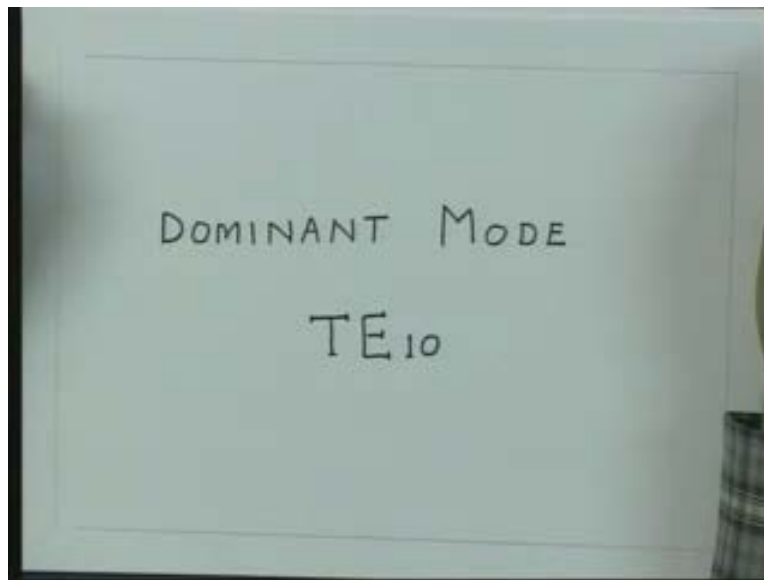


Transmission Lines & E M. Waves
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Lecture – 41

In the previous lecture, we tried to visualize the electric and magnetic fields in side parallel plane wave guide. We also investigated the model characteristics of a rectangular wave guide and we found that the mode which first propagates on a rectangular wave guide is the transverse electric mode with index 1 0 and we call that mode as the dominant mode of rectangular wave guide.

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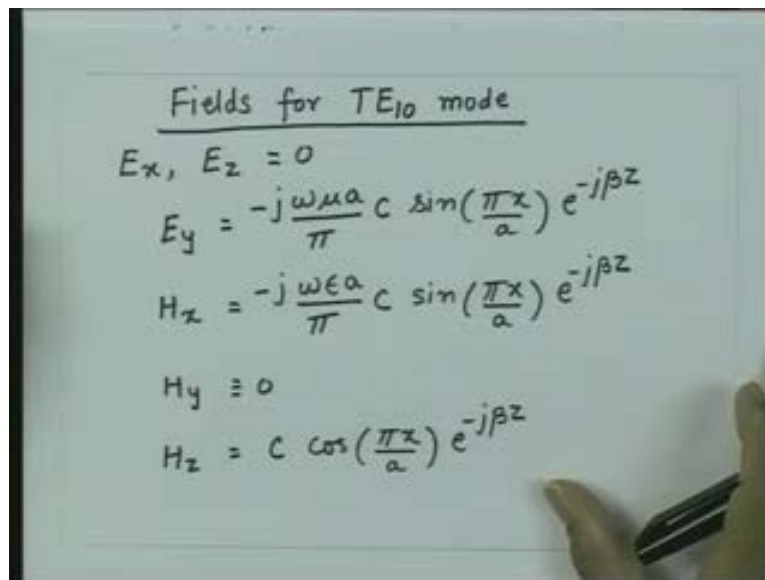


We also argued that most of the time, people want to operate in the dominant mode or in single mode on a waveguide to avoid the dispersion, that is broadening of that signal in time domain, as it travels on a guiding structure. So, this mode which is the dominant mode the T E 1 0 mode is the important mode for rectangular waveguide because most of the time the energy is going to propagate in this mode. So, whether you conduct the

experiment in the laboratory or you go to field, most of the time, you have to deal with this dominant mode which is T E 1 0 mode.

So today, we will see the mode properties of T E 1 0 mode and try to visualize the fields for T E 1 0 mode and then we will go to the calculation of what is called attenuation constant of a waveguide, because whenever we have a practical structure, we never have ideal dielectrics in practice. We do not have ideal conductors in practice; and as a result, there is always a loss in the walls of the waveguide. Also, there is a loss in the medium which is filling the wave guide. So, after visualizing the fields for T E 1 0 mode in rectangular wave guide, then we will go to the calculation of the attenuation constant in a rectangular wave guide. So today, we try to visualize the fields for the dominant mode, that is T E 1 0 mode.

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Fields for TE_{10} mode

$$E_x, E_z = 0$$
$$E_y = -j \frac{\omega \mu a}{\pi} C \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$
$$H_x = -j \frac{\omega \epsilon a}{\pi} C \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$
$$H_y = 0$$
$$H_z = C \cos\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

We have derived these fields and we have written these fields in these components. We have seen for a rectangular waveguide the E_x and E_z components are 0, the electric field has only y component which is given by this and the magnetic fields had two components which was, one was x component, other one was z component. So, there was no y

component for the magnetic field and there was no x components and z components for the electric field. Then, when we try to visualize these fields for the parallel plane waveguide, we have done some simple manipulations.

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$$\begin{aligned}
 E_y &= A \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z - j\pi/2} \\
 \text{Inst field: } \operatorname{Re}\{E_y\} &= A \sin\left(\frac{\pi x}{a}\right) \sin \beta z \\
 &= A \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi}{\lambda_g} z\right) \\
 \operatorname{Re}\{H_x\} &= B \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi}{\lambda_g} z\right) \\
 \operatorname{Re}\{H_z\} &= C \cos\left(\frac{\pi x}{a}\right) \cos\left(\frac{2\pi}{\lambda_g} z\right)
 \end{aligned}$$

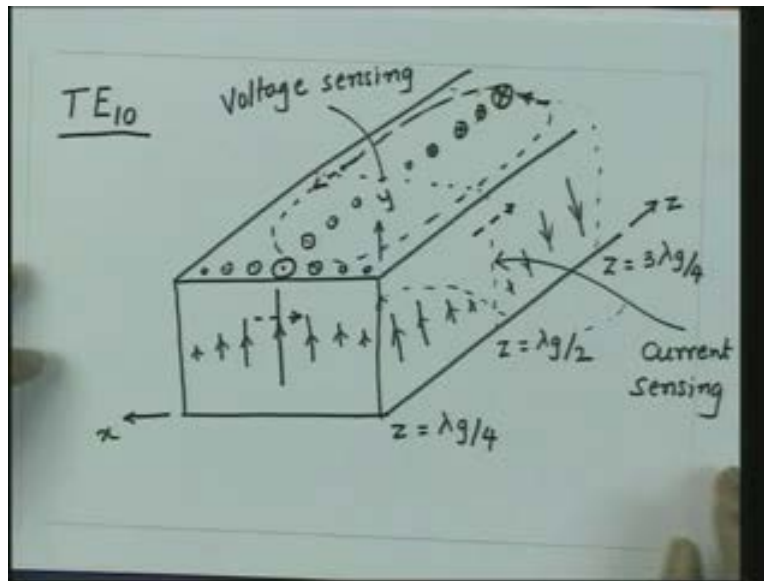
We have said we can absorb this j in the exponent and then we can write down the appropriately these fields; the real part of that, so that you get the instantaneous value of the electric field at a particular location and by doing this essentially, we found the value of the field at some instant of time which we have taken equal to 0, we got the instantaneous fields.

So essentially, we have to now visualize these fields inside the rectangular wave guide. So, we have now a structure which is this, with rectangular cross section. This is the length of the waveguide. Firstly, what we note here is that the field which is the electric field, which is y oriented, that is oriented in this direction. This is y direction, this is x direction and this is z direction.

So, the electric field as a function of z is sinusoidal and at z equal to 0, this field is 0. If I go to a distance of $\lambda_g/4$, then this field will become maximum. This quantity

will become \cos of βz . So let us say, instead of defining the origin here z equal to 0, let us say the origin is defined somewhere here and this location is z equal to λg by 4 so that this quantity is maximum at that location.

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Now the field in x direction is having a sinusoidal variation. It is 0 here, it is maximum half wave and then is zero here again; and it is having a sinusoidal variation in the z direction with maximum at this location, which is λg by 4. And then, if I go distance of λg by 4 from here that means at z equal to λg by 2, this quantity will go to 0. Then the fields will reverse and so on.

So, if I go to a distance of λg by 2 at this plane, the electric field is 0. If I go to a distance of one more λg by 4, this is z equal to $3 \lambda g$ by 4. The field will be maximum again with a reverse sign. So, now if I try to visualize the electric field as the vectors and the field is not varying as the function of y , that means no matter where I go in the y direction, the field amplitude is constant. So, I can represent now this more like a...an arrow. So here, I have arrow which is of large amplitude. As I go on either side, the amplitude of this reduces like that...like that.

If I see from the top, it will appear the large electric field at this location than as I go on either side, the field amplitude essentially dies down. So, we have seen in the earlier case, to show this thing like circles, I can show a bigger circle here and this is smaller circle, smaller circle, smaller circle; all the fields are coming out. Same thing is true here; this, this, this and as I move in this direction, z direction, this field has a sinusoidal variation. So here, the field was maximum. As we go along the z direction, the field amplitude will decrease. This will become smaller, smaller, smaller and become 0 at this location.

So now, the field if I see from the top, it will appear as if there are the arrows which are coming upwards; and the thickness of the arrow which is denoted by the diameter of the circle, that gives me the strength of the electric field at that location. And the electric field does not vary as a function of height; it is everywhere is same. When I go further beyond this point, z equal to $\lambda/2$. Again, the electric field increases, but now the direction of the electric field is reverse.

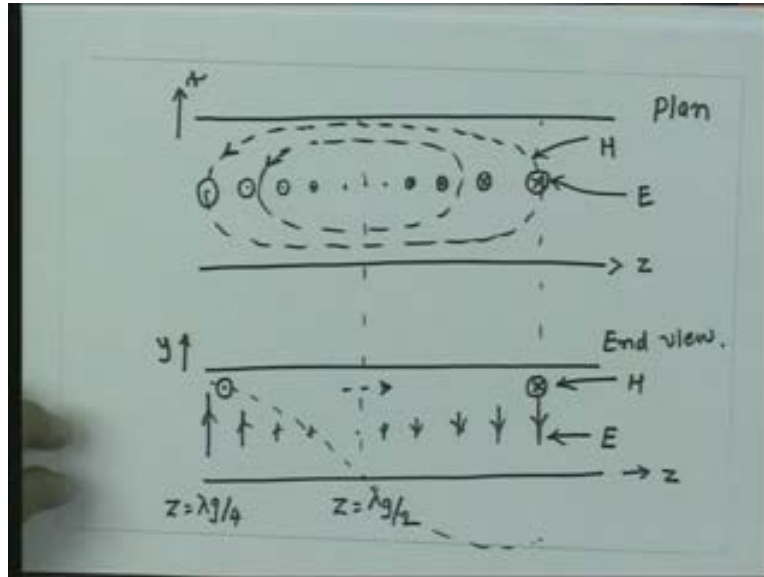
So, again I will slowly start growing this, this. This is bigger, bigger. I take this one. Then, these are the fields which are oriented in opposite direction like that. So, each of this quantity is having a sinusoidal variation starting maximum here, going zero here and again going maximum with opposite direction at this location. So, if I look from the sides, they will appear...the arrows which will be having an amplitude which is sinusoidal.

So, we will start with the amplitude which is maximum like that and then slowly it will die down to 0 and this is become negative maximum. So here, the arrows would look like that, like that, like that and then when I go on the other side, arrows will start increasing and so on. So, if I see in this direction, if I see from this side, the arrows which will be upwards and downwards and their amplitude will be varying sinusoidally.

If I see from the top, I will see the circles where the arrows will be coming upwards or going downwards and the thickness of this will tell me the strength of the electric field. So, I can write down the plan and the side view for this waveguide. So, let us say this is

the plan of the waveguide and these are the side view of the waveguide. This direction is z, this direction is z.

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For this is...we are talking about plan. So, this direction is the x direction. Now, is for plan and for side view, this direction is the y direction. So, we mention this; z equal to Λg by 4. If we take the electric field, it will look like that by go to a distance of z equal to Λg by 2. So, the amplitude will reduce to zero. It will become opposite. So, direction will become like that, like that. So, I have a sinusoidal variation which is like this, going on the other direction. From the top when we see, as we saw, this will look like the circles which will be that; and again, going like this in this wave. So, this is the plan of the wave guide. This is the end view of the wave guide or side view of the wave guide.

So, the visualization of electric field is very simple because we have only one component of the electric field which is y oriented. Same thing now, we can do for the magnetic field and as we have seen, in case of parallel plane waveguide, that this is having a variation which is same as the electric field. That means wherever electric field is maximum, the x

component of magnetic field is maximum whereas electric field is 0; the x component of the magnetic field is always 0. So, the magnetic field, the x component will be maximum here, 0 here, maximum here with the opposite direction and so on.

But the z component of the magnetic field is shifted in quadrature with respect to the x component. So, wherever h_x goes to 0 as z is maximum and vice versa, so that you see from here. So, it is quadrature in x because these two are sin and cos functions, but it is also quadrature in z. So, along the z direction where h_x is maximum, which is this location, as z is zero and z will become maximum here and z will become 0 here, and in the x direction, z is in that quadrature. So, x_z will become maximum here at this location, zero here and maximum here at this plane.

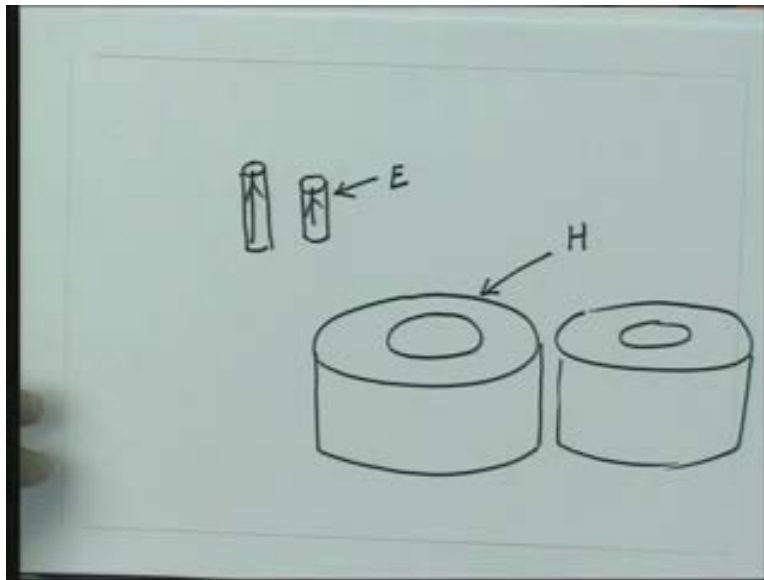
So, now if I look at the magnetic field line, this is e and the h direction should be such that the power should flow in the direction z. So, this is direction e; then the h should be coming right towards so that you get the $e \times h$ which will be in the direction of z, which is the pointing vector. So, we can get the magnetic field lines from here which will be like that and it is...this location is the x direction. When it comes here, we will have a maximum. The magnetic field line will be going like that. At this location, the x component is 0. When it comes here, the magnetic field lines will be going this way. When it comes to this wall, the magnetic field lines will be going this way. It is maximum here. So you see, as z is maximum at this location, h_x is maximum at this location; and as we have seen last time, we can visualize this now as magnetic field lines which are looping like that in this plane.

So, it is actually going to form a loop like that and the variation of the magnetic field is constant in the y direction. So, if I see these magnetic field lines in the elevation was the end view and the plan, the magnetic field lines would look like that with appropriate direction, so that you get $e \times h$ in the direction z. If I see from the sideways, then I will see the magnetic field lines which will be having small amplitude here. This vector is very large. So here, the magnetic field lines will be coming towards us with a large value. So, this is the value. Then slowly the direction changes, magnet and the magnetic field

comes here. It becomes like that when it goes here. So again, the direction has become this. So, this is your h and this is your e .

In this case this is...the lines are h and these circles are the electric field lines. So, if I visualize now this field as a three dimensional structure, the electric field looks like rods of various heights for various diameters, where diameters or the height of the rod represents the strength of the electric field and the magnetic field look like a cut piece of a roll carpet or the transformers stampings.

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So, if I just independently write the electric field vectors everywhere, look like that...no, just like the rods, these are the electric field vectors, whereas if I look at the magnetic field lines, magnetic field lines are like that, like this; and they are stacked one after another in the z direction. So every $\lambda/2$, you get this kind of roll, this of carpet. So, you will be, just be continuing beyond this point and so on. So, this will be just the way the magnetic field lines would be. That is the way the electric field lines would be. Once you get now this visualization of the field at some instant of time, then you can start your clock and say, Okay, let these patterns move with the phase velocity. Inside the

waveguide, if this pattern will start drifting; so every location, you will sometime see this point, sometime will see this point, sometime will see this point.

So, if I see here, the electric field at some instant of time will be maximum here. After a quarter cycle, this point would have moved here. So, 0 point will come here. So at every location along the waveguide, you will see a sinusoidal variation as a function of time and these fields will be distributed in space like this. So, what we now **note**, that the electric field is maximum on the broader wall of the waveguide as we mentioned in the previous lecture also. So, for the T E 1 0 mode, that is where the electric field is going to see maximum, as the wave travels in this direction. But the magnetic field is going to be maximum on this wall. That is where this component is maximum and it is...does not vary as a function of height.

So, no matter where I go in this direction, the magnetic field is going to remain same. So, if I have to excite this waveguide by the electric fields, what is called the voltage probe, then I must excite this waveguide by putting a voltage probe on this wall, broader wall so that the electric field is excited and that electric field will give me the excitation which is T E 1 0. However, if I have a current probe which can excite magnetic fields, then putting the probe on this wall would not help because at this location, the magnetic field is not really good. The magnetic field is here.

So, if I put a current probe which can excite this field, then this will help in exciting T E 1 0 mode inside the waveguide. The same thing is true, the converse is true that if the waveguide was having this mode propagating, and if I want to sense the voltages or the currents from this waveguide. If I have a voltage probe, I must mount the voltage probe on this side. However, if I have a current probe, then I must mount on this wall so that I can get proper detection of this field. So, this wall here should give you the voltage probe. So, on this one will give you voltage sensing whereas this side of the wall, here, we should have current sensing.

So, that is the way the fields inside a rectangular waveguide are detected. By using the voltage and current, probes are excited by giving the signals to these probes which are protruding inside the waveguide and they excite the field inside the structure. So, this visualization of the field for the dominant mode which is TE_{10} , that is quite useful because this also tells us how this excitation of this can be achieved by putting proper probes - whether voltage or current probes - on the waveguide walls.

Once you get this thing now, then the next question arises is, if I have a higher order mode to visualize this for the rectangular waveguide, which is your dominant mode, which is this mode; but now having understood that that is the way the fields are going to be distributed, that means the electric field vector is like rods and the magnetic fields are more like the transformers stampings or roll carpets. We can very easily draw the electric and magnetic fields for various higher order modes.

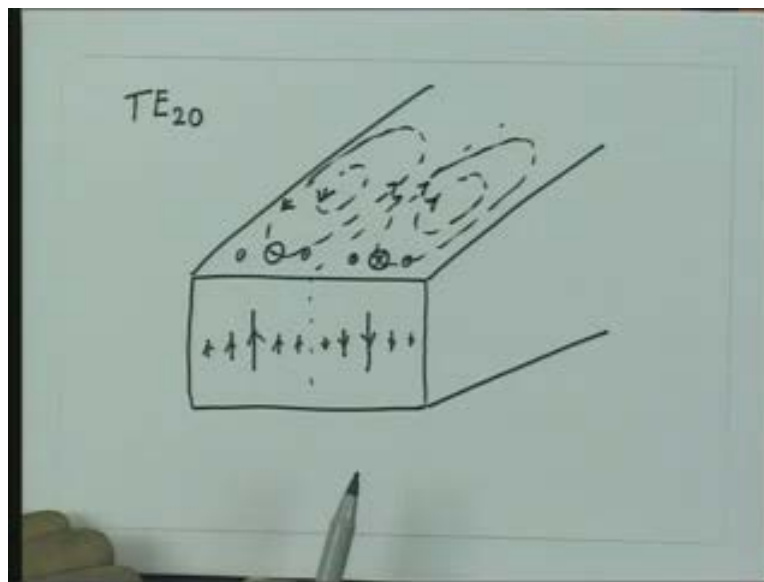
So, let us say if I take, just for sake of discussion, if I take the rectangular waveguide and let us say I want to excite or the TE_{20} mode. So, let us say we have a mode which is TE_{20} . Of course, we can always write down the field expressions - the electric and magnetic fields - and then can do the same thing what we did for TE_{10} , to visualize these fields. However, once I have understood that electric and magnetic fields are in this specific form, I can stretch our imagination little bit to visualize the fields for this mode.

Firstly, the TE_{20} mode is telling you that there are two cycles in the x direction and no variation in the y direction. So, the electric field always lies in the y direction which is like this and there is two cycle variation. That means it is 0 here; the electric field is maximum here. It is maximum here with the opposite direction and then as I go, it should become 0 this should increase again.

So, I got one cycle variation for the electric field in this direction. Again, if I look from the top, we will again see these big circles here. This is coming out, this is going in and then this is going in, going in, this is coming out, coming out, and so on. What about the magnetic fields? We saw the magnetic fields are like stampings but now you are having

two sections here. Each one will have a magnetic field loop. I will have a magnetic field loop like that. That will have a magnetic field loop like this. The direction of the magnetic field will be such that there is a pointing vector which is in that direction. So, at this, this is the way the magnetic field will be oriented. For this, the direction of electric field is reverse and magnetic field direction also is reverse; so here, the field will be like this.

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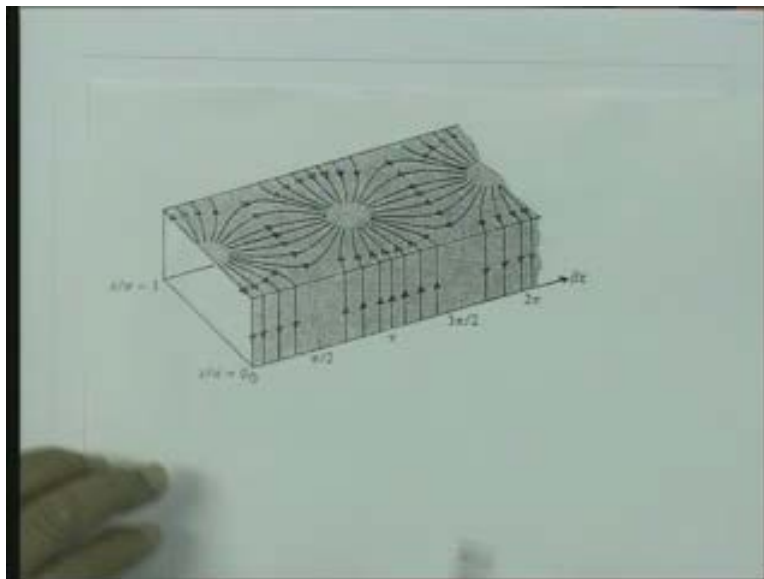
So in this region, all the fields essentially will be going together. So, you will have two roll carpets **start** next to each other in this waveguide, for T E 2 0 mode. So, once this basic understanding of visualization field is developed, then it is very interesting to visualize these fields for various higher order modes. We can leave this as an exercise to the students; that they can imagine any particular mode and try to visualize how the fields, the electric and magnetic fields, could look like for that particular mode.

The next question then we have to ask is, once fields are excited inside this waveguide, now the surface currents will be induced inside the waveguide. Again we comeback to T E 1 0 mode. We have seen that the surface current is related to the tangential component

of the magnetic field. So on this wall, when we go the top wall or the bottom wall, the magnetic field is like that; here it is like this. So that means, the direction of the magnetic field keeps changing. If I go under this wall however, this wall or this wall, then the magnetic field direction is always this way. This magnitude will be changing but it is always along the z direction.

So, if I now calculate the $\mathbf{n} \times \mathbf{h}$ where \mathbf{n} for this wall will be going downwards, for this wall will be going upwards, for this wall going right to left and from this wall will be from left to right. And if I calculate $\mathbf{n} \times \mathbf{h}$ for this one, you will get the current, surface current, which will be flowing perpendicular to this. So, it will be flowing in y direction. If I calculate a surface current here, it will be... \mathbf{n} is like that, \mathbf{h} is in z direction. So again, the surface current will be flowing in x direction now, because normally **is in** y direction. If I go here, then the magnetic field is in x, the \mathbf{n} is in y direction. So, current will flow in z direction.

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So, you will see that on the top surface, the direction of the current will be from like this here and slowly it will change. When it comes here, it will become like this. Then slowly

it will become like this when it comes here. When it comes here, it will become like this and so on; whereas, if I come to the vertical wall, then the surface current will be always flowing in the y direction because normally is x and magnetic field direction everywhere is in the z direction. So, it will appear... Now, if I look at the current distribution which I get from calculating $n \times h$ for all the four walls, the current distribution now will look like that.

So, we see here the magnetic field was in z direction. So, we got the surface current which is y. Here, the normal direction has become y. So, current direction has become x. So, it is like as if the current is just coming out of this from this location, flows this way, remains constant all around this wall. Because it is the function of height, the amplitude of the magnetic field does not change. So, the current amplitude remains constant and the opposite wall again, the current dies down.

So, the current is 0 here at this location, center. Slowly, the current amplitude increases; when I come here, it remains constant on this wall; and on the opposite wall again it decreases and become 0 and the opposite point on the lower wall. So...in fact, the current now is starting from nowhere. There is no source as such here. Slowly, the current grows and again dies down to 0, when I go on to the other side. Obviously, this was the...that must be happening. If the current is flowing this way, there is a moment of charges on the surface of this wall. In the next half cycle, the direction of the current will change. So here, the current is going this way; the next half cycle, current will come downwards.

So essentially, in one half cycle, the current flows upward. That means the charges move downward, the electrons move downwards; and in the other half cycle, the electrons move upwards, the current moves downwards. So that means, there is an accumulation of charges which take place on the two walls; and the charges keep going back and forth and essentially the current flows on the surface of this waveguide. Same thing essentially is going to happen here also. That, every $\lambda/2$, you will have a current island and of created, the current is again 0. Here it grows, becomes maximum again, will become 0

and so on. So, the current flow is like blooming flower. And on the other side, will be sinking kind of feeling you will get for the current.

So, that is the way the currents are going to get induced on the rectangular wave guide. This current direction also helps us in finding out, if I excite this waveguide or if I cut some slots inside this wave guide. We will see later...in antennas, if the currents are disrupted, then there is a possibility of getting radiation from the systems. So, if you know the current directions on this waveguide, then we know where we should cut the slots on this wave guide so that, there is a possibility of radiation. If we cut a slot which does not disturb the current flow, that means, if I cut a slot which is parallel to this and we hardly see any disruption of the current and because of that the radiation possibility will be less.

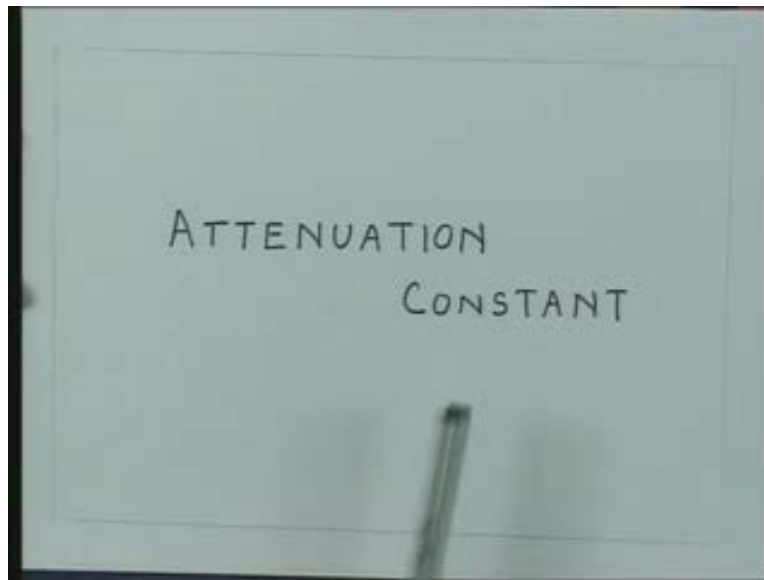
So, the direction of the current flow or visualization of current is very important in this waveguide in structure because if these structures are used for getting radiation, then location of the slots which can give you efficient radiation would be decided by the current flow. So, we should know the current flow.

The other usefulness of finding currents is if this walls are not ideal conductor then these currents are going to create ohmic loss so the power when it propagates inside the waveguide part of the power is going to get lost in heating because of finite conductivity and that will be related to the current distributions on the walls.

So the knowledge of current distribution is useful from finding out how the structure can we made to radiate and also how the losses will change if the walls are not ideal conductors. With this now, we can go to the next important topic in waveguide and that is the loss calculation in a rectangular wave guide. So we have seen that if the structures are not ideal that means if the dielectric which is filling the wave guide is not ideal dielectric if the conductor is not ideal conductor that means the conductivity is not infinite, there will always be loss of energy when the energy propagates through the structure.

So now our effort is to find out what is the loss per unit length of this wave guide and as we know this is measured by a parameter what is called the attenuation constant , we have seen in case of transmission lines that if there is a loss on transmission line the variation will be $e^{-\alpha z}$ where α is attenuation constant. So all the fields exponentially decay as they travel along the structure.

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So here also we assume that the attenuation constant gives me exponential decay of the fields when they travel and we are interested in finding out, what will be the attenuation constant if the conductivity parameters for the walls and the loss with the dielectric is given. However, the problem in this case is little complicated and that is for this simple reason that if I consider arbitrary loss in the wall and arbitrary loss in the dielectric. The model analysis which we have carried out has to be modified now because we have done this field distribution which we got assuming that the dielectric and the conductors are ideal. So in the presence of loss the electric and magnetic fields are going to get modified and modification of electric field and magnetic fields will change the loss because the loss is related to the current distribution.

So we essentially are in a loop that the loss calculation requires the knowledge of the electric and magnetic fields and the electric and magnetic field depend upon the loss. So this problem is very complicated in fact, if you want to solve this for arbitrary loss in the dielectric and arbitrary conductivity of the walls. However, if you assume that the primary objective of this wave guide was to transmit power from one point to another efficiently. We make every effort to get the losses as minimal as possible that means we make a wave guide of a material which has higher conductivity as possible and we fill this wave guide with a dielectric which is as pure as possible.

So normally the losses which take place either in dielectric which is filling the wave guide or the conductivity of the walls is very very small and under that assumption then we can say that as a first order, the fields do not get disturbed significantly because of the losses in the wave guide, what that means is we assume that the field model fields if we got for any TE mode or any mode they are exactly same as the lossless wave guide even in the presence of their small loss. So we say that we have a full knowledge of the electric and magnetic fields and once we say that now the loop is broken.

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Attenuation const $\alpha = \alpha_d + \alpha_c$

\uparrow Dielectric \uparrow conductor walls.

$$\beta^2 = \omega^2 \mu \epsilon_c - \left(\frac{\pi}{a}\right)^2$$

$$\epsilon_c = \epsilon_0 \epsilon_{rL}$$

$$\epsilon_{rL} = \epsilon_r (1 - j \tan \delta)$$

\uparrow Loss tangent.

$$\tan \delta = \frac{\sigma}{\omega \epsilon_0 \epsilon_r}$$

So from the knowledge of electric and magnetic fields, now we can find out what is the current from their we can find out what are the ohmic losses and then, we can calculate the attenuation constant normally what we will do since the attenuation is coming because of the two components, one is the loss in the dielectric other one is the loss in the conductive walls. We separate out these two losses and we say well since the losses are very small, when we calculate dielectric losses we assume that wave guide is made of ideal conductors and when we calculate the conductor losses, we assume that the waveguide is filled with ideal dielectric.

So if I say I have a attenuation constant let us say α attenuation constant, let us say α this α consist of two components and that the first order approximation I can say that this α is sum of the two α s, one is because what is called the dielectric loss, other one is called the conductor loss. So this is because of dielectric which is filling the wave guide, this is the conductor walls.

So as I mention, when I calculate α_c , I assume α_d is 0, when I calculate α_d that time I assume that the walls are ideal conductors and then by calculating the two attenuation constant separately, then I can calculate the total attenuation constant which is sum of these two attenuation constant. For calculation of α_d essentially we use the same approach as we have did in case of transmission line that means we calculate the, the propagation constant β and then from dispersion relation simply replace the dielectric constant by the dielectric constant of the lossless medium.

So let us say now, first we calculate this quantity α_d and the propagation constant for the mode β^2 is $\omega^2 \mu \epsilon$ and in this case the ϵ is ϵ for the lossy medium $1 - f^2$ and let us see, we want to do this derivation only for TE₁₀ mode, so that will be equal to ϕ upon a^2 and this quantity lossy dielectric permittivity that we can write as ϵ_1 will be equal to ϵ_0 into ϵ_r relative permittivity for lossy medium where this relative permittivity for lossy medium ϵ_r as we have seen earlier that is ϵ_r into $1 - j \tan \delta$ where this is the quantity which we have defined earlier, what is called the loss tangent, what one can

do now is we can just replace this epsilon l by this, if the medium was lossless that was the dispersion relation where this was only epsilon.

So what we are doing is in the dispersion relation is simply replaced epsilon by epsilon for lossy medium, we can write in terms of loss tangent and tan delta generally is very small for low loss in the dielectric medium, separate out real and imaginary parts and you get the attenuation constant for the wave guide.

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$$\beta_l = (\beta^2 - j\omega^2\mu\epsilon_0\epsilon_r\tan\delta)^{1/2}$$

$$\approx \beta - j\frac{\omega^2\mu\epsilon_0\epsilon_r\tan\delta}{2\beta}$$

Attn const. due to lossy dielectric filling the waveguide

$$\alpha_d = \frac{\omega^2\mu\epsilon_0\epsilon_r\tan\delta}{2\beta} = \frac{\omega\mu\delta}{2\beta}$$

$$= \frac{6\eta}{2\sqrt{1-(f_c/f)^2}}$$

Intrinsic Imp in the dielectric
 $\sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}}$

So if I do that by substituting essentially I can get beta for the lossy medium, so let me put a suffix here for beta for lossy medium, so beta l that will be equal to beta square which is for lossless medium minus j omega square mu epsilon 0, epsilon r tan of delta square and since this quantity is very small because tan delta is very small, this approximately we can write as beta minus j omega square mu epsilon 0 epsilon r tan of delta divided by 2 beta.

I just take beta square common and take a square root of that and approximately this step is very small, so we retain only the first order term in the binomial expansion I get this

point. So this is the phase constant which is having the phase constant real part and this quantity which is imaginary part of phase constant that means this is now representing the attenuation constant α .

So from here you get the attenuation constant due to lossy dielectric filling the wave guide and that will be α_d that is equal to $\omega^2 \mu \epsilon_0 \epsilon_r \tan \delta$ divided by β . Since, we know that the $\tan \delta$ this quantity $\tan \delta$ here loss tangent is σ upon $\omega \epsilon_0 \epsilon_r$. We can substitute for $\tan \delta$ into this expression and we get $\omega \mu \sigma$ upon 2β using expression for β for the lossless case, hence we have derived for the TE_{10} mode which is related to the cut off frequency of the mode. So β for if I substitute for this that will be equal to $\sigma \eta$ divided by 2 times square root of $1 - f_c^2 / f^2$ whole square, where η is the intrinsic impedance in the dielectric which is square root of μ_0 or μ upon ϵ_0 or ϵ_r .

So knowing the dielectric constant of the of the medium and assuming that the loss tangent is very small that means the losses in the medium are very small. We can calculate the attenuation constant due to the finite conductivity of the dielectric medium by this expression. As one can say for low losses the attenuation constant is proportional to the conductivity of the medium but what we also see is that this now is that related to even this cut off frequency. So when the frequency is much larger compared to the cut off frequency, this expression is very similar σ upon 2η , this is very similar to the transmission line case, if you recall if you take a transverse electromagnetic mode thus load was the attenuation constant for the transmission line that is σ upon 2 multiplied by the characteristic impedance of the medium.

So when we talked about the lossy medium in the unbound medium that time we had got a loss which was this loss, what happens now however is that in the rectangular wave guide it also depends upon how far away you are from the cutoff frequency. So if you are very close to cut off frequency then this quantity becomes close to 0, this quantity become very large. So the dielectric loss becomes very large. So now the dielectric loss is

a function of frequency which otherwise was not a function of frequency, if we take a transverse electromagnetic mode then this was only depending upon the conductivity.

So this dielectric loss is proportional to conductivity in the of the dielectric but it also depends upon how far away you are from the cutoff frequency of a particular mode and as you go closer to the cut off frequency of the mode the dielectric loss increases. So by using this now we can calculate one component of the attenuation constant and that is the dielectric constant. The second component which we want to calculate now is the due to the finite conductivity and this calculation is not as straight forward as this because since, the fields are now inside the wave guide simply modifying the propagation constant then I want to know now, how is the, how do I put this medium as a lossy medium when the losses are going to take place in the walls.

So what you have to do is we have to go from the first principles and calculate the attenuation constant using first principles, what does it mean is that if there is a loss in the medium the e and h both the fields vary as a function of z which is along the propagation in amplitude that e to the power minus alpha z, where alpha is the attenuation constant.

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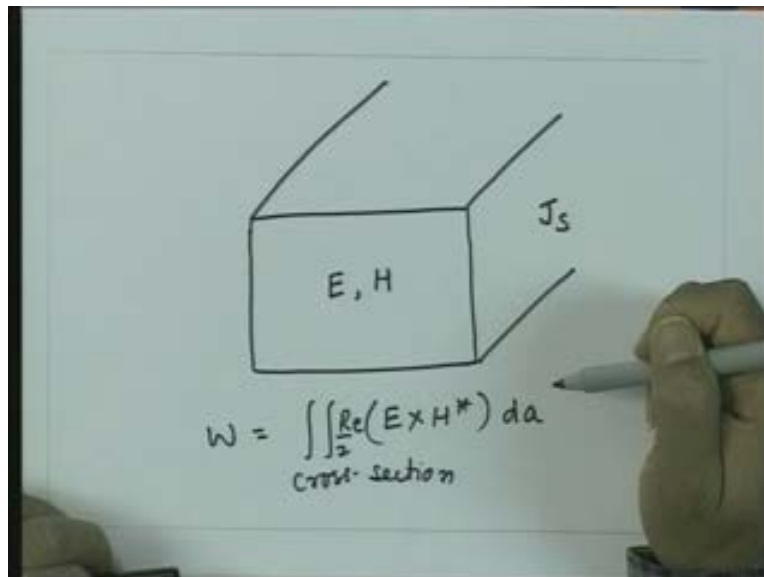
Handwritten derivation of the attenuation constant α from first principles:

$$\begin{aligned} \bar{E}, \bar{H} &\sim e^{-\alpha z} \\ \text{Power } W &\sim e^{-2\alpha z} \\ &= W_0 e^{-2\alpha z} \\ \frac{dW}{dz} &= -2\alpha e^{-2\alpha z} = -2\alpha W_0 e^{-2\alpha z} \\ &= -2\alpha W \\ \alpha &= \frac{-dW/dz}{2W} \\ &= \frac{\text{Power decrease/unit length}}{2 \text{ Total power carried by the WG}} \end{aligned}$$

So the power which is proportional to mode e square or mode h square because power will be e cross h . So the power density or power which the waveguide carries or as the structure carries, let us say that is w that varies as e to the power minus $2\alpha z$. I can differentiate this w with respect to z , so I get dw by dz that is equal to minus 2α e to the power minus $2\alpha z$. So w varies like this, so the dw by dz will vary like that instead of putting equal to let us say proportionality.

So the α attenuation constant in general if we calculate that will be this quantity e to the power minus $2\alpha z$ that is w . So you will see from here I can write it, this is dw by dz upon 2 times w . If we want to write down this this w is equal to say as w_0 , e to the power minus $2\alpha z$. So dw by this thing that will be equal to 2 times minus 2 times $\alpha w_0 e$ to the power minus $2\alpha z$. This quantity is w , so this is minus $2\alpha w$ now from here we get the attenuation constant which is like this. Physically, what does this term mean this is the rate of change of power and negative sign means rate of decrease of power in the direction of the wave propagation and this is the total power guided by the structure.

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So now the attenuation constant can be calculated by two quantities that is power loss per unit length along the wave guide divided by two times the total power carried by the wave guide that gives me the attenuation constant. So this is I can say power decrease per unit length of the wave guide divided by total power carried by the waveguide. So now in general if I want to calculate the attenuation constant I required 2 quantity is to be calculated one is the power loss per unit length and second quantity is the total power carried by the wave guide. So if I go to rectangular waveguide I have to calculate now two things.

So if I now consider the rectangular wave guide, there is a electric field here and the magnetic field here and then there are surface currents which are going to flow on all this 4 walls. So the surface current will give me the loss and I can calculate per length that is the power loss in the waveguide, calculating $\mathbf{e} \times \mathbf{h}$ which gives me the pointing vector and integrating over the cross section that give me the total power flow inside the wave guide. So from here I can get \mathbf{w} which will be integrated over the cross section, $\mathbf{e} \times \mathbf{h}$ conjugate half real part da , where a is the area of cross section that gives me the total power flow inside the wave guide.

Once I know the surface current \mathbf{j}_s on this walls then I have seen that the power loss per unit area is given by half surface resistance multiplied by mode of \mathbf{j} square. So knowing the surface current I can calculate the loss per unit area and since, I know the height I can calculate the loss per unit length of the wave guide.

Once I know these two quantities then using this relation, I can calculate what is the attenuation constant of this wave guide due to finite conductivity of the walls. So in the next lecture essentially by using this basic definition of the attenuation constant, we will derive the attenuation constant for 2 modes; one is for a parallel plane wave guide, the transverse electromagnetic mode that is the simplest mode just to get a feel, how do you calculate this quantity and then we will go to the calculation of attenuation constant of a rectangular waveguide for TE_{10} mode.