Transmission Lines and E.M. Waves
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Lecture-4

Welcome, in the last lecture we derived the voltage and current expressions then we defined the origin on the Transmission Line we defined the origin at the load end and then we defined all the distances on Transmission Line measured from the load point towards the generator which we denoted by variable l.

So now we have this voltages and currents which are given in terms of the length which are towards the generator from the load end.

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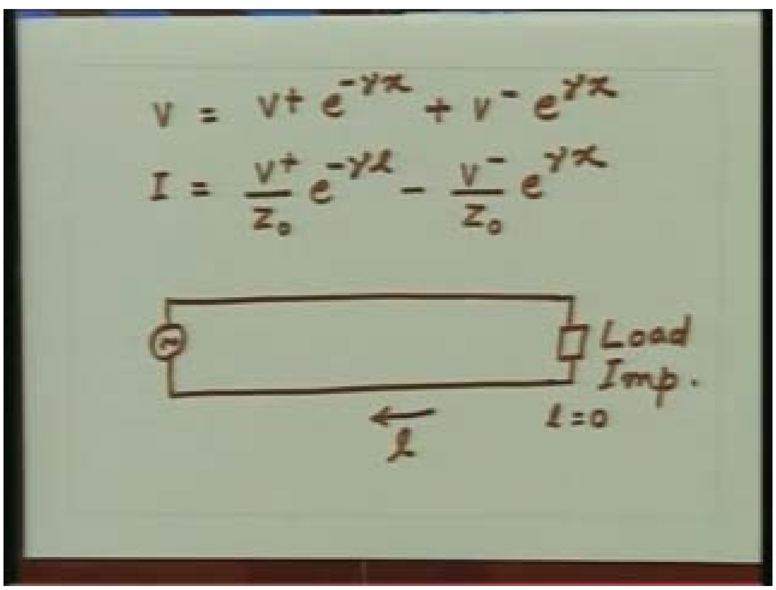

Then satisfying the boundary condition at the load end we got the relationship between the impedance at the load end which is related to the parameter which again was a ratio of $\frac{V^-}{V^+}$

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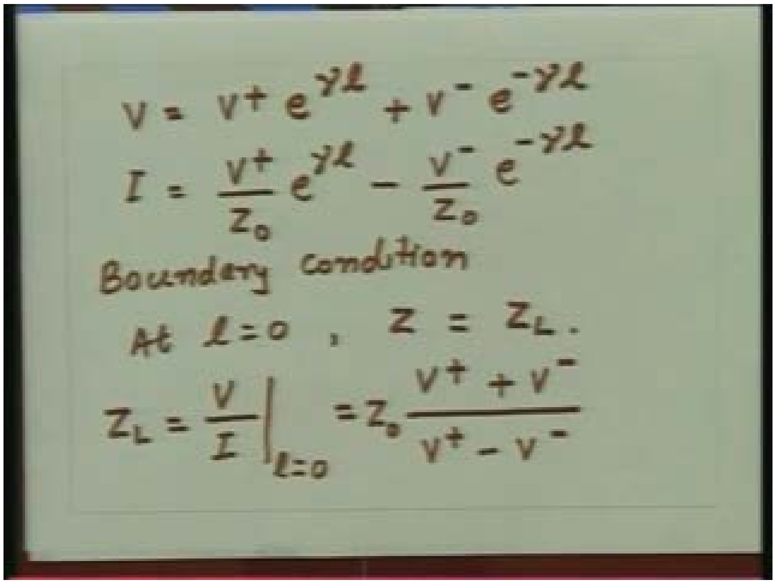

This quantity $\frac{V^-}{V^+}$ is related to the energy reflected from the load end and we define this parameter called the voltage reflection coefficient which was the ratio of the reflected voltage to the incident voltage.

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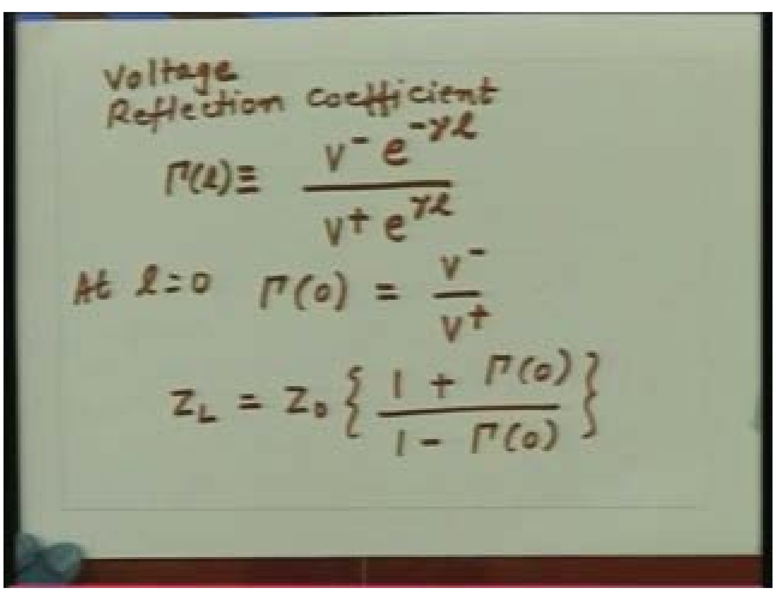

Both these quantities are complex in nature and that is the reason the reflection coefficient at any point on the Transmission Line is complex. Then at l = 0 which means at load end the reflection coefficient which is $\Gamma(0) = \frac{V^-}{V^+}$ so the ratio of the reflected and the incident voltage at the load end is denoted by this quantity $\Gamma(0)$ at l = 0. By substituting this we got the relationship between the load impedance and the reflection coefficient at the load end.

If we now invert this relation we will get the value of the reflection coefficient at the load end and let us call the reflection coefficient at the load end as Γ_L which is nothing but the Γ measured at l = 0 which is again equal to $\frac{Z_L - Z_0}{Z_L + Z_0}$.

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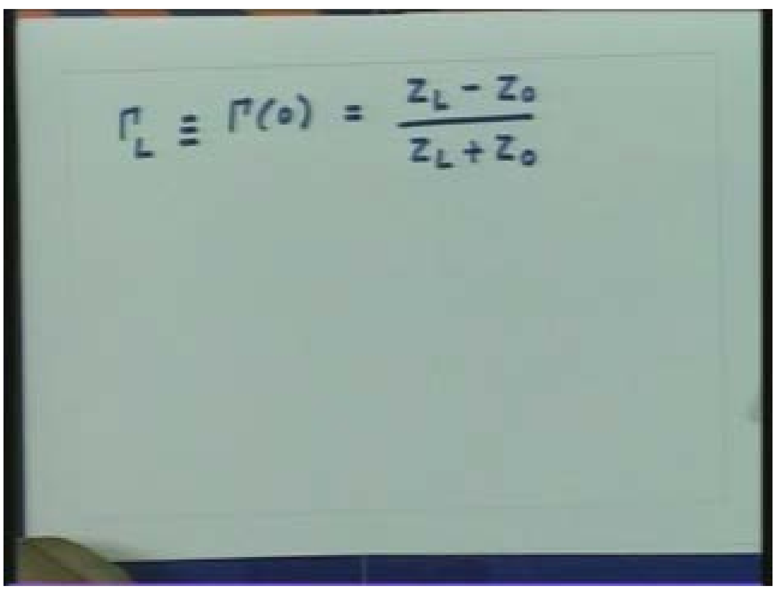

so what we find here is the reflection coefficient at the load end which is the measure of how much energy is reflected from the load end related to the terminating impedance of the line which is Z_L, also it depends upon the characteristic impedance of the line.

What we see from here is that since this quantity is equal to $\frac{V^-}{V^+}$, one thing can immediately strike to us is when $Z_L = Z_0$ means if the terminating impedance on the line is equal to the characteristic impedance the $\frac{V^-}{V^+}$ goes identically to zero. In other words the reflection coefficient $\Gamma_L = 0$ when $Z_L = Z_0$. So we get $\Gamma_L = \frac{V^-}{V^+} = 0$ when the load impedance is equal to the characteristic impedance.

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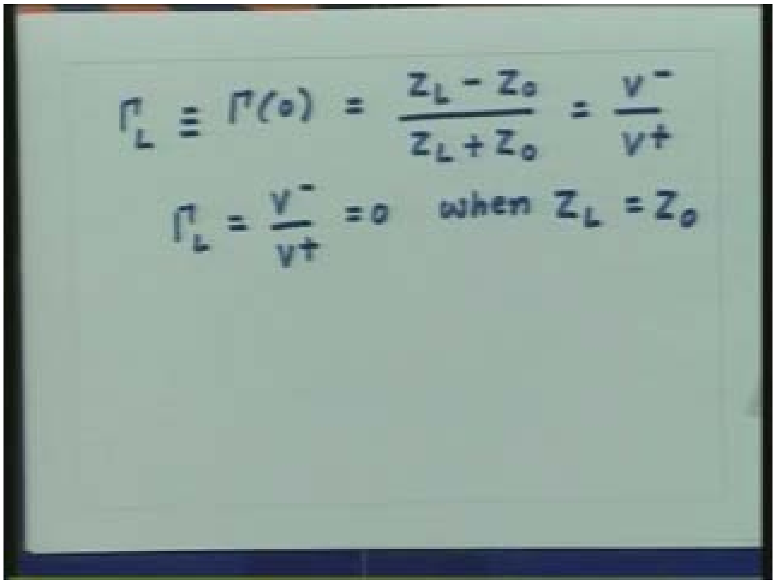

Now this is a very interesting condition, what we see now is if the load impedance is equal to the characteristic impedance since this quantity is zero and V^+ will not be zero because we have put some incident wave that means the V^- would be zero or in other words when the load impedance becomes equal to the characteristic impedance there is no reflection on the line and that is the condition we were looking for because when we initially talked about the purpose of the Transmission Line the objective was to transfer the power from the generator to the load.

However, when we analyzed the Transmission Line we found that there are some reflections on Transmission Line and the entire energy is not transferred to the load.

Now we get a condition under which the entire energy will be transferred to the load because there will not be any reflection on the line if the load impedance is equal to the characteristic impedance. So the characteristic impedance is a very important parameter of the Transmission Line. One may wonder where this characteristic impedance located on Transmission Line, it is not located anywhere. It is a characteristic parameter which cannot be located on a Transmission Line but it governs the power flow on Transmission Line and this condition $Z_L = Z_0$ is called the match condition that means the impedance is matched to the Transmission Line characteristic impedance. The power transfer is hundred percent and there is no reflected energy on the Transmission Line.

This condition is similar to the maximum power transfer condition on the circuits where if the load impedance is equal to the complex conjugate of the generator impedance then there is a maximum power transfer from generator to the load. Now exactly same condition is here for Transmission Line that when the load impedance is equal to the characteristic impedance of the line the whole power get transferred to the load and there is no reflected power on the Transmission Line. So we call this condition as the matched load condition. One may not physically understand what is happening and why there is a reflection on the Transmission Line. The Z_0 represent some kind of a medium which is very smooth on which the power is flowing along the Transmission Line. Suddenly it sees a disturbance on the Transmission Line in terms of impedance because when it reaches to the other end of the Transmission Line the impedance is no more seen equal to the same smooth flowing impedance which is Z_0.

So it is some kind of a step change which the energy flow encounters and because of that part of energy tends to get deflected on the Transmission Line. So for maximum power transfer the energy flow should always see an impedance which is equal to characteristic impedance. If any other impedance is kept other than characteristic impedance there will always be reflection on Transmission Line and the power flow will not be maximum with the load.

Now having understood this one can generalize this and then one can say that the way we have defined the reflection coefficient at any point on Transmission Line once I get the value of $\frac{V^-}{V^+}$ I can find out the complex value of the reflection coefficient at any point on the Transmission Line. And once I define this parameter which is the reflection coefficient then I can substitute this quantity $\frac{V^- e^{-\gamma l}}{V^+ e^{\gamma l}}$ in our voltage and current relation and we can define the voltage and current at any point on Transmission Line in terms of the reflection coefficient.

so now we get the relation of the voltage and current which is V is a function of length as we have defined earlier this quantity the V was equal to $V^+ e^{\gamma l} + V^- e^{-\gamma l}$

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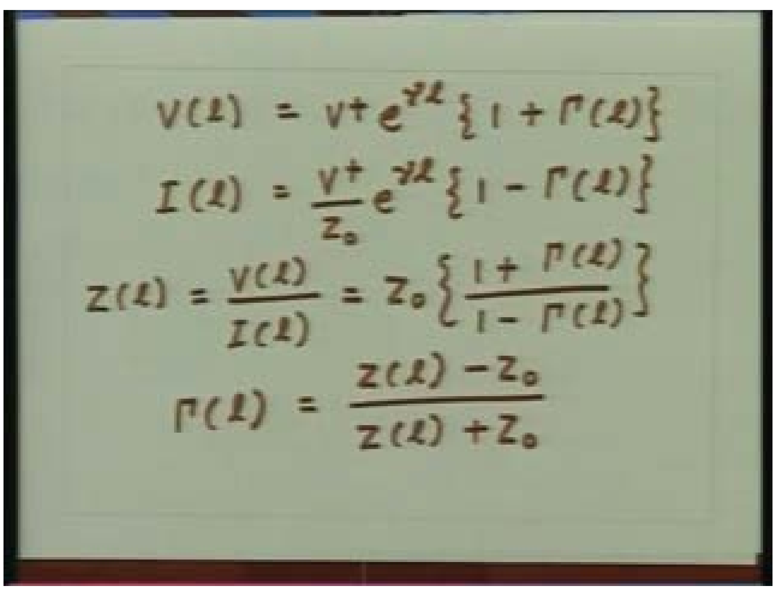

I can take V^+ $e^{\gamma l}$ common so this quantity will become $\frac{V^- e^{-\gamma l}}{V^+ e^{\gamma l}}$ that is nothing but reflection coefficient at distance l. So I get $V(l) = V^+ e^{\gamma l} \{1 + \Gamma(l)\}$ and the current at that location will be $\frac{V_0}{Z_0} e^{\gamma l} \{1 - \Gamma(l)\}$.

Once you get the voltage and current at any location we can now find out what is the impedance at that location. So we can divide this equation by this so we will get the impedance measured at a location l on the line which is V at that location divide by current at that location. V^+ $e^{\gamma l}$ will cancel so you will get $Z_0 \left\{\frac{1+\Gamma(l)}{1-\Gamma(l)}\right\}$.

So the impedance at any location on the Transmission Line is related to the reflection coefficient at that location in the Transmission Line. Inverting the relation we can get reflection coefficient at any point on the line that is equal to $\frac{Z(l)-Z_0}{Z(l)+Z_0}$.

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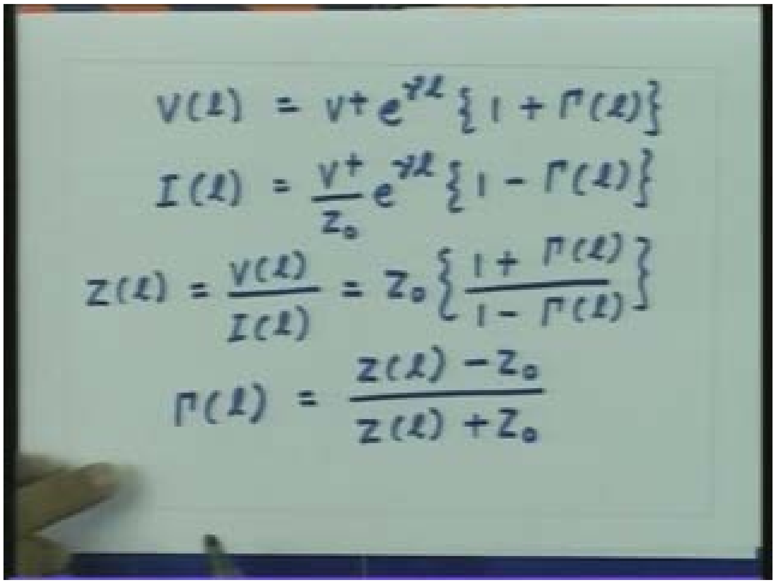

So in fact the impedance at any point on Transmission Line and the reflection coefficient at that location have one to one relationship. So if I know the reflection coefficient at that location I can find out what is the impedance at that point and if I know what is the impedance at that point I can calculate what is the reflection coefficient at that point.

Once I get this I can substitute for reflection coefficient $\Gamma(l)$ and noting that the quantity $\frac{V^-}{V^+}$ is the reflection coefficient at the load end here I can write down the impedance in general form as follows.

If I can get the impedance Z at any location l is equal to Z(0) where $\Gamma(l)$ is $\frac{V^-}{V^+} e^{-2\gamma l}$

So this is nothing but $\left\{\frac{1+\Gamma_L e^{-2\gamma l}}{1-\Gamma_L e^{-2\gamma l}}\right\}$.

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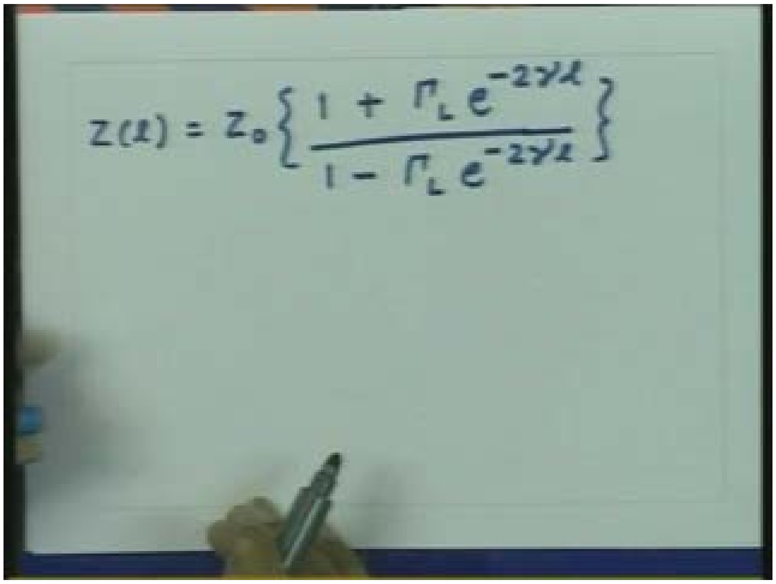

and Γ_L is nothing but the reflection coefficient at the load end so Γ_L as we have already derived is nothing but $\frac{Z(l)-Z_0}{Z(l)+Z_0}$ where Z_L is the load impedance.

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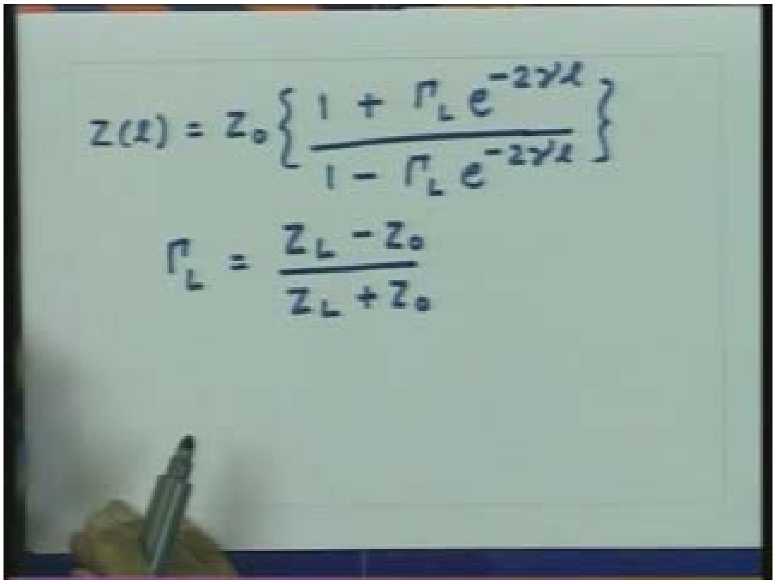

I can substitute for Γ_L into this equation and I can get the impedance at any point on the line in relation to the load impedance so I get a relation Z(l) called the impedance transformation relation which is equal to $Z_0\left\{\frac{1+\frac{Z_L-Z_0}{Z_L+Z_0}e^{-2\gamma l}}{1-\frac{Z_L-Z_0}{Z_L+Z_0}e^{-2\gamma l}}\right\}$.

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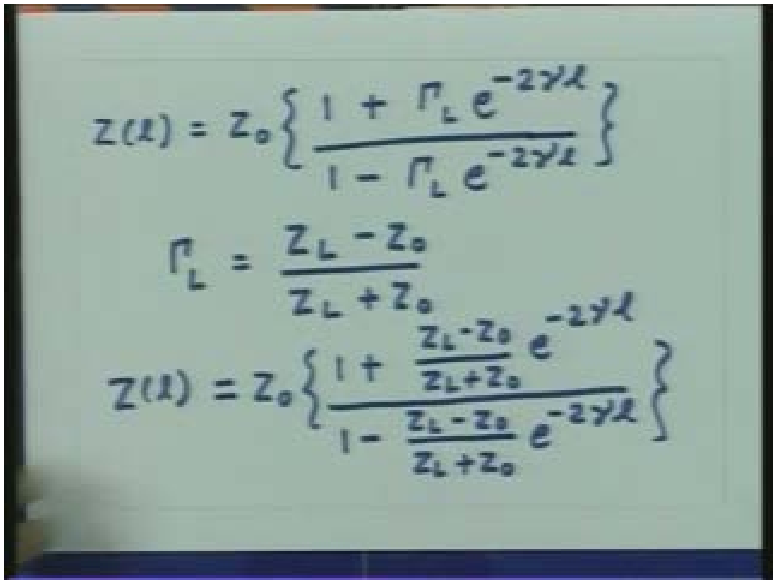

Rearranging this terms and collecting the terms of Z_L and Z_0 separating them out and noting that we are now having the terms which will be $e^{\gamma l}$ I can rearrange and rewrite this expression in terms of the hyperbolic cos and sin functions.

So this expression after reconstitution becomes $Z(l) = Z_0 \left\{ \frac{Z_L \cosh\gamma l + Z_0 \sinh\gamma l}{Z_0 \cosh\gamma l + Z_L \sinh\gamma l} \right\}$ where we have used a relation $\cosh \gamma l = \frac{e^{\gamma l} + e^{-\gamma l}}{2}$ and $\sinh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{2}$.

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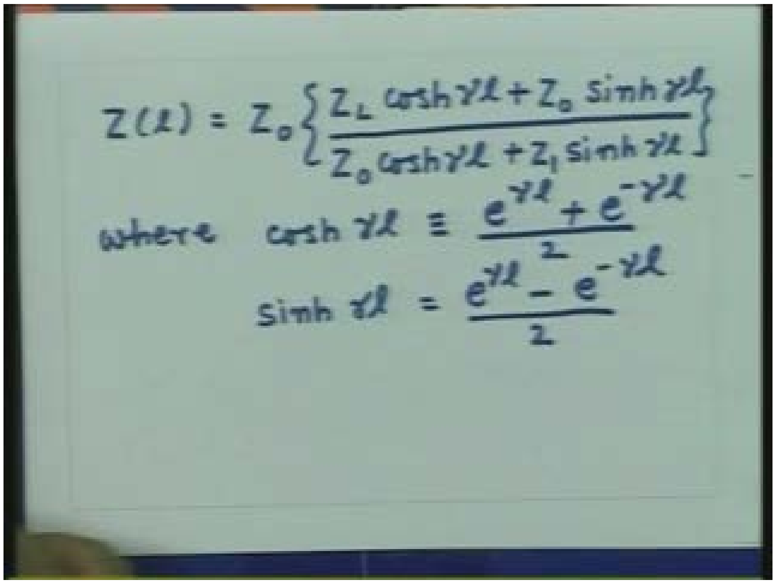

So now this is telling me that the impedance at location l is related to the load impedance through this relation what as we will call as the impedance transformation relation so this relation we call as the impedance transformation relation.

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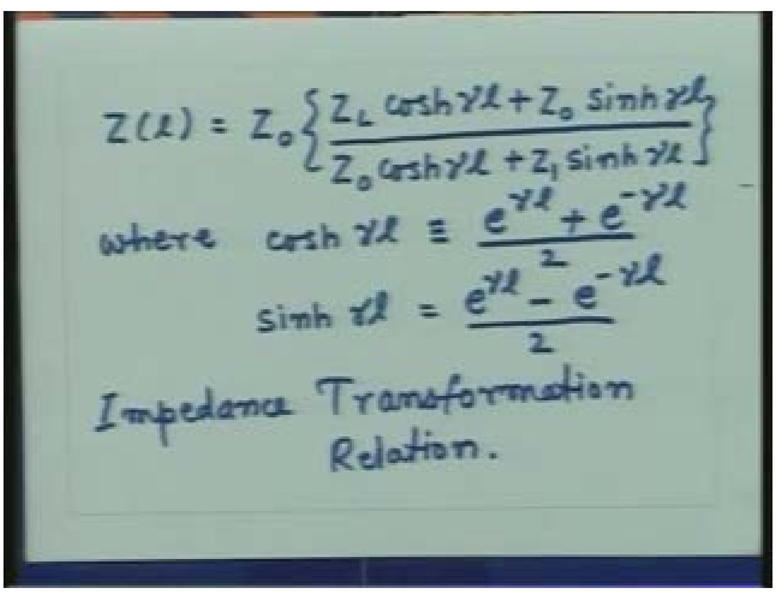

And this relation is a very important relation in Transmission Line calculation because what it tells you is that if a line is terminated in impedance which is equal to Z_L the impedance which you will measure at the input end of the line will not be Z_L it will be different in Z_L and not only it will be different in Z_L it will also depend upon the length of the line. So for the same load which you have connected to the line if the length of the line keeps varying the impedance which you measure at the input of the line will keep varying or in other words if I design a circuit at high frequencies and connect that circuit to a load depending upon what piece of wire or cable I am connecting between the load and the circuit the impedance seen by the circuit will be different.

So for the behavior of a circuit the impedance seen by the circuit is more important. If that quantity keeps varying depending upon the length through which the actual impedance is connected to the circuit the whole circuit behavior keep on varying as the function of length of a connecting wire.

so when we go to high frequencies even length of a connecting cable is important because the impedance transformation which the cable does might change the impedances which are seen by the actual circuits which are connected to the end of the connecting cable. So the important thing to note here is that whenever we are having distributed element then the impedance seen at input end of the Transmission Line deal in general with different than the impedance connected to the load end of the Transmission Line and the impedance transformation will essentially be done by that.

One can also note a important thing here that I can take this Z_0 down here I can take Z_0 common from denominator and numerator then every quantity which I have here is now normalized with respect to Z_0 this quantity is $Z(l)/Z_0$, this is $Z(l)/Z_0$ and this is $Z(l)/Z_0$.

So the expression which I have for impedance transformation essentially can be written in terms of normalized impedances so the same expression I can write down in terms of the normalized impedance and let me define the normalized impedance by bar so any impedance which is normalized denoted by bar is equal to the actual impedance Z divide by the characteristic impedance Z_0. This is what is called the normalized impedance.

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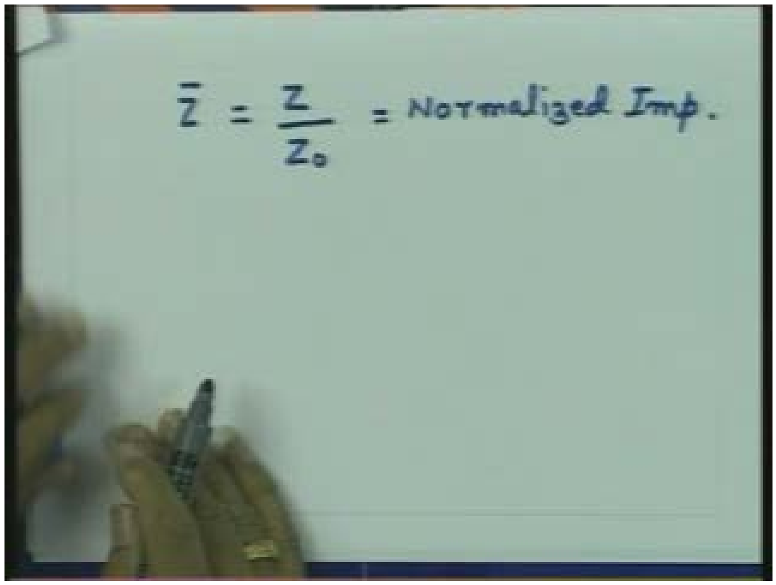

Then my $\bar{Z}_L$ bar is equal to Z_L/Z_0, normalized impedance at any location l is equal to $Z(l)/Z_0$ and so on.

Substituting this in the impedance transformation relation I get the normalized impedances $\bar{Z}(l) = \left\{ \frac{\bar{Z}_L \cosh\gamma l + \sinh\gamma l}{\cosh\gamma l + \bar{Z}_L \sinh\gamma l} \right\}$.

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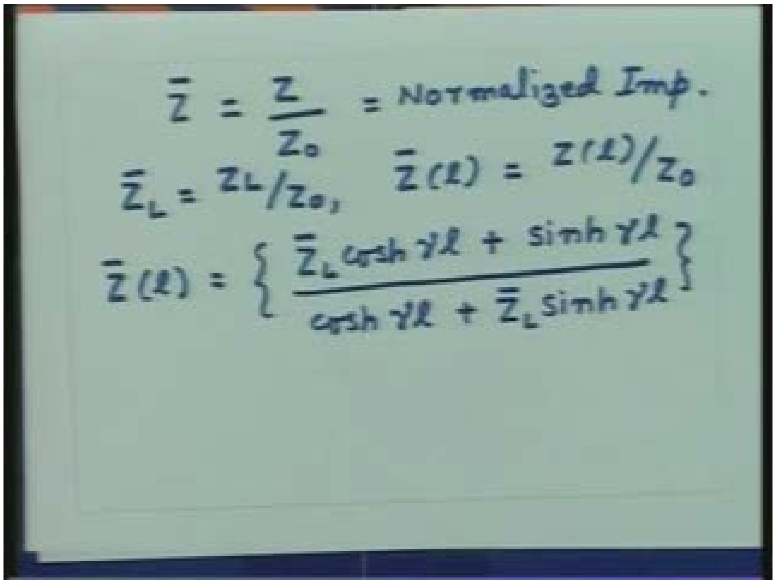

So the important thing is on Transmission Line calculation the absolute impedances do not mean anything the impedances which are normalized with respect to the characteristic impedance has some meaning. what that means is if I have a Transmission Line whose characteristic impedance is fifty ohms a hundred ohm impedance is going to create the same reflection as if the Transmission Line was of characteristic impedance three hundred ohms and the load impedance is of six hundred ohms. So the absolute impedance which you are connecting to the line has no meaning and that is the reason before we start any Transmission Line calculation first you ask what is the characteristic impedance of the line once you know the characteristic impedance then every impedance which you have you normalize those impedances with a characteristic impedance.

So every calculation which you do in Transmission Line is always in terms of the normalized impedances and when require you always un do the normalization and you find out the absolute impedances. But as far the Transmission Line calculations are concerned the impedance has to be always in terms of the normalized terms and this normalization is with respect to the characteristic impedance.

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So what we see is that the characteristic impedance which is not really located anywhere is not seen anywhere, always at the background but is the governing parameter of the energy flow on Transmission Line because every calculation every voltage current whatever relationship we are talking about all are governed by this hidden parameter which is called the characteristic impedance.

So characteristic impedance is a extremely important parameter when we do the transmission line calculation. Now we also notice from here that if the $Z_L = Z_0$ or the normalized Z(l) = 1 then this quantity is 1 this quantity is 1 so this whole term will become equal to 1 so normalize Z(l) will be equal to 1 or the absolute value of Z(l) will be equal to Z_0.

If $\overline{Z}_L = 1$ that is $Z_L = Z_0$ which we call as the matched condition then $\overline{Z}(l)$ at any location on the line is equal to one that is the impedance measure at any point on the line will always be equal to the characteristic impedance. This is extremely important condition.

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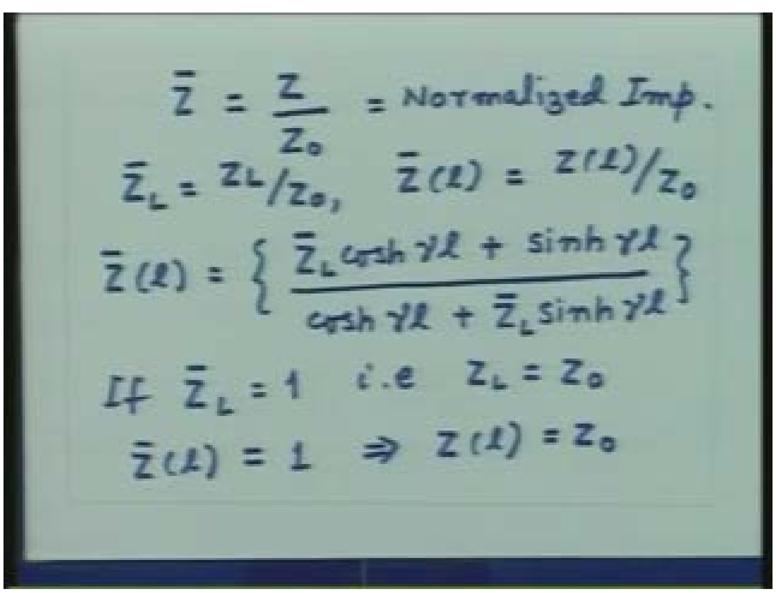

Now what we are saying is that if the load impedance is not equal to the characteristic impedance then the impedance measured at any point on the line will be the function of the length of the line or the input impedance of the line is the function of the length and the impedance which is connected to the other end of the line.

However if I take a special case that is called the match condition then load impedance is equal to Z_0 and the impedance measured at every point on the line is equal to the characteristic impedance. Now I do not have to worry about the length of the line the input impedance will always be equal to the characteristic impedance no matter what is the length of the line. So if the line is terminated in the characteristic impedance then without worrying about the length of the line I can carry out my measurements because the impedance seen at the other end of line will always be equal to the characteristic impedance. So the matched condition is an extremely important condition because it takes away all the worries which one would have because of the length of the line connecting between the load and the measurement point or the generator point.

Now this also gives you some means of providing the definition to the Z_0. What we have done earlier is we have defined this characteristic parameter Z_0 which had dimension of impedance so we started calling it characteristic impedance because this impedance was related to the primary parameters of Transmission Line and the frequency. However we did not have any physical way of defining this quantity now may with this property of the matched condition we may give some physical meaning to the characteristic impedance so I can define the characteristic impedance is that impedance with which if the line is terminated. Then the impedance measured at any point on the line is equal to the terminating impedance then we call that impedance as the characteristic impedance.

Sometimes we give another definition to the characteristic impedance and that again comes from this condition that when we have matched conditions there is no reflected wave and if there is no reflected wave there is only incident wave going on Transmission Line and we would have to seen earlier the incident wave always sees an impedance which is equal to characteristic impedance. If I take a hypothetical situation that the line is of infinite length even if there was reflection on other end of the line it will take infinite time to reach to us that means at any finite time you will always see a wave which will only be a forward traveling wave or the incident wave there will not be any reflected wave.

So the impedance measured on an line which is only now due to forward wave will be equal to the characteristic impedance. So at times we also give a definition of characteristic impedance as it is the input impedance of a line of infinite length. But for that line the reflected wave would not have reached to you so the line sees always a forward traveling wave and forward traveling wave always sees the impedance which is equal to characteristic impedance.

So the impedance measured at the input end of the line will be equal to the characteristic impedance. So there are two ways to define the characteristic impedance. One is for infinite line and other is if the line is terminated in the characteristic impedance then there is no reflection on the Transmission Line.

With this now we can generalize the impedance transformation relationship. Till now what we have done is we have transformed the impedance relation or the impedance which is at the load end at some distance l from the load end. Then one would note here that there is nothing special about load end the load end was coming because we define the origin at load end, if we had defined the origin at somewhere else we could have started measuring the distances from that point and then would get an impedance transformation relationship with respect to that point which we call as the reference point so in fact this relation is between any two points on Transmission Line for impedance transformation.

So if I know the impedance at any point on the Transmission Line I can find the impedance at any other point on the Transmission Line using this transformation relationship.

Now what we are saying is in general if I have a Transmission Line which might be connected into some load and if measure at some location an impedance Z_1 where I am showing the arrow just to indicate that the generator is on the left side so energy flow is in this direction.

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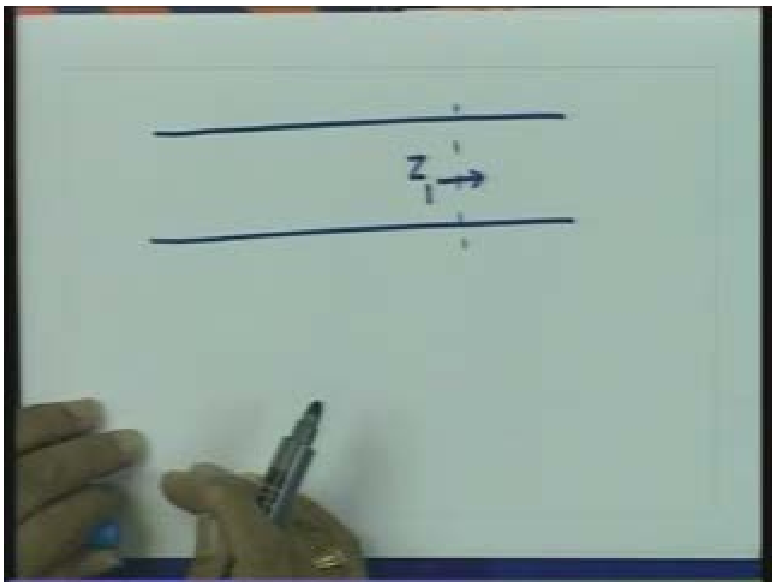

If I go to any other location on the Transmission Line if I measure the impedance is equal to Z_2 then I can use the impedance transformation relationship with this distance l replacing load impedance Z_L by Z_1 and this Z_L will indicate the impedance at this location so from the same impedance transformation relationship here I can write down $Z_2 = Z_0 \left\{ \frac{Z_1 \cosh\gamma l + Z_0 \sinh\gamma l}{Z_0 \cosh\gamma l + Z_1 \sinh\gamma l} \right\}$.

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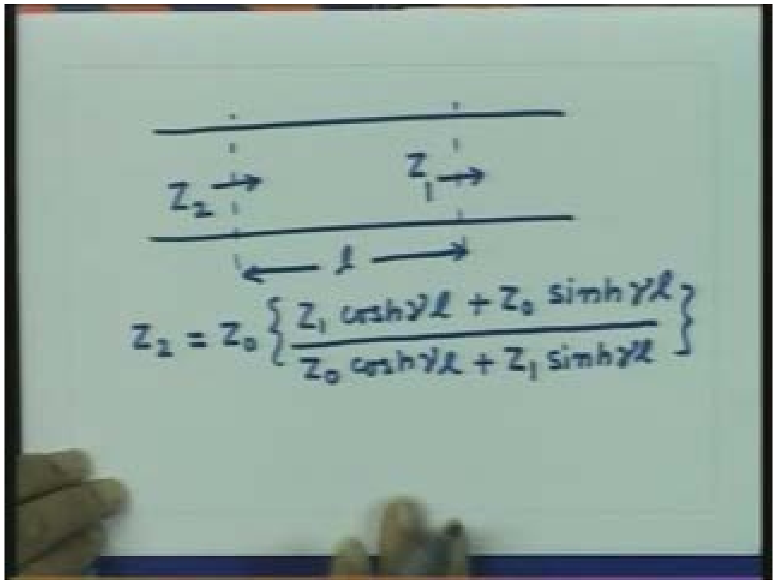

If I invert this relationship means if I find out the value of Z_0 in terms of Z_2 I will get $Z_1 = Z_0 \left\{ \frac{Z_2 \cosh\gamma l - Z_0 \sinh\gamma l}{Z_0 \cosh\gamma l - Z_2 \sinh\gamma l} \right\}$.

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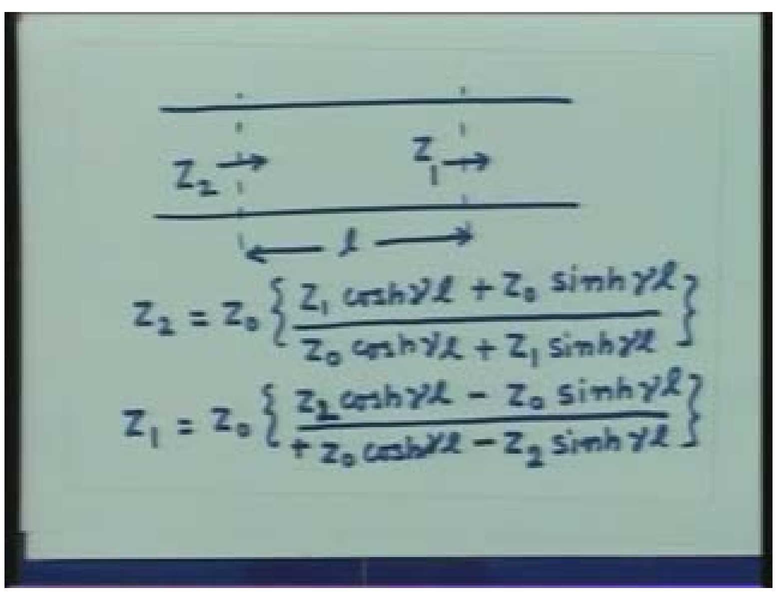

What has changed is if I invert this relationship I will get a negative sign here, I will get a negative sign here this negative sign can be absorbed into the sinhγl.

So this is equivalent to Z_0 into sinhγ(-l), similarly this quantity will become sinhγ(-l). So I have a Z_1 related to Z_2 with effectively l becoming negative because coshγ(-l) is equal to cosh γ(+l) so this sign does not change. So what we are now saying is if I know the impedance at this location I can use the impedance transformation relationship which is this to get a impedance at a distance l from this point towards the generator generated from this side. However the same relation can be used from transforming this impedance to this one with l replaced by -l because the distances which you have taken are taken positive towards the generator so if I transform the impedance from this point to this point then I get the distance traveled towards the generator which by definition is positive.

However if I transform the impedance from this location to this location then I am moving away from the generator and the distance measured by definition is negative. So the impedance transformation relation is the same relation which is this so this relation was same as this. So in this case if I replace Z_L by in general impedance at any location and measure the distances from that location that l if l is moved towards the generator I take the distance l positive if the l is moved towards the load or away from the generator then I take this distance negative.

So I have to remember only one impedance transformation relation which is this then I can transform the impedances from any point on the line to any other point on the line. And once I know the impedance I can find out the reflection coefficient on that point of the line where like we saw earlier there is a unique relationship between the reflection coefficient and the impedance at any point on the Transmission Line. With this understanding of the impedance transformation and the definition of the characteristic impedance now we can go to a little simplified version of this and that is we discuss a special case of the general Transmission Line called a lowloss or a Loss-less Transmission Line.

If you recall the whole purpose of the Transmission Line was to transfer the power from the generator to the load effectively with as little power loss in between that means ideally the structure called the Transmission Line should be as low loss as possible. Therefore in practice every effort is made to reduce the losses on the Transmission Line. So if you take a good Transmission Line this loss should be negligibly small at that operating frequency. Once I have that condition in practice then one can make some simplifications in the analysis and come up with an idea called a low loss transmission line and if the losses are ideally zero on a Transmission Line then we call that Transmission Line as a lossless Transmission Line.

Now if you recall that the Transmission Line has four primary parameters it was the resistance per unit length, the conductance per unit length, the capacitance per unit length and the inductance per unit length. The ohmic elements in these four parameters are only the resistance and the conductance that means the resistance of the two conductors of the line they get heated because of ohmic losses so they consume power the dielectric which is separating the two conductors of the Transmission Line they have the leakage current and again they have the ohmic losses so they consume power. So the power loosing element in the Transmission Line is because of the resistance and the conductance, the inductance and capacitance store energy but there is no loss in the inductance and capacitance.

So ideally a line will be lossless if the resistance and the conductance are identically zero so let us first write down the parameters for an ideal Loss-less Transmission Line then we will go to a more practical line a good line which is called a low loss transmission line for which the losses are much smaller and there is a efficient power transfer from the generator to the load.

So taking the first case called the lossless transmission line by definition a line is called loss less if R = 0, G = 0. By substituting R = 0, G = 0 in the secondary constants of Transmission Line that is the propagation constant γ and the characteristic impedance we get $\gamma = \sqrt{j\omega L \, . \, j\omega C}$ where R = 0 so this will be equal to $j\omega\sqrt{LC}$.

Now by definition $\gamma = \alpha + j\beta$ we just saw the attenuation constant we saw the phase constant so this means for this lossless case $\alpha = 0$ and $\beta = \omega\sqrt{LC}$. As correctly we can see here there are no losses there is no reason for wave amplitude to reduce on the Transmission Line. And as we note it know that α is the attenuation constant which gives you the reduction in the traveling wave amplitude as you travel from the Transmission Line. So when α goes to zero the wave amplitude does not reduce and there is a sustained propagation of a traveling wave on the Transmission Line from here ω which is equal to 2π into frequency and β as we have seen by definition is $2\pi/\lambda$ we can substitute see you get $2\pi/\lambda$ that is equal to 2π into frequency into $\sqrt{LC}$. From here 2π will cancel so I get λf which is equal to $\frac{1}{\sqrt{LC}}$.

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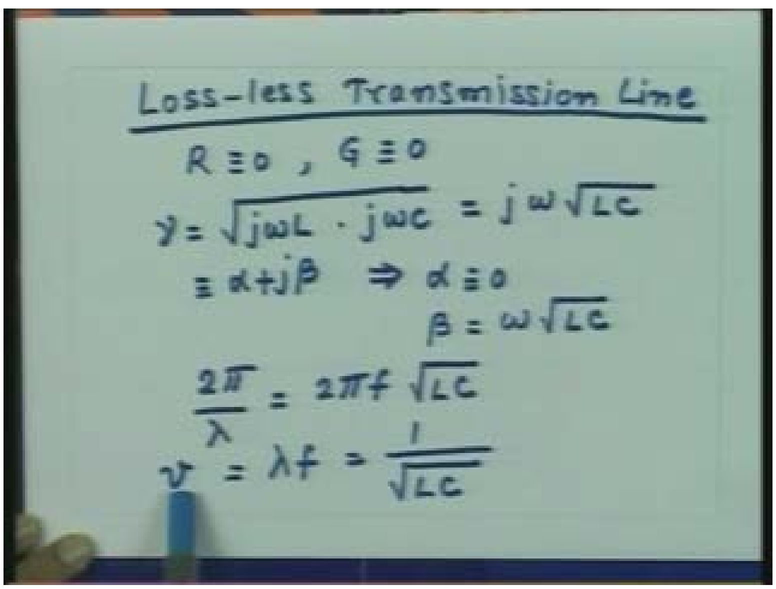

And from our basic physics we know frequency times the wavelength that is nothing but the velocity of the wave the voltage wave on Transmission Line. So the velocity of the wave is related to the inductance and capacitance per unit length on the Transmission Line so once the inductances and capacitances are given the velocity of the wave on the Transmission Line is reached.

Then one may wonder I can arbitrarily choose the inductances and capacitances and can arbitrarily vary the velocity of the traveling wave on Transmission Line in fact the problem is not that the situation is exactly opposite. The velocity is the parameter which is decided by the field distributions and the boundary conditions as we will see little later in this course that means the inductances and capacitances of a Transmission Line are not independent because this quantity v is decided by some other boundary conditions on the Transmission Lines so once the velocity of the traveling wave is fixed by some other conditions this quantity is constant or product of L and C is constant. And this one can say physically if I consider the two conductors of Transmission Line if I vary the separation between these two conductors the mutual inductance is going to vary so the net inductance of the line is going to vary. But the same time since the separation between the conductor is varying the capacitance is also going to vary so when I am trying to vary the inductance the capacitance varies and vice versa. Precisely that is what we are talking about the inductance and capacitances per unit length on a Transmission Line are independent products. The product of that is constant because that is related to the velocity of the propagation of wave on the Transmission Line.

So for a lossless line the velocity is given by $\frac{1}{\sqrt{LC}}$, α is identically zero so the wave has the phase constant β but its amplitude remains constant as it travels on the Transmission Line. Then we can calculate the another parameter which is the characteristic impedance Z_0 for the lossless line and again substituting R = 0, G = 0 this will be equal to $\sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$.

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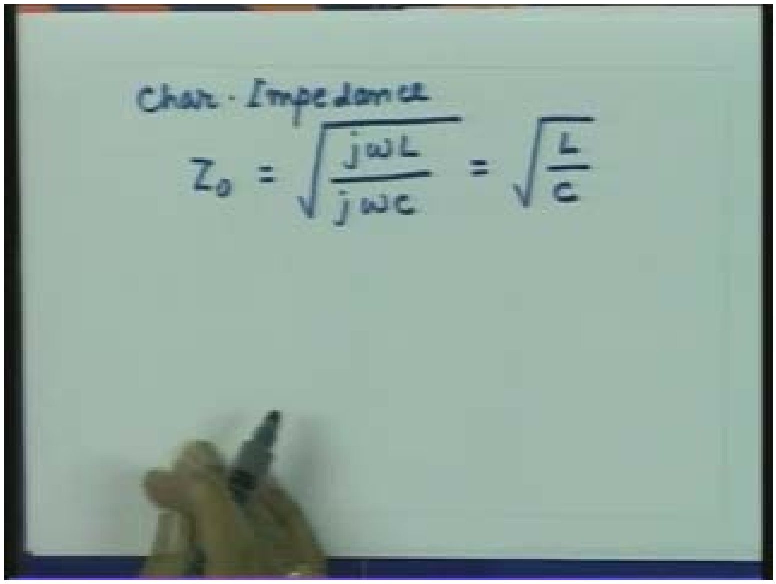

So the characteristic impedance of the lossless line is square root of the ratio of the inductance and the capacitance per unit length. But now the important thing to note is that for a lossless line this quantity is a real quantity Z_0 is real and that is very interesting.

Now I do not have any resistance or conductance on the line that means I do not have any ohmic parameter on the line R = 0, G = 0 but we still do not know where the characteristic impedance is it is some where on the Transmission Line it governs the propagation of energy on the Transmission Line and that quantity is the real quantity. So I do not have any resistive element on the line but the characteristic impedance is the real quantity that means the wave which travels whether it is forward or backward this always sees the characteristic impedance that means it sees an impedance which is equal to the real impedance which is a resistive impedance. So forward traveling wave when it travels on a Loss-less Transmission Line sees an impedance which is a real impedance like a resistance and make sense physically because once you are having a line which is only forward wave this wave is going to go and going to go forever no energy is going to get reflected that means this power somewhere is going to get dumped and that is what this impedance Z_0 is.

So if you are having a real quantity for the Z_0 that means now the power is completely transferred to the line. When you are saying power transferred to the line does not mean that the power is lost in the line, it is simply the power has been carried by the line to the other end and no reflection has come back so for a lossless line the characteristic impedance is the real quantity that means if I measure the input impedance of the line which is terminated into Z_0 I will see an impedance which will be like a resistance and this line can expect power from the generator as if the power is drawn into a resistance.

Once we get the feeling for the lossless case then one can now go to the more realistic lines where the losses are small and then ask how much these parameters deviate the β or the propagation constant γ and the Z_0 how much deviation is there between in these parameters when their losses are relaxed they are not ideally zero but they are small.

So what one can do is one can now define more practical line and we call that line as the Low Loss Transmission Line and we define a Low-Loss Line as when R is much smaller compared to ωL which means when the reactances are much larger compared to the resistances then we call that line as the Low-Loss Transmission Line. So the condition that R is much smaller compared to ωL and G much smaller compared to ωC. If these conditions are satisfied then we say this line is a Low-Loss Transmission Line. Let us see what are the implications of this thing on the propagation parameters and the characteristic impedance.

As we saw this γ is $\sqrt{(R + j\omega L)(G + j\omega C)}$ I can take $j\omega L$ and $j\omega C$ common from here so that is equal to $\left\{ j\omega L\left(1 - j\frac{R}{\omega L}\right) j\omega C\left(1 - j\frac{G}{\omega C}\right)\right\}^{1/2}$.

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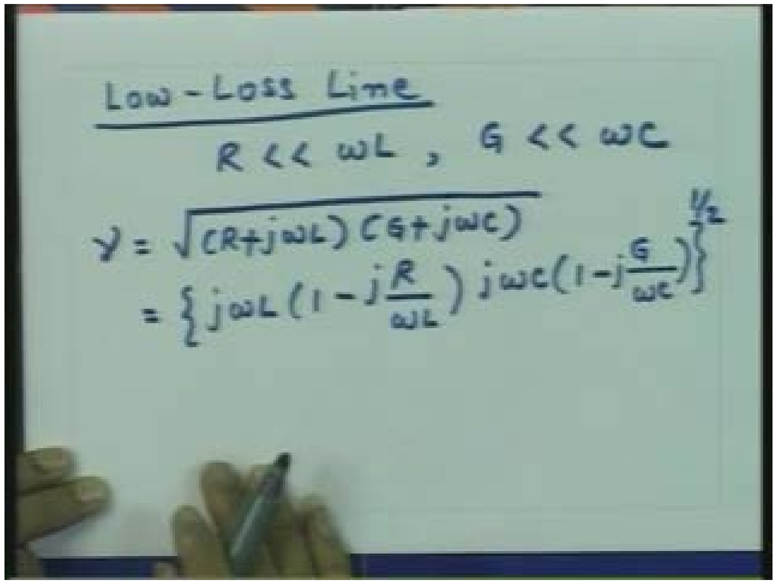

Noting that R is much smaller compared to ωL and G is much smaller compared to ωC and if I retain only the first order terms the product of these will be second order term. So if I retain only the first order term this I can write as $\left\{ \mathrm{j}\omega \mathrm{L}.\mathrm{j}\omega \mathrm{C}\left(1-\mathrm{j}\dfrac{\mathrm{R}}{\omega \mathrm{L}}-\mathrm{j}\dfrac{G}{\omega \mathrm{C}}\right)\right\}^{1/2}$

I can take out this quantity so that is equal to $\mathrm{j}\omega\sqrt{\mathrm{LC}}\left\{\left(1-\mathrm{j}\dfrac{\mathrm{R}}{\omega \mathrm{L}}-\mathrm{j}\dfrac{G}{\omega \mathrm{C}}\right)\right\}^{1/2}$

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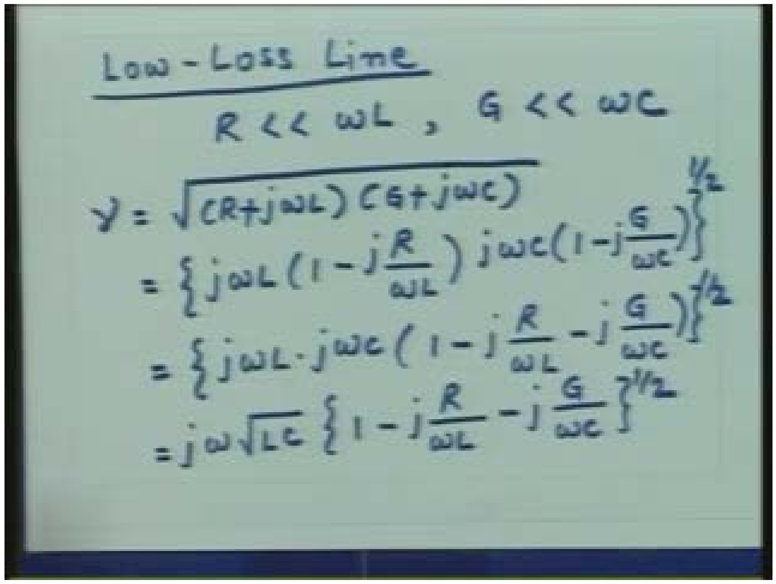

Again expanding this binomial series and retaining only the first order term this can be approximated as the gamma is equal to $\gamma = j\omega\sqrt{LC}\left\{\left(1 - j\frac{R}{2\omega L} - j\frac{G}{2\omega C}\right)\right\}$

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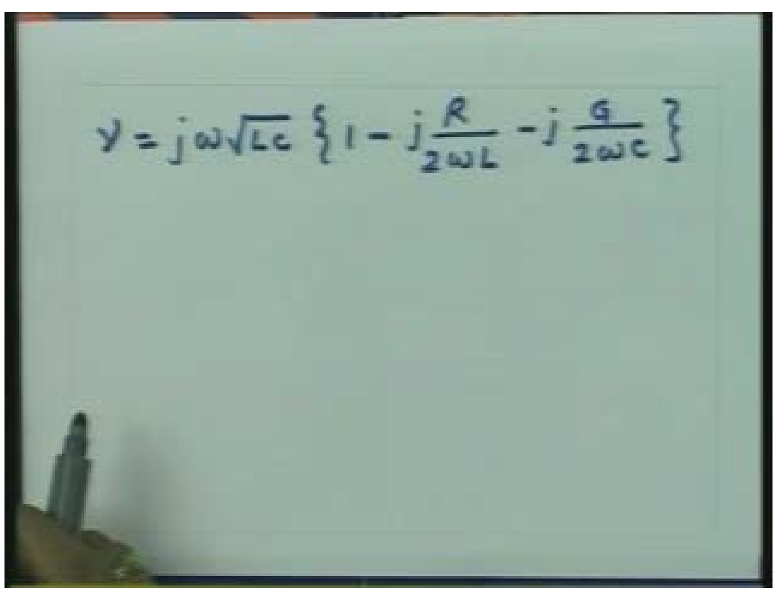

Taking this inside this can be written as $j\omega\sqrt{LC}$ plus I am taking inside this so ω will cancel with this square root L will cancel with this so this will be equal to $\frac{1}{2}R\sqrt{\frac{C}{L}}+\frac{G}{2}\sqrt{\frac{L}{C}}$.

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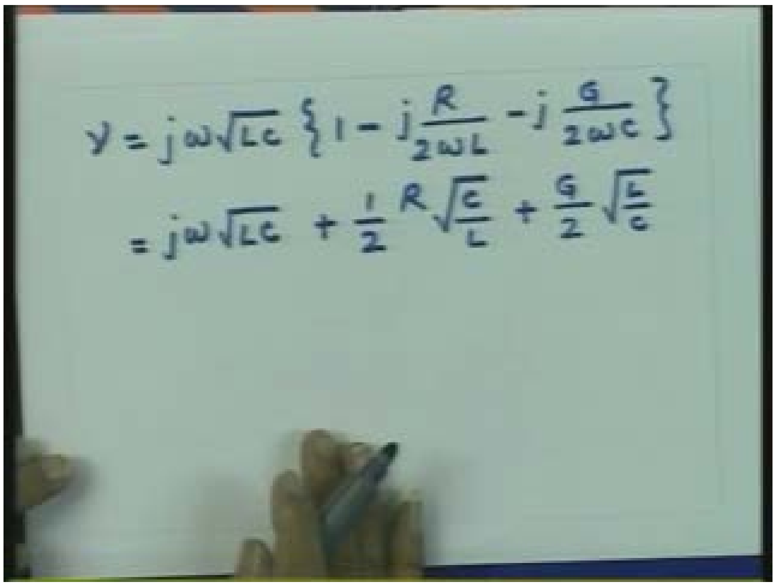

So this quantity here which is the imaginary part of the propagation constant that represents β and this quantity here represents the attenuation constant α.

Look at what we had got for the Loss-Less Transmission Line. When we had a Loss-Less Transmission Line when R = 0, G = 0 we had α = 0 and we had β which was $\omega\sqrt{LC}$.

In this case also when we are having low loss the β is still $\omega\sqrt{LC}$ which means the phase constant to the first order does not change when we introduce a small loss in the Transmission Line that again means if I am interested only the phase calculations on the Transmission Line then that line can be transmitted like a losses line because beta value does not change and I can carry out the analysis of Transmission Line for a low loss

Transmission Line same as if there was no loss on the Transmission Line and the line is a ideal line with R = 0, G = 0.

However in case of a real line even with a small loss you have this value α which is related to R and G so where the wave travels the amplitude reduces slowly as it travels on this Transmission Line but the amplitude does not change very rapidly there is small decrease in the amplitude so over a short distance if I do the calculations the line can still be treated like the lossless line with the phase constant which is equal to $\omega\sqrt{LC}$.

If I look at this quantity here which is $\sqrt{\frac{C}{L}}$ or $\sqrt{\frac{L}{C}}$ is nothing but the characteristic impedance of the line which was lossless. So if I say that my losses are very small I can substitute for $\sqrt{\frac{L}{C}}$ = Z_0 so this quantity can also be written as $\omega\sqrt{LC}$ + $\frac{1}{2}$ $\left\{1-\frac{1}{2}\frac{R}{Z_0}+GZ_0\right\}$.

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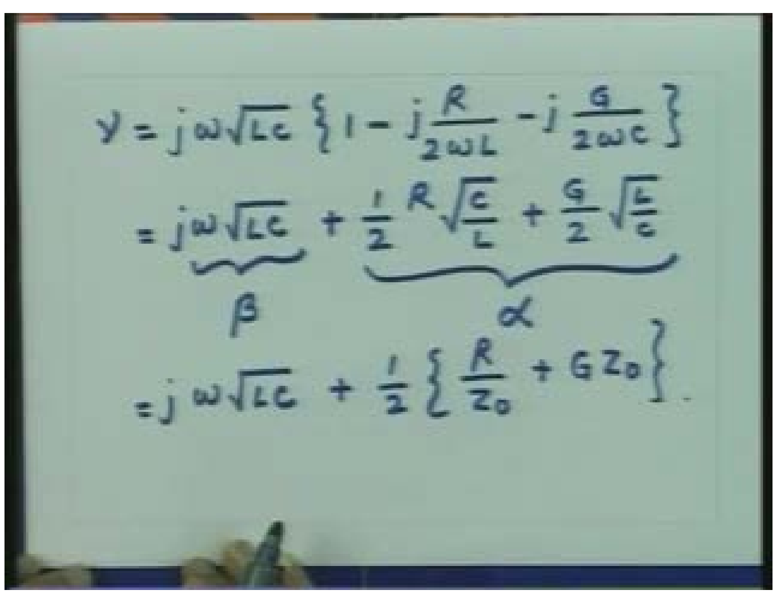

So for the quick calculation if I know the resistance per unit length if I know the conductance per unit length and if I know the characteristic impedance of the line then I can calculate what the phase constant is which is same as the Loss-Less Line and I can calculate the small or whatever the value of the attenuation constant is from this expression. So for a real line one can calculate the value of α and β and then one can proceed for the calculation of the impedance another thing from the Transmission Line.

As we have done the calculation for γ now we can write the characteristic impedance for the Low-Loss Line $Z_0 = \sqrt{\dfrac{\sqrt{\dfrac{L}{C}}\left(1 - j\dfrac{R}{2\omega L} + \dfrac{G}{2\omega C}\right)}{j\omega C\left(1 - j\dfrac{G}{2\omega C}\right)}}$.

Again doing the same thing taking the $j\omega L$ and $j\omega C$ common from these parameters this you can write as $\sqrt{\dfrac{j\omega L\left(1 - j\dfrac{R}{\omega L}\right)}{j\omega C\left(1 - j\dfrac{G}{\omega C}\right)}}$.

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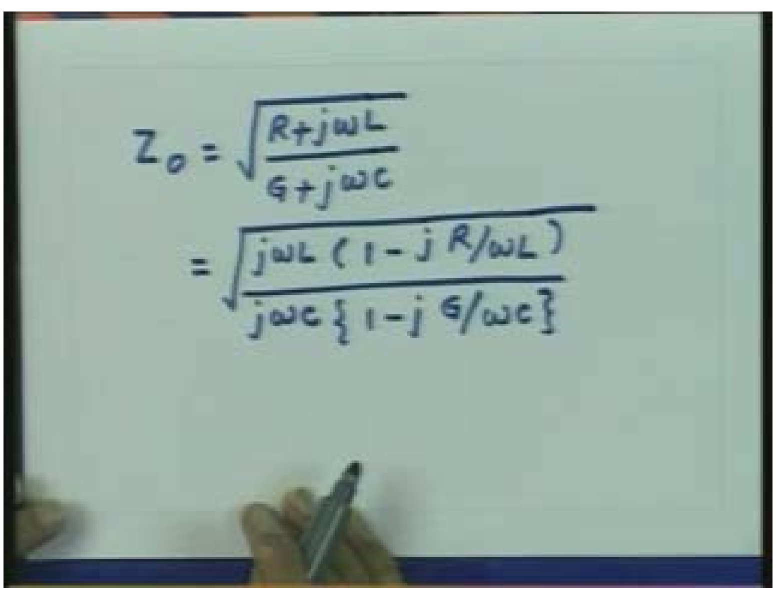

This quantity is $\sqrt{\frac{L}{C}}$ that will be nothing but the characteristic impedance of the ideal Transmission Line I can expand this parameters again and this we can write as $\sqrt{\frac{L}{C}}$ this square root again since $\frac{R}{\omega L}$ is very small this can be approximated by $1 - j\frac{R}{2\omega L}$ this will be $1 - j\frac{G}{2\omega C}$.

I can bring this to the denominator and then approximate it to write as $\sqrt{\frac{L}{C}}\left(1 - j\frac{R}{2\omega L} + \frac{G}{2\omega C}\right)$.

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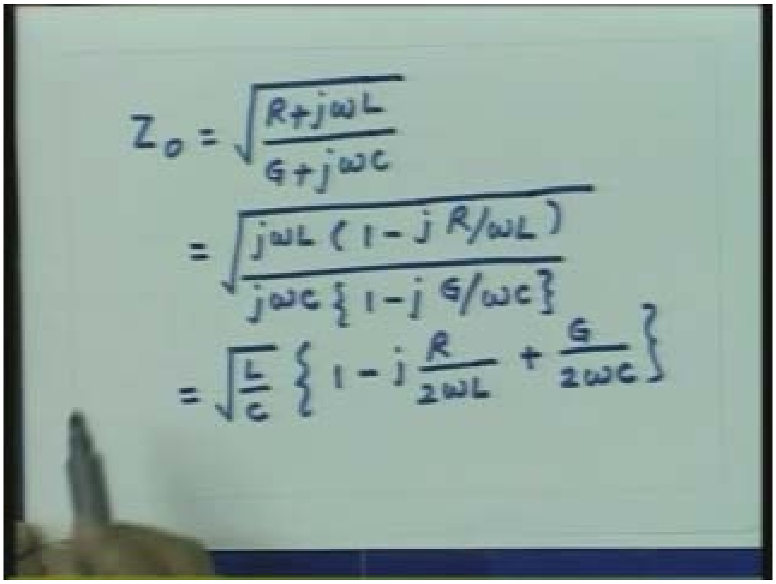

So the characteristic impedance this quantity is real by $\sqrt{\frac{L}{C}}$ which is nothing but the characteristic impedance of the Loss-Less Transmission Line. But now for a real low loss line the characteristic impedance is no more real it has a component which is imaginary coming from this quantity there is some j here. So now we conclude two important things that for a Loss-Less Transmission Line the propagation constant γ has two parts it has imaginary part and the real part.

The imaginary part which is nothing but the phase constant is same as the phase constant of a Loss-Less Transmission Line.

However the attenuation constant has a small value the characteristic impedance of a low loss transmission line is no more real it is complex its real value is almost same as the characteristic impedance of the Loss-Less Transmission Line.

But now you are having a small imaginary part which represents the losses in the Transmission Line with this understanding of the propagation constant and the characteristic impedance of the low loss transmission line.

Now here onwards until and unless specifically we are told to include the losses in the Transmission Line we assume the line to be lossless and carry out the analysis of the impedances voltages and currents and all other things under the assumption that there are no losses on the Transmission Line.

So in the next lecture and on wards we will in detail discuss the behavior and the analysis of the lossless Transmission Lines.