Transmission Lines & E. M. Waves
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Lecture – 38

In the last lecture, we developed a general approach for analyzing wave propagation inside a waveguide. In this lecture, we take specifically a waveguide whose cross section is rectangular and that wave guide is called a rectangular wave guide.

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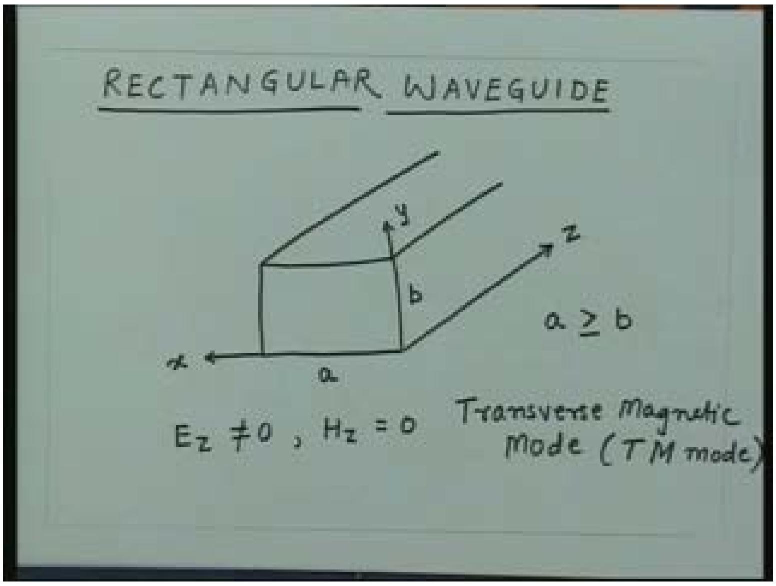

So, we have a wave guiding structure for which we have a hollow pipe whose cross section is rectangular and the energy is going to flow along the length of this metal pipe. We are again considering that this pipe is made of ideal conductor terminal conductivity of this pipe is infinite and the medium which is filling this wave guide is ideal dielectric. That means the conductivity of the medium which is filling this pipe is 0.

So, we are having a very ideal structure that is the pipe is ideal conductor and the medium filling this wave guide is ideal dielectric and we want to now investigate how the electromagnetic energy is going to propagate inside this pipe without losing generality. Let us orient our coordinate system so that the z axis is along the length of the pipe that mean that is the direction in which the energy is going to propagate.

So, let us say this direction is the z direction and then horizontal axis let us say x axis, so this axis is x and this vertical axis is y the origin is located at one of the corners of this cross section. Now, considering in general this cross section of this wave guide is rectangular. Let us say the width of this wave guide is denoted by a and the height of this wave guide is denoted by b and that definition a is greater than or equal to b and why we are taking it convection will become clear when we define the particular type of mode. So, here we are assuming that by definition a should be greater than or equal to b and x axis is oriented along this broader dimension that means along a that is the convention essentially we are taking.

Now firstly, if you try to see whether TM mode exist inside this, the answer is; no, the TM mode cannot exist inside this structure and the reason for this, we will see later after we have understood the TM and TE propagation. So, first we will take the simpler case which is the transverse magnetic propagation that means for this the E_z is not equal to 0 and H_z is equal to 0.

We saw in the last lecture that if the transverse field has to exist, then either E_z or H_z has to be non-zero. So, in this case if I consider H_z is 0 that means the magnetic field now is in the oriented in the transverse plain and longitudinal component z component is only E_z. So, this mode we can call as the transverse magnetic mode or as we called earlier, in short, this is called the TM mode.

So, let us investigate now the problem for the transverse magnetic mode for which the longitudinal component is only E_z and H_z is equal to 0. Of course, E_z and H_z both are all field component for their matter have to satisfy the wave equation inside this structure. So, the approach now is as follows; first we solved the equation for this longitudinal component E_z, then we go to the expression which we have derived last time for the transverse components, substitute here for E_z and H_z equal to 0 and we will get the transverse fields for the transverse magnetic mode.

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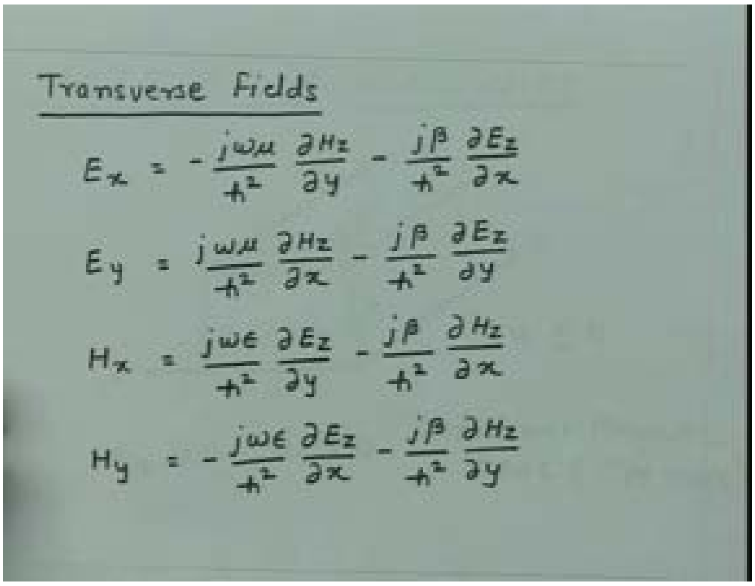

So, problem essentially has to be solved in 2 steps; one is, first finding the solution for this longitudinal component E_z, then finding out transverse components and then applying boundary condition finding proper solution to this structure which is rectangular wave guide.

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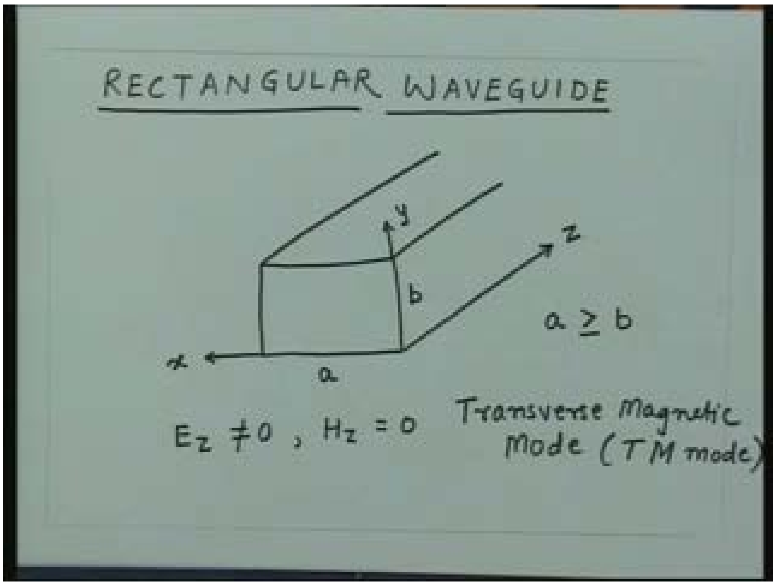

So, if I take the wave equation in Cartesian coordinate system, this E_z is a scalar quantity. So, we can write down the wave equation which is del square E_z plus omega square mu epsilon E_z that is equal to 0. This is the wave equation.

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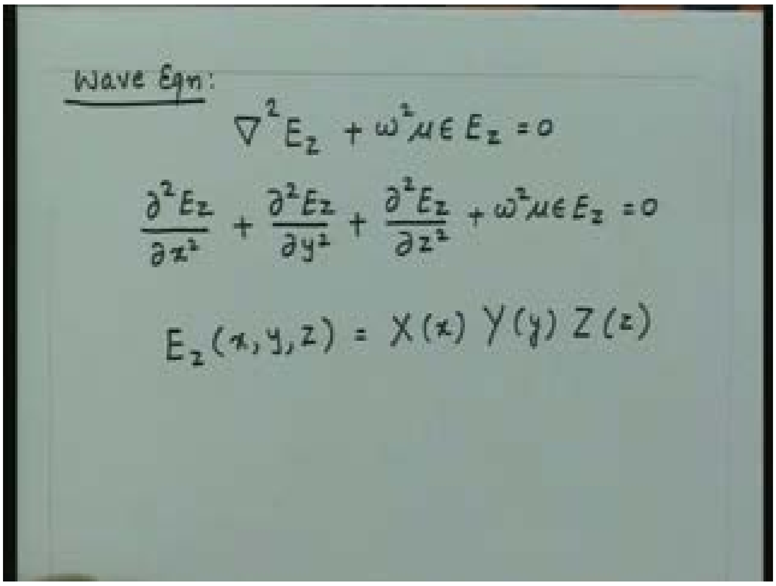

So, this equation is a scalar equation essentially and omega is the frequency, mu is the permeability of the medium and epsilon is the permittivity of the medium which is filling this wave guide. We can expand this del square in the Cartesian coordinate system, so we can get d^2 E_z by dx square plus d^2 E_z by dy square plus d^2 E_z by dz square plus omega square mu epsilon E_z that is equal to 0. Now, we solve these problem essentially just on mathematical considerations.

However, since you have developed the understanding for the wave propagation inside the parallel plain wave guide, we make frequent visits to that understanding. So, whenever we choose some constant or something while writing the solution to the equation, we make sure that whatever solution you get that solution should be consistent with what understanding we have developed from the parallel plain wave guide.

Now, this equation can be solved by separation of variables. So, we can define this quantity E_z which is product of the three functions, each one is a variable of either x, y or z and of course, there is implicit assumption that all these fields are varying as a function of time which is e to the power g omega t. So, we can say that we can apply separation of variable to get a solution which is a function of x, y, z that is some function of x, some function of y, some function of z.

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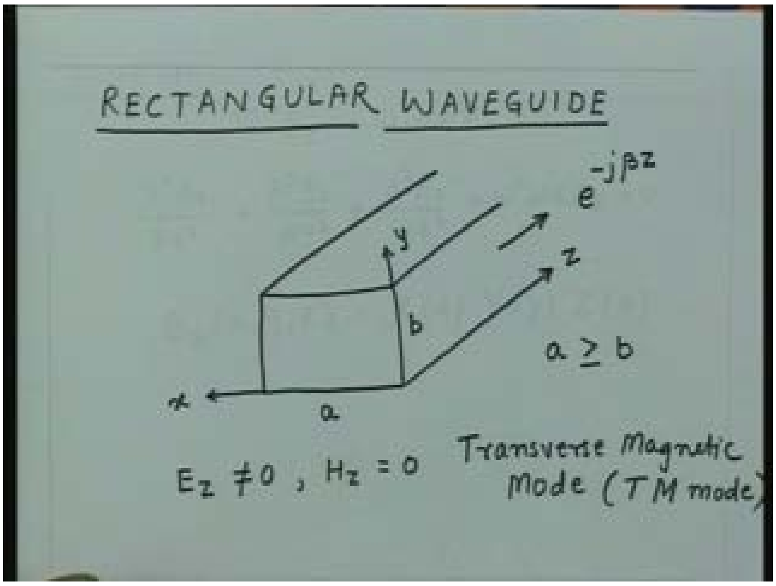

Now, if you recall, the wave is propagating in the z direction that means we are looking for a solution of travelling wave type in the z direction and since we are considering a medium which is completely lossless because the dielectric is ideal dielectric, the conducting boundary of this wave guide is again ideal; so there is no loss anywhere. So, we have a sustain propagation of wave inside this structure. So, we have z variation for a travelling wave in the z direction which will be e to the power minus j beta z. That information we will use when we try to solve this problem that we are looking for a travelling wave solution in this direction.

Also, we a have information from parallel plain wave guide and that is let us say suppose, I make one of the dimensions of the wave guide and push it up, let us say I make b equal to infinity, then essentially I got a structure which is parallel plain wave guide and I have seen for parallel plain wave guide that this can be visualized as the propagation of uniform plain wave inside this parallel plains by multiple reflection on this two plains. So, it creates standing wave kind of atoms in this direction which is perpendicular to the plains and travelling wave propagation along the plains.

So, that means whatever solution we are going to get for this problem must have a standing wave kind of solution in the x direction because if I take a limit when b tends to infinity, the solution must represent the solution for parallel plain wave guide. Same is true for this also that if I take these two plains, then if I take a tending to infinity; again I will get a parallel plain wave guide which is horizontal now and then I must get a standing wave kind of solution which are in y direction.

So, the physical understanding tells me that a solution which we are getting along x direction must be of standing wave type, the solution which we should get along the y direction also must be of standing wave type and the solution which we should get along the z direction must be of travelling wave type because in that direction, the wave is going to, net wave propagation is going to take place. With this understanding, now we are going to solve the problem.

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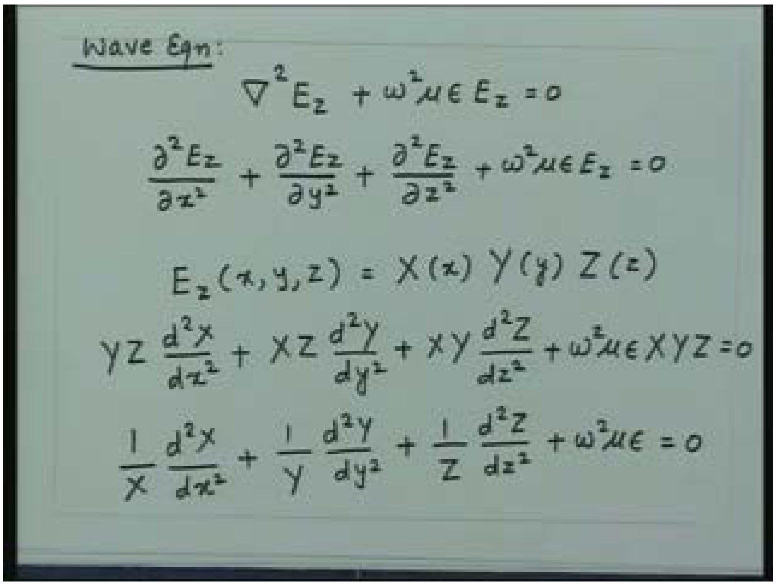

So, first let us just take this and blindly substitute into this equation. So, if I substitute into this, I get YZ d^2 X by dx square plus XZ d^2 Y by d y square plus XY d^2 Z by dz square plus omega square mu epsilon XYZ that is equal to 0. Note here, now, the partial derivatives have been converted to the full derivatives and the reason is X is now a function of x only, Y is a function of y and Z is a function of z. By dividing by xyz, all these terms, essentially we get 1 upon X d^2X upon dx square plus 1 upon Y d^2 Y upon dy square plus 1 upon Z d^2 Z upon dz square plus omega square mu epsilon, it should be equal to 0. Now, this quantity is only a function of X, this quantity is only a function of Y and this quantity is only a function of Z and this is constant.

Now, this equation whatever you have written, it should be valid at every point in space. So, this equation must be satisfied by every point XYZ in the three dimensional space and that can only happen provided each of this term is a constant quantity. So, this should be a constant, this should be a constant, this should be a constant; then and then only this equation can be satisfied by every value of XYZ.

So, what we do is; we just take each of this term and equate them to some constant and the equation essentially reduces to three equations and that is 1 upon X d^2 X upon dx square is equal to some constant and let me just put a constant as minus A square, second term which is 1 upon Y d^2 Y upon dy square, let us put that thing as minus B square and third quantity which is 1 upon Z d^2 Z upon d z square that is equal to minus beta square where beta is the phase constant of the mode of propagation.

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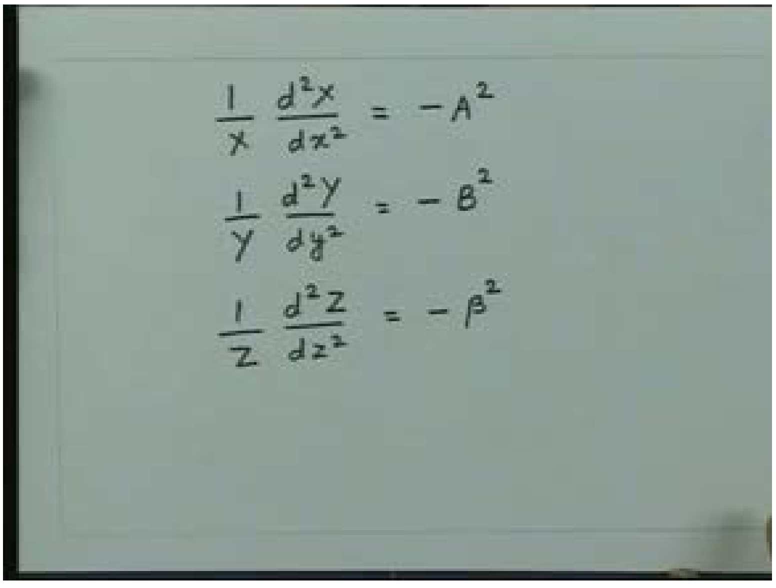

We have chosen here this negative sign appropriately so that the solution which you get for this equation that will be a standing wave kind of solution in x direction, a standing wave kind of solution in y direction and a travelling wave kind of solution, we will see, will be given by this phase constant beta.

So, while writing the expression for X, Y and Z, essentially we make sure that we write a solution which will look like standing wave kind of solution and travelling wave kind of solution as a function of X, Y and Z. So, this equation to solve is very straight forward, so this is I can multiply X on this side and take the term which will be d^2 X upon dx square plus A square X equal to 0, this is second order homogenous equation for which the solution can be written in a straight forward manner.

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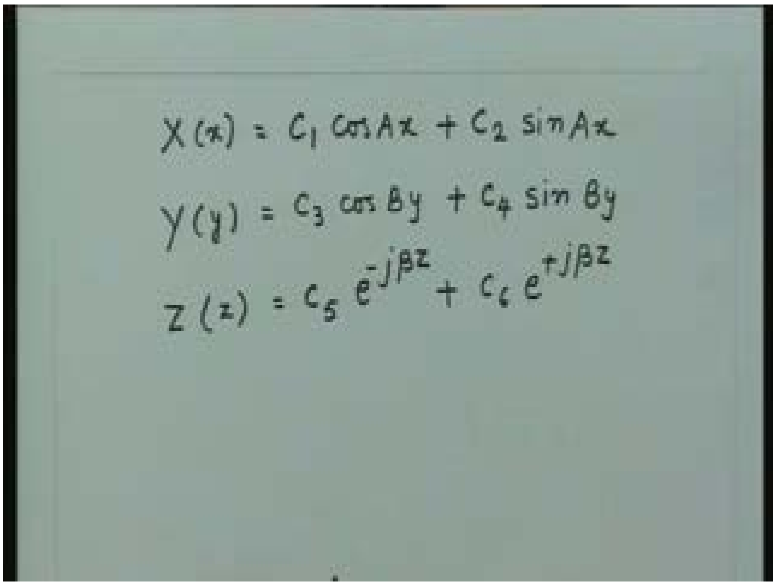

So, we get the solutions for the capital X as a function of x will be some arbitrary constant C_1 cos of Ax plus some other constant C_2 sin of Ax; Y is a function of y will be some constant C_3 cos of By plus C_4 sin of By and Z is a function of z will be C_5 e to the power minus j beta z plus C_6 e to the power plus j beta z.

So, note while writing these solutions; these functions which are cos and sin functions, they show amplitude variation which is the standing wave kind of behavior. Whereas, if I look at this quantity - e to the power minus j beta z, that represents a travelling wave in positive Z direction; if I consider e to the power j beta z quantity, that represents a travelling wave which it is in negative z direction.

So, while writing this solutions, first we use our understanding which you have developed with the parallel plain wave guide that is in the transverse direction between the two conducting boundaries we must have a solution which were like a standing wave kind of solution and along the length of the pipe in which the energy is going to propagate, we have a travelling wave kind of solution.

Now, in general when we have this wave equation solution, of course there are two waves which are travelling on this structure; one in positive z direction and one in negative z direction. However, if I assume that this wave guide is of infinite extent in z direction, then there is no reflected wave on this because we have seen from transmission line that if I take an infinitely long transmission line, there is no reflected wave on this.

So, we can just for simplicity of the analysis, we can assume that there is no wave which is travelling in the negative z direction and we have only one wave which is the forward wave which is on this wave guide. So, this wave as shown is a forward wave travelling in positive Z direction.

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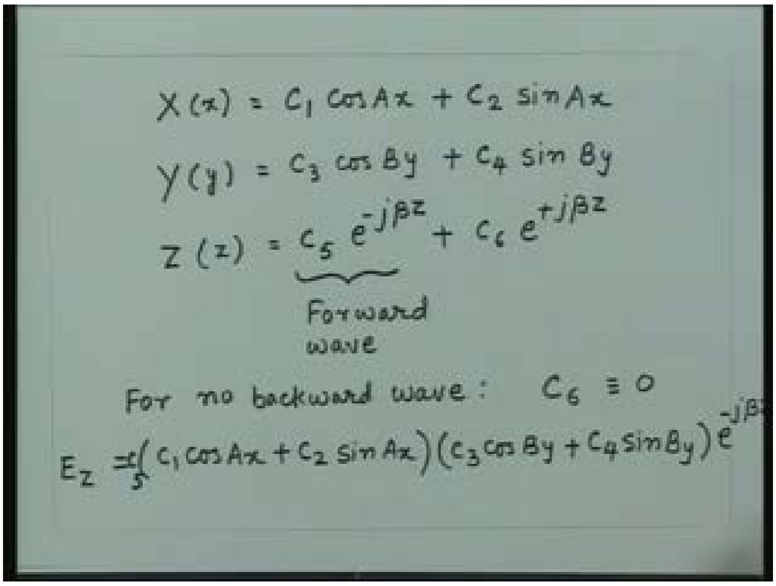

So, let us assume that only one wave which is travelling in positive Z direction exists and there is no wave which is travelling backwards on this structure. So, we can assume that for no backward wave, we can take the C_6 as identically 0. So, the Z solution is only C_5 e to the power minus j beta z and with that understanding, then I can write down the complete solution for the E_z which is the product of this, this and this quantity.

So from here, then I can get E_z which is a function of x, y, z will be equal to C_1 cos Ax plus C_2 sin Ax C_3 cos By plus C_4 sin By e to the power minus j beta z and we put a constant here which is C_5. So, this is now the solution, general solution for the wave equation for this component E_z. Now, we can apply boundary conditions to get these arbitrary constants and in this case, we are having this E_z which is like that in the wave guide, so this component E_z is tangential this wall, it is tangential to this wall, it is tangential to this wall and it is tangential to the lower wall.

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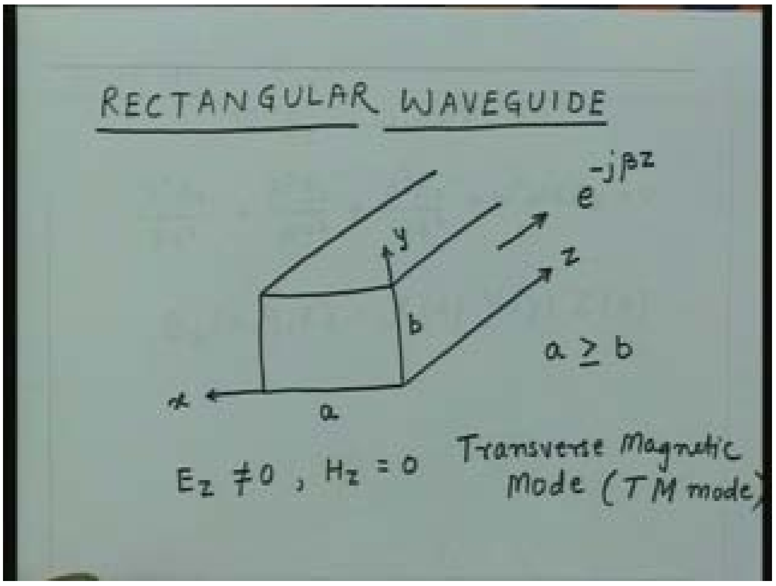

So this component, easier component is a tangential component to all the 4 walls of this and that is why we have 4 boundary conditions; one on this wall which is x equal to 0, one on this wall which is x equal to a, one on this wall which is y equal to 0 and one on this wall which is y equal to b.

So, we have got four boundary conditions that all these four walls where x equal to 0, x equal to a, y equal to 0, y equal to b, this component which is tangential should be 0.

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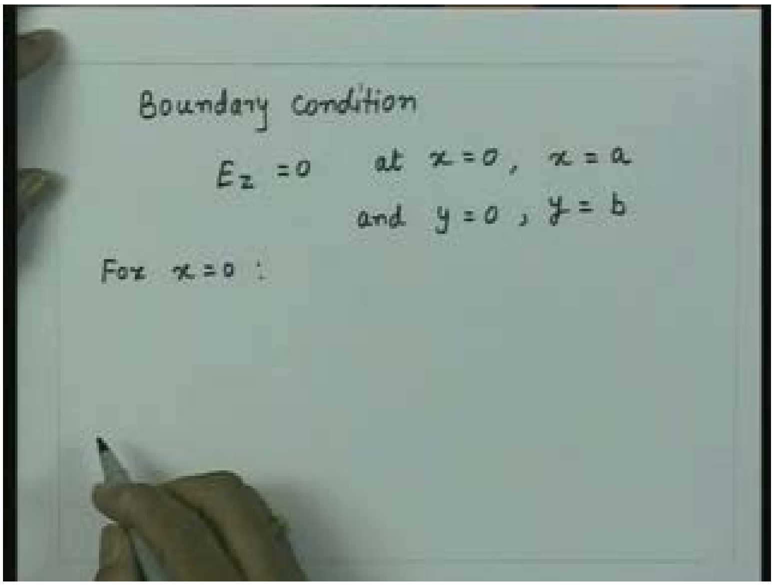

So, we have a boundary conditions now to apply on E_z and that is E_z is equal to 0 at x equal to 0, x equal to a and y equal to 0, y equal to b. So, I just use one by one boundary condition. If I put x equal to 0 in this, this quantity will be 0, sin of Ax this will be 0. Now, E_z is 0 that can happen provided this C_1 quantity that should be equal to 0. So, I get from here, from the first condition that for x equal to 0, this condition we get this arbitrary constant C_1 should be identically equal to 0.

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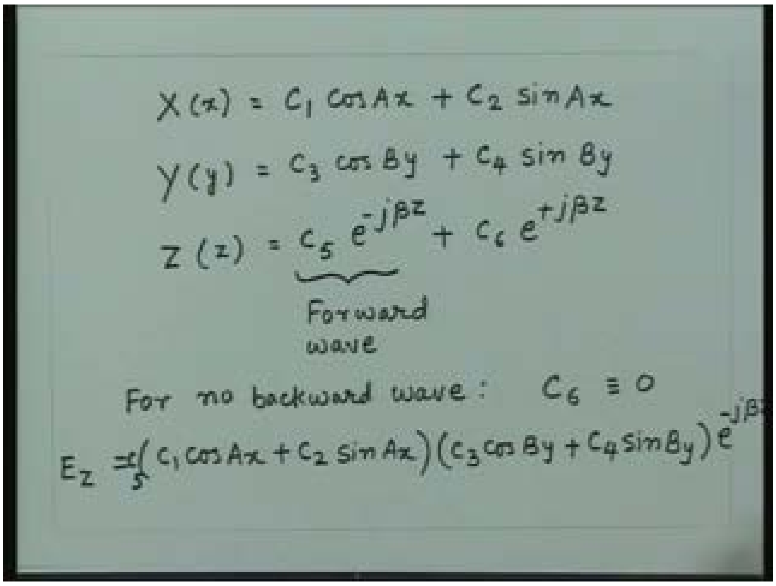

So, this gives me C_1 should be identically equal to 0. Same thing we can do from here that since for y equal to 0, again E_z is 0. So, if I put y equal to 0, this quantity will be 0 and then if E_z has to be 0, this arbitrary constant c_3 should be 0.

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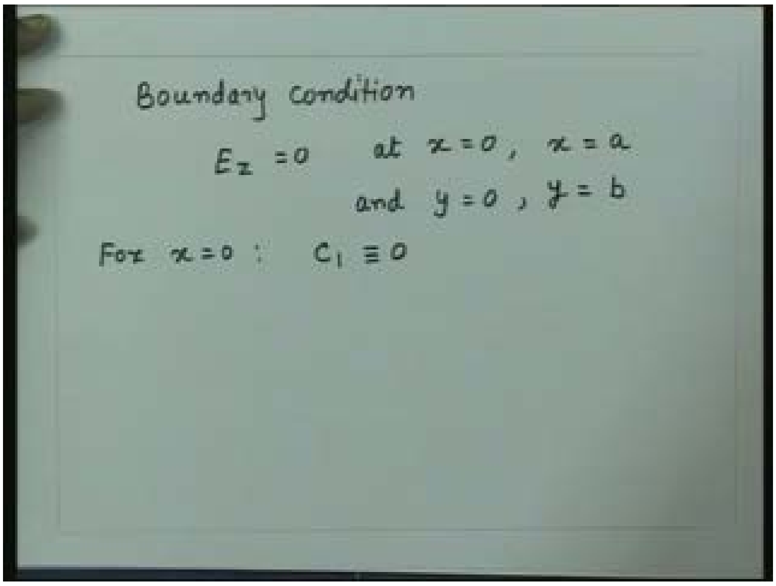

So, we get for y to be equal to be 0, you get the arbitrary constant C_3 should be identically equal to 0.

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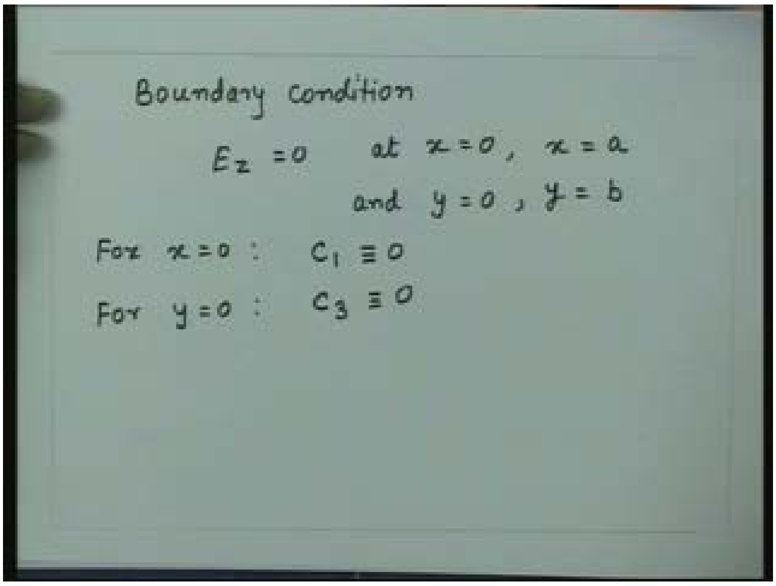

Once you get that, then the solution essentially now has become C_2, C_4, C_5 sin of Ax, sin of By e to the power minus j beta z. So, the general solution which can satisfy this boundary condition x equal to 0, y equal to 0, now will be E_z C_5 C_2 C_4 sin of Ax sin of By e to the power minus j beta z.

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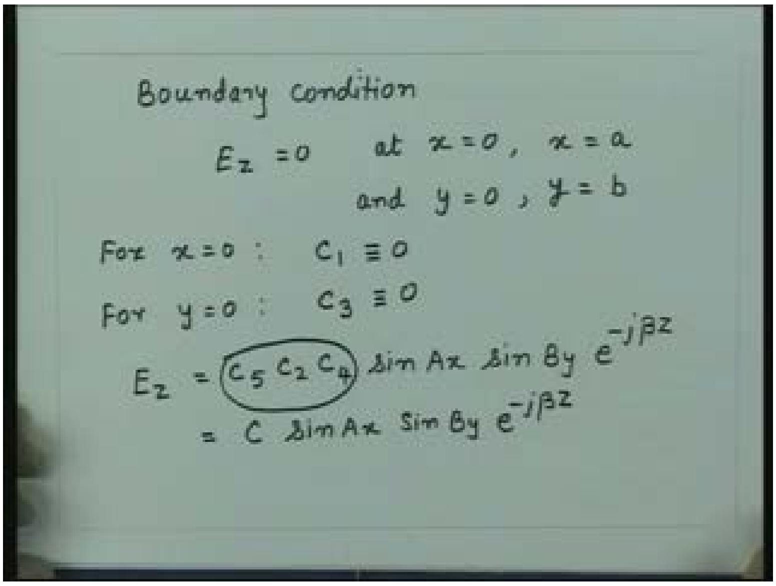

Now, these are the arbitrary constants, so we just combine this into one, so this is just telling me the amplitude. So, I can say this combine, all this thing combine together, let me call this quantity as some C into sin of Ax sin of By e to the power minus j beta z.

Now, apply second boundary condition that when x is equal to a, again this quantity E_z is 0 and that can happen when provided this A times small a, that quantity is multiples of pi. Similarly, from this boundary condition that E_z should be 0 for y equal to b, if this By is multiples of pi, then again this quantity will be 0. So, from these two boundary conditions; x equal to a and y equal to b, we get that A times a that should be equal to some multiples of pi. That means this constant A which we still have to determine that is equal to m pi by a.

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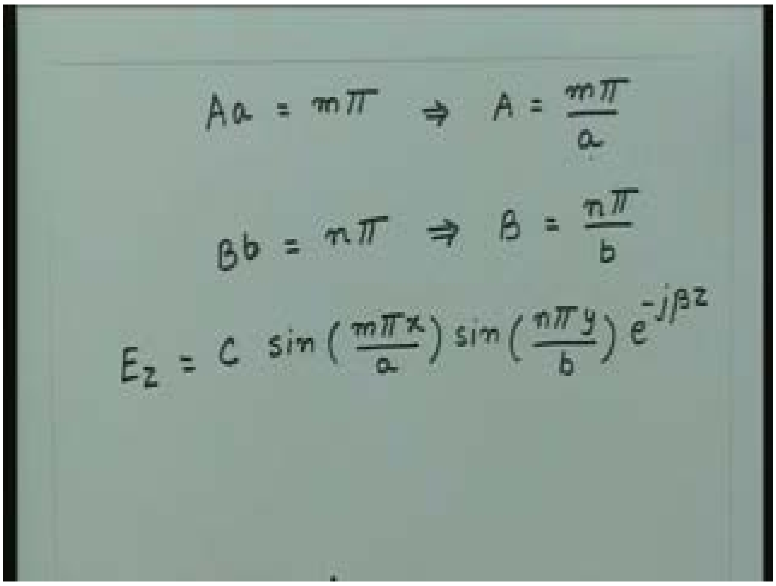

Similarly, B times small b that should be equal to some another integer of pi and that gives B is equal to n pi by b. We can now substitute, once you get these constants a and b which are related to these dimensions, the broader dimension of the wave guide and the smaller dimension of the wave guide and m and n are integers, this longitudinal component field E_z is constant C sin of m pi x by a sin of n pi y by b e to the power minus j beta z.

This is now the complete solution of the wave equation for this longitudinal component E_z. We applied all the boundary conditions, this arbitrary constant C will remain undefined because this essentially represents the amplitude of the electric field and that has nothing to do with the boundary conditions because boundary conditions are satisfied irrespective of power level or the amplitude of the field. So, this parameter this constant, arbitrary constant C will remain as it is at this stage; when we really talk about the power inside a mode, then and only this quantity C will be evaluated.

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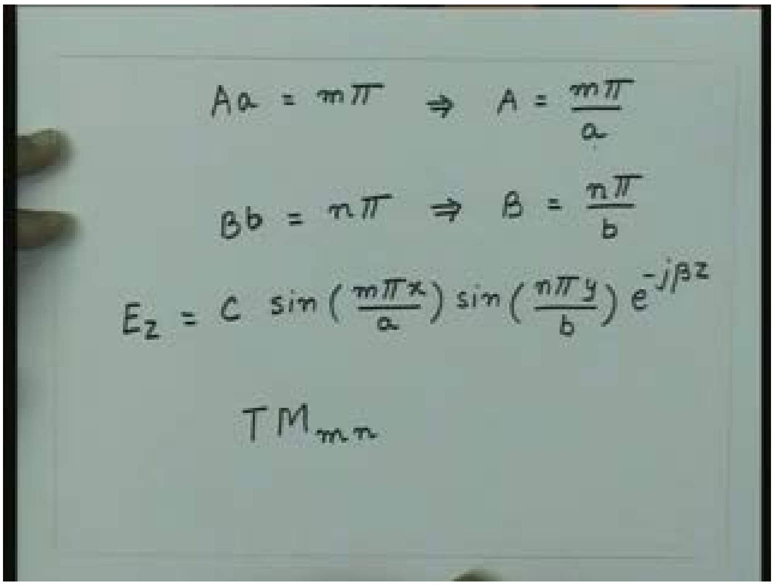

So now, we have got a complete solution and these two quantities now m and n which are integers, they now represent essentially the order of the mode by the same token as we have done for the parallel plain wave guide that there we had a transverse magnetic mode and we had put an index, now we can put the indices which are these two indices for m and n. So, a transverse magnetic mode now can be designated by TM with these two indices m and n.

Now here, the convention essentially comes from handy, why you have taken a convention; the first index which we have here is the index around the broader dimension a. So, that is the reason we taken the convention that broader dimension is called a and that is the direction in which the x is oriented. So, the first index essentially tells me the field variation along the broader dimension of the wave guide which is x direction.

Similarly, this index n tells me the field variation along the shorter dimension of the wave guide which is the y direction. So, when we write a mode TM_{mn}; the first index tells me the field variation along the broader dimension, the second index tells me the field variation along the shorter dimension. Also, we will note here that if m equal to 1, then when x varies from 0 to a, here; you have essentially one half cycle variation.

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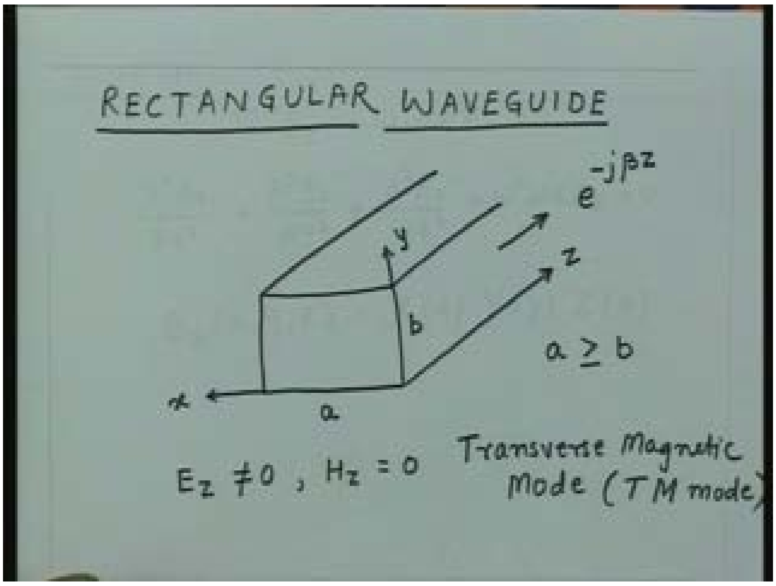

If I take m equal to 2, I have one full cycle variation and then so on and same thing is going to happen in y direction.

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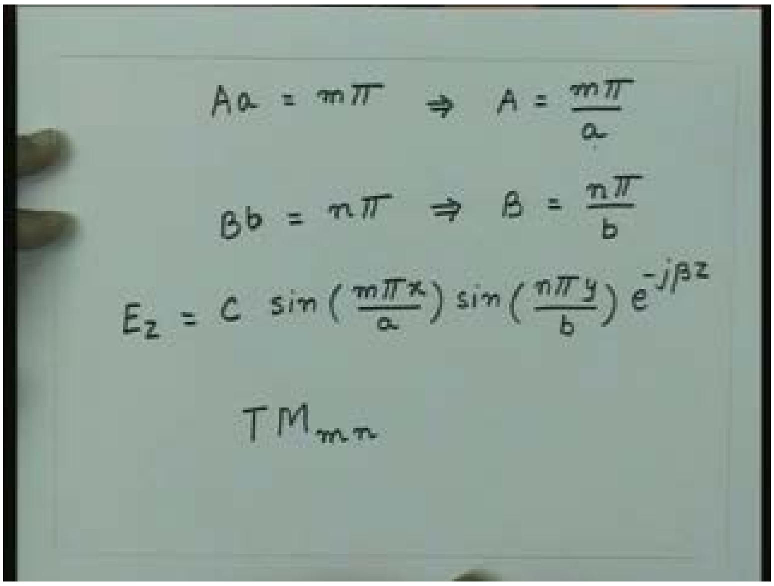

So, this behavior is exactly identical to what we have seen for the parallel plain wave guide that the order or the index which we have for the mode, it represents number of half cycles in the transverse direction and in this case, there are two transverse direction; one is x and one is

y. So, along each of these transverse direction x and y, these indices tells me the number of half cycle variations for the magnitude of the field. So, if I take m equal to 1, then I get one half cycle variation; if I take m equal to 2, I will get two half cycle variations and so on.

So, once you have understood the field behavior for the parallel plain wave guide, visualizing these now for rectangular wave guide is very straight forward because we have already developed a physical understanding of how the field variation is going to be and as we can see in the limit when a or b tends to infinity, this essentially, wave guide will become a parallel plain wave guide and then the expression which we had for E_z or for that matter, we will see later for H_z that is identical to what is the expression for electric field in the parallel plain wave guide.

So, this field expression which we have got now can be reduced to the parallel plain wave guide by properly substituting either a equal to infinity or b equal to infinity. Now, once you have got these constants now - m and n, there are few things which can be noted from here. Firstly, m equal to 0, n equal to 0; if I substitute into these equations, the E_z will be identically 0. What that means is that TM_{00} mode cannot get excited inside a rectangular wave guide.

We have seen for a parallel plain wave guide that the TM_0 mode which was same as the transverse electromagnetic mode was possible in a parallel plain wave guide and that was the mode which was the lowest order mode. However for this mode, we note that when both the indices are 0, the field goes to 0. So, TM_{00} does not exist.

Also we will note, when any of the indices go to 0, then also again this field goes to 0; H_z is already 0 for transverse magnetic. If E_z also goes to 0, then all the fields will identically go to 0 because h is not equal to 0 that is what we have to show.

So first, let us show that for this mode, h is not equal to 0 if I go back to the wave equation and substitute now this quantity from this equation. This quantity as we have defined as minus A square, this quantity we defined as minus B square.

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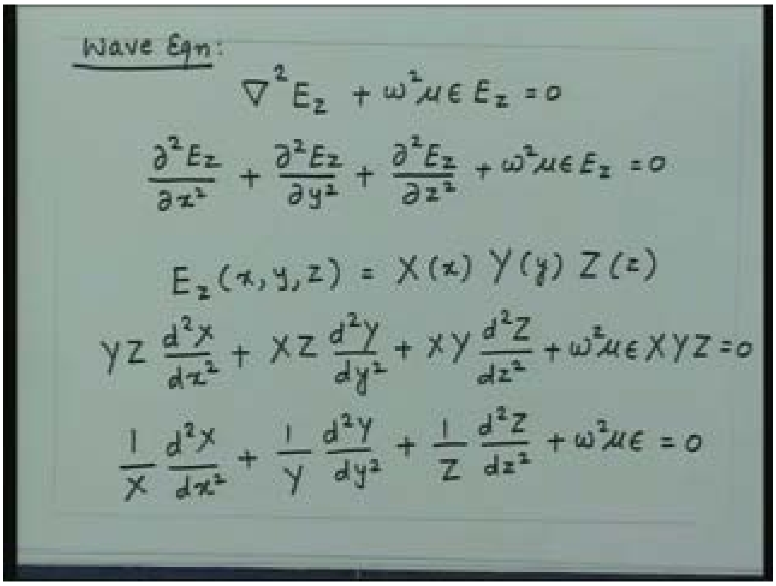

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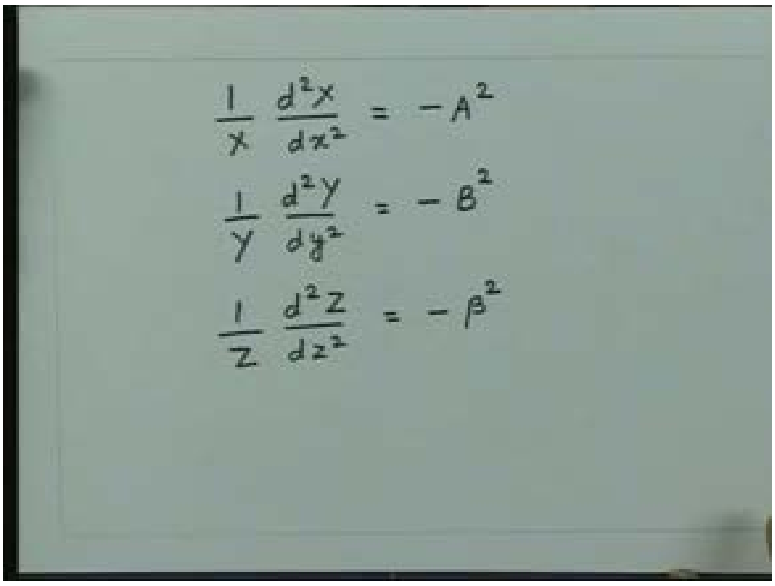

Here, this quantity is minus A square, this quantity is minus B square and this quantity is minus beta square and we have found out now this value of A which is m pi by a, this quantity is n pi by b.

So, you can go back and substitute into this to get a relation which is minus m pi by a whole square minus n pi by b whole square minus beta square plus omega square mu epsilon is equal to 0.

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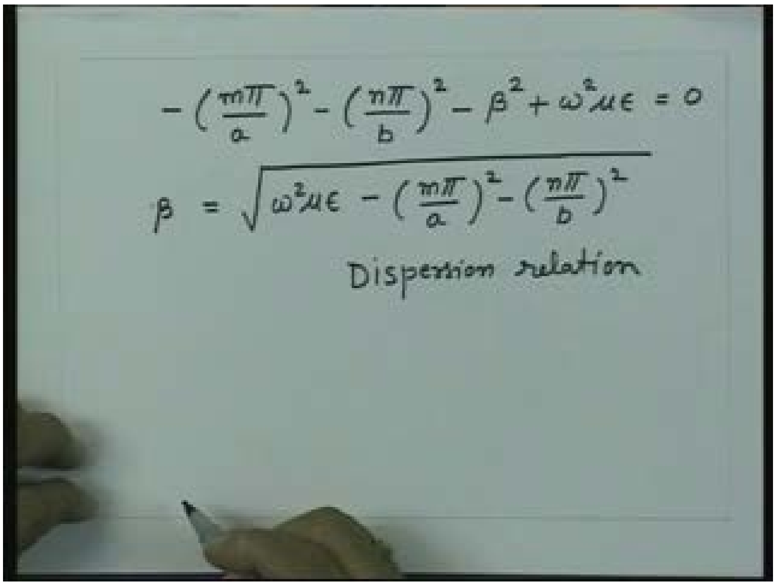

I can take this beta on one side, so the phase constant for this mode will be equal to square root of omega square mu epsilon minus m pi by a whole square minus n pi by b whole square.

Now, this relation is familiar, the similar relation we had seen for the parallel plain wave guide with only one term which are omega square mu epsilon minus m pi by d where d was the height of the wave guide and this expression essentially tells how the velocity or the phase constant varies as a function of frequency on this structure.

So, this expression is identical to what we have derived for parallel plain wave guide and that is why this is the dispersion relation for the transverse magnetic mode on a rectangular wave guide. So, we can call this as the dispersion relation that we saw in the last lecture. So, this is the dispersion relation for TM_{mn} mode where m and n are the two indices which are non-zero.

Now, if I take m and n both equal to 0, we saw the fields are not existing. But if I take m and n, one of them is 0.

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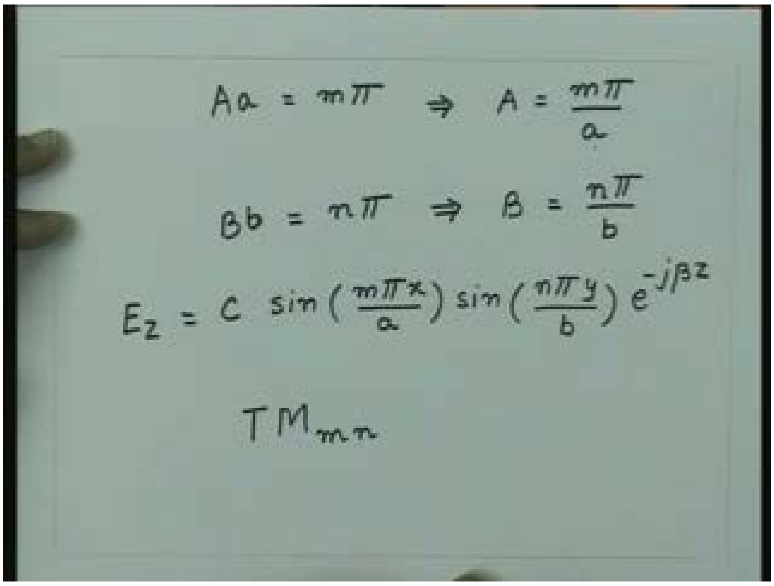

Then, this quantity now, this is h; what is h square now? The h square was defined as omega square minus beta square in our general analysis.

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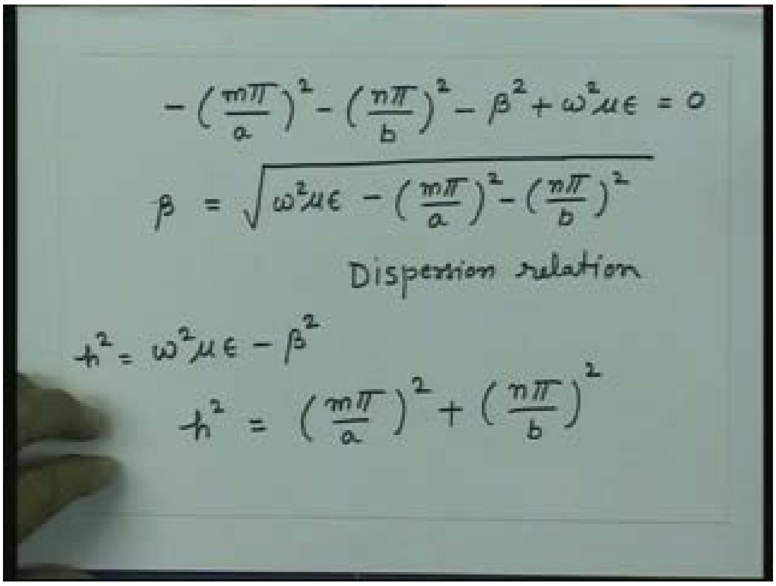

So, we had defined this quantity h square was omega square mu epsilon minus beta square. So by that, comparing this with this, we get essentially for this case; h square is your m pi by a whole square plus n pi by b whole square. So, when m and n both are not 0, the h square is not zero.

So, when the h square is not zero, the field which we have transverse fields which are represented in terms of the longitudinal components, we have seen the transverse component would exist if both of E_z and H_z are 0, provided H is also equal to 0.

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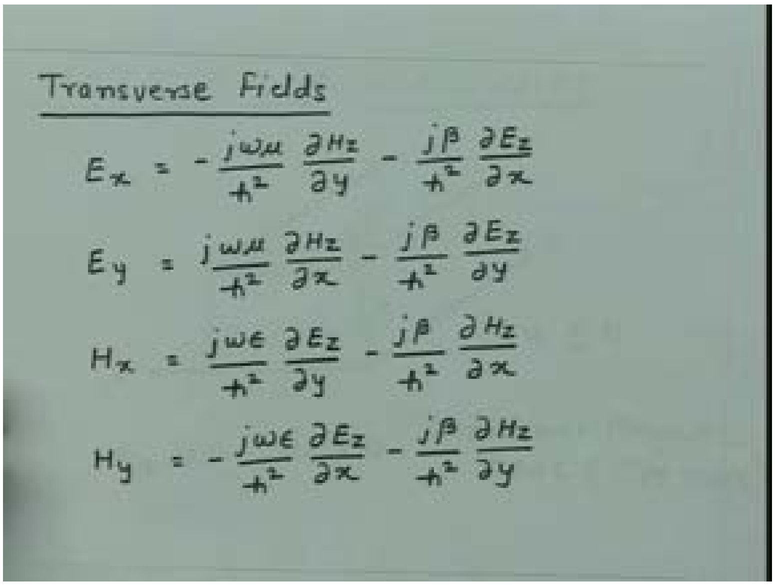

However, now we are seeing that h is not equal to 0 when m, either m or n are not equal to 0 and then if H_z and E_z both go to 0, all transverse field would go to 0 and the mode would not exist. So, we see that for the transverse magnetic case, when either m or n are 0 or any of these two is 0, then again this field will be 0 and since h is not 0, again the transverse field will go to 0 and the modes will not exist.

So now, we have some important conclusion drawn from this analysis for transverse magnetic mode. So, we have a mode TM_{mn} mode.

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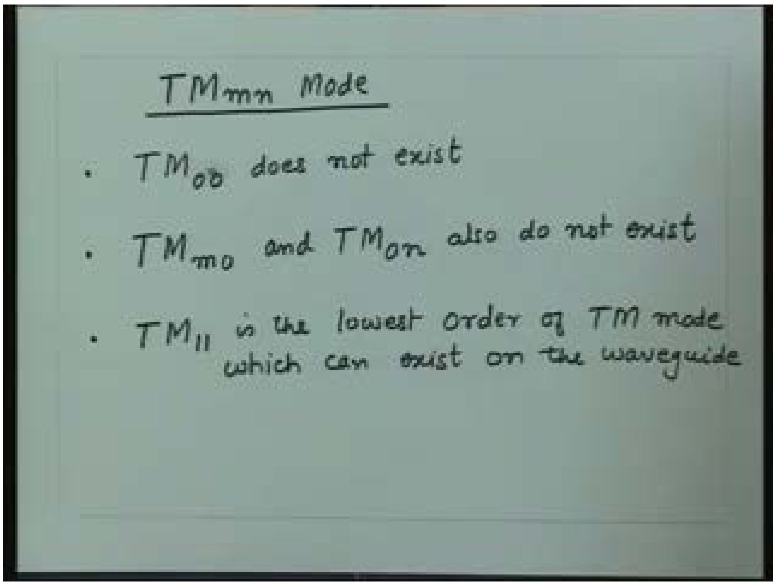

So, the first conclusion which we have is TM_{00} does not exist. So, we also conclude that TM_{m0} and TM_{0n} also do not exist. So, the lowest mode which can exist, the lowest index that is when both these indices are non-zero, that means TM_{11} is the lowest order of TM mode which can exist on the waveguide.

So, by doing this general analysis, we come to the very important conclusions that if the TM mode has to be excited on a rectangular waveguide, the fields must vary in both the directions x and y on the rectangular waveguide and that we can see physically as follows. Since the index m and n is telling the variation, one half cycle variation along the x and y direction respectively; when m is 0, there is no variation of the field along x direction and when n is 0, there is no variation of the field along y direction.

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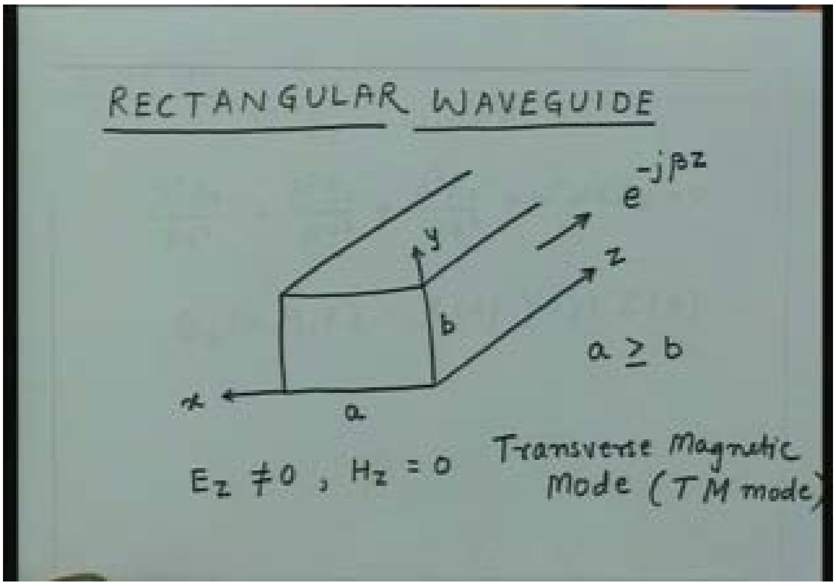

But the field has to be 0 here, this is E_z component; so it has to be 0 here, it has to be 0 here and there should not be any variation in the x direction if m is equal to 0. That is only possible provided the field is identically 0 everywhere; the same is true for these two boundaries that when n is equal to 0, the field should not have any variation in the y direction but it should be 0 here, it should be 0 here and that again can happen when the field is identically 0 everywhere.

So, physically it makes sense that yes, this field E_z cannot exist inside this without a variation in x and y direction, it must vary. Also, we assume from the solution in this Cartesian coordinate that the variation is always sinusoidal and it is always number of half cycles; you can have one half cycle or two half cycles or three half cycles and so on.

So, essentially we get now the variation in this E_z is always 0 on these two and depending upon the value on m and n, you will have the variation in x and y direction which is sinusoidal variation. So, the field in the transverse direction always varies sinusoidally in the Cartesian coordinate system and they must vary because if they do not vary in any of the

Of course, your conclusion might be different when we go to a transverse electric case but these are the conclusion which we can draw now for a transverse magnetic case. Once we get now this E_z, then we can substitute for E_z into these components and then we can write down the transverse components for the TM mode. So, you can substitute now the general solution which we got and now if I write for Ex, H_z is 0; so this term is 0, this term is 0.

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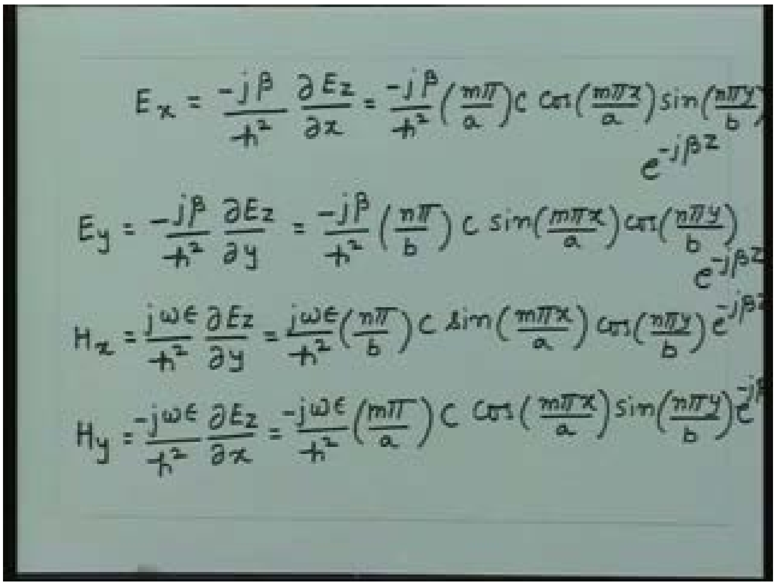

So, I get now E_x which is equal to minus j beta upon h square where h square is already have been calculated; this is d E_z by dx. So, that will be equal to minus j beta upon h square m pi by a into that arbitrary constant C cos m pi x by a sin n pi y by b and multiplied by e to the power minus j beta z. Same thing I can do for E_y that is equal to minus j beta upon h square d E_z by dy which will be equal to minus j beta upon h square n pi by b C sin of m pi x by a cos n pi y by b and put this phase term e to the power minus j beta z.

The magnetic fields on the same lines, we can get as j omega epsilon upon h square d E_z by dy that is equal to j omega epsilon upon h square n pi by b C sin of m pi x by a cos of n pi y by b e to the power minus j beta z and H_y will be equal to minus j omega epsilon upon h square dE_z by dx that is minus j omega epsilon upon h square m pi by a C like this cosine of m pi x by a sin of n pi y by b e to the power minus j beta z.

So now, we got the complete fields for the transverse magnetic modes with the understanding that the lowest order mode which is going to propagate on this structure will be the TM $_{11}$ mode. So, few things can be seen from here and that is the E_z component which is for which we obtained the solution. That essentially is 0 at these 4 walls it is there in the sinusoidal variation, so we had a E_z variation which is this.

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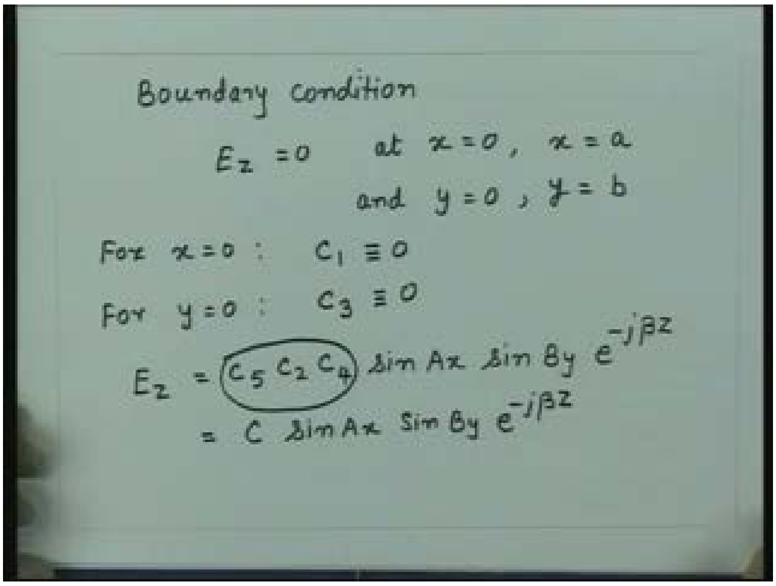

So, the E_z was 0 x direction at x equal to 0, at both the walls, the E_z was 0, E_z in y direction also having half cycle variations.

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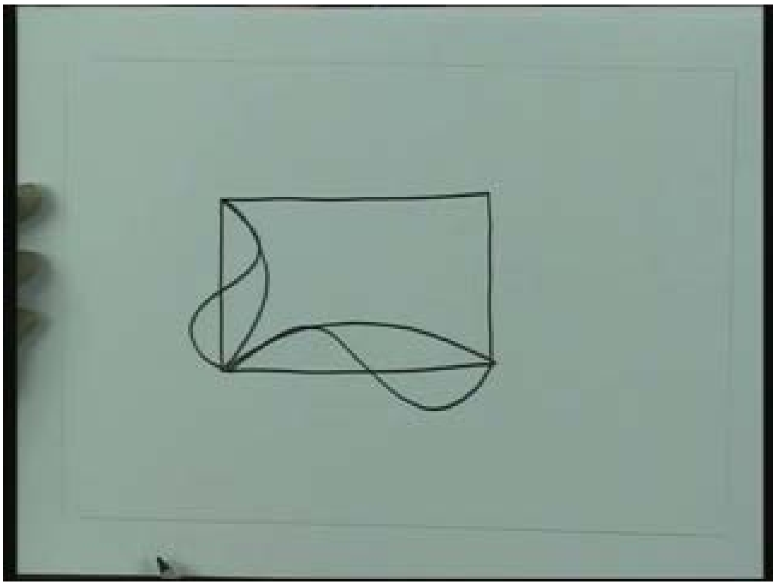

If I look at the cross section like that, the wave guide; the E_z is 0 here, 0 here, 0 here. So, it had half cycle variation like that or it could have a variation which will be like that, the same is true for half cycle variation in this direction or it could be like that. So, the E_z is 0 at this point and the maximum here if m is equal to 1, maximum here along this if n is equal to 1 and so on.

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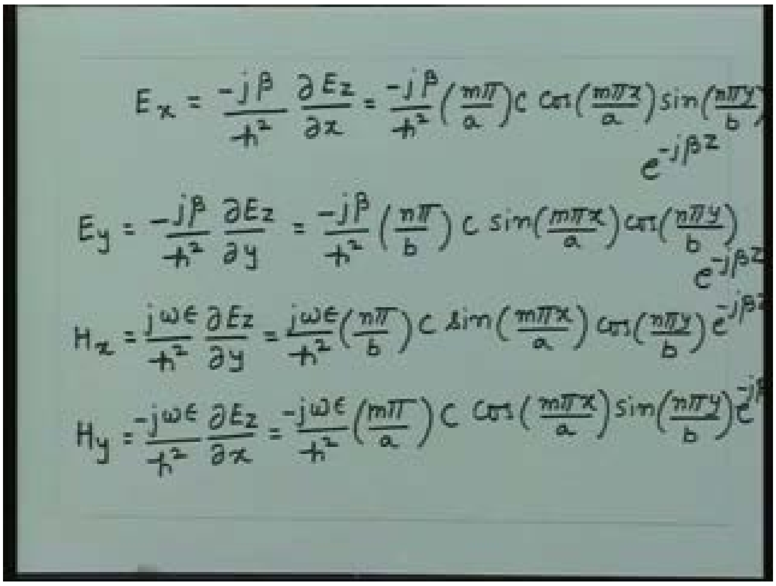

However, if I look at this field now which I have got here, E_x E_y H_x H_y, the E_x is having a variation which is cosine variation as a function of x. So, the component E_x which is this, x component which is oriented this way, it is having a variation cosine variation along x direction. That means it is maximum here, 0 here and another maximum here; again it is half cycle but it is not 0 here. In other words, it is shifted like a quarter cycle in space with respect to the E_z component. So, the E_x component if I look at; it will be like that.

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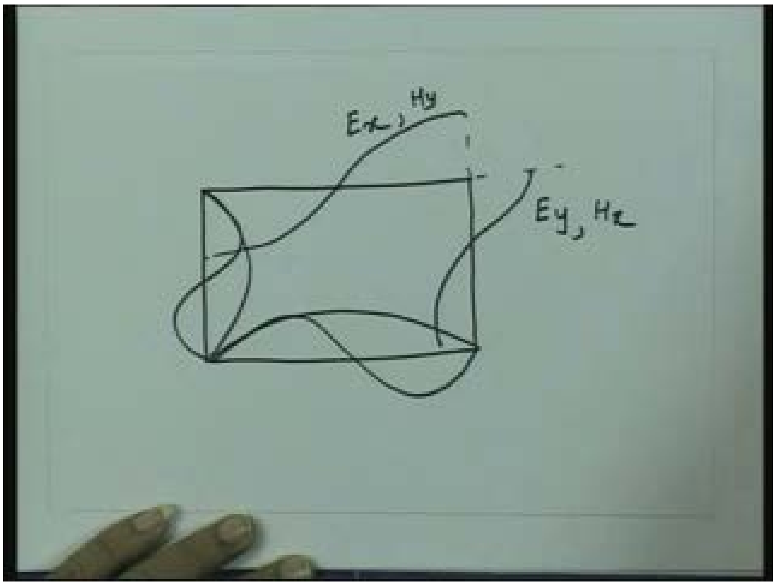

This will be the component which is E_x; so that is maximum here, 0 here, maximum here and H_y component if I look at, it will be again a cos variation along y direction. So, it will be in this direction; if I look at, the field variation would be like that for the E_y component.

So now, later on we will try to visualize these fields actually inside this pipe but at this point it appears that component E_x E_y H_x H_y depending upon the boundary conditions, they will be staggered in space in the transverse plain with respect to the longitudinal component which is the E_z component.

Once we get this understanding that the electric field which is like that is maximum on this boundary and if I look at the magnetic field which is let us say I take x oriented magnetic field which is having a variation which is sin variation in the x direction and cos variation in y direction; so if I consider now a magnetic field which is x oriented, then it is cos variation in y direction that means it is maximum here, 0 here, maximum here because E_x and H_y, they have the same behavior and E_y and H_x have the same behavior. By comparing the expressions, this is what essentially we see.

So, what we observe from here and off course from rigorous analysis also we can do that; but what we observe from here is that H_y component which is this is maximum on this wall and on this wall. That means when the magnetic field is tangential to the boundary, that is where it is maximum and off course it is having a variation which is sinusoidal variation with number of half cycle depending upon the order of the mode.

Similarly here, when I consider H_x which is going to be maximum here, maximum here; that means magnetic field is maximum when it becomes tangential to the conducting boundary and an electric field is 0 when it becomes tangential to the conducting boundary. Of course, the tangential magnetic field whatever is there on the boundary is balanced by the surface current, so it is a boundary condition on the tangential component of magnetic field.

So essentially, by using this observation that the magnetic field is maximum; tangential component of magnetic field is maximum on the conducting boundary and having understood that the solution gives me a sinusoidal variation in the transverse direction, now we can readily write the solution for the transverse electric mode without going to the same analysis. Of course in a routine way, we can do the same thing that if you want to analyze a transverse electric case, then we take E_z is equal to 0, we take H_z not equal to 0.

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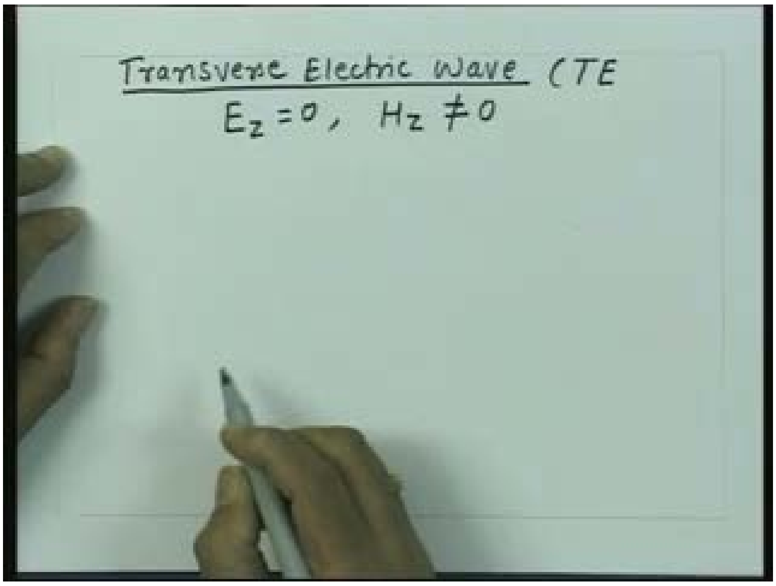

So, we consider a case which is transverse electric case or a TE wave.

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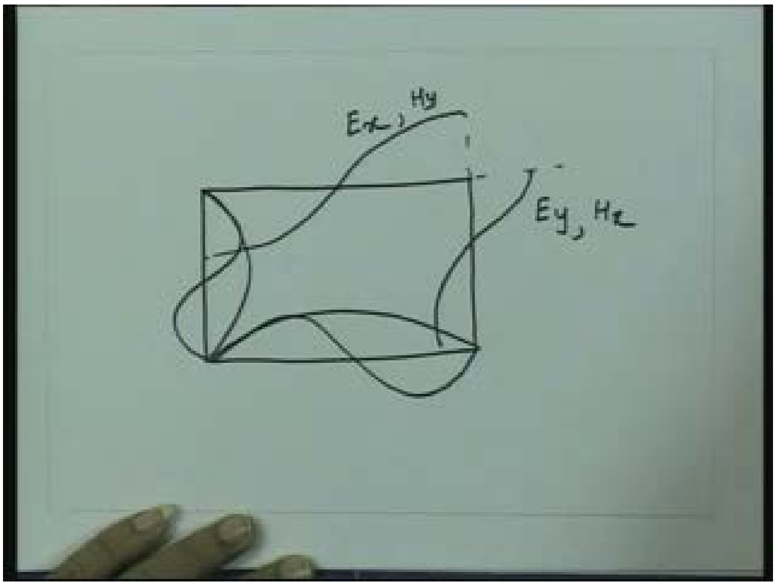

So in this case, the longitudinal component which we have is H_z which is like that and what is observed from this is that this component which is going to be tangential to this wall, this wall, this wall and this wall should be maximum on the walls and it should have a variation which is sinusoidal variation in the transverse direction.

So now, having understood all these analysis, we can write the solution without going to the same steps same algebraic steps which we have done, we can get the solution for H_z which will be some constant cosine of m pi x by a cosine of n pi y by b e to the power minus j beta z.

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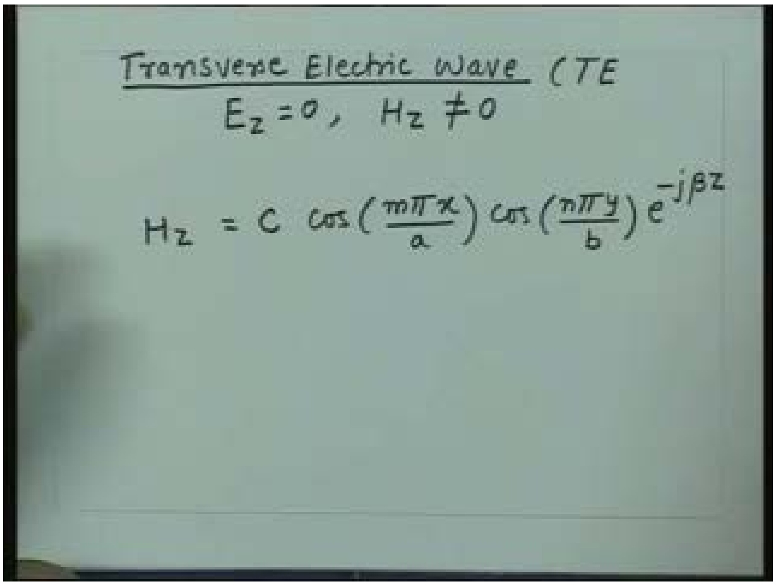

So, without repeating the algebra which we have gone through for solving the wave equation; of course, I can go exactly same way, I can solve the wave equation now for H_z, then apply the boundary conditions but note; directly on H_z, there was no boundary condition.

So, if I have to go from the same routine way, then I have to find out general solution for H_z, find out a transverse component then find out the component which are tangential to the boundaries and then apply the boundary conditions on those components because tangential component of magnetic field, there is no boundary condition. So, in this case, TE case, if I go by routine way, there will be two steps involved; first, find out a general solution for H_z, substitute into the transverse component expressions, find transverse components and then apply the boundary conditions on the transverse components because intrinsically there is no boundary condition on H_z.

However, without going to the same routine steps essentially the understanding which we developed at the magnetic field when become tangential to the conductor is maximum, I can get the expression for H_z and one can go and verify, verify that if I had done the analysis, essentially I would get the same expression.

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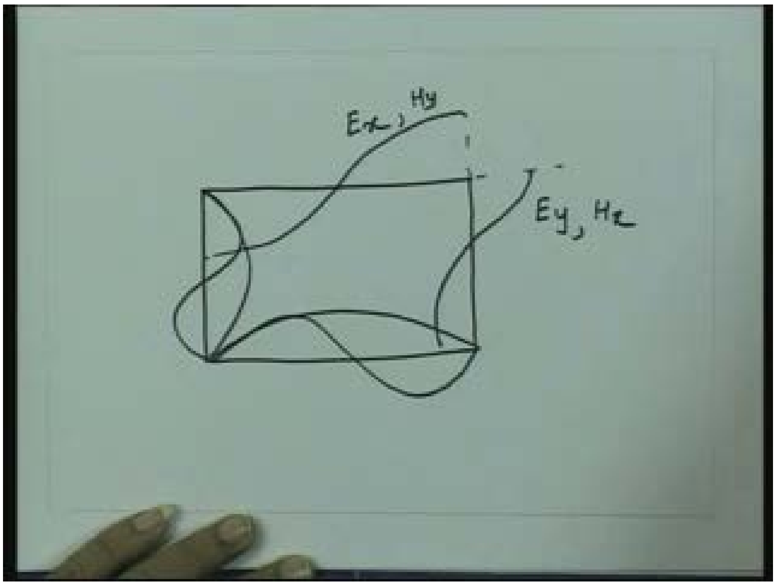

Now, this you can verify when x is equal to 0, the field is maximum; when x is equal to a, again the field is maximum; y is equal to 0 and again y equal to b, again the fields are maximum.

So, these fields are having the variation which essentially we are looking for; the tangential component of magnetic field must become maximum. Using this expression for H_z, now we will investigate the transverse electric mode and see the characteristics of transverse electric mode and then compare the behavior of this mode with the transverse magnetic field. So, we will continue the discussion on the transverse electric and transverse magnetic mode inside the rectangular wave guide and see more properties for these modes.