

**Transmission Lines & E M Waves**  
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**Lecture – 36**

We are investigating propagation of an electromagnetic wave inside a parallel plane waveguide. That is if you have two conducting boundaries which are parallel to each other, the way energy will propagate between these two conducting boundaries; that is the investigation we are carrying out. Last time we tried to launch a wave which had perpendicular polarization between these two conducting boundaries and as a result, we got a propagation which we called transverse electric propagation.

Then we also introduced the concept of mode that means the discrete electric and magnetic field patterns which propagate between these two conducting boundaries. Today we can ask a question - if instead of perpendicular polarization, if we had launch a uniform plane wave between these two conducting boundaries with parallel polarization; what kind of propagation will take place? Would the characteristics be identical to what we had in the previous case that was for perpendicular polarization? What are the differences between these two and so on.

So, in this case, essentially we can take a conducting boundary. Below this boundary, we have conductivity infinite and above this boundary, the conductivity is 0. So, there is a dielectric medium above this boundary. Again, we take the same coordinate system that the x is oriented upwards this is a direction z and the y is coming out of the plane of the paper.

Let us now launch a uniform plane wave again at an angle  $\theta$  with respect to the normal to the boundary which is X axis, this will be the reflected wave, again it a same angle just  $\theta$  but now we are considering the polarization which is the parallel polarization.

Now, since this is the transverse electromagnetic wave, uniform plane wave which is incident on the boundary, if the electric field vector lies in the plane of incidence, then the magnetic field vector would lie perpendicular to plane of incidence that is perpendicular to plane of the paper;

then without losing generality, we can say that we had assumed that the magnetic field is oriented coming upwards, coming out of the plane of the paper. So, let us say this is given like this, so this is the magnetic field and without losing generality, I can assume that this magnetic field also is coming upwards. So, this is the magnetic field; so this is the incident magnetic field, this is the reflected magnetic field.

Then using the pointing vector, we can find out the direction for the electric field. So, since the wave is coming this way and the  $H$  is coming upwards, the electric field vector should be this way; so it will be oriented in this direction. So, this is the electric field which is  $E_i$ . For this wave, since the wave is going in this direction to get a pointing vector this way, the electric field should be oriented in this direction. So, this is the direction of the reflected electric field  $E_r$ .

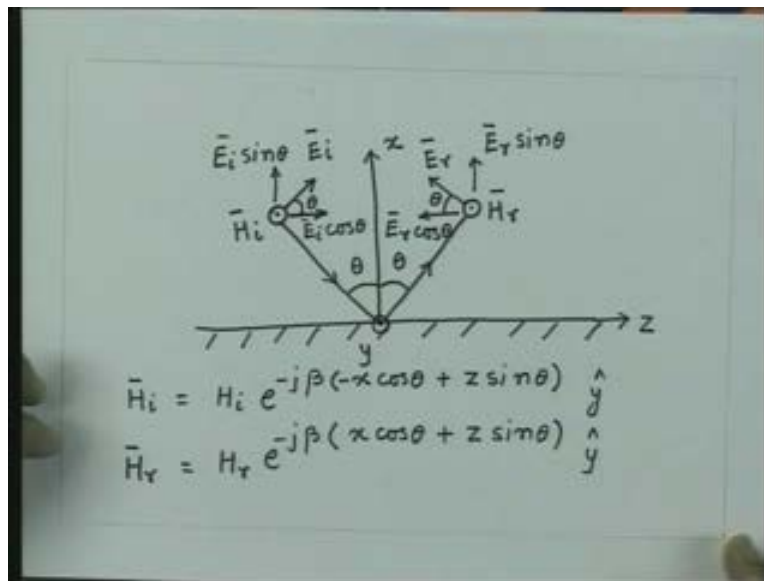
Then the problem is identical to what we had in the previous case; we just find out the components of the electric and magnetic fields and apply the boundary conditions. Now again, taking the two components of the electric field, this is tangential component and this is the normal component, same here, tangential and normal component; if this angle is  $\theta$ , this angle is also  $\theta$ . So, this angle is  $\theta$ , this angle is also  $\theta$ .

So, this component is  $E_i \cos \theta$  and this component will be  $E_i \sin \theta$ . Similarly, this component here will be  $E_r \cos \theta$  and this normal component will be  $E_r \sin \theta$ . Then I can apply the boundary condition that the tangential component of the electric fields should be 0 at the conducting boundary that means if I write down the field expressions as we did in the previous case and if I take  $x$  equal to 0 and apply the boundary condition at  $x$  equal to 0, the tangential component of the electric field should be equal to 0.

So, as we have done in the previous case, we can write down again the expression for the incident and the reflected wave for the electric and magnetic fields. So, in this case, you have  $H_i$  will be some amplitude  $H_i$  with the phase function  $e^{j(\omega t - \beta x \cos \theta + \beta z \sin \theta)}$  where  $\beta$  is the phase constant in this medium and this is the wave is coming this way, so with  $x$  axis as we saw last time, this makes an angle of  $\pi - \theta$ . So, that is  $x \cos \theta + z \sin \theta$  and this field is oriented in  $y$  direction, so we put unit vector here  $\hat{y}$ .

Same thing I can write down for the magnetic field also, for the reflected wave which is  $H_r$  that is the reflected wave amplitude  $e$  to the power minus  $j$  beta. Now, this wave makes an angle  $\theta$  with  $x$  axis, so direction cosine will be  $\cos$  of  $\theta$ . So, this will be  $x \cos \theta$  plus  $z \sin \theta$  in  $y$  direction.

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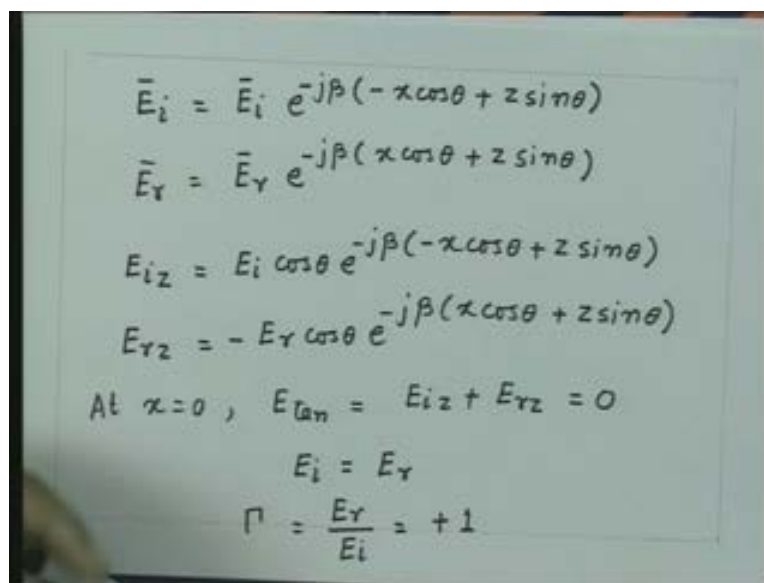
Same thing I can do for the electric field also and I can write this component and specifically since we are going to apply the boundary condition for the tangential component of the electric field, let us write down this component.

So first, the incident electric field  $E_i$  will be some amplitude  $E_i$ , then  $e$  to the power minus  $j$  beta minus  $x \cos \theta$  plus  $z \sin \theta$  and  $E_r$  will be amplitude  $E_r$   $e$  to the power - this quantity will be vector now because you have to take the component of this electric field in this two direction  $x$  and  $z$  - so this will be minus  $j$  beta  $x \cos \theta$  plus  $z \sin \theta$ . Then, taking the component of the incident electric field in the  $z$  direction will be  $E_i \cos$  of  $\theta$ , so I can get the  $z$  component  $E_{iz}$  that will be  $E_i \cos$  of  $\theta$   $e$  to the power minus  $j$  beta minus  $x \cos \theta$  plus  $z \sin \theta$  and the component in  $z$  direction for this electric field which will be opposite to positive  $z$  direction, so  $E_{rz}$  will be minus  $E_r \cos \theta$   $e$  to the power minus  $j$  beta  $x \cos \theta$  plus  $z \sin \theta$ .

Now, we apply the boundary condition on the electric field that is  $x$  equal to 0, some of these two electric fields should be 0, so that means at  $x$  equal to 0,  $E$  tangential which is  $E_{iz}$  plus  $E_{rz}$  should be equal to 0. See, if I substitute in this expression  $x$  equal to 0, this the phase condition  $e$  to the power minus  $j\beta z \sin \theta$  which is a common term both of this. So from here, I will get  $E_i$  is equal to  $E_r$ . What are the other words? The reflection coefficient which we have defines, which is the ratio of  $E_r$  and  $E_i$ , that quantity  $\Gamma$  will be  $E_r$  upon  $E_i$  that is equal to plus 1.

If you recall, when you had the perpendicular polarization, we had the reflection coefficient minus 1 and we said the time boundary is behaving like a short circuit because the reflection coefficient is minus 1. So, looking at the transmission line analogy of this particular configuration, we concluded that this conducting boundary behaves like a short circuit and that is why the reflection coefficient is minus 1.

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$$\begin{aligned}\bar{E}_i &= \bar{E}_i e^{-j\beta(-x \cos \theta + z \sin \theta)} \\ \bar{E}_r &= \bar{E}_r e^{-j\beta(x \cos \theta + z \sin \theta)} \\ E_{iz} &= E_i \cos \theta e^{-j\beta(-x \cos \theta + z \sin \theta)} \\ E_{rz} &= -E_r \cos \theta e^{-j\beta(x \cos \theta + z \sin \theta)} \\ \text{At } x=0, E_{\text{tan}} &= E_{iz} + E_{rz} = 0 \\ E_i &= E_r \\ \Gamma &= \frac{E_r}{E_i} = +1\end{aligned}$$

What is happening in this case? Here we get a reflection coefficient that is equal to plus 1. Does that mean that the boundary is now behaving for this polarization as an open circuit boundary? If you go from transmission line analogy, the reflection coefficient for plus 1 means open circuit condition. This is not however true because if you look at the wave, consider the electric and

magnetic fields, the direction of electric fields for incident and reflected wave are already opposite in direction. So, this reflection coefficient which you are getting minus 1 or plus 1, it essentially means that the direction of the reflected electric field is opposite compared to the incident electric field.

So, in the previous case, since we have considered the electric field which you are coming out of the paper or incident and reflected wave, we got reflection coefficient minus 1 that means this field is in opposite direction with respect to the incident field. However, in this case we have already taken a field to satisfy the pointing vector appropriately and they are in opposite direction. So, the negative sign has been observed already while defining the direction of the vector electric field and that is the reason why the reflection coefficient is appearing as plus 1.

So, one should keep in mind that since we are now dealing with vector quantities here, just looking at the sign of the reflection coefficient would not give you the correct idea what the boundary is. The boundary is still behaving like a short circuit, we have the conductivity is infinite here, so it is like, whatever fields come here, the voltage is 0 at this location, so the boundary is still a short circuited boundary. But you get the reflection coefficient plus 1 because the sign appropriate as observed into the direction of the electric field.

Once we get that and then using the relation that the magnetic fields are related to the electric field by the impedance of the medium, so we have  $E_i$  upon  $H_i$  is equal to  $\eta$  which is the ((... Refer Slide Time 14:01)) as the medium; same thing is true for reflected wave also, so  $E_r$  upon  $H_r$  equal to  $\eta$ . We can now write down the magnetic field and electric field in terms of this quantity  $E_i$  and  $E_i$  equal to  $E_r$  we can substitute and then find out total electric field and the magnetic fields.

See, if I do that, I get now the total electric field which is superposition of the incident and the reflected field in medium one and same is true for the magnetic field. So, the total fields we are going to get as the electric field which has two components; one is in x direction which is combination of these two and one in z direction which is combination of these two.

So, the electric field will have two components  $x$  and  $z$  and the magnetic field will have only one component that is  $y$  which will be oriented this way. See, if I do that and as I did in the previous case, I can combine these fields and I can get now the electric field for this  $E$ , so I can get  $x$  component of the electric field which will be  $2 \text{ times } E_i \cos \theta \sin \theta \cos \beta x \cos \theta$  and I will get the  $z$  component of the electric field  $E_z$  that is  $2 \text{ times } j E_i \sin \theta \cos \theta \sin \beta x \cos \theta$  and the magnetic field which will have only  $y$  component,  $H_y$  that will be  $2 \text{ times } E_i \sin \theta \cos \theta \sin \beta x \cos \theta$  and the condition which was satisfied for the perpendicular polarization, the same arguments are going to be true in this case also that is if I look at the electric field which is tangential, this quantity is going to be 0 at  $x$  equal to zero that is what we started with this boundary condition that the tangential component should be zero here.

However, the electric field tangential component will also be zero whenever this quantity is multiples of  $\pi$ . So, the condition as we had obtained for the perpendicular polarization case, we have identical condition in this case also that is this quantity, electric field  $z$  will be zero. So, we have  $E_z$  zero when  $\beta x \cos \theta$  is equal to  $m \pi$  where  $m$  equal to 0, 1, 2 and so on. And from here, we again find out the  $\theta$  that is the angle at which the parallel polarized wave can be launch inside the structure and those angles will be  $m \pi$  divided by  $\beta x$ .

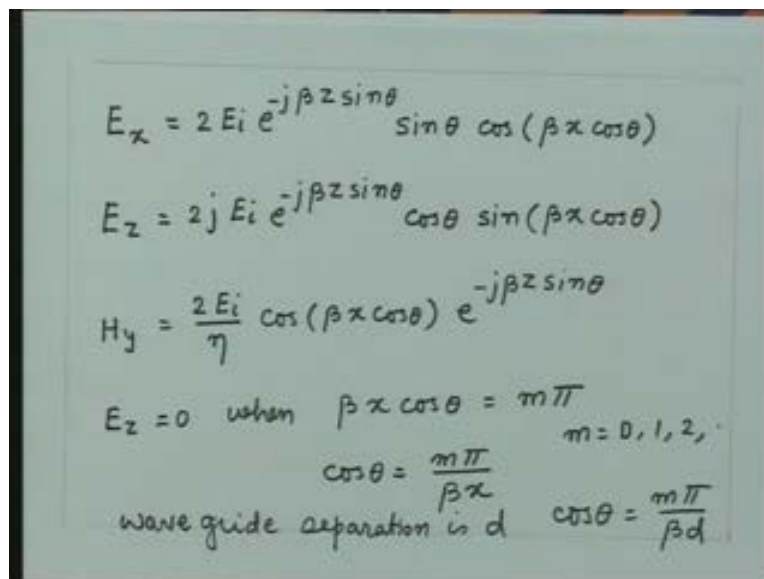
So, as we discussed in the previous case, if I have this  $x$  which is the separation between the two conducting plane given to you, so this is quantity  $SD$ ; then we can launch the wave at discrete angles which satisfy this condition. Then and then only the wave propagation will take place inside this parallel plane waveguide. So, whether we take a polarization which is perpendicular or whether we take a polarization which is parallel, the angles at which the wave can be launched inside the structure are same.

So, at a same discrete angle, you can launch a parallel polarized wave you can launch a wave which is perpendicularly polarized. So, all the argument which you had in the previous case that is you have finite number of angles at which the waves can be launched, you have minimum value of frequency which is required; all the arguments are now applicable to this field

configuration also. So, saying that the waveguide separation is  $d$ , we have the condition as we had in the previous case;  $\cos$  of  $\theta$  is equal to  $m\pi$  divided by  $\beta d$ .

So now, the field which we are going to get in this case, if I look at these fields now; the magnetic fields for this case is oriented this way, the electric field as a component which is either this or that and the field expressions indicate that these terms are representing the variation of the field in the  $x$  direction which is more like a standing wave kind of behavior and this term gives you a behavior which is a traveling wave behavior which is in  $z$  direction.

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$$E_x = 2 E_i e^{-j\beta z \sin\theta} \sin\theta \cos(\beta x \cos\theta)$$

$$E_z = 2 j E_i e^{-j\beta z \sin\theta} \cos\theta \sin(\beta x \cos\theta)$$

$$H_y = \frac{2 E_i}{\eta} \cos(\beta x \cos\theta) e^{-j\beta z \sin\theta}$$

$$E_z = 0 \text{ when } \beta x \cos\theta = m\pi \quad m = 0, 1, 2, \dots$$

$$\cos\theta = \frac{m\pi}{\beta x}$$

waveguide separation is  $d$        $\cos\theta = \frac{m\pi}{\beta d}$

So, in this case also, the fields would travel in direction  $z$  with a phase constant which is  $\beta \sin\theta$ . So, we have a field pattern generator in this case also between the two conducting boundaries which are going to travel with a phase constant  $\beta \sin\theta$ . Now, since the wave is traveling in the  $z$  direction and the magnetic field now is perpendicular to this direction of propagation, then it does not have any component along the direction of propagation, the magnetic field is always transversal to the direction of the net wave propagation.

So, magnetic field will always remain like that, the wave will propagate. So, the scenario is exactly identical to as we had in the previous case; we have a parallel plane waveguide, the uniform plane wave comes like that, like that, like that, like multiple reflection between the two conducting boundaries but the magnetic field now is oriented always this way, this is the direction of the magnetic field and the electric field will have two components because it is oriented this way. So, this is the electric field.

Now, since the magnetic field remains always transversed to the direction of net wave propagation which is this direction which is  $z$ , we designate this mode as the transverse magnetic mode. So, we have in this case, what is called transverse magnetic mode and in brief we denote that as the TM mode. Again, following the same convention that this quantity  $m$  defines the order of the mode and we saw this essentially gives me how many number of half cycles variation we have between the two conducting boundaries for a particular field, that defines the order of the mode. So, if  $m$  equal to zero, the fields are constant; if  $m$  equal to 1, there is one half cycle variation;  $m$  is equal to 2, 2 half cycle variations and so on.

The same arguments are true in this case also. So, we can put an index, as we put for T case, we can put an index here which is TM mode. So, for a given value of  $d$ , what value of  $m$  we have chosen to excite the fields, that is what will decide the order of the mode of this propagation. So now, what we see? We have two types of propagation inside the parallel plane waveguide; one is the transverse electric mode for which the electric field remains transversed to the direction of net wave propagation and we have transverse magnetic mode for which the magnetic field remains transversed to the direction of net wave propagation.

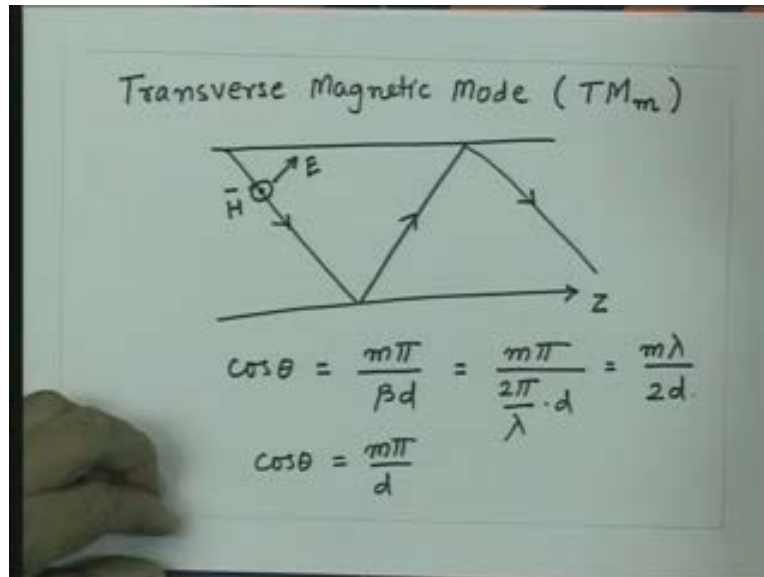
Up till now, this behavior is very identical, they have similar conditions satisfied for  $\theta$ , field expressions are different but otherwise principally nothing is changed; whether it is a transverse electric mode or transverse magnetic mode. So, this mode also will have same kind of cut of condition as we got in the previous case.

However, if I look at now this quantity, this field expressions here and now  $\beta \cos \theta$  which we get from here that will be equal to  $m \pi$  upon  $d$ . So, as we have done in the previous case, we



have this cos of theta that is equal to m pi divided by beta d and I can substitute for beta which is  $2\pi/\lambda$ . So, this is m pi divided by  $2\pi/\lambda \cdot d$ .

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So, that gives me cos of theta is  $m\lambda/2d$  or beta cos theta is m pi divided by d. Then we get with this also, m pi by d. So, I can substitute now for beta cos theta in this to get m pi by d into x, same here and same here. See, if I substitute explicitly, I can get now the fields for this transverse magnetic mode  $E_x$ , that will be 2 times  $E_i$  and just substituting now for beta cos theta into this into sin of theta cos of m pi x by d e to the power minus j beta z sin theta,  $E_z$  will be 2 times j  $E_i$  cos theta sin of m pi x by d, same term e to the power minus j beta z sin theta and  $H_y$  will be 2 times  $E_i$  upon eta cos of m pi x by d e to the power minus j eta z sin theta.

So, for a given dimension of the waveguide, now these are the field and of course cos theta, we can substitute again from here and once we know cos theta, we can get sin theta also. So, the mode which you are now having  $TM_m$  mode, it is suffix small m and that m essentially gives you the order of the mode that gives you a number of half cycle variations in the transverse direction that is in the x direction.

Now, in case of transverse electric case, we have seen that if I put  $m$  equal to 0, then all the fields identically go to 0. Then we concluded that the T in zero modes cannot exist inside a parallel plane waveguide. The same question we can ask in this case also that if I put  $n$  equal to 0 in this case, first thing we will note that if I put  $m$  equal to 0 in this, so if we take  $m$  equal to 0, then  $\cos \theta$  will be 0. So,  $\theta$  will be equal to 90 degrees. That means the wave now will be going, this angle will be 90 degree, so wave essentially going to go increasing to the conducting boundaries. So from here, you get  $\theta$  equal to  $\pi/2$ . So, that means  $\cos \theta$  is 0 and  $\sin \theta$  is 1.

See, if I substitute now  $m$  equal to 0 in this expression, this quantity is 1, this quantity again if I put  $m$  equal to 0, this quantity is 1, this quantity is 0 but this quantity is not 0, this quantity is again 1. So, in this situation, what I find that I have  $E_x$  that is 2 times  $E_i e^{-\alpha z}$  because  $\sin \theta$  is 1 and  $E_z$  is 0 because  $\cos \theta$  is 0. So, the magnetic field  $H_y$  will be 2 times  $E_i$  upon  $\eta$ . Again, this quantity is 1, so  $e^{-\alpha z}$  and the phase constant which you had in the case, which was in the  $z$  direction, now the phase constant is same as it is in the intrinsic medium which is filling this parallel plane waveguide that is the material which is filled between the two conducting boundary.

So, first thing to note at this point is that in this case when  $m$  goes to 0, all the fields identically do not go to zero. That means  $TM_0$  mode does exist. Then, in contrary to T zero mode,  $TM_0$  mode exist and that is a special mode which we will discuss little later.

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$$\begin{aligned}
 E_x &= 2 E_i \sin \theta \cos \left( \frac{m\pi x}{d} \right) e^{-j\beta z \sin \theta} \\
 E_z &= 2j E_i \cos \theta \sin \left( \frac{m\pi x}{d} \right) e^{-j\beta z \sin \theta} \\
 H_y &= \frac{2 E_i}{\eta} \cos \left( \frac{m\pi x}{d} \right) e^{-j\beta z \sin \theta}
 \end{aligned}$$

If we take  $m=0$ ,  $\theta = \pi/2$ ,  $\cos \theta = 0$ ,  $\sin \theta = 1$

$$\begin{aligned}
 E_x &= 2 E_i e^{-j\beta z} \\
 H_y &= \frac{2 E_i}{\eta} e^{-j\beta z}
 \end{aligned}$$

$TM_0$  will exist

But conclusion from here is that in this case, the  $TM_0$  mode will exist and then subsequently we will have  $TM_1$  mode,  $TM_2$  mode and all that. Now, the phase constant, since we are having this which is same for T and TM mode, essentially we have the relation that beta bar which is the phase constant in z direction, this is the contradictive which we defined last time, here is the phase constant in z direction that was equal to beta into sin theta. I can substitute for sin theta, so this is beta square root of 1 minus cos square theta.

Taking beta inside, this will be square root of beta square minus beta cos theta whole square but beta cos theta as you have got this case which is m pi by d, I can substitute here for m pi by d the whole square. So, whether I take a transverse electric case or transverse magnetic case, the phase constant for the net wave propagation which is in z direction is given by this. Now, when I put m equal to 0 in this case, this phase constant become equal to beta and for any other value of m which is non-zero, then the net phase constant will not be same as beta, it will be always different.

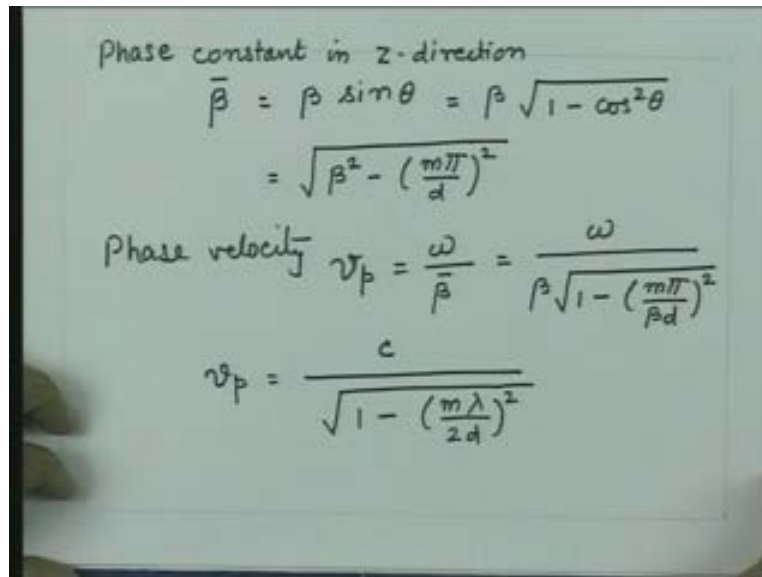
Now, once we know this value of the phase constant in the direction of the net wave propagation, then one can ask with what velocity this particular mode will be traveling. So, for a given value

of  $m$ , what is the velocity with which this modal pattern or the field pattern travel in the  $z$  direction and that we can either define by, as we said by group velocity or by phase velocity. If you want to define phase velocity, then we do the same thing; we take the phase combined with time and so from the first principle, we can find out what is the total phase which is the combination of phase in time and then make the phase dictionary as a function of time and we get a quantity what is called the phase velocity.

In this case, since the wave is traveling in the  $z$  direction and the phase is constant, so the phase velocity will be  $\omega$  divided by the phase constant in that direction. So, we get the phase velocity which will be in this case will be in  $z$  direction, phase velocity -  $V_p$ ; that will be  $\omega$  divided by  $\beta_z$  or that is the phase constant in  $z$  direction. So, this will be equal to  $\omega$  divided by square root of, quantity we can write here or we can use this expression, so this is  $\beta \sqrt{1 - \cos^2 \theta}$  and  $\cos \theta$  will be  $m \pi$  upon  $\beta d$ . So, this will be  $m \pi$  upon  $\beta d$  whole square.

Now,  $\beta$  is the phase constant in the medium filling the two conducting planes or that is the medium which is filling this parallel plane waveguide. So, this is the  $\omega$  upon  $\beta$  is nothing but the phase velocity of a wave in an unbound medium having same property as the medium filling with parallel plane waveguide. So, that velocity is has been denoted earlier; this is velocity of light or uniform plane wave in an unbound medium. So, we can output  $\omega$  upon  $\beta$  that is equal to  $c$ , so we have the phase velocity is  $c$  divided by this quantity and here  $\beta$ , I can substitute for  $2 \pi$  by  $\lambda$ , so I can get  $1 - \cos^2 \theta$ , so putting  $2 \pi$  by  $\lambda$  this will be  $m \lambda$  upon  $2 d$  whole square.

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Handwritten mathematical derivations on a whiteboard:

Phase constant in z-direction

$$\bar{\beta} = \beta \sin \theta = \beta \sqrt{1 - \cos^2 \theta}$$
$$= \sqrt{\beta^2 - \left(\frac{m\pi}{d}\right)^2}$$

Phase velocity  $v_p = \frac{\omega}{\bar{\beta}} = \frac{\omega}{\beta \sqrt{1 - \left(\frac{m\pi}{\beta d}\right)^2}}$

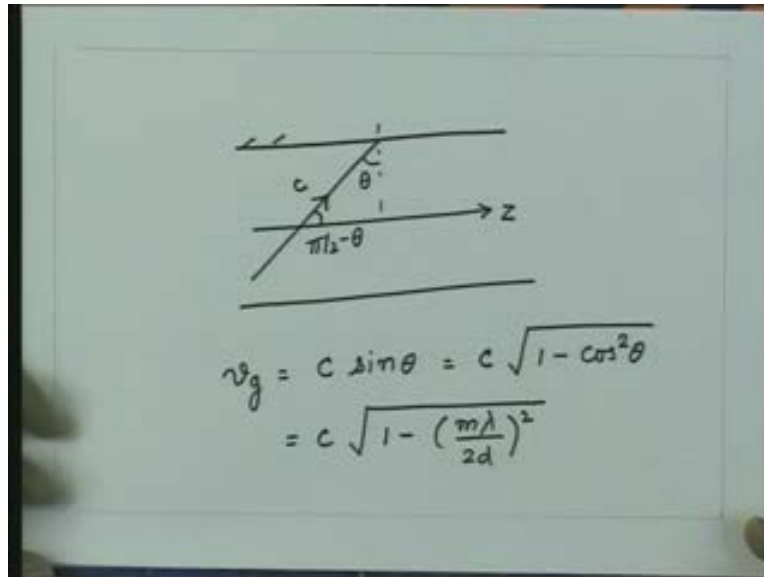
$$v_p = \frac{c}{\sqrt{1 - \left(\frac{m\lambda}{2d}\right)^2}}$$

For group velocity, either I can use the property that the product of phase in group velocity is equal to c square or I can say that in the parallel plane waveguide, if the wave is going at an angle like that, just theta; the component of this wave in this direction which is the z direction, that gives me the group velocity.

See, if I really take the velocity in this direction which is c and resolve that in this direction which is the z direction, I get the group velocity of the wave along the parallel plane waveguide. So, since this angle is theta, this angle is pi by 2 minus theta and velocity of this wave in this direction is c; so I will get c cos of pi by 2 minus theta, so c into sin theta. So, I will get the group velocity which is the component of the velocity in z direction that will be  $V_g$  equal to c into sin of theta.

Again, substituting for sin theta which is c square root of 1 minus cos square theta, I can get the expression for substituting for cos theta, I can get the expression for the group velocity which is c square root of 1 minus m lambda upon 2 d whole square.

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We can verify, as we discussed earlier, the phase velocity given by this and the group velocity is given by this. So, the product of the phase velocity and the group velocity is equal to  $c$  square which we had established earlier. So, I have mentioned, I could have found out the group velocity by using that property that the product of  $V_p$  and  $V_g$  should be equal to  $c$  square or as we have done here, we can resolve the velocity vector in the direction of wave propagation and that gives me the velocity of the energy which is the group velocity.

So, two things should be noted from these expressions and that is if I look at now this phase velocity expression, what we note is that if  $m$  is not equal to 0 and that is the case which you have discuss later that is the special case; but if  $m$  is not equal to 0, the phase velocity is a function of wavelength. So, for a given mode, when  $m$  is not equal to 0: as the frequency changes, the phase velocity of that particular mode changes.

Now, this property that the velocity of wave changes at the function of frequency is what is called dispersion. So, what we then find is that when you have a bound medium like this parallel plane waveguide, the structure has become a dispersive structure. That means when the

electromagnetic wave tries to move on the structure, the velocity becomes a function of frequency, a function of wavelength.

So, though the medium which you are considering, intrinsically the medium which is filling this waveguide is not dispersive; the conducting boundaries which we talked about, they are ideal conductors, so the energy is not propagating them. So, neither the boundaries where dispersive nor the medium which is filling the waveguide is dispersive. But when we put this finite region over which the wave is propagating, this bound medium becomes a dispersive medium.

So, first thing important to note here is when we have the bound structures; in general, we may expect dispersion on the structure. That means the velocity of wave, whatever form in the wave traverse form in the structure and as you have seen this traverse in the form of modes, there velocity varies as a function of frequency. So, this phenomenon is what is called dispersion and that is the very important thing to note that for a bound structure; in general we have dispersion, though the media which are involved in creating the bound structure intrinsically may not have any dispersion.

Then this relation which we got here that  $\beta$  which is the effective phase constant is related to this and the size of the waveguide and the mode index, this relation then is called the dispersion relation for a particular mode. So, this expression we call as dispersion relation. So, dispersion relation essentially tells you the variation of the velocity as a function of frequency or a function of wavelength and the conclusion is that whenever you having a bound structure; in general, the velocity will vary as a function of frequency.

Now when this quantity; for that wavelength, when this quantity becomes equal to 1, that we defined as the cut of wavelength. So when this quantity, well  $\beta$  becomes equal to this, this will be 0 and the wave propagation will seem for a frequency lower than that, the wave propagation will not take place; for the frequency higher than that, the wave propagation will take place as we discussed yesterday.

We had now the cut off frequency concept above which the more propagation takes place. But if I come from the propagating side, as I approach to the cut off frequency that means when these two terms approach each other, the phase constant  $\beta$  becomes 0 or this expression, this quantity becomes equal to 1 as a cut off frequency, the phase velocity of the approaches infinity. So, what we conclude now that as we approach cut off frequency,  $V_p$  approaches infinity.

So, for a particular mode, you will have a cut off frequency and for that cut off frequency the phase velocity will approach infinity. At the same time when the phase velocity approaching 0 at the cut off, this quantity becomes 0. So, the group velocity approaches 0. So, that means at cut off, the energy flow ceases because there is group velocity is approaching 0 and as I go to frequencies which are very high compared to the cut off frequencies that means  $\lambda$ , now it has become very very small compared to the cut off, this quantity will be negligible.

Then the group velocity will approach to  $c$  - intrinsic velocity in the medium, the phase velocity also will approach  $c$  because this quantity will be negligible compared to this. So, when we go very far away or higher frequencies compared to the cut off frequencies, then both group and phase velocities would approach to the intrinsic velocity in that medium.

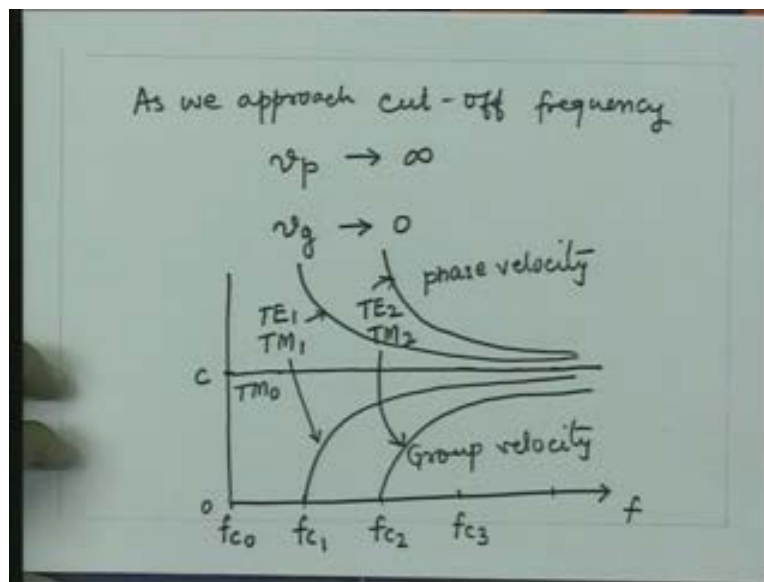
See, if I plot the group and phase velocities as a function of frequency, I have cut off frequencies for various modes; so, this is a cut off frequency for some more and this is the velocity, this is  $c$ . So, if I take a mode,  $TM_1$  mode; so this is the cut off frequency for  $TM_1$  mode, so let us call it the  $fc_1$  mode, this is for  $fc$  second mode, this is  $fc$  third mode and so on. And, for  $m$  equal to 0, as you have seen when this quantity is 0, there is a cut off frequency for this. So, this is the cut off frequency for  $fc$  zero mode which is only true for TM because  $TE_0$  does not exist.

So, for when  $m$  equal to 0; the phase velocity is always equal to  $c$ , the group velocity also is always equal to  $c$ . So, this is the line which you will always get for  $TM_0$ . Whereas, if I go for  $m$  equal to 1, I have two possibilities; I have  $TE_1$  mode or  $TM_1$  mode, both will have the same cut off frequencies and then at this cut off frequency, the phase velocity will go to infinity group velocity will go to 0 and as the frequency becomes much larger compared to the cut off frequency, the velocity will tend to  $c$ .



So, I will get a typical plot which will look something like this which will start from 0. This is the group velocity which is always below  $c$ , the phase velocity is always above  $c$  and here we have phase velocity and here we have group velocity. This is for... this is the case for  $TE_1$   $TM_1$  mode, these two. If I go to the next mode, then the cut off frequency at this will be like that, will start from here, go like that and so on. So, this will be the case for  $TE_2$  and  $TM_2$ , this one and this. So typically, if I get the phase and group velocity plot for different modes, the plot essentially would look like that and these are the cut off frequencies for the different modes.

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And, as you have seen for  $TM_0$  mode, there is no cut of frequency because when  $m$  is equal to 0, the  $\beta_z$  is always equal to  $\beta$  which is the phase constant in the intrinsic medium. Now, with this now, then let us go back to cover the special case which we were talking about and that is  $m$  equal to 0; and if I take  $m$  equal to 0, this case, we had got the fields which are now  $E_x$  and  $H_y$ ,  $H_z$  was 0 in this case and this special mode is the  $TM_0$  mode.

So, let us say specifically, talk about this mode  $TM_0$  mode; now since for this mode, this is the waveguide, the wave is now launched parallel to the interface because  $\theta$  is equal to 90 degrees and the electric field is  $x$  oriented that means the electric field for this wave is  $x$  oriented,

that is  $E$  and the magnetic field is  $y$  oriented that is this  $H$  and the wave is propagating in  $z$  direction with phase constant  $\beta$ . So, the net wave is propagating this way, this way with phase constant  $\beta$ .

We will also note from this that if I take a ratio of  $E_x$  and  $H_y$ , that quantity will be equal to  $\eta$ . So, for this mode, we also have  $E_x$  upon  $H_y$  that is equal to  $\eta$ . So, first thing we note is that this electric and magnetic fields which we have here, they are now perpendicular direction of propagation which is  $z$ . So,  $E$  is also transversed to  $z$ ,  $H$  is also transversed to  $z$ . So, though this mode we are calling as transverse magnetic mode with zero index; in fact this mode is same as the transverse electromagnetic case because here in this case, both electric and magnetic fields are transverse to the direction of wave propagation. So, this mode we also can call as a transverse electromagnetic mode.

Though this started with  $TM_0$  mode but essentially  $TM_0$  means electric and magnetic field both have become transversed. So, this mode is same as a transversed electromagnetic mode and for this mode, the ratio for the electric and magnetic field, intrinsic impedance of the medium which is filling the parallel plane waveguide; that way essentially we are having a transverse electromagnetic wave which is passing through these conducting planes. So, it have all the property which have uniform plane wave had unbound medium.

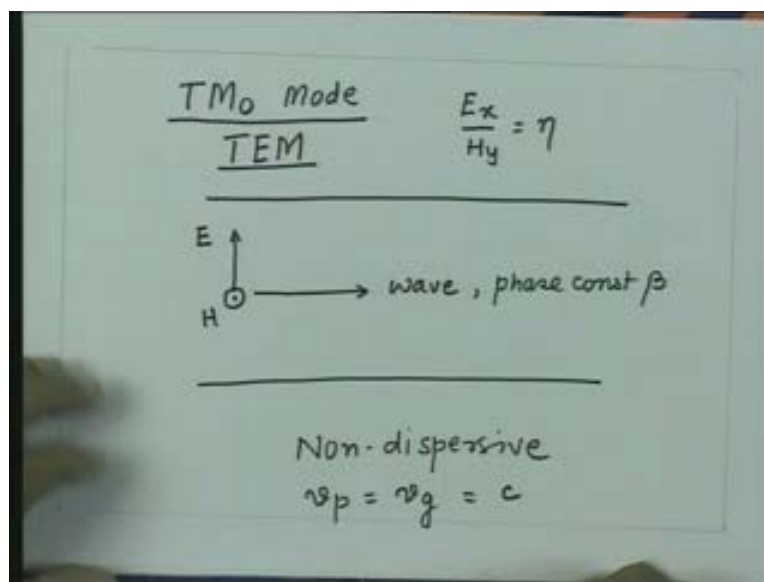
So, its behavior is exactly like a uniform plane wave in an unbound medium. One can wonder; when I am having this situation, aren't the boundaries affecting the wave propagation? What is the special about this case that the boundaries are just not existing for this mode? Where even if the boundaries were not there, the wave would have travel exactly like this in transverse electromagnetic case, it will be uniform plane wave. Here also we are having exactly like uniform plane wave and its characteristic like ratio of electric and magnetic field should be intrinsic impedance and so on or exactly identical. So, aren't boundaries playing any role?

And, if you look very carefully, you will see that yes, the boundaries are not plane any role and the reason is when the wave is launched now like this, the electric field is this way which is perpendicular to the boundaries and there is no boundary condition on normal component of

electric field if the normal component of electric field always can be balance by the surface charges on the conducting boundary.

Similarly, when I have a tangential component of magnetic field, I can always a surface current on the boundaries and I can have the magnetic field. So, I can have a uniform magnetic field, I can have uniform electric field and the wave passes through this parallel plane waveguide as if this boundaries are not modified the electric field because whatever electric and magnetic fields we have, they can reduce appropriately surface charges and surface currents and these field do not get modified.

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So essentially, this mode propagates in the parallel plane waveguide and since the  $m$  is equal to 0, this mode is non-dispersive that means for this mode, the velocity  $V_p$  is same as group velocity is equal to  $c$ , the cut off frequency for this mode is 0 as you have seen when  $m$  is equal to 0, the cut off frequency is 0 that means this mode can propagate down to the zero frequency. Precisely that is what we have seen; when you are having a two conducting system, any lowest possible frequency voltage can be apply to this and the energy can be transported. However, if you go to higher order mode, then you require a minimum frequency for transporting the energy.

So, what we find is that in a parallel plane waveguide, this  $TM_0$  mode which is also transfer electromagnetic and that is the mode which essentially propagates at the lowest frequencies but as we go to higher frequencies, we require higher order modes or we will get higher order modes in the energy propagation.