Transmission Lines & E M. Waves
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Lecture #35

Up till now we discuss the wave propagation in an unbound or semi bound media. We investigated specifically the refraction and reflection of electromagnetic wave across the dielectric boundaries. Lastly we also saw the reflection of an electromagnetic wave from a conducting boundary. Now we take a very important topic of electromagnetic waves and that is waveguides that the name suggests waveguide is a structure which can guide electromagnetic energy along it. We have variety of waveguides in practice. For example the coaxial cable which you are seen as a transmission line also the wave guiding structure because it can guide electromagnetic energy from 1 point to another the parallel wire transmission line is also a wave guiding structure. However as we go to higher frequencies, these waveguides have excessive loss. And that is way you get the waveguides which are like hollow pipes which are more efficient for transmitting energy at higher frequencies.

So when you go to the frequencies like gigahertz or 10s of gigahertz, the time the coaxial cable are the parallel wire transmission line though they are waveguide in structures, become more lossy and then we get the waveguides like rectangular waveguides which is like hollow pipe, whose cross section is rectangular or a circular pipe who cross section is circular. They can transmit energy from 1 point to another more efficiently. So in fact in communication the waveguide structure play the very important role. The communication engineer is always in search of would waveguide in structures. So that he can transfer energy at higher frequencies very efficiently from 1 point to another. So ideally it is waveguide structure should have as minimal loss is possible so that while transmitting the energy from 1 point to another, there is no loss of power.

Also these structures should have certain characteristic like they should be small in size; they should be compact; they should be moldable in the shapes and sizes if you want so you can carry the energy very easily from 1 point to another. Of course whenever you have a structure like this, we always start with the basics. That is, the Maxwell equations and solve the Maxwell equation for given boundary conditions and then we get the solution for the particular problem. However without going into that approach which is a routine kind of approach what we will try to do first we will try to understand, if you still confined our self the uniform plane wave propagation and try to capture this uniform plane wave between the boundaries, can we get a structure like a waveguide structure?

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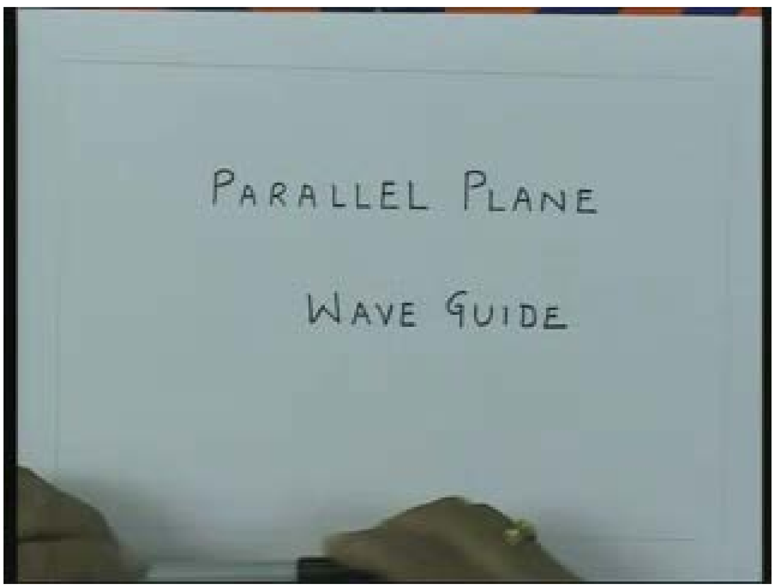

At the sort precisely we are going to do here that with the understanding which you have develop from the reflection of a plane a wave from a conducting boundary. We will try to develop an understanding for a waveguide what is called the parallel plane wave guide. As the name suggests this waveguide essentially consist of 2 parallel planes which are infinite extend and the electromagnetic energy is trapped between these 2 planes. So the energy essentially propagates along this planes trapped between 2 planes. So we now in this lecture we essentially discuss what are called the parallel plane waveguide.

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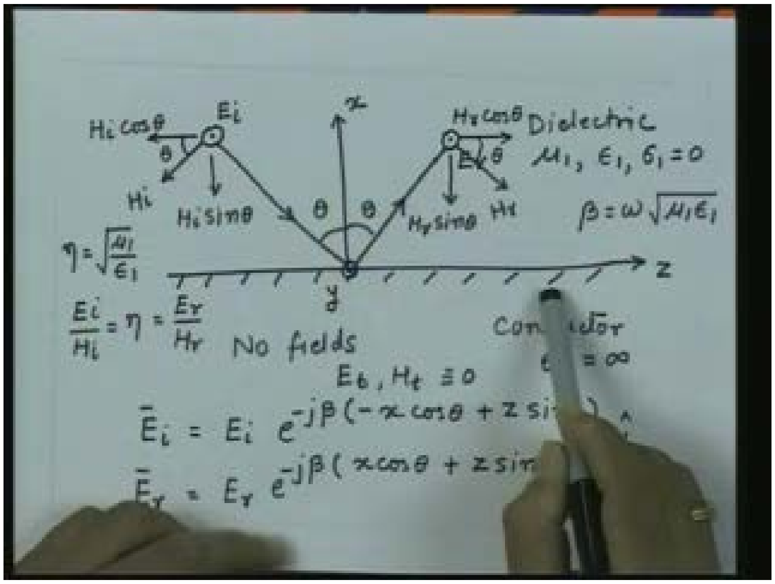

As we have seen in the previous lecture we have taken a conducting boundary and on which the uniform plane wave are incident at some angle theta and there is a reflected wave. And then by satisfying the boundary condition we saw the reflection coefficient for the electric field is minus 1. So this boundary was behaving more like a short circuit boundary and then the fields which you having in this media is superposition of the incident field and the reflected field.

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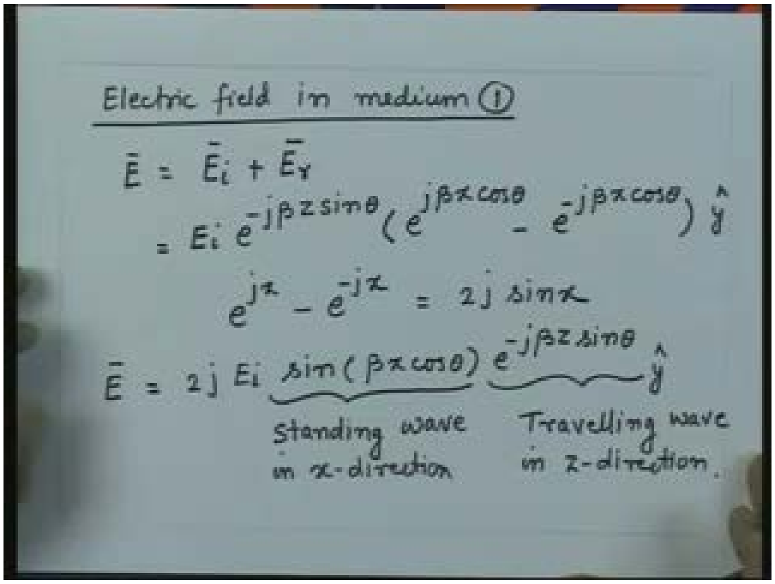

So by superposition of these 2 fields essentially we obtain the electric field in the medium above the conducting surface and that essentially was given by this. So we saw that it consists of 2 types of fields 1 is the standing wave kind of field which is in x direction. And there is a traveling wave of field which is in z direction. So we had seen there is a pattern whose amplitude varies the electric field amplitude varies as a semisolid function in the x direction that is perpendicular to the boundary and along the boundary you are having a traveling wave which is having a phase constant which is beta times sin theta. So this boundary as this demonstrates has a capability of guiding the wave along the z direction because we having a net traveling wave which is traveling along z direction.

Precisely that is the phenomena essentially we are going to make use of to get a more guiding structure. That means I will track the energy from both sides. Now at the moment the energy is not trap energy traveling into this semi infinite media we will try to trap the energy even from this side. So that energy propagates in the 2 planes something like this which we call as the parallel plane waveguide. And then we said that the electric field which is tangential to the interface in this case has to be 0 at the interface that means Zx equal to 0. The total electric field should be 0 and that is what we got this condition from.

So when beta x cos theta is 0 that means when x is equal to 0, the time the electric field goes to 0 since satisfy the boundary condition.

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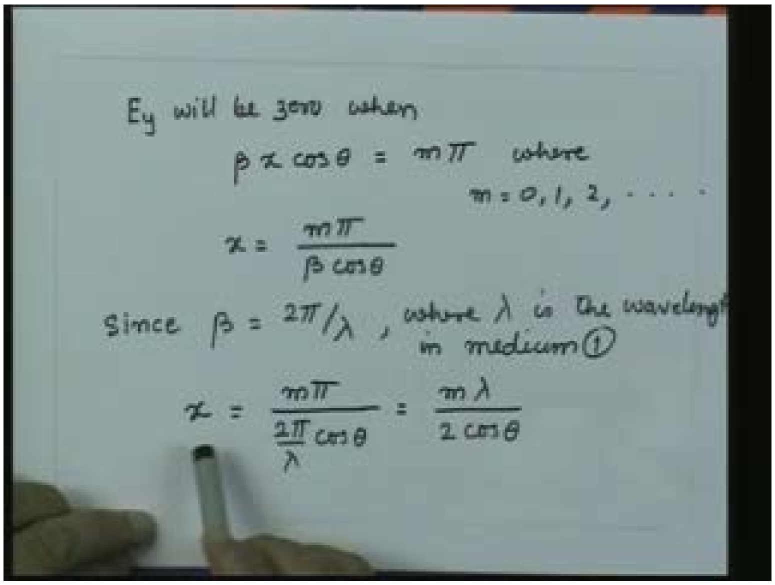

However what we also saw that only x equal to 0 but there are some other values of x also for which the tangential component of electric field will go to 0. So whenever we have a beta x cos theta is multiples of pi that m is an integer. So when beta is cos theta is equal to m pi or x equal to minus divide by beta cos theta for those values of x again the electric field would be 0 everywhere. And since the electric field is oriented like this perpendicular to the plane of the paper which is the y direction, if you put a conductive sheet here which is tangential to this electric field which is 0 at this point it will not affect the boundary conditions. Because the electric field is 0 here and boundary condition demand that this tangential component should be 0 at the conducting boundary.

That means at this value of x if I insert a conducting sheet of infinite extend like this the fields will remain unaffected. What that the means is that not only the value x equal to 0 where the condition will be satisfy, but I will have some another value of x another value of x another value of x at which if I insert a conducting boundary like this. These fields whatever we have got will remain unchanged I will get the same field distribution. So I substituting the value for beta which is 2 pi by lambda here lambda is the wavelength in medium 1. I can get this x which is m lambda divided by 2 times cos theta. So if I am equal to 0 or 1 or 2 or 3 I get different values of x at which this boundary can be inserted without affecting the electric or magnetic field distribution.

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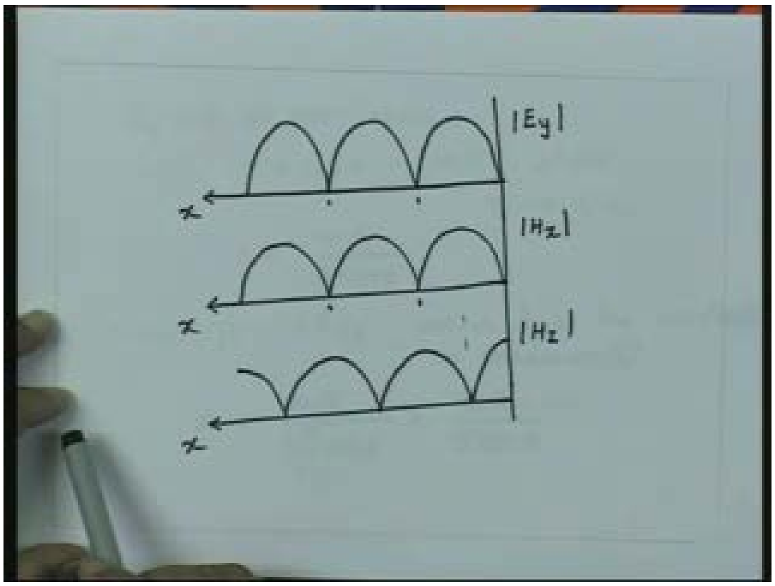

So these boundary locations essentially as we saw the plot of the magnitude of electric field it would go 0 here, it would go 0 here, it would go 0 here, here, here. So I can insert a boundary at this location this location this location and the field distribution will be mentioned. However by inserting a boundary at this distance, essentially now the wave is trap between these 2 boundaries. So I have created a structure which is more close structure it is no more a semi-infinite medium. Now it is become a bound medium in x direction. So this was 1 in interface this is another interface and now the field disturb between these 2 boundaries. Similarly I can put the plane here so I can get a boundary like this or I can get here. And now the energy electromagnetic energy is trap between these 2 parallel planes. Precisely that is the structure we are trying to investigate and the separation between this planes essentially is given by this condition.

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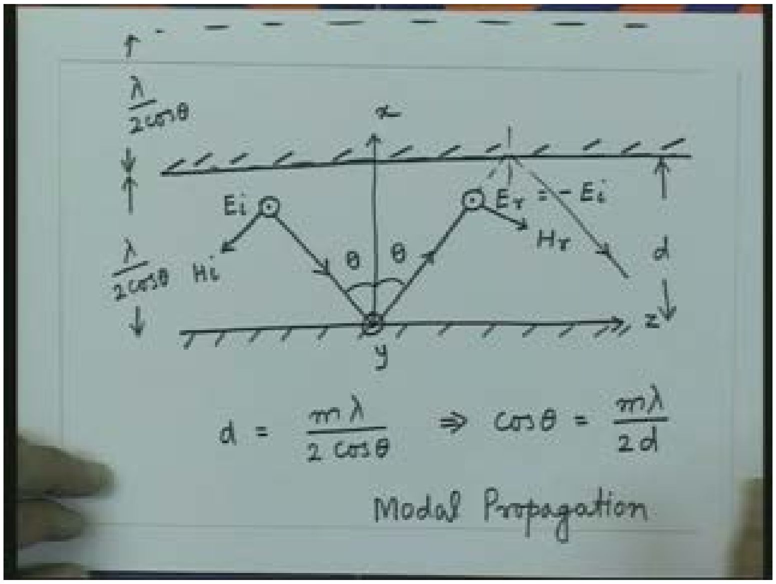

So let me redraw of the structure this is the original boundary which we had below which we have a conducting medium this is the x direction this is the z direction. The y-direction is perpendicular to the plane of the paper and let us say, the wave is incident at an angle which is theta so the reflected wave also will be at angle theta. And we are considered a case here where the polarization is perpendicular that means the electric field was this. So this is Ei and then we have got here Er which was equal to minus Ei because the reflection coefficient was minus 1. So that these this reflected electric field has same magnitude as the incident field only thing is its negative that means its direction is reverse. So if this field was coming upwards these fields essentially should be going downwards and then the magnetic fields for this where this was Hi and this is Hr. And for these term we had write down the components of the electric field or magnetic field.

So we head 3 components for this case the electric field which was y oriented and the magnetic field which will be either z oriented or x oriented. So magnetic field now lies in the plane which is x-z plane it has 2 components and the electric field is y oriented which is perpendicular to the plane of incidents. Thus the situation essentially we are considering. And now we have got this condition here that x is equal to m lambda upon 2 cos theta at that location again the tangential field will go to 0. So we will have 1 location here where field will go to 0. I may have 1 location here or I may get next location where there which will be double of this and so on. So this distance is lambda upon 2 cos theta from here to here where this field will again go to 0 or if I go to double of that it will be lambda upon cos theta. So this distance again lambda upon 2 cos theta this distance.

So this electric field which is tangential to this interface is 0 here, it is 0 here, it is 0 here and so on. So in fact I have multiple locations of planes where this component tangential component of electric field will go to 0. So what we are saying now is that if the introduce a plane at this location then the field distribution will remain unaffected. However only thing which is going to happen is that when this planes for not there the fields are distributed everywhere in the semi-infinite space. However when I introduce now a boundary at this location the fields are only confined to this region, we do not know what is going to be in this. So essentially we will have propagation of electromagnetic wave only in this region or in other wards 1 can save that the wave which was coming here at an angle theta, it goes and 1 it goes here it reaches this boundary again it will be making an angle which is theta with normal.

So we have gets reflected from here that is same angle. So it will appear as if the wave is moving coming here getting reflected, again getting reflected, when it reaches here again it is the boundary, then reflected. So the fields which are going to survive in this structure can be visualizes as multiple reflections of uniform plane wave. So wave simply bouncing back and forth between these 2 planes which are conducting boundaries. So I conducting boundary here now between these 2 boundaries by multiple reflections the wave by this exact pair is going to travel along the structure for along this planes which are conducting planes. And the height over the separation of these planes is given by this. What now 1 get do is, 1 can invert the problem and 1 can say well here knowing the angle theta we could find out what should location of these planes. What should be separation between these planes, so that this field remains unaffected? If I invert this relation say well separation between the places is given and let us say that is equal to D. Then what are the angles at which if the wave is launch the propagation will be sustained.

That means it will satisfy the boundary condition and the wave will propagate by this multiple reflections around the boundary. See if I say that I have 2 conducting boundaries now and let us say separation between them is given by d. Then this d should be equal to m lambda divided by 2 times cos theta. Or if I invert this expression I will essentially get cos of theta that is equal to m lambda by 2 d. Now this is very interesting because m is a discrete number it is 1, 2, 3, 4, so it is the integers. So this quantity is discrete, what that means is that for given value of d that means for given separation between this conducting boundaries? This angle theta is no more continuous were launch at any arbitrary angle will not sustain in this structure. Only if the, it is launch at particular angles which are discrete, because m is discrete, then only you can have a sustain propagation of electromagnetic wave in this structure.

Now this is a very significant departure from our original understanding of wave propagation. Up till now when we talked about unbound media there was no restriction on the launching angles. We could launch ray in any direction. When we had a semi infinite medium even then we could have any angle of incidence and we had some field distribution in the 1 medium or both media depending upon what is media wave. However now what we have saying is if we make the structure bound in this direction then I cannot launch an electromagnetic wave at an arbitrary angle of incidence. If I do that this structure is going to reject that way it is not going to except that way. If the wave is launch at these angles which satisfy this condition which are discrete now then and then only you can have a sustain propagation of electromagnetic wave. This behavior of the bounds structure is capture by what is called the model propagation. So what we are saying is that is, the wave which can be launch at discrete angles.

Superposition of this incident and reflected wave they are going to create certain electric and magnetic patterns in this. And you having a unique pattern created for electric and magnetic field for a given value of m. Say m equal to 0 and get 1 pattern is m is equal to 1 I will get another pattern if m equal to 2 I will get another pattern and so on. So we have this discrete electric and magnetic field patterns which you can survive inside this bounds structure. That is what is called the model propagation of electromagnetic wave.

So we have migrated now from the continuum domain of theta to discrete domain and that is the significant departure which you have done in the wave of propagation. And then these discrete angles are going to create discrete electric and magnetic field patterns which we call as the modals patterns and this propagation in this bound media is called the model propagation.

There are few things which we can which we observe at this point. One is since for physical angle theta the cos theta has to be between 0 and 1 for a given value of d I have only limited number of m's. For example I can put m equal to 1 I can put m equal to 0 I can put m equal to 1, 2, 3 and whenever this quantity becomes greater than 1 that does not represent the physical angle theta. That means not only that the waves can be launch inside this structure, a discrete angle. But you also not that this discrete angles are finite number. That means for a given spacing between the boundaries only finite number of uniform plane wave can be launch at discrete angles which are given by this.

Or in other words this is going to create discrete finite number of electric and magnetic field patterns inside the bound structure which is what is called a parallel plane waveguide. So this model propagation essentially is the core of all waveguide structure whether you take a parallel plane waveguide or you take a rectangular waveguide or you take optical fibers or dielectric waveguides. Whenever we have a bound medium like this the electromagnetic energy is going to travel in definite patterns. And that is what is called a model pattern and that propagation is model propagation. So model propagation is a very important aspect of electromagnetic wave of propagation in a bound structure which is called a waveguide. Once we understand this then of course finding out the electric and magnetic fields that analysis becomes straight forward.

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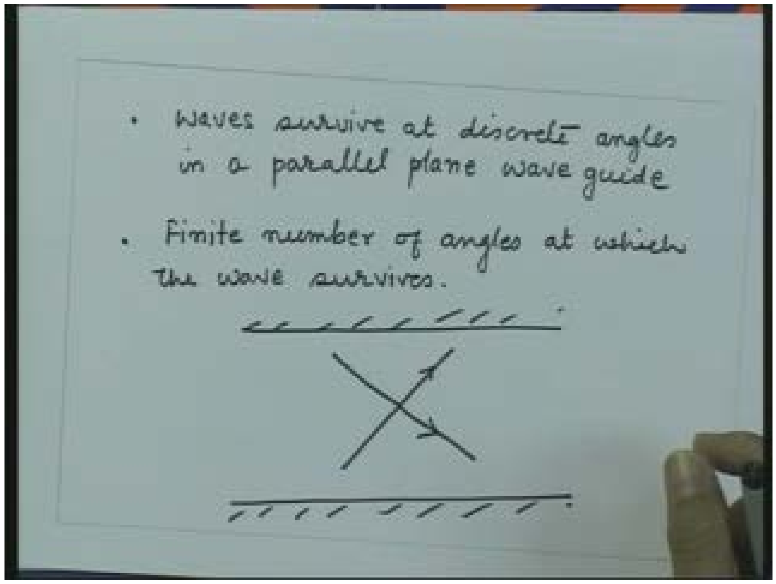

So let me right down the conclusion which we got from here that 1 wave survive at discrete angles in a parallel plane waveguide and there of finite number of angles at which the wave survives. Now in the original discussion we say this is the incident wave and this was the reflected wave. However when we have a structure like this which is a parallel plane structure this way which is reflected from here goes and meets this boundary at this time. So as for as this boundary is concerned, this becomes say incident wave and this will become a reflected wave. As for as this boundary is concerned, this is incident wave and this is reflected wave, so there is once we are having 2 boundary there is nothing like a incident wave and reflected wave depending upon which boundary you are considering.

The wave can be called the incident wave or a reflected wave. What that essentially means is there is nothing like what is you should call incident or reflected there are 2 steps of wave which are move. One set of wave which is moving like this which is having the wave vector which are parallel which are in this direction, other set of wave which are moving like that. And superposition of interference of these 2 waves plane waves 1 moving this wave and 1 moving this wave. Create the field distribution inside this bounds structure that is what is called the model pattern. So what we have essentially is these are the 2 boundaries which are conducting boundaries and there is 2 sets of waves 1 moving this wave other set of wave which moves this wave.

And super positions of the fields are these 2 waves create the patterns which are the model patterns. So essentially the propagation of electromagnetic wave inside this bound structure can be visualize as superposition of the 2 waves which are traveling like this. Let us same angle with respect to this direction which is parallel to the boundary or perpendicular to the boundary. Once we get this then we can get now the field expressions very easily for this pattern. However before we get into that let us take 1 more observation about these fields.

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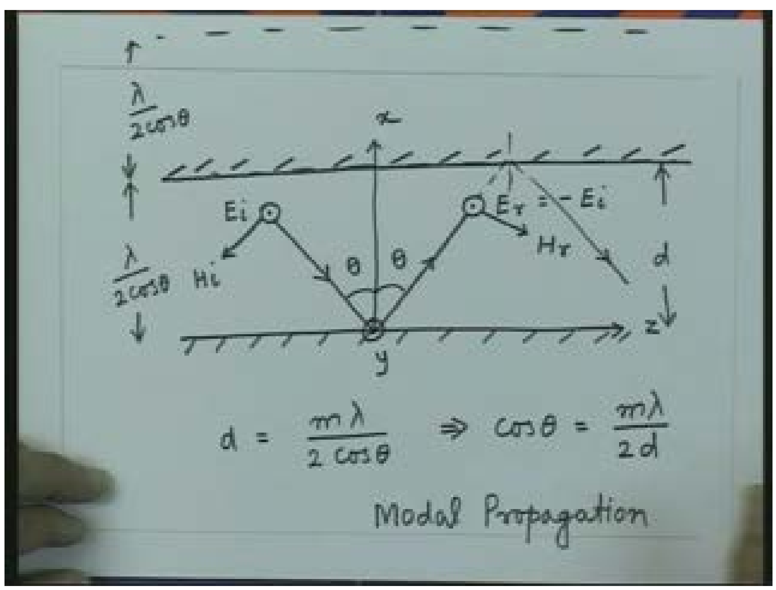

And that is if I look at now this propagation where the multiple reflections are taking place and the net propagation of energy is in this direction which is along the z direction. The electric field is now perpendicular to this plane of incidence what we call which is the plane of this paper. So when the reflection takes place, the electric field vector comes like this it get reflected go like that it get the like that and so on. So effectively we are having a wave of propagation now which is in z direction and the electric field is like that everywhere inside this medium where is the magnetic field is like this and this. So it is a direction is like that when it comes here the direction get change it become like this and direction get change it will become like this. So the magnetic field if I look inside the space between this plane waveguides has 2 components. One is perpendicular to this that mean in the x direction 1 is in z direction and since the net propagation is taking place in z direction.

The magnetic field essentially has components in the direction of net propagation and in the direction perpendicular to it. Whereas, if I look at electric field the electric field is always perpendicular to direction of the propagation; no matter where I go I will always find this electric field is perpendicular to the net direction of propagation. That means electric field is having a special nature in this particular case. It always remains transverse to the direction of the net wave propagation. That is the reason we designate this propagation as the transverse electric propagation. So we call this mode whatever the pattern which are going to set up inside the structure. Since this patterns will have electric field which will be transverse to the direction of let wave propagation. We call this mode as the transverse electric mode.

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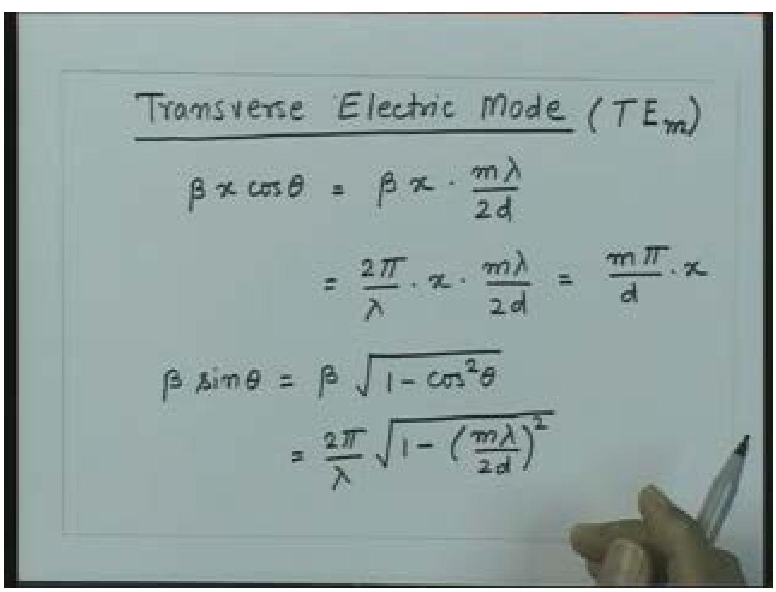

So we have now propagation what is called the transverse electric mode. That means the electric field always remains transverse to the direction of net wave propagation inside the structure. And in short this is denoted TE. So that mode for the electric field is transverse to the direction of net wave propagation we call at that as a TE mode. Now if I look this structure again this thing here we can get either value of let us say m equal to 1 that gives me this spacing which will be lambda upon 2 cos theta. If I can take m equal to 2, I can get this spacing for a given launching angle that means either I can have a set of fields which will be in this or I can have a set of field which will be in this. And for this region m will be equal to 1 if I go here m will be equal to 2 and so on.

That means though the fields are having the same nature that means electric field is transverse I can have a different possibilities either I can have a region corresponding to m equal to 1. Or I can have a case which is m equal to 2 or m equal to 3 and so on whatever is maximum value per meter. That is the field distribution which you are going to get in this is characterize by this integer number m. So that is the reason when we talk about the transverse electric mode, we put an index to this mode which is this value m as a suffix and we say that this transverse electric mode corresponds to this value of m which could be 1, 2, 3, 4 and so on. So we say that for a parallel plane waveguide this mode will be TEm mode where we will see little more the physical meaning what represents.

But at this point it appears that is m is going to tell at what angle this wave is launched for a particular value of m, so that we have a sustain propagation of the electromagnetic wave. Once we get this then we can go back to our fields which we have derived in the last lecture. Now the electric field is essentially given by this and cos theta now is equal to this quantity m lambda upon 2 d. So from here I can get first of all this quantity beta x cos of theta which is coming in the field expression for the electric and as well as in the magnetic field. So I can substitute for now cos theta which is m lambda upon 2 d. So I can write here beta x into m lambda upon 2.

Now beta we know is the phase constant in medium 1 and now this no medium 1 as such it is the medium which is filling this waveguide that means the medium which is filling the phase between the 2 conducting planes. So that phase constant is 2 pi by lambda x to m lambda divide by 2 d. So lambda will cancel 2 will cancel so this quantity will essentially become m pi by d into x. So I have got this beta x cos theta which is m pi by d into x. So this field expressions I can reply this quantity by m pi x by d. What about this quantity now since the cos theta is given by this I can get sin theta because I require sin theta here.

So from that beta sin theta that will be beta square root of 1 minus cos square theta. And cos theta is m lambda by 2 d. So I can write here this is 2 pi by lambda square root of 1 minus m lambda by 2 d square. So I have got now the phase constant which is in z direction which is this. That is what we get from this field expression. That is the phase constant beta times sin theta and beta x cos theta can be replace by this quantity. Once I get that then I can go to my electric and magnetic field which we derive last time and substitute these values for theta and we can get the fields for the transverse electric case.

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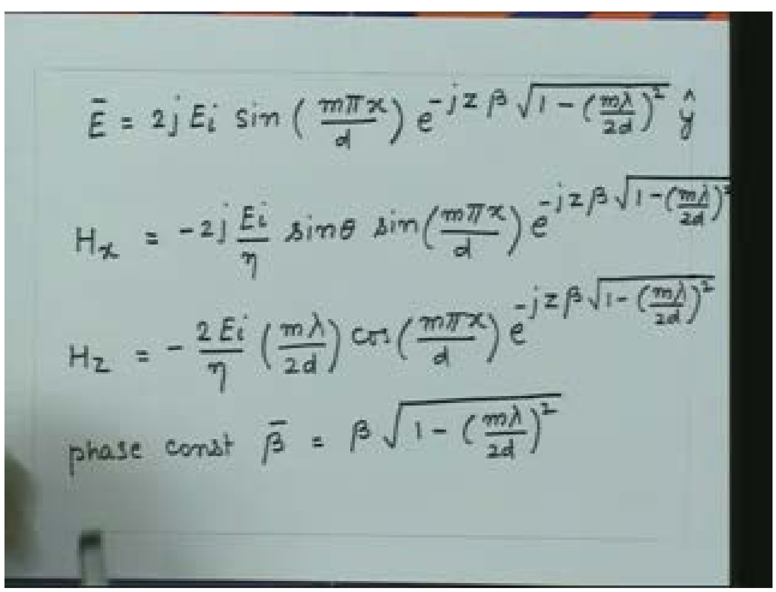

And that will be now the electric field E that will be 2 j Ei sin of m pi x by d e to the power minus jz beta square root of 1 minus m lambda by 2 d square. Similarly I can write down expression for the magnetic field which we have got last time. So we get, this is the y oriented. So this field is essentially oriented in y direction and I can get the magnetic field which will be having component Hx which is minus 2 j Ei upon eta sin of theta which is which you get from here this quantity. So sin of theta will be square root of 1 minus m lambda upon 2 d whole square I can write that will be just keep it for time being sin theta. Sin of this quantity which is same as this.

So sin of m pi x by d and this quantity same e to the power minus jz beta square root 1 minus m lambda by 2 d whole square and then I have the magnetic field z component which will be H of z will be minus 2 Ei upon eta cos theta which will be m lambda upon 2 d. So this is m lambda upon 2 d cos of this quantity cos of m pi x by d in this term is same e to the power minus jz beta square root 1 minus m lambda by 2 d whole square. So in this configuration now we get the 3 field components which are given by that. So this is the y component and this is the magnetic field x 2 component 1 is x and other 1 is z. So the field which can survive in a parallel plane waveguide for an electric field vector which is perpendicular to the plane of incidence will have this 3 field components.

Now as we saw that this quantity here we have this is now showing you the amplitude variation in x direction whereas this is the 1 which is showing you the traveling nature of the wave which is in z direction. So we have a phase constant in the z direction in which the net wave is propagating that is nothing but this quantity which is beta into square root of 1 minus m lambda upon 2 d whole square. So when the net wave propagation takes place now this quantity beta does not have a meaning because beta is the phase constant of a uniform plane wave which is bouncing back and fore between these 2 conducting planes. What phase constant essentially we are going to see for the net propagation that will be this quantity the total quantity.

So we say now that the phase constant for this mode which is superposition of 2 uniform plane wave as a phase constant which is this quantity. So we say that the mode is going to propagate with a phase constant which is this 1. So we say now for the phase constant for the mode and normally it is represented as beta z. So let is called beta z or let me just put as beta bar per beta bar gives me the propagation constant of the model fields along the boundaries and this quantity is nothing but beta into square root of this quantity 1 minus m lambda upon 2 d whole square. And this quantity as we know is essentially sin of theta, so we have beta bar upon beta that is nothing but a sin of theta. That is what the expression would be.

Now first thing 1 should immediately note from this expression is that if I put m; equal to 0 in this expression. Then the electric field will go to 0 because this quantity will make 0 for any value of x this quantity will go to 0 this quantity will go to 0. So the E y will go to 0 Hx will go to 0 Hz will not go to 0 because of this term but then when m equal to 0 this quantities 0. So these field will go to 0 so what that means is that for these fields to survive inside the structure I must have value of m which is integer but non 0. Because for, m equal to 0 all this fields will identically will go to 0 that means these fields cannot survive inside the structure. What does m equal to 0 mean if I go from here if I put m equal to 0 in these expressions that corresponds to this angle theta of pi by 2. That means if I put a wave which is parallel to the boundaries if the wave goes parallel to the boundaries. This wave cannot propagate inside this structure. So if I have electric field likes this and if the wave goes like this.

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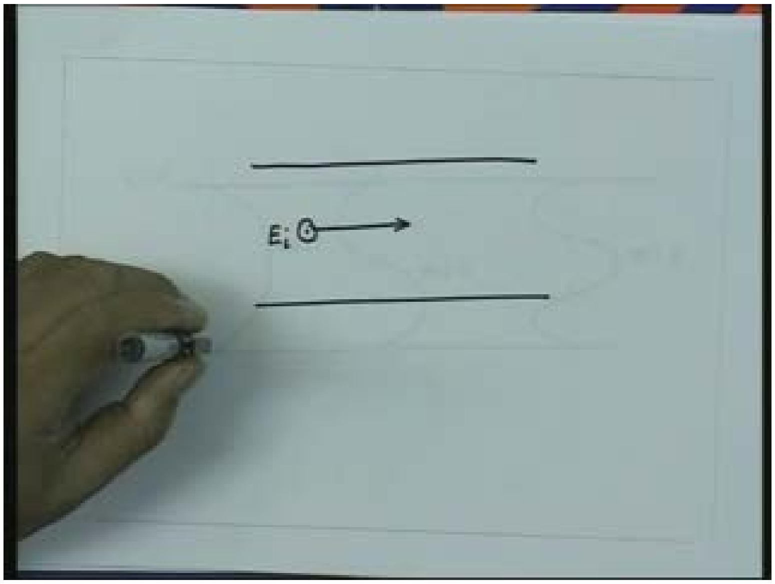

See if I have a situation that is these are the 2 parallel plane boundaries and if I try to launch a wave inside this which is having an electric field which is perpendicular to this okay. So I am trying to put the wave inside the structure electric field is like this and m is equal to 0. So this is my Ei if m is equal to 0, this wave cannot survive inside the structure it cannot propagate this field cannot get excited inside this structure. What that happens we can see little more physically by that happens and then happens because what this quantity m essentially telling. The m is telling you that if I take m equal to 1 then as x varies from 0 to d which is the spacing between the 2 planes. I get 1 half cycle variation of this field.

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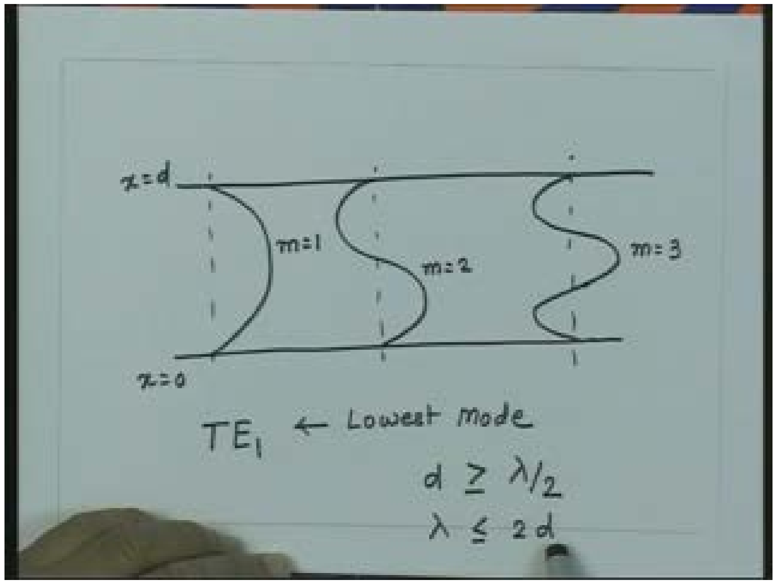

So what the m is telling you is inside a structure if I look at the field distribution amplitude variation of the field the m equal to 1 would corresponds to half cycle. So this is x equal to 0 this is x equal to d. So from here an x is equal to d by 2 this quantity will be pi by 2 when m equal to 1. So I will get the field maximum at this location which is d by 2. So I will get an amplitude variation for this which will be like that for, m equal to 1 if I take m equal to 2 then it will represent 1 full cycle variation as x varies from 0 to d. So if I take m equal to m equal to1, to b. So at x equal to d by 2 here when m equal to 2 this quantity will be pi so again sin pi will be 0 which will be this. So this will represent m equal to 2.

Similarly as I go to higher values of m I can get more and more cycles in that. So that will be so this will represent m equal to 3 and so on. So what this m what we have now the index for this field tell you is how many half cycle variation of the electric field amplitude or for that matter magnitude field amplitudes would be the in the directions transverse to the direction of the net wave propagation. So if m equal to 0 there is no variation. That is what this would mean. So m equal to 0 means the constant field; m equal to 1 means half cycle m equal to 2 means 2 half cycles m equal to 3 means 3 half cycles and so on. So m equal to 0 means constant. So that means if I consider now a field for which m equal to 0, the field is constant in the transverse direction it does not vary.

And in this case the electric field is oriented like this. Now we know since these are the conducting boundaries the electric field must be 0 here it must be 0 here because the tangential component to the conducting boundary. So for, m equal to 0 we do not want any variation of the field and it should be 0 here and here that can only happen if the field is identically 0 everywhere. And that is what precisely these expressions are telling you that when you put m equal to 0 these field cannot survive. So the lowest value of m which can survive the field which can survive for that lowest value of m, they will be corresponding to m equal to 1.

So that means the modes which can survive for transverse electric mode the lowest mode which can propagate in this structure is the TE 1 mode this is the lowest mode. So for a given phasing of these conducting planes we must have this value m which is at least 1 then and then only the fields will exist inside the structure. If m equal to 0 then the field will not exist inside the structure and the wave will not. So, depending upon the size that the saw there be finite number of modes, so you can have first mode to propagate which is TE 1 with the size is sufficient you may get TE 2 TE 3 and so on. So, on a given structure you can have a multiple modes propagating depending upon how many angles can be satisfied for this m.

But for the lowest mode when m equal to 1, one can ask what is the wave which can survive in this. That means d this is the largest value we can have which is equal to 1 for theta equal to 90 degrees. So if I take this is 1 then lambda by 2 will be equal to d, so the separation between the planes is lambda by 2 for propagation of this TE 1 mode. It should be at least lambda by 2 if the separation is less than lambda by 2. I cannot have an angle d is less than lambda by 2 for; m equal to 1 this quantity will be greater than one. I do not have a physical angle at which the wave can be longed.

So what that means for this lowest mode to propagate inside the structure my d should be greater than or equal to lambda by 2. And that is a very important conclusion what that means is once the separation between this 2 planes is given. That means the height of the parallel plane waveguide is given. I require a wave length which is small enough, so that this condition is satisfied or if I invert this essentially I get my lambda should be less than or equal to 2 d. So only those wave lengths since this has to be less than this.

That means the frequency has to this above certain value, so that this condition is satisfy then and then only the wave propagation will take place. If lambda is greater than 2 d or frequency is less than certain value then the wave propagation cannot take place inside this tangent. So from here we get a important concept of what is called cut off of a mode. That is when this condition is satisfied d is equal to lambda by 2. That frequency or lambda is equal to 2 d for the equality condition. That is the lowest frequency which can propagate on a waveguide whose separation ST.

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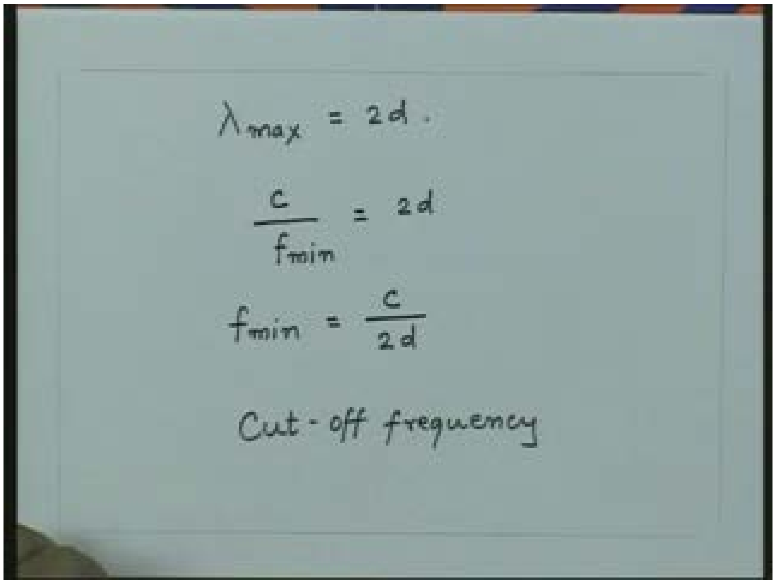

So from here essentially we can get that lambda maximum is the largest lambda which can propagate on this that should be equal to 2 d or if I write in terms of the velocity and the frequency is velocity of the wave in that medium. Let us say denote by c divided by the minimum frequency which can propagate on this that is equal to 2 d. So from here I get f minimum is equal to c divided by 2 d below this frequency the wave does not propagate and that is the reason this minimum frequency for this mode for m equal to 1 mode. We will call as the cut off frequency of that mode. So for a particular mode to propagate the frequency must be above that value and that is that value is what is called the cut of frequency of that mode?

So we have concept what is called cut of frequency and that is the lowest frequency which will propagate inside the structure for a given mode number given m as we can see as the mode number increases when become T 2 then the frequency will be double and so on. So that is the reason we are saying this; the lowest mode because that is the lowest frequency which can propagate in this structure. And that will be in this in the mode which will be TE 1 mode. Now if I put this condition that this quantity your d equal to lambda by 2 if I put in our propagation constant expression. This is the phase constant which I hired for these fields and if I put m equal to 1 and lambda upon 2 this quantity here is equal to d then m lambda upon 2 d that will be equal to 1.

So this quantity beta bar essentially will go to 0. So at this frequency when this condition is satisfy what we call as the cut off frequency. At cut off frequency the phase constant of this wave goes to 0. And below this frequency this quantity is greater than 1 this quantity is imaginary. So beta becomes imaginary that means it is no more a phase constant. But it becomes attenuation constant and the wave losses the wave nature it represents only the field which exponentially dying down because behave only attenuation constant and there is no phase constant. So when you have a frequency below this cut off frequencies, it represents the fields which are exponentially decaying fields but there is no propagation of electromagnetic wave.

So what we see from this analysis is that when we have a bounds structure like a parallel plane waveguide firstly there are discrete angles at which the uniform plane waves can be launched inside this. Superposition of this uniform plane waves which are bouncing back and forth between 2 parallel planes gives you the field distribution which will be like this. And those field patterns we call as the model patterns and we also see that for a given mode number or to have a particular mode pattern excited inside the structure the frequency has to be about certain value what we call as a set of frequency.

So depending upon which order of the mode you want to excite you require a minimum frequency then and then only that particular mode will excited. So even if the waveguide is capable of supporting different values of m it will depend upon the frequency whether that value of m will be acceptable or not because whether this condition the frequency is greater than set of frequency is satisfied or not. So this is now a basic picture of model propagation in a bounds structure like a waveguide. So we will elaborate on this and then following this essentially we will try to see another mode when we meet in next lecture and then we will go to the more compact structures like the rectangular waveguides.