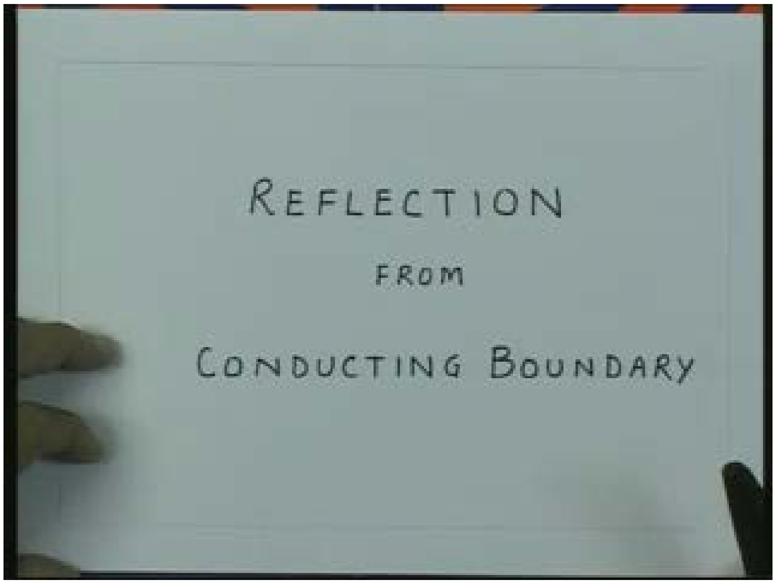
Transmission Lines & E M. Waves
Prof. R.K.Shevgaonkar
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture #34

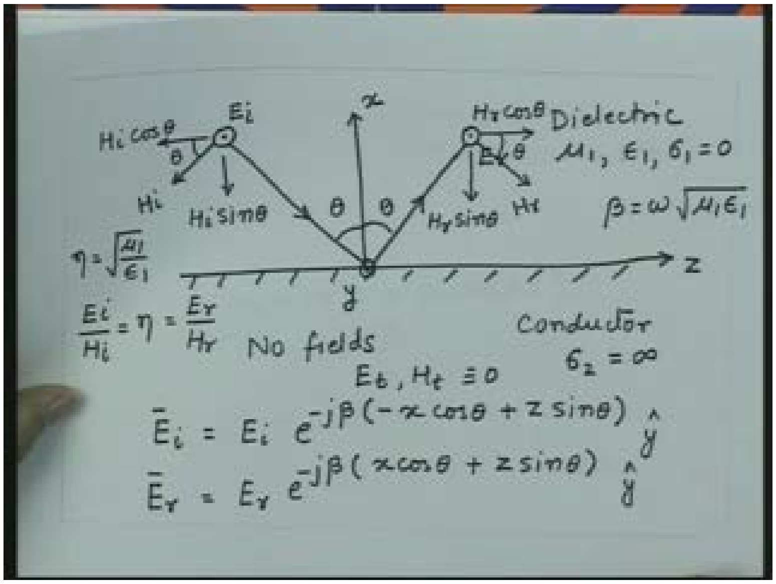
In this lecture we discuss reflection of a uniform plane wave from a conducting boundary. You will see that this will essentially make a foundation for a structure what is called a wave guide which can guide electromagnetic waves along its length. So in this lecture we still consider a media which is divided into 2 parts; on one side of which is a dielectric and other side of which is an ideal conductor.

(Refer Slide Time: 01:55)



So we talk here essentially the reflection from a conducting boundary. So we divide the phase into 1 part and in this case let me just draw the figure ((…)) in different way.

(Refer Slide Time: 02:07)



So let us say the horizontal line is the 1 which is dividing the space into 2 parts. And also orient the coordinate system which is differently and they have reason for that will become clear as we proceed in our discussion. So let us say now the infinite medium is divided into 2 parts by this plane. Below this plane the conductivity of the medium which infinite above this interface the conductivity is 0. So in this side we are having an ideal dielectric and below the interface we are having an ideal conductor. So we have here a conductor this side we have dielectric. So we have here some permeability mew 1 permittivity epsilon 1 for sigma 1 is 0 for this. Whereas in this case since we are having an ideal conductor the sigma 2 is equal to infinite.

So this boundary then we can call as a dielectric conductor boundary and the wave is incident on this boundary. Now from the dielectric side the wave cannot come from the conducting side because your time varying fields will be 0 inside the conductors. So there is no wave propagation inside the conductor. So only wave can come from a dielectric side and get incident on this dielectric conductor interface. Let us choose now the coordinate system. Let us say this is my x axis. So direction perpendicular to plane of incidents in coming outwards is that is say that is y axis. And the direction along the interface now is z axis. So the wave is incident on this at some angle let us say that angle is theta I am not putting the suffix here.

Because now we are dealing only with 1 angle which is this angle there is no angle in the second medium. We are the wave is not propagating the second medium because of infinite conductivity. And the angle of incidents and reflection is same which you already seen. So this angle is also theta. So this is a reflected wave so we have an incident wave and we have a reflected wave and both this waves are going to make an angle theta with the normal to the interface. And that is what we have seen earlier this condition is angle of incidents is equal to angle of reflection which is the law of reflection. So now we have incident wave which is incident at an angle theta. So the angle of incident is theta and the angle of reflection also is theta and there is no way of now I the second medium. So there is no transmitted wave.

We can consider again 2 polarizations 1 is perpendicular polarization and 1 is parallel polarization. And do the analysis on the line identical to what we have done for a dielectric interface. However this case is rather simple case because we do not have the 3 waves to match the boundary conditions. We have only 2 waves for matching the boundary conditions. So in fact this case is the simpler version of the case which you have already discussed. So let us say we take a polarization which is perpendicular polarization. So the E vector is oriented in y direction. So let us say this is my incident field and as we have done in dielectric case, again without losing generality we can say that even reflected electric field has orientation in y direction. So that is also perpendicularly polarize then by using pointing vector we can get a direction of the magnetic field.

So E cross should give me the direction of the wave propagation, so the magnetic field must be this direction. So this is H i and since this wave is going in this direction again E cross H should give me the pointing vector so that should be the direction of the magnetic field Hr. We can resolve the magnetic field into 2 components. 1 component which is parallels to the interface other 1 which is perpendicular to the interface. So if this angle is theta this angle is also theta similarly we can do for these 2 components. So this angle is also theta. So this component will be Hi cos of theta, this will be Hi sin of theta. Similarly this will be Hr sin of theta and this will be Hr cos of theta. So in this case the electric field is tangential to this boundary while it is like this.

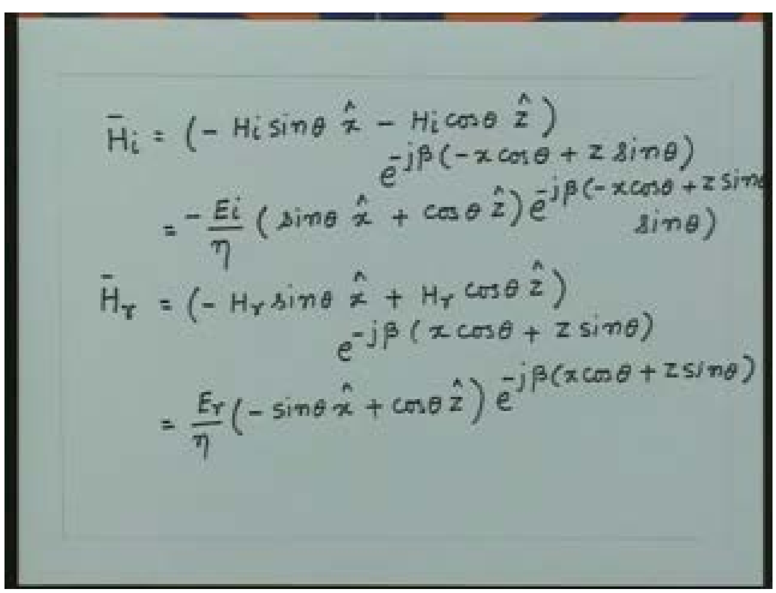
So when the wave is incident on this interface which is this; the electric field is tangential to the interface. The component which is tangential to the interface for magnetic field is this Hi cos theta these; a component which is tangential to the interface. So Hi cos theta and Hr cos theta will be the tangential components to the interface and Hi sin theta and Hr sin theta are the normal component to the interface. Since we are having a conducting boundary, now we will have the surface currents. So either I can use the boundary condition for the tangential component of the magnetic fields with appropriate surface current. Or if I want to be on safer side, as you always use the boundary condition which is always applicable without worrying about surface current and that is the normal component of magnetic field.

So since there are no wave of propagation are there are no fields in the conducting medium the time varying fields we do not have no fields in this region. So that means my Et and Ht are identically 0 in. This means so the boundary conditions. Now how to be satisfied only by these 2 waves which is the incident of the reflected wave. But before we do that, let us now write down explicitly the expression for the incident and the reflected electric and magnetic fields. And then we take the appropriate components of these fields and satisfy the boundary conditions. So if I write the incident electric field is Ei that is having some amplitude Ei and we will have a phase function which is e to the power minus j beta. We are not using the term suffix 1 here because since the way of propagation is going to be only in this medium.

And there is no wave propagation in medium 2. For simplicity let us say that the propagation constant beta in medium 1 is denoted by beta. So without putting in a suffix 1 this is the phase constant of wave propagation in medium one. So here for this medium your beta is equal to omega square root mew 1 epsilon 1. Then we can write the expression here this wave is traveling this wave this is direction x. So the angle which the wave makes the x axis is pi minus theta. So you will get x cosine of pi minus theta which is minus x cos of theta the angle which this makes with z axis is pi by 2 minus theta so that will give me plus z sin of theta and direction of this electric field is in y direction. So let us put a unit vector for this which is y.

Same thing I can do for the magnetic field for 2 components. The phase function for the magnetic field for this will be similar to this. And for this from we can write so let us say first we write for the electric field for the reflected wave. So let us say this is Er that is some amplitude Er e to the power minus j beta. Now this wave the angle which makes with x axis is not pi minus theta; but it is only theta. So this quantity will be x cos of theta. So it is x cos of theta plus triangle with this makes the x axis pi by 2 minus theta. So that will remains z sin theta and we can assuming that this is oriented in positive y direction so we have unit vector y. So we can write down just once we know the angle of incidence we can generate this phase function and then we can write down the expression for the incident and the reflected electric fields. Same thing we can do magnetic field also.

(Refer Slide Time: 13:10)



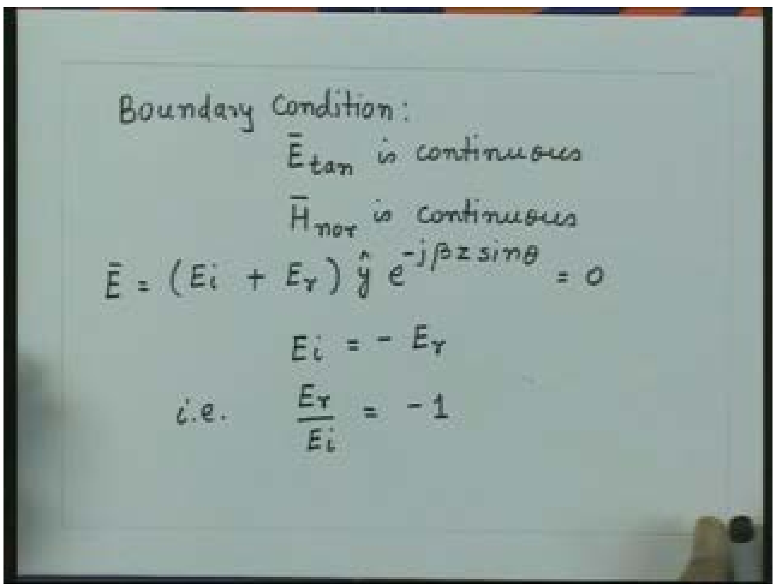
So we have now the incident magnetic field Hi. That is, the Hi is Ei upon eta and eta is the intrinsic impedance of this medium. So we have in this medium eta which is equal to square root of mew 1 epsilon 1 and we have a condition Ei upon Hi is equal to eta. And same is true for this case also so Er upon Hr are also equal to eta. So we have from here Ei upon Hi that is equal to eta and same is true for this case also so that is also equal to Er upon Hr.

So in medium 1 the ratio of electric and magnetic field for a incident wave is same is same as the intrinsic impedance and same is true for the reflected wave because the both of them are the plane wave in medium one. Once I get this then I can write down here the vector magnetic field and that begin have 2 components. One is the component which is along the x direction which will be a minus Hi sin theta. So this will be equal to minus Hi sin theta there is no y component of the magnetic fields so that component is 0. And that this component will be opposite to the z direction. So it will be minus this will be along x axis. So this is a unit vector x minus Hi cost theta unit vector z and both this field we have a phase function which will be identical to this. So this will be multiplied by e to the power minus j beta minus x cos theta plus z sin theta.

We can write down the reflected magnetic field also. So we have here Hr that is equal to from here this is component is Hr sin theta what is opposite to the x direction. So that is minus Hr sin theta x direction. But this component Hr cos theta is in positive z direction. So this is plus Hr cos theta z direction with the phase function which is e to the power minus j beta and the phase function for the reflected wave which is this function which is x cos theta plus z sin theta. Now as we mention this quantity Ei and Hi are related to the intrinsic impedance of this medium 1. So we can write this Hi in terms of Ei. So this expression can also be written as I can take a minus sign common. So this is minus Ei upon eta this will be sin theta x plus cos theta z and this phase function e to the power minus j beta minus x cos theta plus z sin theta.

And similar thing I can do for this also. So I can substitute for Hr which is Er upon theta so this thing is Er upon eta minus sin theta x plus cos theta z and this phase function which is e to the power minus j beta x cos theta plus z sin theta. So now I got the expression for the electric and magnetic fields for the incident and the reflected waves. And now the problem is simple just make the boundary conditions which are appropriate for this interface and as we mention earlier. We satisfied the boundary condition which are always applicable and that is the tangential component of electric field should be continuous and the normal component of magnetic fields should be continuous across an interface. So we take here now 2 boundary conditions.

(Refer Slide Time: 19:23)



That is the tangential component E tangential at the interface is continuous and also the H normal is continuous. So in this figure as we have seen the tangential component is the electric field is tangential to the interface. At this some of these 2 fields if I put x equal to 0, that is the total field at the interface. So the some of the 2 electric fields at x equal to 0 should be equal to 0 because the fields should be continuous and there is no field in second medium. So some of these 2 electric fields at x equal to 0 should be equal to 0. Similarly some of these 2 normal components of magnetic fields at x equal to 0 again should be 0 because the normal component should be continuous at the boundary. So what we get from here that if I just take Ei and Er input x equal to 0 in this.

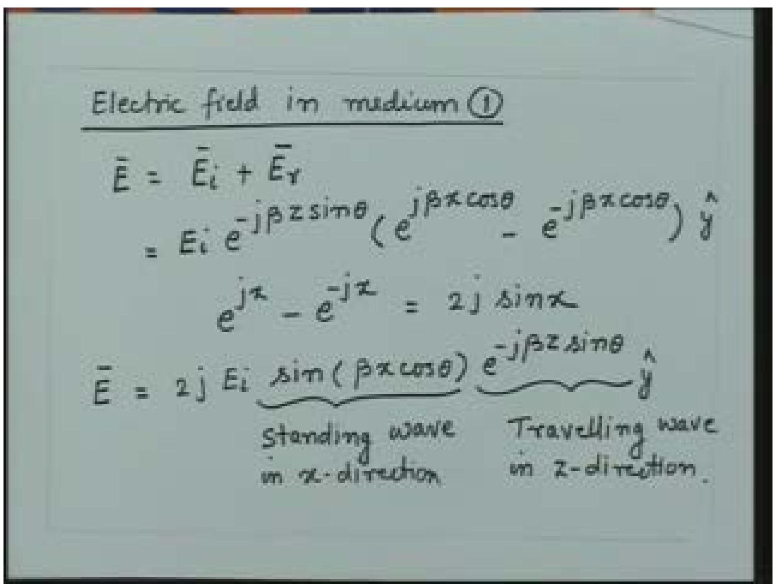
Then we get E which is Ei plus Er y e to the power minus j beta z sin theta that should be equal to 0. This is for tangential component of the electric fields. So this is the incident field which is y oriented as the phase function and its x equal to 0 this is the phase function same is true for the reflected field. So this is the total field at x equal to 0; that is at the interface that should be equal to 0. So from here essentially we get that relation the Ei is equal to minus Er, that is the reflection coefficient in this case which is Er upon Ei is equal to minus 1. It has a same condition I can get if I apply to the continuity of the normal component of the magnetic field.

So I do not have the really use this boundary condition just by using the tangential component of the electric field continuity I can find out the reflection coefficient for the electric fields. So in this case the electric field reflection coefficient is always equal to minus 1. What that means is that initially we have taken direction of the electric field which was same as this. But now the reflection coefficient is negative that means the direction of the electric field should be going inverse for the plane of the paper. And 2 amplitudes are equal so that means the reflected electric field is equal in amplitude with the incident field but oriented in the opposite direction so some of these 2 at the interface is 0.

If you recall when you are discussing transmission line we had a reflection coefficient minus 1 for a short circuited load. That means the conducting boundary essentially the identical to the short circuit condition on transmission line. So one side we are having a dielectric medium which is like a transmission line on which the wave is propagating. When it reaches to this ideal conductive boundary the electric field is completely reflected from the boundary with a phase reversal that means the phase difference of 180 degrees. So this boundary essentially behaves like a short circuit in the transmission line terminology. Once we get that then other analysis very straight forward its substitute. Now this value of electric fields into the expression for electric and magnetic fields.

And now ask what is the total field which will be existing in this medium? Because of nothing much to find out about the transmission and reflection coefficient the reflection coefficient is become equal to minus 1 and there is no energy gone to the second medium. So the finding out the reflection and transmission coefficient problem is a very simple problem in this case. However what you would like to now we would like to study is that when this electric field is completely reflected. What kind of field the attend will be created in this medium which will be superposition of these 2 way. That is what is of interest now. So let us say I take this electric field and total electric field will be some of these 2 electric fields at any point in medium 1. So I can find out what is the total electric field which is some of these two.

(Refer Slide Time: 25:11)



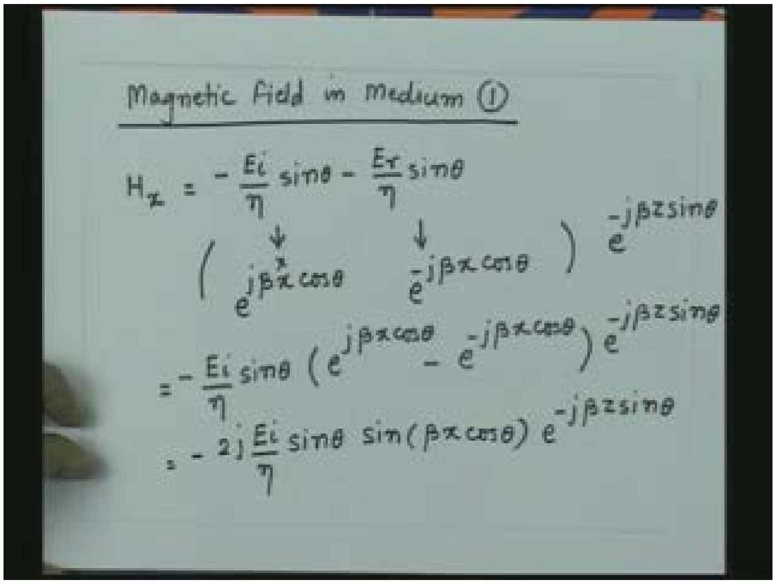
So with this condition then we can get now the electric field in medium 1 which is E is equal to Ei plus Er. And now we are doing get any arbitrary point and space I just take this 2 expressions put Er equal to minus Ei and find the some of these 2 fields. So this will be equal to where Ei this quantity e to the power minus j beta z sin theta is same for these 2. Only the sign of these different this will be plus j beta x cos theta just will be minus j x beta cos theta. So these 2 are different. For these 2 waves but this quantity is identical for these 2 waves. So I can take it common let Ei e to the power minus j beta z sin theta then for this wave what we have is e to the power plus j beta x cos theta.

Now we have e to the power j beta x cos theta and for the reflected wave this will be minus j beta x cos theta. And Er is equal to minus Ei, so this will be minus e to the power minus j beta x cos theta orientation of this is the y orientation. So the total electric field which I have in medium 1 which is superposition of the incident and the reflected field and Er is minus Ei. We can write the total field like this. Now this thing is we can combine we know that e to the power jx minus e to the power minus jx is equal to 2 times j sin of x. So we can use this represent this a sign function so we get now the total electric field in medium 1 which is 2 times j Ei sin of this quantity which is beta x cos theta.

If this e to the power minus j beta z sin theta and that the oriented in y direction. So we have now the electric fields which have this sinusoidal variation in the x direction. And it is having a phase term which is only z direction. What that means is that you are having an electric field. Now which is having a some kind of a standing wave behavior in the x direction because this function is does not have phase but it is an amplitude variation and which can go from 0 to 0 to 1 which is the nature of a complete standing wave. So this term now represents something like a standing wave which is x direction and a traveling wave which is given by this term which is in z direction.

So this term gives me a standing wave in x direction and you get a traveling wave in z direction. So the waves which are going to have which is a composite phenomenon of the incident and the reflected wave is a complex wave. Now which is the combination of a standing wave in a direction perpendicular to the interface and a traveling wave which is in the direction of the interface? Same thing we can do for the magnetic fields also so we can take the 2 magnetic fields and get the expression for the magnetic fields also so when we combine these 2. So we can get x component we can combine that we can get z component and we can combine that separately.

(Refer Slide Time: 30:48)

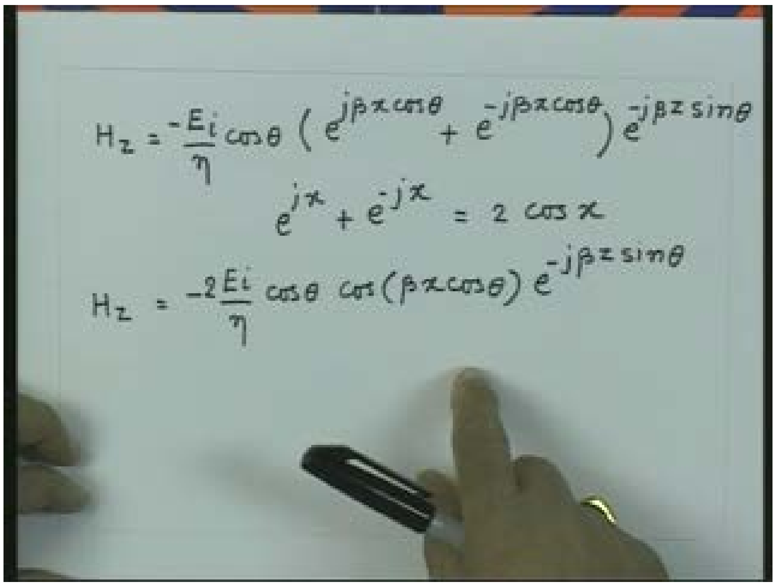


So we can get now the finally the expression for the magnetic fields also. So we get here magnetic fields in medium 1. And let us take 2 components of the magnetic fields. So I take x component so I get H of x that is some of this 2 components here. So this will be minus Ei upon eta sin of theta plus Er upon eta minus sin of theta. So I will get from here minus Ei upon eta sin of theta minus Er upon eta sin of theta. And the phase term for these 2 this will be having plus j beta x cos theta this term will be again common. We would appropriately multiply by this term by this phase term which is e to the power j beta x cos theta. So this has to be multiplied by e to the power j beta x cos theta whereas this is to be multiplied by e to the power minus j beta x cos theta. And then the whole thing has to be multiplied by the traveling wave term which is e to the power minus j beta z sin theta. So this has to be then finally multiplied by e to the power minus j beta z sin theta.

Again if I substitute Er equal to minus Ei then I can get the expression which is I can get this this becomes plus Ei. So I can take minus Er upon eta common. So it is minus Ei upon eta sin theta I can take common sin of theta multiplied by e to the power j beta x cos theta minus e to the power minus j beta x cos theta e to the power minus j beta z sin theta.

Again as we did in the previous case we can combine these 2 terms using this so this will become 2 times j sin of x. So this quantity will be minus 2 times j Ei upon eta sin theta sin of beta x cos theta minus j beta z sin theta. So the x component first of all has again similar behavior. That is, it has a standing wave component which is in x direction and it has a traveling wave component which is in z direction. The same thing I can do for the other component which is the z component of the magnetic field.

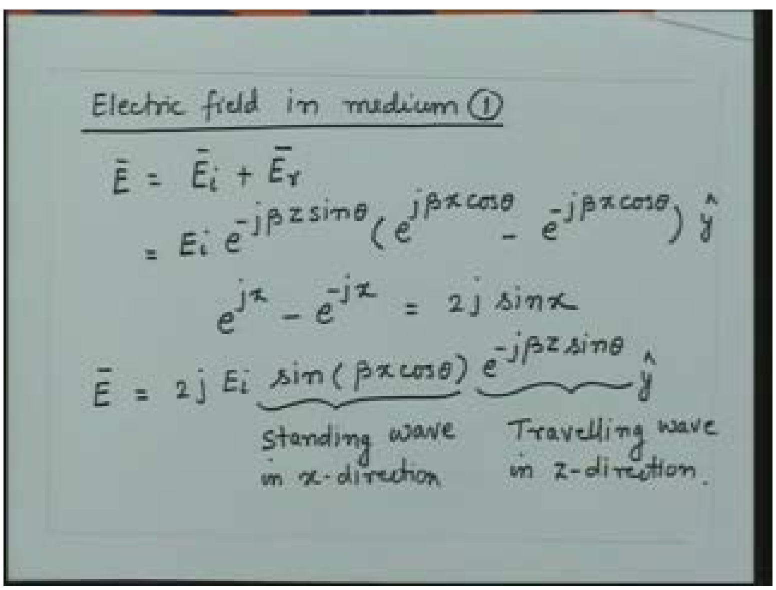
(Refer Slide Time: 35:13)



I get here H of z that is this quantity here minus Ei upon eta cos theta and this is Er upon eta cos theta but Er is minus Ei. So essentially I get here minus Ei upon eta cos of theta e to the power j beta x cos theta plus e to the power minus j beta x cos theta e to the power j beta z sin theta. Now I can again combine this 2 that e to the power some jx plus e to the power minus jx is 2 times cos of x. So Hz can be written as minus Ei upon eta cos theta 2 here cos of beta x cos theta e to the power minus j beta z sin theta. So now we have got a complete discussion of the fields that is we got the electric field which is given by this which is having a standing wave of component and a traveling wave of component.

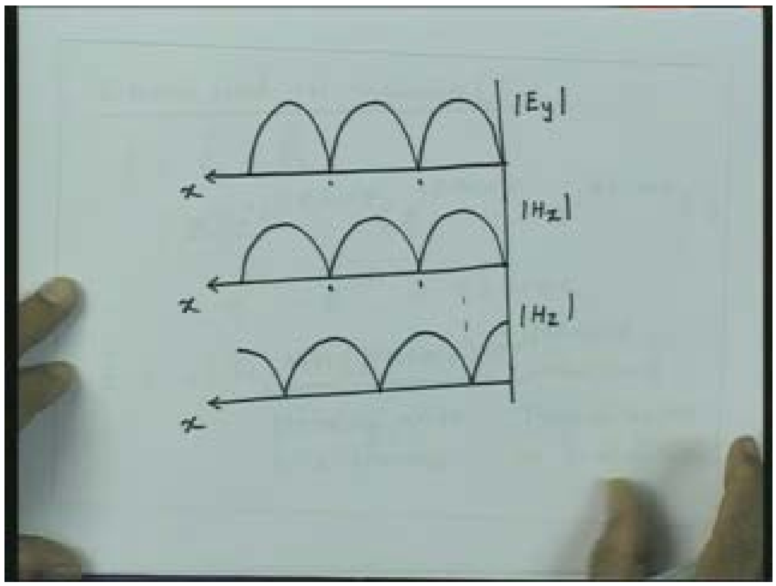
The x component of the magnetic field it has the standing wave of component and a traveling wave of component and the z component of the magnetic field which has a standing wave of component and a traveling wave of component. So in general, then we are having now the fields in medium 1 to be combination of traveling wave and a standing wave. And all of these fields are having a wave which is traveling in the z direction positive z direction. That means it is traveling along the interface. So we are having the standing wave in a direction perpendicular to the interface. But all this fields are having a traveling which along the interface which is in z direction. Now we can have made some observations from 3 expression which we got for the electric and magnetic fields.

(Refer Slide Time: 38:20)



Firstly if I plot the amplitude of the electric field as a function of x then x is equal to 0 this quantity is 0. So the field is 0 and same thing happens even for the x component of the magnetic field well x is equal to 0 the magnetic field will be 0. So whenever the electric field is 0 the x component of the magnetic field is also 0 in fact the amplitude behavior of Hx and electric field which is Ei is identical as a function of x. And the magnetic field component as z is a cos functions. That means it is shifted by a quarter cycles in the x direction. So wherever Hx is 0 Hz is maximum and wherever Hx is maximum the Hz is 0. See if you plot this this amplitude of the electric and magnetic fields essentially we get as follows.

(Refer Slide Time: 39:29)

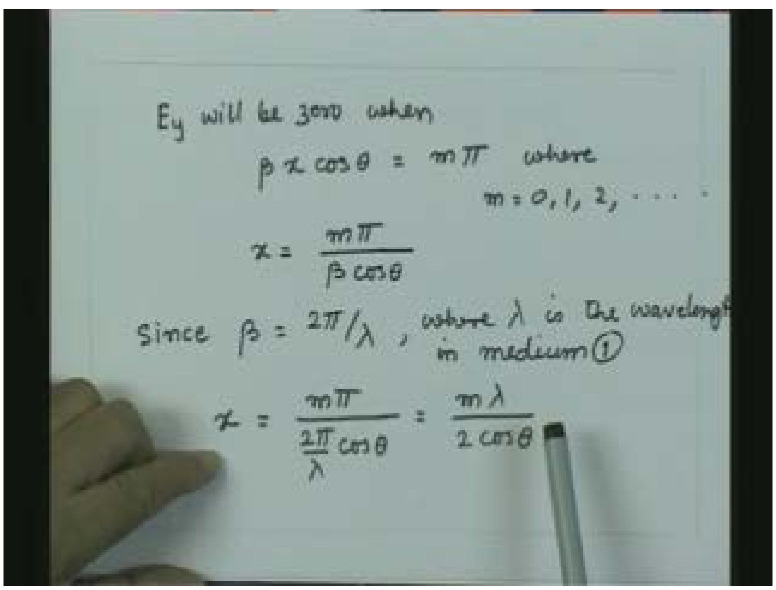


So let us say this is multiplication boundary and apply this. So let us say this is mode of electric field which is Ey the electric field behavior will be like that. Exactly identical behavior I will get for the x component of the magnetic field. So Hz also has same behavior. Whereas if I look at the magnetic field z component that is shifted with respect to this because this function is a cos function. So you have a magnetic field z component that will be starting with maximum will go 0 here. So we start from maximum goes to 0 here then and so on. So now we can make certain observations this pattern which is the standing wave pattern which is created in x direction this is x, x, x.

The Ey and Hx patterns are aligned in space whereas the Hz pattern is shifted by quarter cycle with respect to these 2 patterns. So wherever Hx is maximum Hz is 0 and vice versa. This is the interface which is the conductive interface and since Hz which is the tangential component is not 0 essentially we have the surface currents on the interface and the magnitude of surface current will be equal to tangential component of the magnetic field. So now when the plane wave is incident on the conducting boundary the surface currents are going to get reduce on this surface which is due to this tangential component of the magnetic field as we have seen and the normal component of the magnetic field will be 0.

And a tangential component of electric field will be 0 as the boundary condition needs. Now if we go back to the expression for the electric field. The electric field is 0 at this point x equal to 0. And so it will be 0 whenever this quantity is multiples of pi. So I have when this quantity is 0 the field is 0 and this quantity is pi if it is again 0 the when this 2 pi the field is again 0 and so on. So if I go to this pattern here these are the locations where this field is going to become 0, so this electric field will become 0 for those values of x for which beta x cos of theta will be multiples of pi.

(Refer Slide Time: 42:43)



So we say that Ey will be 0 when beta x cos of theta that will be equal to multiples of pi. So it is say that is m pi where m is an integer 0, 1, 2, 3 and so on. So now we have got this value of x which is given by m pi divided by beta cos theta. And what is beta? Beta is the phase constant of the uniform plane wave in medium 1 which is nothing but 2 pi by lambda for uniform plane wave in medium 1. So we can substitute since beta is 2 pi divide by lambda where lambda is in the wavelength in medium 1. We get the value x will be m pi divided by 2 pi by lambda cos theta that is m lambda divided by 2 cos theta.

So at this distance x from the interface if I go the electric field will be 0 and that will depend upon this distance will depend upon at what angle the wave is launch on the interface. See if I consider a situation here the wave is launch at this angle. I may. Fine. Some distance here x at which the field will be 0 double of that again the field will be 0 and so on. So essentially now we have got the planes which are parallel to this boundary parallel to the conducting media interface where the electric field will again be 0. So in fact we are going to have the multiples planes here in which the electric field will be identically 0.

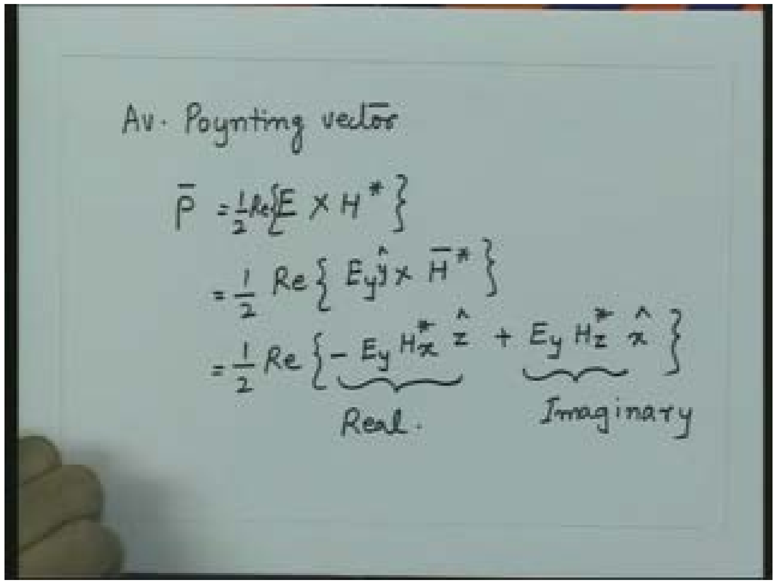
And since the electric field and the x component of the magnetic field have the same behavior in those planes both this quantity will be 0. See Ey will be 0 and Hz will be 0 and in those planes Hz will be maximum. If I go by quarter cycle away then we will see that the z will be 0 and these 2 quantities will be maximum. So essentially now we are defining some kind of a wavelength in the direction which is perpendicular to the interface which is lambda upon cos theta and multiples of this lambda upon cos theta. That effective wavelength if I go lambda by 2 of that. At that location essentially the field will be 0. So what we find that when the wave is reflected from a conducting boundary we have created.

Now the planes parallel to the conducting boundary where the electric field will be 0 in this situation. And the whole wave essentially going to travel along the direction which is along the interface, we can get this electric and magnetic fields and ask the question. Now one, of course we know that the wave is traveling along the z direction. So there must be a power flow in the direction. But we can also verify this from our pointing vector argument that if you calculate the average pointing vector for this complex field which you have got, we must get the pointing vector in the direction of the net power flow. So since we are having the electric field which is y oriented and this quantity is 2 j Ei the magnetic field Hx if I take.

So if I look at this expressions for E and Hx this quantity for electric field this 2 j Ei and this thing is minus 2 j Ei upon eta. See if I calculate the cross product of E by and Hx it is in z direction since we are having both of them j this 2 will be in phase with negative sign. So Ey cross Hx gives me a real value of the pointing vector which is in z direction. So I get Ey across Hx which will be minus Ez and since there is a minus sign here that gives me the net power flow which is in z direction due to this product. If I consider Ey and Hz component you are having 2 j here for the electric field and this thing, there is no j. That means the phase different between the electric field and the z component of the magnetic field is 90 degrees.

So the cross product of Ey and Hz gives me only an imaginary term. So the real part of the cross product of these 2 will be 0. So Ey and Hz which would give me the power in the x direction that is purely imaginary power, so there is no power flow in the x direction. But there is a net power flow in this z direction because these 2 quantity Ey and Hx. These essentially give me the pointing vector which is in z direction. So when we calculate the cross product of E and H in this case we get average pointing vector which is P.

(Refer Slide Time: 49:40)



That is, we have electric field which is so we have here half real part of E cross H conjugate and from here I get half real part of Ey cross H which are both the components. So this is Ey y cross H conjugate so that is half of real part of minus Ey Hx z direction plus Ey hz conjugate x direction. And as we have seen this quantity gives me a power flow which is purely imaginary this wave quantity which is a real quantity. So we get a power flow which is in the z direction. There is no net power flow in the x direction and that next time, that once you are having a boundary which is a conducting boundary. The power is not going to go inside this conducting boundary.

So whatever power is incident essentially has to go back in medium one. So this thing can be visualize as follows the wave is incident which is in this direction which is essentially having a component of propagation in this direction. And something which is coming normal to the interface the wave which is coming normal to the interface it is completely reflected. So you get a standing wave which is created here. So whatever power flow was in this direction is completely balanced by the power flow which is in reverse direction because there is a complete reflection so the net power flow in this direction which is next direction is 0.

So this essentially gives you some kind of a reactive power in this direction. But in this direction where the there is a net flow of the wave you have a traveling wave in z direction. And there is a net power flow along the z direction and that is what precisely the pointing vector gives you that it gives you the pointing vector in the z direction which is same as the net direction of the traveling wave. And there is no power flow in the direction perpendicular to the conducting boundary. What that means is that if you have a conducting boundary then the boundary can be used to guide the energy along with it. So conducting boundary has a capability of guiding the electromagnetic energy, so you launch a wave at any arbitrary angle and what we will find is the net power flow is always along the surface of this interface.

Essentially this is the thing which is used in creating what is called a wave guide. So in a wave guiding structure we use the conducting boundaries so that the electromagnetic energy is guided along these boundaries. So now we will see when we are discuss later that in those planes where the electric field was going 0 which was given by location of this we have created, now a structure that you have a boundary. Then there are certain distances to these planes where again the electric field goes to 0 and if the electric field is 0 there I can insert a conducting boundary there without affecting the field is distribution. But by doing this essentially I have created a structure which is bound from both the sides that the moment I have a structure which is only bound from this side. But this side the space is open. But if I introduce another boundary here as a distance this is given by this then the boundary condition will be satisfied at that boundary in inherently fields will not get modified.

But I could get a structure which will be a bound structure and that structure will be what is called a parallel plane wave guide. So, using the reflection from the conducting boundaries in the field expression which you have got here, essentially we have created a ground for developing a structure which is a structure like a parallel plane conducting geometry within which the electromagnetic waves can be tracked. And we can have a net propagation of electromagnetic energy along these planes, along these interfaces. So with this understanding, now we are prepared to make a transition to a more realizable structure what is called a parallel plane wave guide. So next lecture when we meet essentially discuss the propagation of electromagnetic waves in a parallel plane waveguides and again we will see some other characteristic of a parallel plane waveguide.