Transmission Lines & E M. Waves Prof. R.K.Shevgaonkar Department of Electrical Engineering Indian Institute of Technology, Bombay

Lecture #33

In last few lectures we have been investigating the behavior of uniform plane wave across the dielectric medium. We studied 2 cases 1 with parallel polarization and other with perpendicular polarization and saw the reflection and transmission across a dielectric boundary. We also investigated a special case what is called total internal reflection across a dielectric boundary. Today we will discuss the important aspect of electromagnetic wave that is the polarization across the dielectric interface.

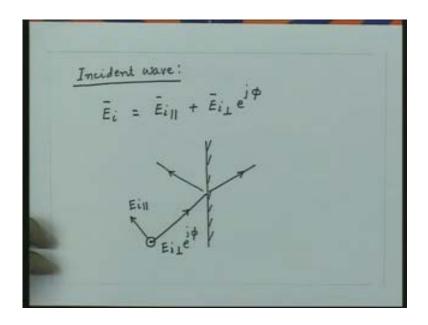
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So in this lecture essentially we take a wave which is arbitrarily polarized. That means its electric field vector meets arbitrary angle with respect to plane of incidence also it might be varying as a function of time. And then we will see how this behavior changes when the wave is reflected from a dielectric interface. So the problem do investigate here is that if the incoming electromagnetic wave has certain polarization what would be the

polarization of the reflected wave and what would be the polarization of the transmitted wave. As we discussed earlier an arbitrary state of polarization can be decompose into 2 orthogonal polarizations and in this case we take 2 orthogonal polarizations which are linearly polarization; one which is perpendicular to plane of incidents and other with parallel to the plane of incidents. So in general an electric field when the wave is incident in the media interface can be represented by the vector some of 2 electric fields that is the perpendicular and parallel polarization.

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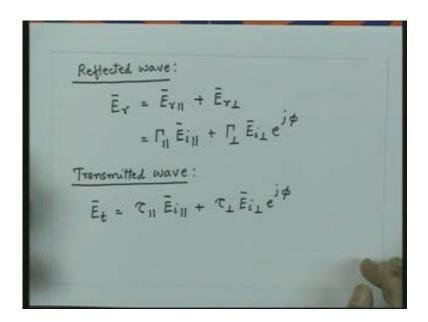


So we have in this case, now the incident wave, let us say this is denoted by Ei and this is vector quantity. So this is now given by the incident wave for parallel polarizations. So this Ei with parallel polarization plus the incident wave for perpendicular polarization again a vector quantity. And these 2 depending upon the state of polarization these 2 orthogonally linearly polarize waves might away phase difference. So let us say that is denoted by e to the power j phi. So we have 2 electric fields. Now one which is in the plane of incidence other which is perpendicular to plane of incidents and the phase difference between these 2 is phi when the wave incident that the dielectric interface. So the situation is if this is the dielectric interface the wave is incident on this. So I have 2

components now 1 is this electric field which is given by Ei parallel other 1 is the electric field which is perpendicular this which is given by Ei perpendicular e to the power phi.

So any now arbitrary state of polarization is a combination of these 2 fields. And if you separately solve the problem as you mention earlier for the 2 polarizations the parallel and perpendicular, then we can combine them back to find out the polarization of the reflected and transmitted wave. So in general gain the transmitted wave also will have 2 electric fields and the reflected wave also will be having 2 electric fields. So what we do? Essentially we find out now the reflected field for the parallel polarization on the transmitted field for parallel polarization and same we do for perpendicular polarization and then we combine them together?

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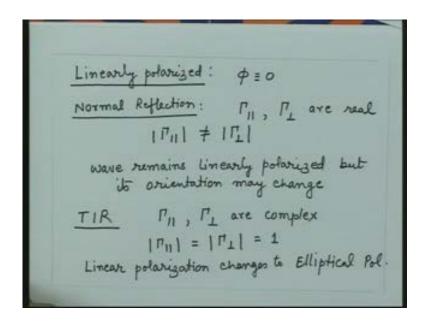


So in general then we can get the reflected wave that is lets say denoted by Er that will be equal to Er parallel plus Er perpendicular. And if I denote the reflection coefficient for the parallel and perpendicular wave with gamma parallel in 1 or perpendicular as you have done earlier. Then this quantity can be written as gamma parallel Ei parallel plus gamma perpendicular Ei perpendicular e to the power j phi. Similar thing I can do for the

transmitted wave also where Et the transmitted wave of total wave will be again a combination of the 2 polarizations. So that is tau parallel Ei parallel plus tau perpendicular Ei perpendicular e to the power j phi. Now depending upon the case whether it is a normal reflection or total internal reflection, these quantities gammas they could be either real quantities or they could be complex quantities.

We are seen earlier if you take a normal reflection then the reflection coefficient is a real quantity and it could have a positive or negative sign. But if you take the total internal reflection then these quantities gamma they become complex. Though they are magnitude becomes equal to 1. So now you are having various possibilities and that is what essentially we will see some special cases just suppose if the wave was linearly polarize and if the reflection was a normal reflection what would happen to polarization. If you have total internal reflection then what would happen to polarization and if the wave was circular what would happen and so on. So let us take now the some special cases.

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That first case let us say if the wave is linearly polarized. That means the electric field vector does not change the direction as a function of time. For the incident wave it makes

some angle like that with of incidents. So this is the plane of incidents that is the angle with electric field vector makes but these directions remain same independent of time. And as we know a linearly polarize wave is represented by combination of 2 orthogonal polarizations with no phase different. So if the wave is linearly polarize then the phase different between these 2 components is zero. That means for this case the phi is equal to 0. Once we have this then of course we get investigate, now 2 cases whether it is normal reflection or the total internal reflection. So if I consider the normal reflection case your gamma parallel and gamma perpendicular are real.

So if this quantity is real and if this phi this angle is zero, essentially these quantities which are these 2 electric field this 1 and this 1. If the initial phase different phi 0 between them and if the reflection coefficients are real the phase different between them again remains 0 or 180 degree depending upon the direction of the electric field. But in the both the cases the polarization remains linear polarization because the 1 eighty degree phase would essentially means reversal of the electric field. So the orientation of electric field will change but the polarization will remain linear polarization. So in this general case phi parallel phi perpendiculars are real and magnitude of phi parallel is not in general equal to the magnitude of gamma perpendicular. So that means a linearly polarized wave will remain linearly polarize because the phase difference between the 2 components is not going to change.

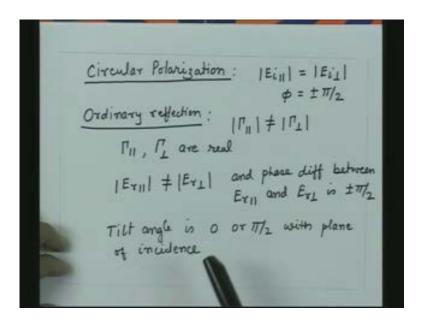
But there relative amplitude for the reflected wave might change depending upon whether this condition is satisfied or in some very special case these 2 are equal. Then these 2 reflected amplitudes also will be same. That means the ratio of the 2 components of the reflected wave are not same as it was for the incident wave. That means the direction of the electric field vector now is changed. That means the orientation of linearly polarized wave will change; but the wave will remain linearly polarized. So in this case we have an important thing that is wave remains linearly polarized but its orientation might change. So plane of polarization might change depending upon when these 2 are not equal. But the nature linearly polarize nature of the wave will be maintain if this is a normal reflection. On the other hand if I having total internal reflection then this quantities

gamma parallel and gamma perpendicular are not real these quantities are now complex quantities.

But the magnitudes of these 2 are equal now. So in this situation gamma parallel and gamma perpendicular is complex. But gamma parallel is equal to gamma perpendicular is equal to 1. So the ratio of the 2 electric fields whatever these quantities now mod of them are 1. But these quantities are now complex. So the phase different between these 2 components is going to change though the amplitude ratio which was for incident wave same thing is maintain for the reflected wave. But since now the phase different could be arbitrary, in general we will get a polarization which will be elliptical polarization. So in this case in general a polarization linear polarization changes to elliptical polarization.

So the important conclusion is if the wave is linearly polarized and if the reflection is ordinary reflection. Then the linear polarization of the wave remains unchanged whereas if the reflection is total internal reflection then the wave becomes an elliptical polarization. The same thing essentially applies to the transmitted wave also because in this case also we are going to get the similar kind of combinations. So a linearly polarize wave will remain linearly polarized after transmission also because the tau parallel and tau perpendicular will be the real quantities for the normal reflection. We can take another special case. That is the circular polarization.

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That if the incident wave is circularly polarize, then what will happen to this wave at ordinary reflection and at total internal reflection. So as the circular polarization as we know that these 2 components Ei parallel equal to Ei perpendicular the 2 components are equal and also the phase different between them is plus minus phi by 2. So the phi in this case will be plus minus phi by 2. So now in general when the wave is incident of this type and if I have ordinary reflection, again your gamma parallel magnitude is not equal to gamma perpendicular. But these quantities gamma parallel and gamma perpendicular are real. That means the phase different between the reflected wave still remains phi by 2 plus minus phi by 2. But the amplitude ratio for the reflected wave for perpendicular and parallel will not be equal because these 2 coefficients are not equal.

So in general then we have in this case Er parallel us not equal to Er perpendicular. But the phase different between these 2 with this is still plus minus phi by 2 and phase difference between Er parallel and Er perpendicular is plus minus phi by 2. And we have seen in this case that when the amplitudes are equal. But if the phase different between them is 90 degrees the wave becomes elliptically polarize. But in this case the major and

minor axis aligns with the coordinate axis for the ellipse. So in this case when the wave was incident on this like that. Initially it was circularly polarize it was rotating like this.

Now after reflection the 2 amplitudes are no more equal. So the wave is become elliptically polarize but since the phase difference between them is 90 degree still or ordinary reflection. The major and minor axis of the reflected wave ellipse will lie in the plane of incidence and perpendicular to plane of incidence. So you will have the ellipse which will be either drawn like that or ellipse which will be drawn like that. But the axis with major axis makes with the plane of incidents will be either 0 or will be 90 degrees. So the tilt angle for ellipse with respect to plane of incidents will not be arbitrary. But it will be either 90 degrees or it will be either 0. So in this case essentially we get the axial ratio will depending upon these quantities but the tilt angle is 0 or pi by 2 with plane of incidence. What will happen to total internal reflection if I take total internal reflection then the magnitude of these will be equal but the phase difference will not remain pi by 2.

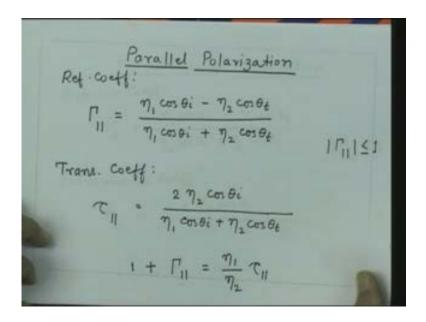
So in general again I will get a wave which will be elliptically polarized. So circularly polarize wave at ordinary reflection will become elliptically polarize with its tilt angle 0 or pi by 2 with respect to plane of incidence. Or, a total internal reflection this wave in general will become unelliptically polarized. Of course if I have a wave which is in general incident wave which elliptically polarized. Then in general we can make a statement then its state of polarization will change and depending upon the parameters right even electrically polarize wave might become circularly polarize right. Or it can become linearly polarize. So essentially what we see from this exercise that the media interface can be used to alter the state of polarization of an electromagnetic wave that is the important message essentially we should get from here.

So that means if I have a certain state of polarization for the incoming wave then by launching the wave at an arbitrary angle we should be able to change the state of polarization to the desire state. One of the special cases of this is that if the wave is some arbitrary polarize wave, is it possible that the reflected wave will be linearly polarized.

Because there are many applications where we require a wave which is linearly polarized the sources which you might have may not generate intrinsically linearly polarized wave.

So we look for some mechanism by which an arbitrary polarize wave can be convert into a linearly polarize wave. And since we saw that the dielectric interfaces can change the polarization of the electromagnetic wave. This could be 1 of the interesting situations to investigate whether an arbitrary polarization can be converted into a linear polarization. So let us now take the special cases that if the wave is incident or the certain angle. We have investigated again 2 cases 1 was the perpendicular polarization for way the reflection coefficient was given by this.

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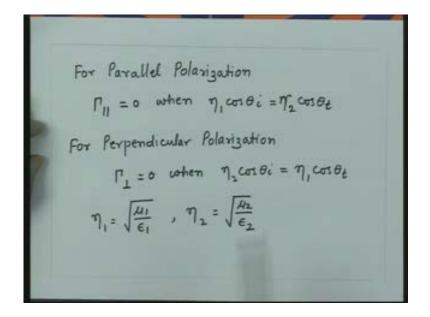
I will also investigate the parallel polarization for way the reflection coefficient was given by that. One can now ask a question at for a given parameter if this quantity becomes 0 for parallel polarization or this quantity becomes 0. That means if the reflection coefficient becomes 0 then whatever wave is incident on the dielectric interface is completely transmitted to the second media. There is no reflection at the boundary that is what reflection coefficient 0 could mean that means now if I put a perpendicularly

polarize wave at an angle when this quantity become 0. Then this perpendicular polarization will completely pass through the medium; there is no reflection.

Similarly if I take a wave which is parallel polarize and if I launch at angle we satisfy this condition then the parallel polarization will completely pass through and there will not be any reflection or this wave. So now if I have a arbitrarily polarize wave and if I launch at an angle. We satisfy this condition then parallel polarization will pass through but the same condition the perpendicular polarization will not completely pass through. So it will have a reflection. So the reflected wave will have only perpendicular polarization will pass through and the reflected wave will have only parallel polarization.

In both the situation essentially the polarization will be a linear polarization. So we have now the interesting cases that are for given parameters look for the angles for which this quantity goes to 0 or this quantity goes to 0. And at those angles 1 of the polarization will be reflected the other polarization will be completely transmitted to the second medium. So an arbitrary state of polarization can be converted into a linear polarization. So that is what essentially the investigate.

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So let us say for parallel polarization the gamma parallel becomes 0 when eta 1 cos theta i is equal to eta 2 cos theta t. So these are the reflection coefficient expression I just equate this to 0. So when eta 1 cos theta becomes eta 2 cos theta t at that point I have the gamma parallels here. Similarly for perpendicular polarization I have gamma perpendicular is equal to 0 when from this expression the gamma perpendicular goes to 0. That means eta 2 cos theta I is equal to eta 1 cos theta t. And let me remind you these quantities the eta 1 and eta 2 are the intrinsic impedance of medium 1 and medium 2. So eta 1 is square root of mew 1 upon epsilon 1 and eta 2 is equal to square root of mew 2 upon epsilon 2. And then you also have the Snell's law which relates theta i and theta t we are considering only the ordinary reflection. So by applying the substituting for theta t in terms of theta i essentially we can get the angles theta I the angles of incidences for these 2 polarization where the reflection coefficient would be 0. See if I do substitute in the do some smaller algebra manipulation.

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$$\tan \theta_{B1} = \frac{\beta_2}{\eta_1} \left\{ \frac{\eta_1^2 - \eta_1^2}{\beta_2^2 - \beta_1^2} \right\}^{1/2}$$

$$\tan \theta_{B11} = \frac{\beta_2}{\eta_2} \left\{ \frac{\eta_1^2 - \eta_2^2}{\beta_2^2 - \beta_1^2} \right\}^{1/2}$$

$$\theta_B \Rightarrow \text{Brewster Angle}$$

$$\beta_1 = \omega \sqrt{\mu_1 \epsilon_1} , \beta_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

Essentially we get the expression for this that would be tan of theta will be called that some B perpendicular. I will explain what this theta t means that is equal to beta 2 upon eta 1 eta 2 square; minus eta 1 square upon beta 2 square; minus beta 1 square, square

root. And for parallel polarization I will get tan of theta B parallel that is equal to beta 2 upon eta 2 eta 1 square minus eta 2 square; upon beta 2 square; minus beta 1 square, square. This angle at which the reflection coefficients go to 0 is called the Brewster angle. So when the parallel polarization goes to 0 we say this is the Brewster angle for parallel polarization, then perpendicular polarization reflection coefficient goes to 0 we will say that angle is the Brewster angle for perpendicular polarization. In general if you take a medium which could be even magnetic medium then the Brewster angle for both polarization exist.

So at that Brewster angle for parallel polarization the reflected wave will have only perpendicular polarized wave and for a Brewster angle which is for perpendicular polarization the reflected wave will have only parallel polarization. So B essentially this quantities here theta B is called the Brewster angle. So theta t perpendicular is the Brewster angle for perpendicular polarization and theta B parallel is the Brewster angle for the parallel polarization. So at this angle the parallel polarize wave will completely get transmitted for this angle the perpendicular polarize wave will get completely transmitted to the second medium. Now since we know this quantities beta 1 and beta 2 which is beta 1 is square root of mew 1 epsilon 1 and beta 2 will be omega square root mew 2 epsilon 2 and eta 1 and eta 2 we already got here. So if you substitute these quantities into the expression for the Brewster angle.

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$$\theta_{B\perp} = \tan^{-1} \left\{ \sqrt{\frac{\mu_{2}}{\mu_{1}}} \left[\frac{\mu_{2}\epsilon_{1} - \mu_{1}\epsilon_{2}}{\mu_{2}\epsilon_{2} - \mu_{1}\epsilon_{1}} \right]^{l/2} \right\}$$

$$\theta_{B\parallel} = \tan^{-1} \left\{ \sqrt{\frac{\epsilon_{2}}{\epsilon_{1}}} \left[\frac{\mu_{1}\epsilon_{2} - \mu_{2}\epsilon_{1}}{\mu_{2}\epsilon_{2} - \mu_{1}\epsilon_{1}} \right]^{l/2} \right\}$$
If we take a Dielectric media interface
$$\mu_{1} = \mu_{2}$$

$$\theta_{B\perp} \quad \text{Does not exist}$$

$$\theta_{B\parallel} = \tan^{-1} \left(\sqrt{\frac{\epsilon_{2}}{\epsilon_{1}}} \right)$$

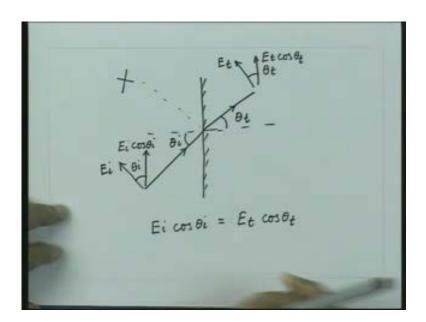
We essentially get the Brewster angle for 2 cases theta B perpendicular. That will be tan inverse square root mew 2 upon mew 1 mew 2 epsilon 1 minus mew 1 epsilon 2 upon mew 2 epsilon 2 minus mew 1 epsilon 1. And theta Brewster for parallel polarization that would be tan inverse square root of epsilon 2 upon epsilon 1 mew 1 epsilon 2 minus mew 2 epsilon 1 upon mew 2 epsilon 2 minus mew 1 epsilon 1 square root. So as we can see if the medium is general. That means if you have the permeability and permittivity different for the 2 media than both this angles would exist and at these angles 1 of the polarization essentially will get completely transmitted to the second media. However if I consider only the dielectric media that is the permeability for this 2 media are same as the free space then the mew 1 is equal to mew 2.

So if we consider only dielectric media if we take a dielectric media. Then we have mew 1 is equal to mew 2 so this quantity will be 1 the mew 1 is equal to mew 2. So this can be taken out so essentially we will get here theta B the epsilon 1 and epsilon 2 this epsilon 2 minus epsilon 1. So this quantity will become any imaginary quantity. So you will we can see here we can take here mew 1 and mew 2 which is same we can take it common. So this will become epsilon 1 minus epsilon 2 this will be epsilon 2 minus epsilon 1. So this

quantity will become minus 1 so you will get this is square root of minus 1 so which is the imaginary angle. What that means is that if we consider only the dielectric media on both sides if the permeability is same, then the Brewster angle for perpendicular polarization does not exist.

So for in this case theta B perpendicular does not exist. So there is no Brewster angle for perpendicular polarization if we consider the media which are the dielectric media. In this case the Brewster angle for parallel polarization would exist because if I take mew 1 equal to mew 2 this quantity will be same so this quantity becomes 1 now. So you will get theta B parallel which will be tan inverse square root of epsilon 2 upon epsilon 1. So in this case theta B parallel will be equal to tan inverse of square root epsilon 2 upon epsilon 1 and since the tan of theta has a range from minus infinity to infinity. So it can take any possible value from 0 to infinity for any dielectric media any values of epsilon 2 and epsilon 1 I will always great a booster angle. So we have an important conclusion that if I consider the dielectric media interface; than the Brewster angle for parallel polarization always exist. But for perpendicular polarization there is no Brewster angle. And this we can see physically as follows why for perpendicular polarization it does not exist and for parallel polarization exist.

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That let us say this is the dielectric interface and let us say the electric field was perpendicular to the plane of incidence so it was like this. Now irrespective of what is the direction of the incident wave this will always going to be like this and its amplitude is fixed. So when I apply the continuity of the electric fields I will get this field plus reflected field plus a transmitted field some of this 2 should be equal to this B. And since these amplitudes are not depending on the launching angle you will never get a condition where some of these 2 will become exactly equal to this. So I always will need the reflected wave to satisfy the boundary condition at the interface where if I consider a parallel polarization which is like this. Then the component which has the satisfy the boundary condition is the tangential component.

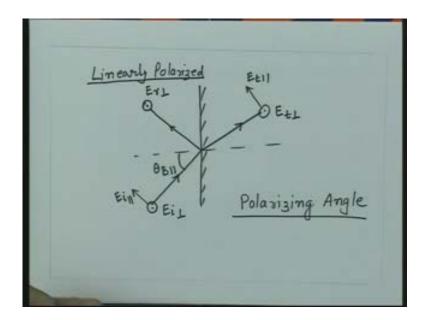
So if I take a way like that and this is the electric field this component has to satisfy the boundary condition. So if this angle is theta i, this angle is theta t then the electric field will be like that for this is the component which tangential component to the interface. So this is theta i this will theta i this will be theta t this is electric field Et this is Ei the boundary condition has to be satisfied by the tangential component of electric field. And since the tangential component is a function of theta i this is Ei cos of theta i and this is Et cos of theta t. As we change the angle it is possible at some particular angle these 2 component become equal. So Ei cos theta i it becomes equal to Equal to cos theta t then I do not require electric field due to the reflected wave to satisfy the boundary condition.

That means this wave will not be needed any more to satisfy the boundary condition at the interface. So when this condition is satisfied that Ei cos of theta i is equal to Et cos of theta t. That time this wave is absent I do not have any reflection. So the wave which is incident on this is completely transmitted to the second media. So in this case since the tangential component varies at the angle of incidents as certain angle these 2 component just become equal and then I do not have this third component to satisfy the boundary condition. However for perpendicular polarization this situation is not there. Because I have this electric field which is like this we do not depend upon the direction of the wave. So we always require this field so that some of these 2 becomes equal to the field in the second media.

Of course, if the media was a magnetic media and if the permeability for the 2 media were not same, then argument which you have for electric field will be applicable to the magnetic fields and then we will have Brewster angle for the perpendicular polarization also. But, if you take the media which are dielectric media, then the Brewster angle would exist only for the parallel polarization and the Brewster angle for perpendicular polarization could not exist. I can write this in terms of refractive indices also because its no square root of the relative permittivity is called the refractive index. So this is also equal to tan inverse of the refractive index of medium 2 divided by refractive index of medium 1.

Generally since with deal with the dielectric media we can say that we have the Brewster angle for the parallel polarization and 1 can make use of this Brewster angle for converting a arbitrarily polarize wave into a linearly polarize wave. So now let us say if the wave was arbitrarily polarized like this we can decompose this wave into the parallel polarization and perpendicular polarization. If the wave is launch at the Brewster angle then the parallel polarization will be completely transmitted to the second medium. So there is no reflection for parallel polarization. But for that angle since this is not a Brewster angle for perpendicular polarization. The perpendicular polarize wave will get reflected. So in the reflected wave we will have a polarization which will be only perpendicular polarization.

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So if I take now a media interface that and launch this way at the Brewster angle for parallel polarization. So in this case we have both the electric fields this is Ei parallel. This is Ei perpendicular and this angle now is theta B parallel. So the reflected wave which will get from here will be having only perpendicular polarization. So this wave is perpendicularly polarization so you get only Er perpendicular because parallel polarize wave is completely transmitted to the second medium. So in the second medium we will get a wave which is in general an elliptically polarize wave because we have both the components same this. So this will be Et parallel Et perpendicular. But for reflected wave we will have only perpendicular polarization which is Er perpendicular.

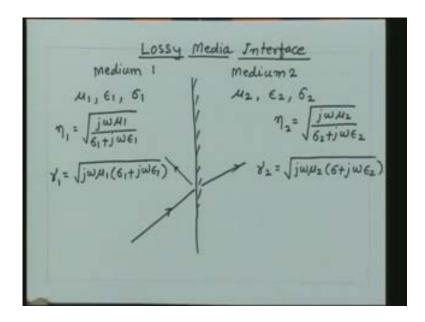
That means without worrying about what is the state of polarization of the incoming wave whether it is linearly polarized at an arbitrary angle or elliptically polarize or circularly polarize. If the wave is launch at the Brewster angle then the reflected wave will have a polarization which is linear polarization. And the orientation of that will be perpendicular to the plane of incidents. So for any arbitrarily polarize wave the reflected wave will be linearly polarize like that. That is the reason this angle is also called the

polarizing angle. Because at that angle if the wave is launched the wave reflected wave is polarize in the linear polarize fashion. So this angle is also refers to as a polarizing angle.

So at polarizing angle if the wave is launch at the dielectric interface the reflected wave is always linearly polarized wave. It finds application in polarizing the light being in the lasses are internally if the polarization was arbitrary then we can always use the concept of Brewster angle. So that the reflected wave from the boundary is always linearly polarize. So what we conclude from this discussion essentially is that by using the dielectric interfaces 1 can change the state of polarization of an electromagnetic wave is been routingly done for the optical beams. But given at other radio frequencies 1 can make use of this phenomena for changing the state of polarization of an electromagnetic wave. We will see little later when we talk about the reflection and refraction from the conducting boundaries.

How the state of polarization will get affected from the incident to the reflected wave we have seen here that if you take a circular polarization, then it may become elliptical polarization I have to become linear polarization and so on. In case of conducting boundaries in the case of little simpler, but you will find something more interesting phenomena that then we talk about the reflection from this boundaries. So this essentially gives you an idea of how to make use of the dielectric boundaries for changing the state of polarization of an electromagnetic wave. With this understanding of the dielectric boundaries now we can ask a general question. Suppose I do not have now a dielectric boundary but suppose I have a general medium media boundaries; that means the conductivity of the 2 media are not 0.

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So if I take a medium interface in general where in medium 1 and medium 2 I have permeability mew 1 epsilon 1, then also the conductivity of this medium sigma 1 but sigma 1 is not infinity it does not ideal conductor. Similarly for medium 2 we have permeability mew 2 connectivity epsilon 2 and the conductivity sigma 2. Once we have this then we have the medium parameter which we know the probation constant and the characteristic impedance or intrinsic impedances of the media. So we have for this medium eta 1 which is now square root of j omega mew 1 divided by sigma 1 plus j omega epsilon 1. You already discuss this earlier similarly for this medium I have the intrinsic impedance eta 2 which is square root of j omega mew 2 upon sigma 1 plus j omega epsilon 2. And propagation constant now is in general it is not only beta by it is gamma.

So gamma 1 is equal to square root of j omega mew 1 sigma 1 plus j omega epsilon 1; and gamma 2 equal to square root of j omega mew 2 upon sigma plus j omega epsilon 2. And now we can write down for this interface if the way is incidence on. This what will happen to the reflected in transmitted wave the interface. Recall now when we solve this problem essentially we satisfy the boundary condition in the interface. And we are

satisfies the boundary condition for the tangential component of electric field and tangential component of magnetic fields even if the conductivity is non-zero. But it is not infinite still we do not have surface currents here and because of that I can still applied the boundary condition which you applied for the dielectric cases. So the expression which we got for transmission and reflection coefficients, they are still valid for this case are the parallel case except long that you have to change this quantities et a 1 and beta 2 appropriately by this quantities where you are taken the conductivity into consideration.

So since we are still applying the boundary condition which we applied for dielectric media nothing the change then the analysis in fact we got the similar expression only these quantities eta 1 and eta 2 have become complex. And because of that in general given for the normal or the ordinary reflection also this quantity reflection coefficient is become complex. And same will happen for the transmission coefficient also. So in general media which is the losing media the analysis remains exactly identical and we can use the same expression which we derived for dielectric media by refluxing eta 1 and eta 2 by this complex quantity its takes into account the conductivity of the media. So this we can say is a lossy media interface.

So in fact appropriately choosing this intrinsic impedance in the media which are complex the analysis of lossy media is identical to what we have done for dielectric media. However there is a some conceptual questions may ask at this point if the medium was lossy that in the presence of loss this quantity gamma is a complex quantity. That means it has a real part which use you the attenuation constant same is true for this 1. Earlier when the plane wave are incident in the media interface it did not matter somewhere the wave was originated. Since the wave amplitude does not change even if you have travel infinite distance the amplitude of the wave, we same when it is incident on this medium.

However if you assume that this uniform plane wave also originated at a infinite distance on the interface, and if this quantity is complex then the wave amplitude exponentially decreases as a travels. So after traveling infinite distance the amplitude will reaches here will be 0. Similarly if I consider that the wave had finite amplitude at this point and it is a plane wave. So which if it has started from minus infinity and travel this distance then it should have amplitude which is infinite at the infinite distance on this plane. See would mean that in case of lossy medium probably you will require in finite energy. To start with if the wave starts some infinity then when the wave reaches here it is have a finite amplitude. Well this is the very hypothetical situation without getting into the issue where the wave got actually originated.

We can address this question only saying that if the wave when it reaches to the media interface has certain amplitude, then the amplitude of the reflected wave just this side of the media interface and just this side of the media interface will be given by the reflection and transmission coefficients which you have got here. When the wave of travels, now the backwards from here because there is a loss in the medium this wave will slowly at any wave. This wave amplitude will growing because it has certain amplitude here it would be exponentially growing as you go opposite to the wave of propagation. And this wave which is transmitted to the second medium also will be exponentially decreasing in the amplitude. So this situation is very straightforward.

That in this case you are having a traveling wave with propagation constant which is this. So wave of travels with slowly decreasing amplitude with depends upon the loss in the medium which is this sigma 2. Whereas when you come to medium one, the field which was going to see are superposition of the incident and the reflected wave. So this wave grows as we go away from the interface this you decase that you away from the interface. So we see super position of these 2 and these cases identical to lossy transmission line that as we go away from this junction or termination of transmission line. The reflected wave becomes weaker and weaker on a lossy line and the transmitted wave of grows larger. So, essentially as you go away from this interface you will see the phenomena which will be a only traveling kind of phenomena.

But when become close to the boundary then you will see the interference of these 2 waves and you will see some kind of a standing wave behavior. So this is exactly

identical to what we have seen a lossy transmission line. So the conclude the analysis of a lossy media interface where the conductivity is finite but not it is not neither 0 nor infinity. So that mean non-of the media or ideal conductors or either they are ideal dielectrics the problem can still be handle exactly in a same way as we did for the dielectric media. We can apply the same boundary conditions and we can apply the same expression for the reflection and transmission coefficients. Then substituting appropriately for the intrinsic impedances for the 2 media we can get the expressions for the transmission and reflection coefficient for a loss media.