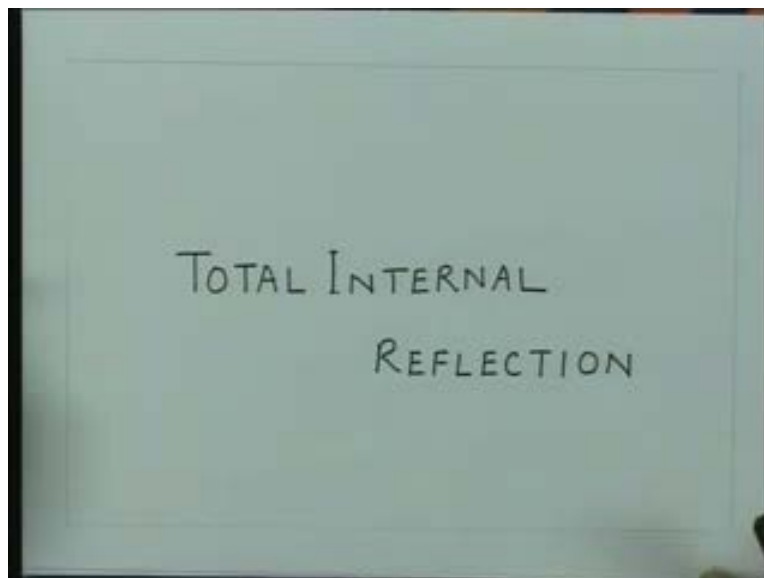


Transmission Lines & E M. Waves
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Lecture #32

We are discussing propagation of plane wave across a media interface. In the last lecture we derive the reflection and transmission coefficients for uniform plane wave across a dielectric interface for 2 polarizations in the parallel and perpendicular polarization. We are seeing that any arbitrary polarization can be decomposes into 2 orthogonal components. One, which is perpendicular to plane of incidence and other in the plane of incidence and then we can find out the reflection and transmission coefficients for these 2 cases. So that we can combine these 2 for any arbitrary polarization could I we discuss a very important case what is called the total internal reflection across the dielectric boundary.

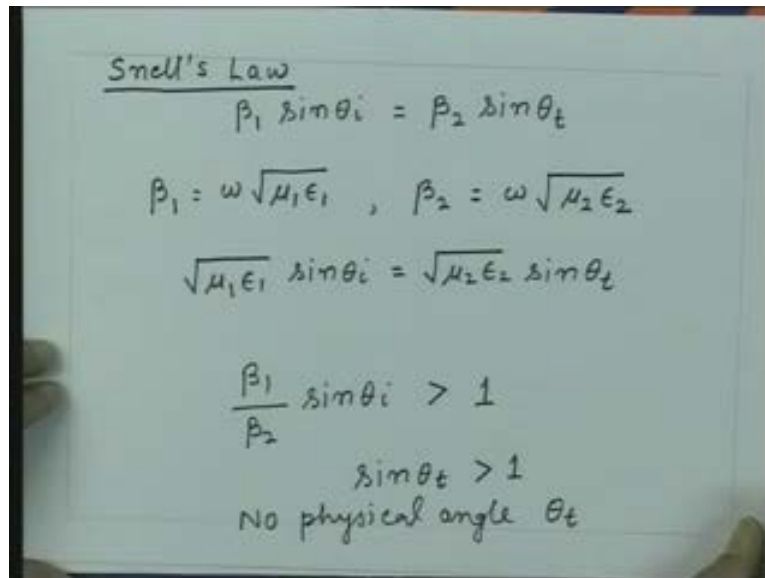
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We have studied total internal reflection at the high school level where we say when the wave is incident at an angle greater than the critical angle the power is reflected back into the

same medium. However that understanding is rather a superficial understanding. Today we will do little detail understanding of the phenomena what is called total internal reflection. We start from the Snell's law and now we are derive the Snell's law we says.

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Handwritten notes on a piece of paper showing the derivation of Snell's Law and the condition for total internal reflection.

$$\text{Snell's Law}$$

$$\beta_1 \sin \theta_i = \beta_2 \sin \theta_t$$

$$\beta_1 = \omega \sqrt{\mu_1 \epsilon_1}, \quad \beta_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$\sqrt{\mu_1 \epsilon_1} \sin \theta_i = \sqrt{\mu_2 \epsilon_2} \sin \theta_t$$

$$\frac{\beta_1}{\beta_2} \sin \theta_i > 1$$

$$\sin \theta_t > 1$$

No physical angle θ_t

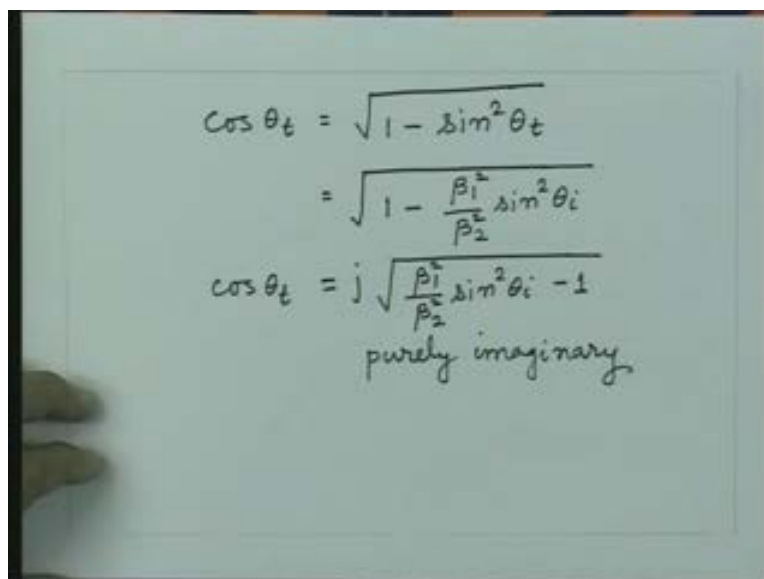
Here $\beta_1 \sin \theta_i$ that is equal to $\beta_2 \sin \theta_t$ where θ_i and θ_t are the angle incidents and angle of transmission. And β_1 and β_2 are the phase constants in the 2 media if you write down this β_1 and β_2 explicitly in terms of medium parameters we have seen that β_1 is $\omega \sqrt{\mu_1 \epsilon_1}$ and β_2 is $\omega \sqrt{\mu_2 \epsilon_2}$. So the Snell's law which we have derived from matching the phase condition for the incident reflected and transmitted waves is valid for the fields in the 2 media for any value of $\mu_1 \epsilon_1$ or any frequency ω . See if I substitute this β_1 and β_2 we can get in terms of these medium parameters the Snell's law which is $\mu_1 \epsilon_1 \sin \theta_i$ that is equal to square root of $\mu_2 \epsilon_2 \sin \theta_t$.

Now while doing this simply we match the phase condition and you also said that the electric and magnetic fields have in the transfer nature. However we have said explicitly

that this wave should be traveling in the medium 1 or medium 2 and this is what the very important thing which essentially will be discussing in this lecture. See imagine a situation that is suppose this quantity beta 1 divided by beta 2 sin theta I, if this become greater than 1 and which really happen for a given value of theta i, I will have a certain value for sin theta i. And if the medium parameters were such that beta 1 by beta 2 with sin theta i that quantity is greater than 1, then this sin of theta t will be greater than 1. What that means is there is no physical angle theta t for which this condition is satisfies.

Now this angle which you have got sin theta t that is a direction of the wave propagation. So what that means is that if this condition is satisfied that beta 1 sin theta i upon beta 2 is greater than 1 is greater than 1. Then there is no physical angle in medium 2 that means there is no way of nature remaining in medium 2 because there is no direction in which the wave is propagated this angle is imaginary angle. But for these angle also the phase condition is satisfy. So whatever fields we have written they are still satisfy the boundary conditions; only thing is there is no direction in medium 2 in which the wave is propagated and that is the very interesting case. See if I have this condition satisfy then sin of theta t is greater than 1. So there is no physical angle theta t. See if we take this condition then we note that's in this quantity beta 1 upon beta 2 sin theta is greater than 1.

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$$\begin{aligned}\cos \theta_t &= \sqrt{1 - \sin^2 \theta_t} \\ &= \sqrt{1 - \frac{\beta_1^2}{\beta_2^2} \sin^2 \theta_i} \\ \cos \theta_t &= j \sqrt{\frac{\beta_1^2}{\beta_2^2} \sin^2 \theta_i - 1} \\ &\text{purely imaginary}\end{aligned}$$

And I require in my reflection coefficient expression a quantity which is \cos of θ_t . This we can write as square root of $1 - \sin^2 \theta_t$. I can substitute for \sin of θ_t from this relation. So from here I can get square root of $1 - \beta_1 \text{ upon } \beta_2 \sin^2 \theta_i$ since this quantity now is greater than 1 just a condition we are talking about in this particular case. So this quantity will be always negative. So this quantity will be always imaginary. So I got I can do is I can take the imaginary constant j explicitly out of this and I can change the \sin of the terms inside the square root bracket.

So we can write $j \sqrt{\beta_1 \text{ upon } \beta_2 \sin^2 \theta_i - 1}$ where this quantity now is a real quantity because $\beta_1 \text{ upon } \beta_2 \sin^2 \theta_i$ is greater than 1, so that is why this quantity is positive. So this total square root is a real quantity. So, \cos of θ_i is an imaginary quantity. So we see if this case is quantity \cos of θ_t that is purely imaginary. So \sin of θ_t for this case is greater than 1 \cos of θ_t is purely imaginary both essentially are telling you that there is no physical angle θ_t in which the wave will be propagating.

However, as you have said that for this angle θ_t and θ_i still will be having the refraction and transmission coefficients. So I can now go back and substitute these values for θ_t in the reflection coefficient expression and we have seen the 2 reflection coefficient cases. One was the perpendicular case for which reflection coefficient must be given by that and we have a parallel case or where the reflection coefficient is given by this at the time we are pointed out that essentially it is the interchanging of η_2 and η_1 otherwise the expression for the 2 polarizations is similar.

So for parallel polarization is $\eta_1 \cos \theta_i - \eta_2 \cos \theta_t$ whereas for perpendicular polarization is $\eta_2 \cos \theta_i - \eta_1 \cos \theta_t$. So in both these expressions if I substitute, now for $\cos \theta_t$ which is a purely imaginary quantity, the reflection coefficient terms they become complex. So I can write explicitly for these 2 cases I get from here.

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Ref. coeff for || polarization:

$$\Gamma_{11} = \frac{\eta_1 \cos \theta_i - \eta_2 \cos \theta_t}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}$$

$$= \frac{\eta_1 \cos \theta_i - j \eta_2 \sqrt{\beta_1^2 / \beta_2^2 \sin^2 \theta_i - 1}}{\eta_1 \cos \theta_i + j \eta_2 \sqrt{\beta_1^2 / \beta_2^2 \sin^2 \theta_i - 1}}$$

$$= \frac{a - j b}{a + j b} = 1 \angle -2 \tan^{-1}(b/a)$$

Total Internal Reflection

The reflection coefficient for parallel polarization and that will be gamma parallel at will be equal to eta 1 cos theta i minus eta 2 cos theta t upon eta 1 cos theta i plus eta 2 cos theta t. And if I substitute for cos of theta t from here this will be equal to eta 1 cos theta i minus j times eta 2 square root beta 1 square upon beta 2 square sin square theta i minus 1 divided by eta 1 cos theta i plus j times eta 2 square root the same quantity which is beta 1 square upon beta 2 square sin square theta i minus 1. The similar expression we will get for perpendicular polarization also, we simply eta 1 eta 2 interchange. So the important thing the note here is that these expression is of the form a minus jb divided by a plus jb; where a is this quantity in this case eta 1 cos theta i and B will be the quantity eta 2 and this with square root of beta 1 square upon beta 2 square sin square theta i minus 1 and a and b are the real quantities in this case.

So the impedance thing you note now is that this quantity always has the magnitude which is 1. Because magnitude of the numerator is square root of a square plus b square same is true for the denominator. So this will be always having the magnitude which is 1 and an angle which will be tan inverse of minus b by a 2 times of that; so that will be minus 2 times tan inverse of b by a. So since the modulus of reflection coefficient is 1

whatever power is incident on the media now is fully deflected. Because magnitude of reflection coefficient is 1, as we have seen earlier this tells me the reflection coefficient of electric field. That means the amplitude of the electric field of the reflected wave with respect to the incident wave and pointing vector is proportional to E square. So the reflection coefficients square if I take that is proportional to the pointing vector.

So this quantity if this magnitude is 1, that means the pointing vector is relating pointing vector with respect to incident wave that is now 1. That means whatever power is incident that the dielectric interface the entire power now is reflected back into medium 1. Now this was a very special case because we are not seen this case earlier whenever we talked about the reflection and refraction in the previous cases we had a reflection coefficient which was always greater than 0 and less than 1. So we had not seen a case where the reflection coefficient is either 0 except when the 2 media are same. Or the reflection coefficient is 1 when the entire power is reflected. So in this case even if you have a dielectric boundary we get the entire power reflected that in medium 1 of course with the phase change which is given by this.

So since there no power transmitted to the second medium and the entire power is reflected back in the first medium. This phenomenon is called the total internal reflection. So this quantity and gives you what is called total internal reflection. So as I mention this condition is satisfied for even the perpendicular polarization because you have similar expression except ϵ_1 and ϵ_2 for interchange. So total internal reflection phenomena will take place for both polarization. That means for any arbitrary state of polarization. Then we can ask what are the conditions under which this total internal reflection phenomena will take place. And what are the implications of this total internal reflection phenomena for the wave propagation across the boundary. So 1 thing is very clear if total internal reflection has to take place then this condition has to be satisfied. That means $n_1 \sin \theta_i$ should be greater than 1.

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Handwritten notes on a whiteboard:

$$\frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}} \sin \theta_i > 1$$

1. If $\sqrt{\mu_1 \epsilon_1} > \sqrt{\mu_2 \epsilon_2}$
Then we can get TIR
For non-magnetic materials $\mu_1 = \mu_2$
 $\sqrt{\epsilon_1} > \sqrt{\epsilon_2}$
 $n_1 > n_2$

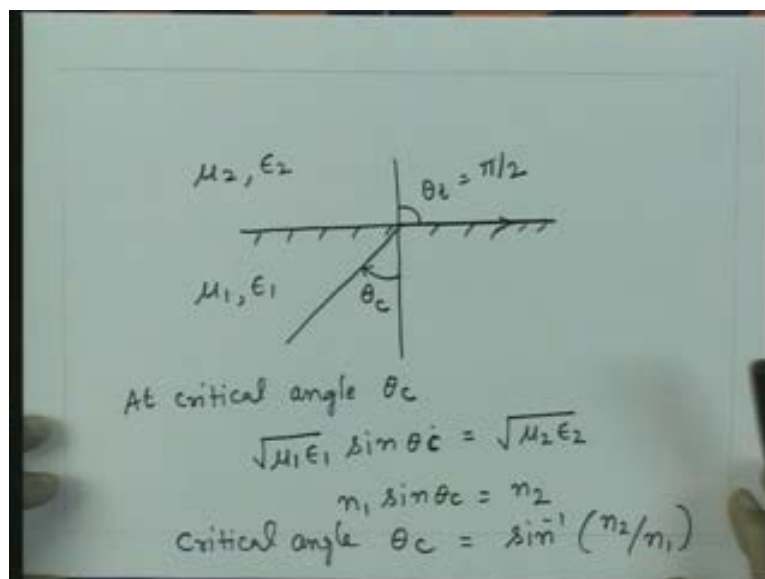
So for substitute explicitly for beta 1 and beta 2 essentially what we are looking for condition $\sqrt{\mu_1 \epsilon_1} \sin \theta_i$ should be greater than 1. Now since θ_i can go maximum up to 90 degrees at when this quantity is maximum value is 1. So the condition under which the total internal reflection will take place or its possible when this quantity is square root of $\mu_1 \epsilon_1$ is greater than square root of $\mu_2 \epsilon_2$. So we want first a condition at if square root of $\mu_1 \epsilon_1$ is greater than square root of $\mu_2 \epsilon_2$.

Then there is a possibility of total internal reflection that means there then there is an angle θ_i for which we will have total internal reflection then we can get total internal reflection. If you recall this is the condition essentially same as saying that the medium 1 is a denser medium compare to the medium 2. So when the wave goes from a denser medium to a rarer medium then there is a possibility of total internal reflection. If you lawn the ray at an angle which is greater than certain value. For a pure dielectric when the permeability μ_1 and μ_2 are equal, essentially this condition reduces 2 square root of ϵ_1 is should be greater than square root of ϵ_2 .

So for non-magnetic material μ_1 is equal to μ_2 . So we get the condition from here a square root ϵ_1 should be greater than square root ϵ_2 and we have seen square root of ϵ_1 which is ϵ_0 into relative permeability for medium 1 and square root of that quantity is a refractive index of the medium. So this gives me the condition and n_1 should be greater than n_2 where these are the refractive indices of the 2 medium. That is the condition normally we have talked about in our high school optics that if the medium 1 is denser compare to medium 2, then there is a possibility of total internal reflection.

And there we talked only about the refractive indices which were n_1 n_2 which was square root of relative permittivity of medium and the relative permittivity of medium 2. However now we know that the refractive index is not defining square root of ϵ_1 , it is actually related to the relative permeability and relative permittivity. So this is more like a general statement saying that if this condition is satisfied then there is a possibility for total internal reflection. Now at an angle when this quantity is equal to 1 that means the \sin of θ_c has become equal to 1. That means θ_c have become equal to 90 degrees at that angle we call the critical angle and beyond which if the rays is launch then there is no ray existing into the second medium.

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So phenomena is here we are having medium which is μ_1 ϵ_1 here you have medium μ_2 ϵ_2 and if I launch a ray such that this angle is θ_c what is called the critical angle. So at critical angle θ_c I have the equality $\sqrt{\mu_1 \epsilon_1} \sin \theta_i$ or θ_c now critical angle that is equal to square root of $\mu_2 \epsilon_2 \sin \theta_t$ which is equal to 1. So the ray is essentially going in the direction along the dielectric interface. So this is the electric interface at the critical angle the ray essentially travels along the dielectric interface.

So this angle θ_t which is equal to $\pi/2$ and as we mention here beyond this, if the angle is increase beyond θ_c then this wave does not exist. So we do not know what does happen to this wave. In fact the angle θ_t becomes imaginary. So we do not know where the rays gone but systematically if I am change the angle from 0 to θ_c systematically the refracted wave or the transmitted wave will change systematically from here up to $\pi/2$ and beyond that the wave nature in medium 2 will be lost with mod no what does happen to that way. So at critical angle essentially this condition is satisfied and then that is what we have seen for a dielectric media when $\mu_1 = \mu_2$ again we get the same my condition for the refractive indices that is $n_1 \sin \theta_c$ that is equal to n_2 or θ_c critical angle is equal to $\sin^{-1}(n_2/n_1)$.

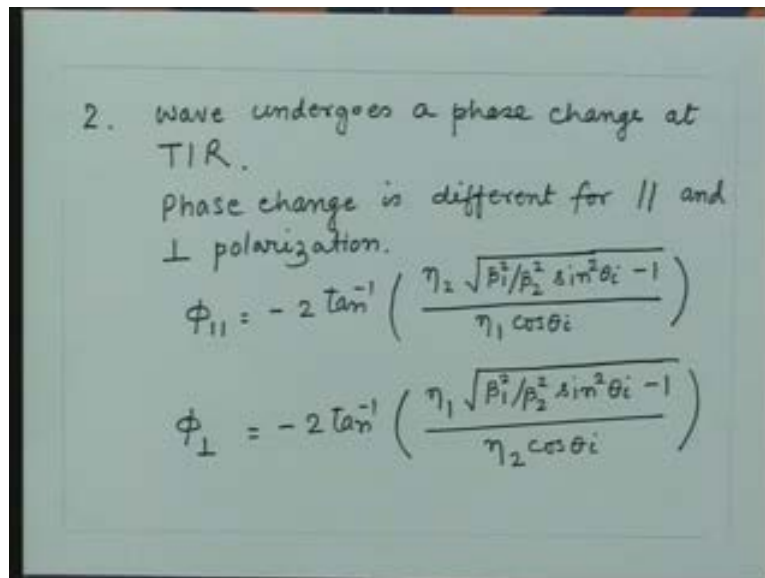
So from this general derivation which you have done we can get this special cases which we have been discussing past in the in our high school. What we had not discussed however then was that when the total internal reflection takes place and if the angle is greater than critical angle that will happen. The wave which is reflected undergo the phase change what we talk there only about the directions and we said the whole power is reflected and that is what the phenomena which was total internal reflection phenomena. However now you want to see little deeper to this phenomena and 1 of the important thing which we note here is that when total internal reflection takes place we get a phase change abruptly at the dielectric interface.

So there is a phase change introduce between the incident wave and the reflected wave abruptly at the junction which is the dielectric interface. And this quantity since b and a

are related to the angle of incidents and the medium parameters beta 1 and beta 2. Essentially the phase change is a function of the angle of incidents and the medium parameters. So in general then we can say that by choosing appropriate parameters for the media and the angle of incidents we can generate an arbitrary phase difference between the incident and the reflected wave. And this will happen for both the cases for the parallel polarization and perpendicular polarization.

However important in the note here is that for parallel polarization behaves eta 1 here and eta 2 here. Whereas for perpendicular polarization eta 2 is here eta 1 is here. So the 2 quantities b and a are different for parallel and perpendicular polarization. What that means is that the phase change with the wave undergoes for 2 polarizations are different. So parallel polarization undergoes certain phase change perpendicular polarization could undergo some other phase change. But 2 polarizations will not have the same phase change except when this quantity is 0. So in general then we can say that the first point we have seen for having total internal reflection we must satisfy this condition.

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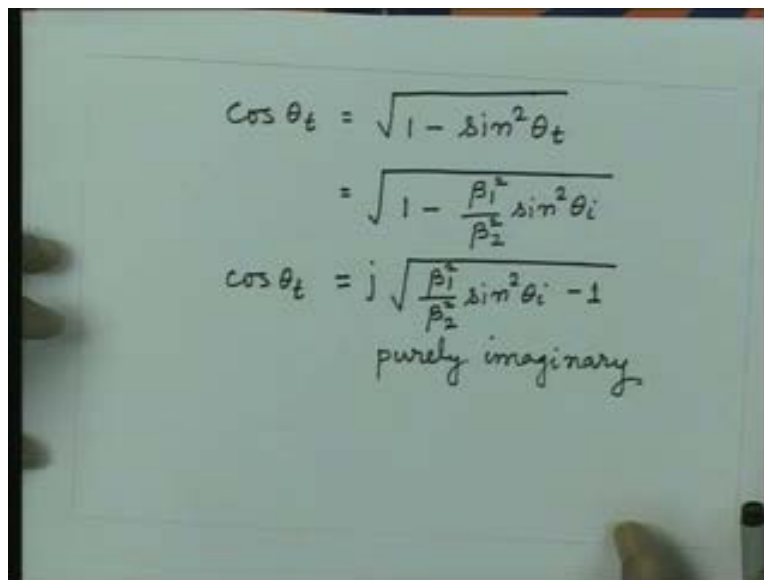
2. wave undergoes a phase change at TIR.
 Phase change is different for // and \perp polarization.

$$\phi_{||} = -2 \tan^{-1} \left(\frac{\eta_2 \sqrt{\beta_1^2 / \beta_2^2 \sin^2 \theta_i - 1}}{\eta_1 \cos \theta_i} \right)$$

$$\phi_{\perp} = -2 \tan^{-1} \left(\frac{\eta_1 \sqrt{\beta_1^2 / \beta_2^2 \sin^2 \theta_i - 1}}{\eta_2 \cos \theta_i} \right)$$

The second thing about total internal reflection we can now notice the wave undergoes a phase change at total internal reflection and phase change is different for 2 polarization phase change is different for parallel and perpendicular polarization. If you write down explicitly the phase changes I will get the phase change ϕ_{\parallel} that will be minus 2 times tan inverse of η_2 square root of β_1 square upon β_2 square sin square θ_i minus 1 divided by $\eta_1 \cos \theta_i$ and ϕ_{\perp} perpendicular could be minus 2 times tan inverse is η_1 square root β_1 square upon β_2 square sin square θ_i minus 1 divided by $\eta_2 \cos \theta_i$. So the second important thing that the 2 polarizations do not undergo the phase change as we will see later will have the implications about the change of data polarization. When the total internal reflection takes place at the dielectric boundary, the third important thing is that when this condition is satisfied and $\cos \theta_t$ will be derived is now given by this which is purely imaginary quantity.

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$$\begin{aligned}\cos \theta_t &= \sqrt{1 - \sin^2 \theta_t} \\ &= \sqrt{1 - \frac{\beta_1^2}{\beta_2^2} \sin^2 \theta_i} \\ \cos \theta_t &= j \sqrt{\frac{\beta_1^2}{\beta_2^2} \sin^2 \theta_i - 1} \\ &\text{purely imaginary}\end{aligned}$$

I can ask what kind of field variation we are going to have in the second medium. So let us say we write the field expression for third things.

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3. Fields in medium 2.

$$\begin{aligned}
 E_t &= E_t e^{-j\beta_2(x \sin \theta_t + z \cos \theta_t)} \\
 &= E_t e^{-j x \beta_2 \sin \theta_t - j z \beta_2 \cos \theta_t} \\
 &= E_t e^{-j x \beta_1 \sin \theta_i \pm z \beta_2 \sqrt{\beta_1^2 / \beta_2^2 \sin^2 \theta_i - 1}} \\
 &= E_t \underbrace{e^{-z \sqrt{\beta_1^2 \sin^2 \theta_i - \beta_2^2}}}_{\text{Decaying field}} \cdot \underbrace{e^{-j x \beta_1 \sin \theta_i}}_{\text{Travelling wave in } x\text{-direction}}
 \end{aligned}$$

The fields in medium 2 as we have done earlier this is the transmitted field which is having some amplitude E_t and we will have the phase which is minus $j \beta_2 x \sin$ of θ_i plus $z \cos$ of θ_t this was second medium \cos of θ_t . This was the phase expression which we had for a transmitted wave in medium 2 this is the amplitude. And now what we can do is we can substitute for \cos of θ_t and in terms of θ_i and same thing we can do for \sin of θ_t also. So we have got a condition from the Snell's law that $\beta_2 \sin \theta_t$ is same as $\beta_1 \sin \theta_i$. So this is $E_t e$ to the power minus j that if you write down explicitly. So this is $x \beta_2 \sin \theta_t$ minus $j z \beta_2 \cos \theta_t$. See if I substitute for $\cos \theta_t$ in this then this is quantity is no more can be imaginary quantity. So it does not represent phase.

Of course when we got here the square root sin here we should ideally choose the both the signs plus or minus and choose the sign appropriately. So that it correctly represent in field variation in the medium 2. And it will be become clear what should be the sign chosen so that we get the correct field variation. See if I substitute now for $\sin \theta_t$ and $\cos \theta_t$ this will be $E_t e$ to the power minus j and this is $\beta_1 \sin \theta_i$. So this becomes $x \beta_1 \sin \theta_i$ and this quantity will become plus minus I substitute here for

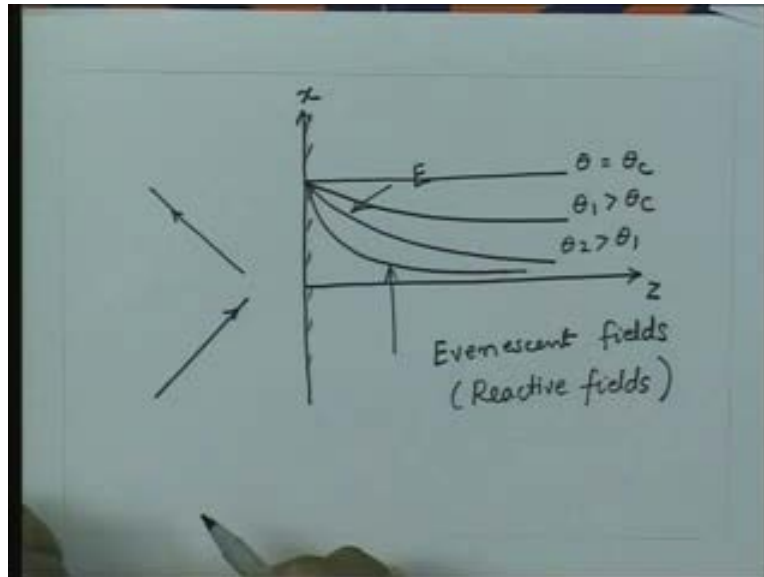
this so that will be $z \beta_2 \sqrt{\beta_1^2 \sin^2 \theta_i - \beta_2^2}$. I can take this β_2 inside to a simplify; but the important into note here now is that this is the term which is representing the phase.

But this is does not have any phase it is showing you now the variation of the amplitude of electric field as a function of z . That is, as I go deeper into medium 2 the field is not only changing getting change in phase but also its amplitude is going to change. Now since this quantity is real quantity either depending upon the sign which we chose here either positive or negative, either this field will grow exponentially as we go deeper inside the medium for it will die exponentially as we go deeper inside the medium. Since the energy source is coming from medium 1 which is the incident field, there is a possibility of this field growing in definitely as we go inside the medium 2.

So the positive sign is not the appropriate sign because it will tell me that the field amplitude is going to infinity as we have gone deeper infinitely inside medium 2. So we should o appropriately the sign which is the negative sign which correctly represents the field which are exponentially decaying field inside medium 2. So what from here then we get that the amplitude term. Now is original amplitude times e to the power minus z and β_2 inside. This is $\sqrt{\beta_1^2 \sin^2 \theta_i - \beta_2^2}$ and the phase term e to the power minus $j x \beta_1 \sin \theta_i$. So the fields are exponentially lying down as a function of z . And they are having a phase variation now only in the direction x .

What does it mean? It means that this term when we combine with time phase which is ωt gives me a traveling wave nature only in the x direction. And there is no traveling wave in z direction. Earlier if we start from here as we know if I did not have total internal reflection it has the phase variation in x direction it has a phase variation in z direction. So we had essentially a traveling wave of nature in both in the directions x and z . However now we see that the traveling wave of nature is only in x direction and the fields are not traveling z direction. So these 1 essentially represent the decaying fields and this term gives you a traveling wave in x direction.

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If I plot this fields in the medium the situation is like this this is your dielectric interface. It is not worry about how the fields are going to be medium 1 it just focuses only on medium 2. I a certain value of the electric field at the interface and now this field is exponentially lying down as we move deeper in z direction. Do this is my, the electric field value and this variation decaying field in z direction are traveling in x direction. So this is the direction in which the fields are going to travel in the x direction also what we note from these expression is when these 2 terms are equal and that is the condition corresponding to the critical angle condition the field does not down exponentially anymore. So the amplitude of the field remains constant in medium 2.

Again it is not traveling but the amplitude of this field is remaining constant in medium 2. So it critical angle the field remains constant. So this is θ is equal to θ_c and as I increase angle θ beyond θ_c . The decay of the field becomes more and more sharp. So this is you may have field decay with like this. So here I have θ_1 greater than θ_c this is θ_2 greater than θ_1 and so on. So as the angle of incidence increases more and more beyond the critical angle the fields essentially get more and

more confined towards the dielectric interface. So you get here the fields which are getting more confined because this function is lying down more rapidly.

So it critical; angle the field essentially extend up to infinitely with constant amplitude and as we go beyond the critical angle then the field start getting more and more confined towards the dielectric interface. But the important thing is that in no situation the field will go to 0 in medium 2. These fields will be always non zero in medium 2. And theoretically whatever small amplitude it is these fields will be extending up to infinity in the directions z . So when the total internal reflection takes place from medium 1 at this interface. Maybe earlier talked about total internal reflection high school level it use to given impression when the entire power is reflected in medium 1 nothing exist in medium 2.

And in fact there was no description at all about what are the fields or what is the behavior what is the role this medium 2 as to play in total internal reflection. Once we have the critical condition satisfied can be removes the medium from beyond this point. Now answer is no. We cannot do that because these fields which require continuity at this point and a functional form which is like this for total internal reflection or as important as the fields in this region. So when total internal reflection phenomena take place we always concentrate only on these fields which are in the region where total internal reflection has taken place. But the field in the second medium or as important as these fields because these are the fields which are supporting this total internal reflection phenomena.

If any disturbance is created to these fields the total internal reflection phenomena also will be disturb. And there will not the total internal reflection there will be always power which will which will be thrown to medium 2. So therefore now it is important that if you wanted to have a good total internal reflection phenomenon. Firstly we must have these fields as confined to the media interface as possible. That means we must launch a wave within angle which is as larger compare to critical angle as possible. So if the field die down very rapidly. Secondly we must provide certain region from the dielectric interface

ideal of the infinite but in practice it will never happen. So certain region beyond the dielectric interface where these fields are protected. If that condition is guaranteed, then we will have a total internal reflection phenomena in medium 2.

Otherwise theoretically unless I provide an infinite medium here and these fields properly protected we cannot have an ideal total internal reflection phenomenon. So we have important conclusions that when the total internal reflection takes place, these fields exist. That means the transmission coefficient is not 0. So you should also note important thing that when the total power is reflected the mode of reflection coefficient is become equal to 1 does not mean a transmission coefficient is become 0. You may still have transmission coefficient okay which is non-zero. But these transmission coefficients do not always mean there is a power flow in the medium 2. Say, as we have seen these fields which are lying down here is the do not constitute any power flow because the wave is not traveling inside the medium the wave is still traveling in this medium with along the interface.

So there is no power flow in the z direction. But the fields exist. So we should very clearly understand the difference between having fields and no power flow and having field and having power flow. We may have the electric and magnetic fields and they may constitute a power flow as we have seen in the earlier cases. However we may have a situation like this, where we have electric and magnetic fields but there is no power flow electric and circuit terminology we can call these fields then like a reactive field that they have the energy stored in this and no net power flow. But this energy stored at least in the transient phase when the field circuit in set up. So the picture 1 can imagine visualize this as follows. When the wave is incident on this in the transient phase.

When the waves are getting set up there is some power flows to the second medium that power flow get stored in the energy the second medium in this fields which are decaying fields. And then once these fields are set up in the steady state there is no net power flow into the second medium and then the power flow essentially medium 1. These fields then are also referred as the evanescent fields. So these fields are reactive fields. So these fields will exist in medium 2 only they will not constitute any power flow the power flow

will be along the interface because in that direction there is a traveling wave. Now what is happening in medium 1? Now we had a wave which is incident in this direction and then we had a reflected wave which came here. The reflection coefficient is now 1 and there is an arbitrary phase change at this point. See if I look at now the fields in medium 1 field in medium 1.

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Field in medium 1

$$E_i = E_i e^{-j\beta_1(x \sin \theta_i + z \cos \theta_i)}$$

$$E_r = E_r e^{-j\beta_1(x \sin \theta_i - z \cos \theta_i)}$$

$$E = E_i e^{-j\beta_1 x \sin \theta_i} \left\{ e^{-j\beta_1 z \cos \theta_i} + e^{j\beta_1 z \cos \theta_i} \right\}$$

Travelling wave Fully standing wave

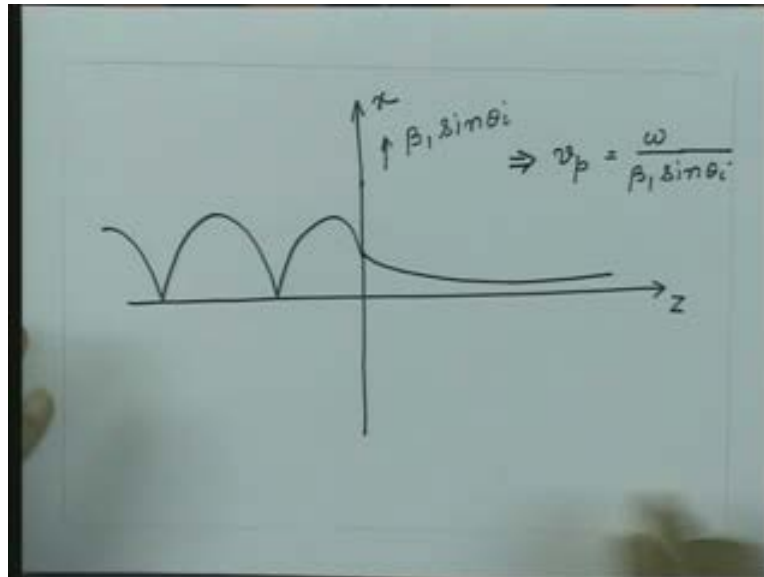
$E_r = E_i e^{j\phi}$

So I have E_i where is amplitude E_i e to the power minus $j\beta_1 x \sin \theta_i + z \cos \theta_i$. And reflected wave E_r that is $E_r e$ to the power minus $j\beta_1 x \sin \theta_i - z \cos \theta_i$. Now the wave is this traveling back wards compare to the z axis that is why we have negative sign. And as you have done earlier we can write down this electric and this electric fields total electric field then in medium 1 E that will be some of these 2. And now E_r mode of E_r is same as mode of E_i because the reflection coefficient magnitude is 1 a total internal reflection only thing it has a phase change. So we can write this quantity E_r is equal to E_i with some phase change so let us say e to the power $j\phi$ where depending upon the polarization this ϕ could be parallel or it could be perpendicular. So I can write here so this will be your E_i this term is same.

So I can take it common so this will be $e^{-\alpha z} \cos(\beta_1 x \sin \theta_1 + \phi)$ this quantity $e^{-\alpha z} \cos(\beta_1 x \sin \theta_1 + \phi)$ plus $e^{-\alpha z} \cos(\beta_1 x \sin \theta_1 + \phi + \pi)$ so $e^{-\alpha z} \cos(\beta_1 x \sin \theta_1 + \phi)$ minus $e^{-\alpha z} \cos(\beta_1 x \sin \theta_1 + \phi)$. If you now recall this term but the amplitude is equal to 1, so this is the standing wave there is a phase difference here. So depending upon the phase difference what will be the amplitude at z equal to 0 that will change but this is always going to create a complete constructive or destructive interference. So these terms are identical to what you have seen in transmission line case when the reflection coefficient was equal to 1. So when if the element which is terminating in transmission line is reactive it used have a phase of the incident and reflected waves.

But these 2 amplitudes for the 2 waves were equal and that is why we had a complete constructive and destructive interference. So this 1 essentially now is representing the fully standing wave and this term is representing a traveling wave. So in medium 1 if I look at this phenomenon of total down reflection what is going to happen? We will have a fully developed standing wave in z direction. But a traveling wave in x direction. Then I look at this phenomenon in medium 2 it is exponentially decaying fields in medium 2 and a traveling wave with the same phase constant $\beta_1 \sin \theta_1$ in the x direction. So if I plot this now how the fields will look like in the 2 media.

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Let us say this is z direction now depending upon the phase of the reflection I may have a certain value for this quantity at this point. Let us say this and then I have a fully standing wave so I may get the field variation which might have small value here. So here and from the continuity of the field just inside this dielectric interface in medium 2 the field are same but they are exponentially diag. So if I now look at the field distribution the field distribution will be like a corrugated surface in medium 1 exponentially decaying thing in medium 2. But what is the constant phase plane now the constant phase plane is decided by only this term.

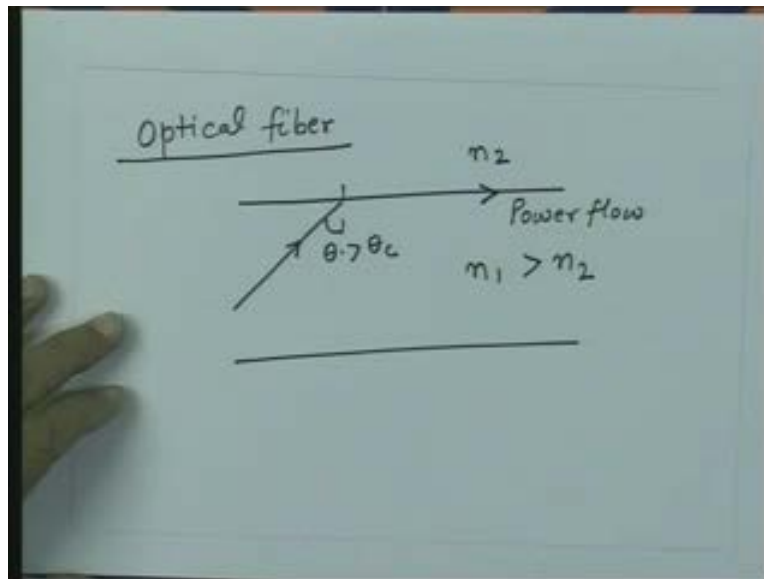
So the constant phase plane is the y - z plane. So now the important thing is that the constant amplitude planes are these planes just were the amplitude is constant. But the constant phase plane is this plane and the wave is traveling and this direction. So this very complex amplitude distribution of the waves travels in the x direction with this with a phase constant which is equal to $\beta_1 \sin \theta_i$. If I want to see now whether the total picture of total internal reflection phenomena, essentially whenever total reflection phenomena sets up you have this complicated distribution created the amplitude

distribution in the space are 1 side, you are having exponentially decaying field, other side you are having standing wave kind of fields.

And these amplitude distribution travels in this direction like this with a phase velocity which is given by that. So this phase constant essentially we give me the phase velocity along x direction that will be ω divided by β which is $\beta \sin \theta$. So this is the phase velocity along the x direction. So now if I ask which direction the net power flow is these fields do not constitute any wave. So there is no power flow in the z direction if there is no power flow in this medium in z direction there cannot be any power flow in the z direction in medium 1 also. Because if the power had come in this direction and if nothing has gone in this medium along z direction, there is no way here to consume power we are talking about completely lossless media. So that means whatever power came in this direction the power essentially get reflected by just what is standing wave that showing you.

So the fully standing wave of nature essentially tells me that there is no power flow perpendicular to the interface along the normal if I calculate there is no power flow here. But there is power flow along the interface you are having the phase velocity you are having the traveling wave. So when the total internal reflection phenomena take place, the net power flow is along the interface or in other words what we are now seeing is this dielectric is capable of guiding electromagnetic energy. We can make use of the dielectric interface for guiding wave electromagnetic energy along with them. Precisely, thus what is use in the structure what are called wave guides and especially when we talk about dielectric media, that is what happens in optical fibers. That optical fiber is nothing but a structure which is having a dielectric interface and that the structures if I consider optical fiber like this.

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I have some refractive index here n_1 another refractive index here n_2 and n_1 is greater than n_2 . So if I now consider an electromagnetic wave launch at an angle like that and if this angle is greater than the critical angle $\theta > \theta_c$. Then there will be total internal reflection phenomena in this medium the field outside this will be exponentially decaying either net energy flow will be along the interface along this direction the power flow. So if I create some structure like this which is optical fiber it can carry the electromagnetic energy over a very long distance without any loss as such because this is the total internal reflection phenomena. So energy is confined to this region and it can propagate along this for a very long distance. So total internal reflection phenomena has played a very important role in the modern optical communication and where in the design of this structure what are called optical fibers.