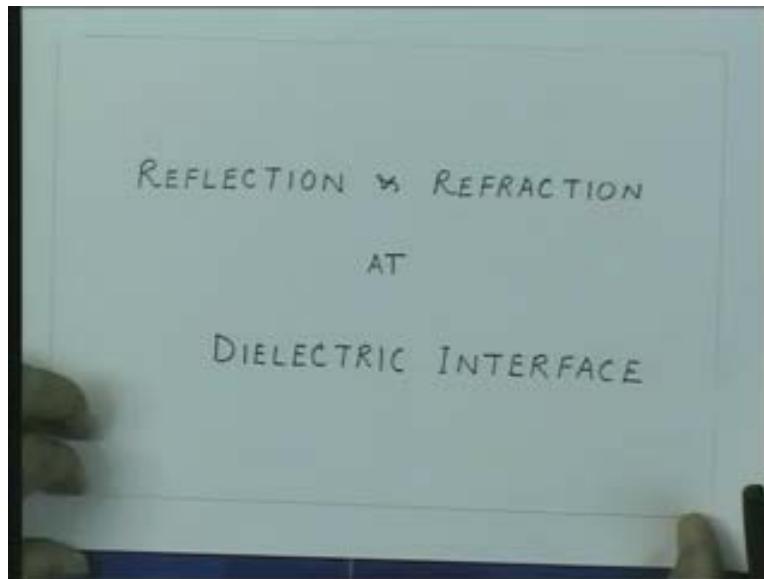


**Transmission Lines & E M. Waves**  
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**Lecture #31**  
**Reflection and refraction at media interface**

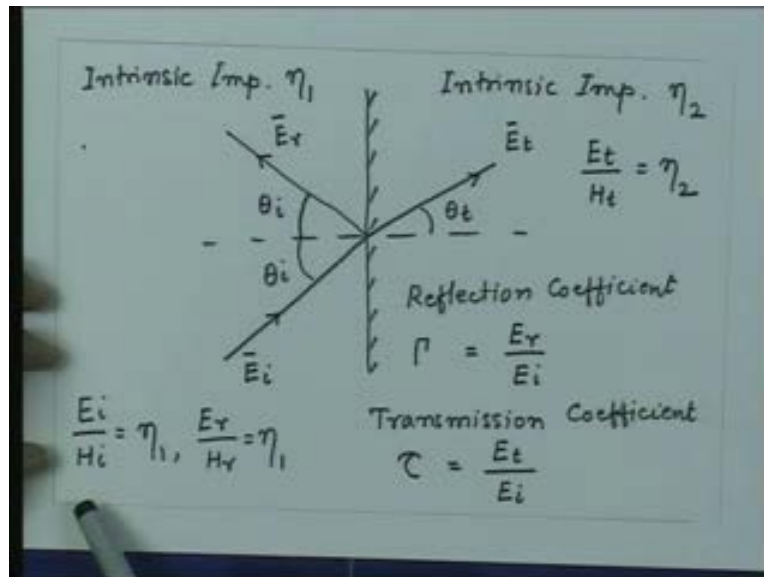
In the last lecture we started investigating the behavior of plane wave at media interface. Matching the phase for incident for the other waves which are reflected and transmitted wave at the media interface, we got the laws of reflection and laws of refraction. Now following that, today we will investigate how much energy is transfer to the second medium and how much energy is reflected from a dielectric interface.

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So today essentially we discuss a problem of reflection and refraction at the dielectric interface. As we took in the previous lecture we are still considering the media which are lossless media that means the conductivity for both the media is 0. But the 2 media can have different permeability and different permittivity. So in the situation we are asking the question how much power get transferred from 1 medium to another and how much power the reflected from the media interface.

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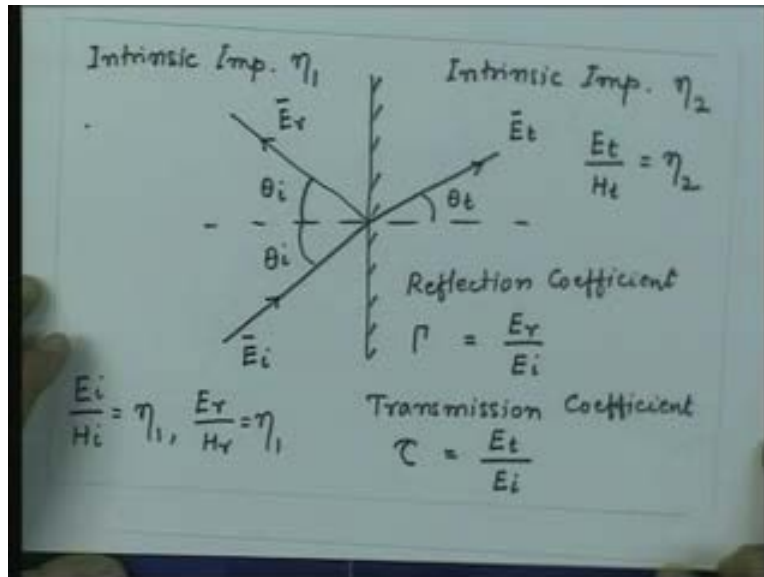


As we discussed the problem essentially now is that you have a media interface we are having different medium properties on the either side of this interface. And then we are interested in now finding out 2 quantities. 1 is what is called the reflection coefficient which we defined as the amplitude of the reflected wave to the amplitude of the incident wave and the transmission coefficient which is the ratio of the amplitude of the electric field or the transmitted wave to the electric field for the incident wave. These quantities we have define for the electric field and as we know since the wave natures still remain the plane wave nature as when require we can always find out the magnetic fields for all the 3 waves. The incident reflected and the transmitted wave. So now the analysis very straight forward essentially we write down the electric fields and the magnetic fields in the 2 media for the 3 waves and then we apply the boundary conditions at the interface for the electric and magnetic fields.

And then by doing some algebra essentially you get this quantity which is  $E_r$  by  $E_i$  which is the reflection coefficient and  $E_t$  by  $E_i$  which is the transmission coefficient. However as we mentioned earlier, that when we are interested now in these quantities the electric field is a vector quantity. So when the wave is incident on the dielectric interface, the

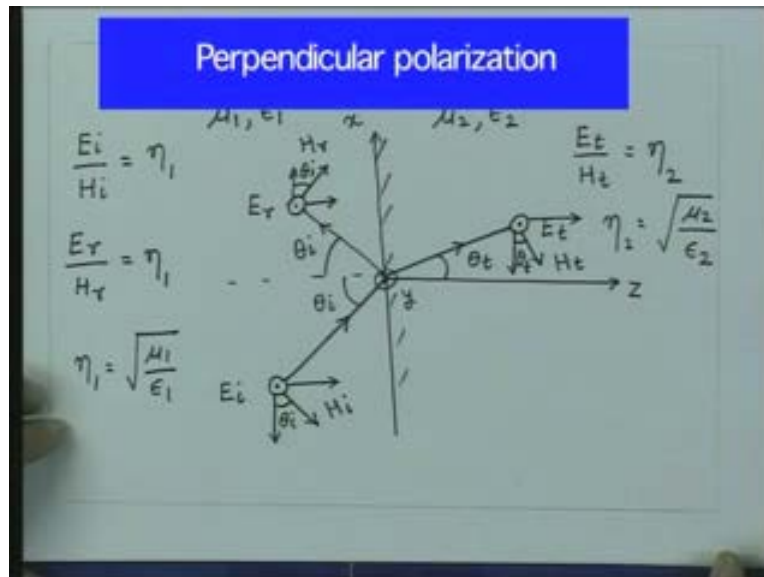
electric field vector in general can make an angle arbitrary angle with respect to the plane of incidence. So if you solve these general problems, that when the wave vector is making arbitrary angle with respect to the plane of incidence the problem is rather complicated. So what we do generally we split this problem into 2. So essentially we take the 2 components of the electric field vector. One is perpendicular to the plane of incidence and the other 1 is in the plane of incidence and then we have 2 cases for which we can find out the reflection and transmission coefficients. And if you get these quantities, for these 2 cases then we can always combine the 2 to find the any arbitrary direction for the electric field. So the problem of reflection and refraction at dielectric interface decomposes into 2.

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1 is when the electric field is perpendicular to the plane of incidence and we call that as the perpendicular polarization. So essentially what we are doing now is, whatever is the polarization of the incident wave, as we have seen this polarization can be decomposed into 2 or 7 a linear polarizations. So we are decomposing this into 2 linearly polarized waves. 1 which is polarized perpendicular to a plane of incidence other 1 which is polarized in the plane of incidence and then you find out the reflection and transmission coefficient for these 2 cases. So let us first investigate that the reflection and refraction coefficient for a perpendicular polarization.

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So as we have taken earlier let us say this is the interface and as we took this direction is the x direction say this is the origin this is the z direction the y direction is coming out of the plane of the paper. So this is y-direction. Will let me remind you we have choosing this direction such that the right handed coordinate system that rule is satisfied. Now let us say the various incidents on this. So this is direction of the way of vector which is making an angle. Let us say theta i with the normal to the dielectric interface. And this way of vector is lying in the plane of the paper that means the plane of the paper itself is plane of incidents on the polarization for this wave is perpendicular polarization. So either the electric field might be coming out of the plane of the paper or it might be going inverse from the plane of the paper. So let us say without using generality let us say the electric field is coming out of the plane of the paper.

So this is the incident electric field which is now oriented in y direction. Then let us say this is the transmitted wave which makes an angle theta t and there is a reflected wave which will make an angle same as the incidents angle which is theta i. So this is the reflected wave. Now one can argue that since we have to satisfy the boundary conditions at the interface and one of the boundary conditions we can satisfy is the tangential

component of the electric field should be continuous across the boundary. If this field is  $y$  oriented then both these fields also must be  $y$  oriented. Not necessarily in positive  $y$  direction it could be positive or negative. But it is oriented in the  $y$  direction. The reason is very simple that if I have to satisfy the boundary condition, then some of these two electric fields which are tangential to the interface this should be equal to the electric field which is for the transmitted wave. If I have field for this transmitted or reflected wave which is not perpendicularly polarize.

If you lies, if you have some component in the plane of incidence then the 2 component which are lying in plane of incidence will satisfy the boundary conditions. But it will not have any role to play with the incidence electric field. What that means is that, you might imagine its equation there where there 2 electric fields which are for this and this way. But these are not because of the field which is incident of the boundary. So as for as the effect of this incidents where this concerned, the electric field must lie perpendicular to the plane of incidents so that the boundary condition is satisfied. So then without using generality we can assume that all the 3 electric fields are  $y$  oriented at may they are oriented in positive  $y$  direction. If our convention was wrong then we will get negative sign for the electric field for transmitted in the circuit wave that means the wave will be going inverse in the plane of the paper. So without using generality we can say this also is positive  $y$  oriented. So this is  $E_t$  this is also positive  $y$  oriented. So let us say this is  $E_r$ .

So all the 3 electric fields, now coming out of the plane of the paper. Now we have to write down the magnetic fields and for writing the magnetic field essentially we use the pointing vector argument. And that is we must choose the direction of the magnetic field such that the pointing vector must be in the direction of the wave properties that means it must be along the wave of vector. So in this case in the electric field is coming outwards if my fingers go from electric field to magnetic field thumb must point in the direction of the wave propagation and  $\mathbf{E} \times \mathbf{H}$  must be perpendicular to each other. So it is very clear since  $\mathbf{E}$  are for all the 3 waves are coming perpendicular to the plane of the paper the magnetic field vectors for all the 3 waves must lie in the plane of the paper that is they must lie in the plane of incidence. So they are perpendicular, they lie in the plane of

incidence whether they would like this way or this way that will be decided by the pointing vector. So in this case since the E is coming outwards my fingers should go like this, so that thumb uses me the direction of the power. So this must be the direction of the magnetic field.

So this is the  $H_i$ , so then I get  $E \times H$  which will be the direction of the pointing vector. Same thing I can do then for here that this also should give me H in such a way that pointing vector in this direction. So again we will get this direction which is  $H_t$ . In this case however, since the wave is now going in this direction and electric field is coming outwards. The magnetic field should be in this direction so that  $E \times H$  gives me the power which is in this direction. So I get for this wave, a magnetic field will be oriented in this direction. So what we do? First we assume the direction for the electric fields then by using the pointing vector arguments we write down the directions for the magnetic fields in the vector form. And also we know since these waves are transfers electromagnetic waves the ratio for this is given by the intrinsic impedance of the medium.

So as we saw, here we have  $E_i$  upon  $H_i$  that is equal to  $\eta_1$  which is the intrinsic impedance of the medium 1.  $E_r$  upon  $H_r$  that is also equal to  $\eta_1$  which is intrinsic impedance of the medium 1. And here  $E_t$  upon  $H_t$  that is equal to  $\eta_2$  which is intrinsic impedance of medium 2. So as we have seen if the medium parameters are  $\mu_1$  epsilon 1 for medium 1 and for this side it is  $\mu_2$ , epsilon 2. Then we have this  $\eta_1$  which is intrinsic impedance of the medium which is square root of  $\mu_1$  upon epsilon 1 and  $\eta_2$  for this medium is equal to square root of  $\mu_2$  upon epsilon 2. So once I not the medium parameters I can find out the intrinsic impedance. Once I know the intrinsic impedance, I know the magnetic fields from this relation. Now essentially we decompose the magnetic field into 2 and then we can apply the boundary conditions appropriately electric field note inherently parallel to the interface.

That means, it is tangential to the interface that this location. So we can satisfy the boundary conditions. So first thing is we write the functional form for this electric and

magnetic fields including the phase function, then we apply the boundary condition the tangential or normal component of electric and magnetic field should be continuous across the boundary. So if this angle is theta i, I can decompose the 2 components 1 is this component and this component. And this will be equal to these angle is theta i. So this angle also will be theta i, this angle will be theta i, I resolve this in 2 components then this angle will be theta t. So this component which is tangential component of a magnetic field will be Hr into cos theta i this will be Hr into sin theta n same is true for the other 3 waves.

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Inc. Wave:  $E_i = E_i e^{-j\beta_1(x \sin \theta_i + z \cos \theta_i)} \hat{y}$

Ref. Wave:  $E_r = E_r e^{-j\beta_1(x \sin \theta_i - z \cos \theta_i)} \hat{y}$

Trans. Wave:  $E_t = E_t e^{-j\beta_2(x \sin \theta_t + z \cos \theta_t)} \hat{y}$

From the boundary condition at  $z=0$   
 Tan. comp. of  $E$  should be continuous

$$E_i e^{-j\beta_1 x \sin \theta_i} + E_r e^{-j\beta_1 x \sin \theta_i} = E_t e^{-j\beta_2 x \sin \theta_t}$$

$$\Rightarrow E_i + E_r = E_t \quad \text{--- (1)}$$

Let us now first write explicitly the expression for the electric fields. So if I let us say I take the incident wave the electric field  $E_i$  is having a some magnitude  $E_i$  and it have a phase function which is  $E$  to the power minus  $j$  beta 1 into as we saw last Time: when we wrote the phase function this will be equal to  $x$  times  $x$  times sin of theta i plus  $z$  times cos of theta i. Now this will be  $x \sin \theta_i$  plus  $z \cos \theta_i$  for the reflected wave  $E_r$  that is amplitude  $E_r$  e to the power minus  $j$  beta 1  $x \sin \theta_i$ . But now this wave is now going in the opposite direction with respect to  $z$ . So either I can find out the direction cosine which is this angle which is put the  $z$  axis for theta i. But for this wave the angle is actually pi minus theta i.

Say it will be direction cosine of  $\pi$  minus  $\theta_i$  corresponding to that so that will be  $-\cos \theta_i$  and then we have a transmitted wave which is  $E_t$  sub magnitude  $E_t$  to the power minus  $j\beta_2 x \sin \theta_t$  plus  $z \cos \theta_t$ . Then all these waves are fields are  $y$  oriented. So I can put if you want is unit vector for all this which is  $y$  oriented. Now if they have to satisfy the boundary condition and since these all 3 fields are continuous and tangential to this boundary the  $E_i$  plus  $E_r$  should be equal to  $E_t$  at the dielectric interface which is  $z$  equal to 0. So this plane which is the dielectric boundary is defined by  $z$  equal to 0 because we have taken the origin at the interface. So at  $z$  equal to 0 these some of these 2 fields should be equal to the transmitted field. So essentially I have from the boundary condition that tangential component of  $E$  should be continuous we get from here  $E_i$  plus  $E_r$  is equal to  $E_t$ .

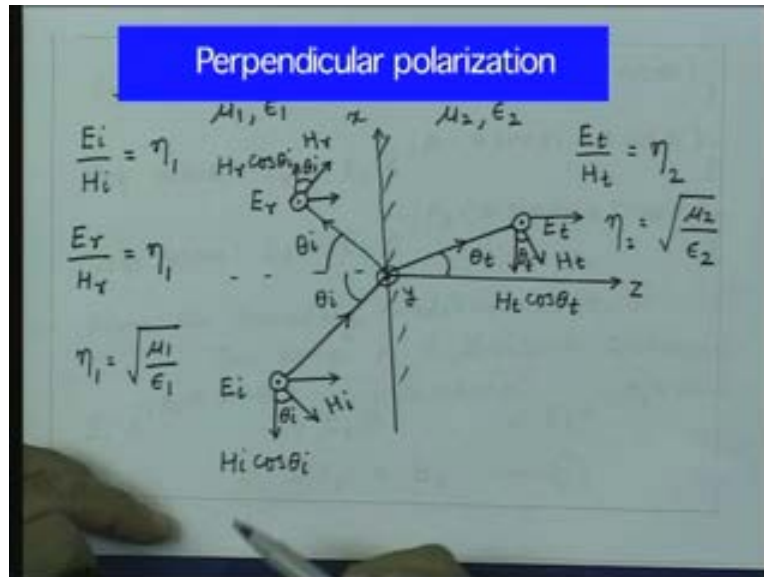
See if I substitute I will get the 3 phase functions this quantity which will be this total function plus this quantity plus this at  $z$  equal to 0. So I get from here  $E_i e^{-j\beta_1 x \sin \theta_i}$  plus  $E_r e^{-j\beta_1 x \sin \theta_i}$  that is equal to  $E_t e^{-j\beta_2 x \sin \theta_t}$ . So this is the boundary condition at  $z$  equal to 0. Now we already establish the Snell's law. We say  $\beta_1 \sin \theta_i$  is equal to  $\beta_2 \sin \theta_t$ . So from the Snell's law we have  $\beta_1 \sin \theta_i$  is equal to  $\beta_2 \sin \theta_t$ . That means this phase function is same for all the 3 waves okay. So essentially this term is a common term because the Snell's law satisfy that this quantity is equal to this quantity. So what they are giving you is that  $E_i$  plus  $E_r$  that is equal to  $E_t$ , and that you will see this phase matching condition essentially true for all the components whether you have normal component or a perpendicular component or normal component or tangential component.

This phase term is same so what will happen is that the component which you take for electric or magnetic field that will be either its quantity multiplied by  $\cos \theta_i$  or  $\sin \theta_i$  depending upon whether we take tangential component or normal component. But the phase function is same for all the component which is which is given by this. So what are boundary condition we satisfies, this phase function will always same for the incident and the transmitted wave. So what that means then the boundary conditions have to be



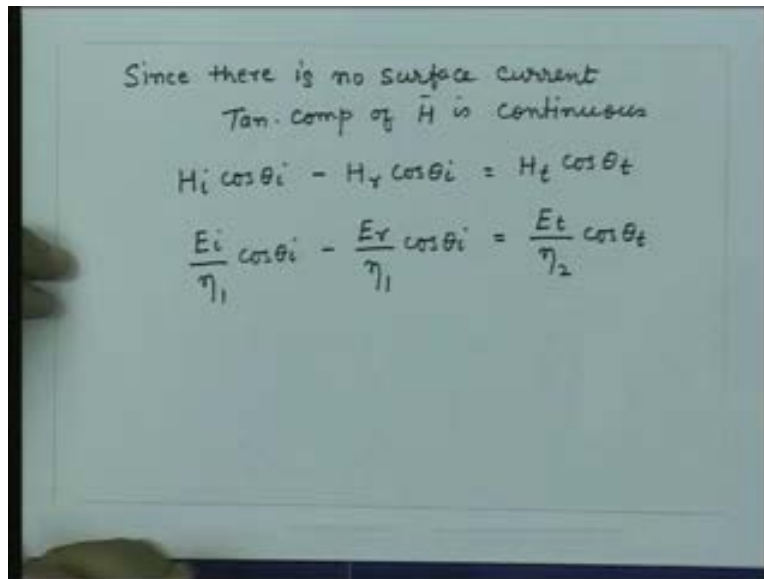
applied only on the amplitude term which is the amplitude of the incident reflected and transmitted wave. So we have 1 relation between these and that is equation 1 which we can use later on for the finding out the quantities as we mention the reflection and transmission coefficient.

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The second boundary condition which we get either by from the continuity of the normal component of the magnetic field or in this case since we are talking about dielectric media there are no surface currents. So we can always also use the continuity of the tangential component of the magnetic field. So as we have seen earlier when we talked about boundary conditions the tangential component of magnetic field cannot be applied if there is a possibility of surface current. However as we have seen the surface currents or for ideal conductors. So if you have a dielectric boundary like this there is no surface currents and then even the tangential component of magnetic field can be applied. So we have tangential component of magnetic field which is  $H_i \cos \theta_i$ . That is the tangential component. So this is  $H_i \cos$  of  $\theta_i$ , this component is  $H_r \cos$  of  $\theta_i$  and this component tangential component this is  $H_t \cos$  of  $\theta_t$ . So we can apply the continuity that this minus, this because they are in opposite direction that should be equal to the tangential component with  $H_t \cos \theta_t$ .

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Since there is no surface current  
Tan. comp of  $\vec{H}$  is continuous

$$H_i \cos \theta_i - H_r \cos \theta_i = H_t \cos \theta_t$$
$$\frac{E_i}{\eta_1} \cos \theta_i - \frac{E_r}{\eta_1} \cos \theta_i = \frac{E_t}{\eta_2} \cos \theta_t$$

So we can apply the since there are no surface current. The tangential component of  $H$ , is continuous at the boundary. So from here we get  $H_i \cos \theta_i$  minus  $H_r \cos \theta_i$  that is equal to  $H_t \cos \theta_t$ . Instead of here this term minus this term that is equal to this term. And we now know the relation between  $E$  and  $H$ . So I can substitute for  $H$  which is  $E_i$  upon  $\eta_1$   $H_r$  is  $E_r$  upon  $\eta_1$  and  $H_t$  is  $E_t$  upon  $\eta_2$ . So this thing I can write as  $E_i$  upon  $\eta_1 \cos \theta_i$  minus  $E_r$  upon  $\eta_1 \cos \theta_i$ , that is equal to  $E_t$  upon  $\eta_2 \cos \theta_t$  that is your second equation which relates the electric field component across the boundary. Now remember we are interested in now finding out 2 quantities 1 is the reflection coefficient which is  $E_r$  upon  $E_i$  and other 1 is transmission coefficient. So using these 2 equations I can now find out these 2 quantities I have 1 equation which is this, the other equation, which is this the other equation which is this and then by just simple algebra manipulation I can find out the reflection and transmission coefficient for this wave and that will be.

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Reflection Coeff:

$$\Gamma_{\perp} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} \quad \& \; |\Gamma_{\perp}| \leq 1$$

Transmission Coeff:

$$\tau_{\perp} = \frac{2 \eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_2}$$
$$1 + \Gamma_{\perp} = \tau_{\perp}$$

So I get now reflection coefficient and we saw this is denoted by gamma. But now in this case we are having the polarization which is perpendicular polarization. So we called this is the reflection coefficient gamma perpendicular and that is the will be equal to just if you solve this equations we get eta 2 cos theta i minus eta 1 cos theta t divided by eta 2 cos theta i plus eta 1 cos theta t. And the transmission coefficient tau and again we put this suffix perpendicular for perpendicularly polarize wave that is equal to 2 times eta 2 cos of theta i that divided by eta 2 cos of theta i plus eta 1 cos of theta 2. We can verify that from this equation 1 if I divide this equation by  $E_i$  I will get  $1 + E_r$  upon  $E_i$  equal to  $E_t$  upon  $E_i$ . So that will give me  $1 + \text{gamma}$  is equal to tau. So we can verify that it will satisfy a condition  $1 + \text{gamma}$  is equal to tau. So this is perpendicular.

Greater So just after the boundary if I find out what are the fields, the electric field then the reflection coefficient gives me just beyond the boundary what will be the value of the transmitted field. And just this side of the boundary left side of the boundary in medium 1 what will be the amplitude of the electric field? And once you get these quantities  $E_r$  and  $E_t$  then of course we have these phase function so we can find out the wave at any location any point in space in medium 1 and in medium 2. So in medium 1 essentially we

have super position of the wave which is 1 which is incident and the wave which is reflected from the boundary. So any at any point in space you have to find super position of these 2 whereas if you go to the medium 2 then we have only this way. So we can find out the wave at any location in medium 2. Few things can be noted from this expression for reflection and transmission coefficient 1 is the reflection coefficient is always less than or equal to 1.

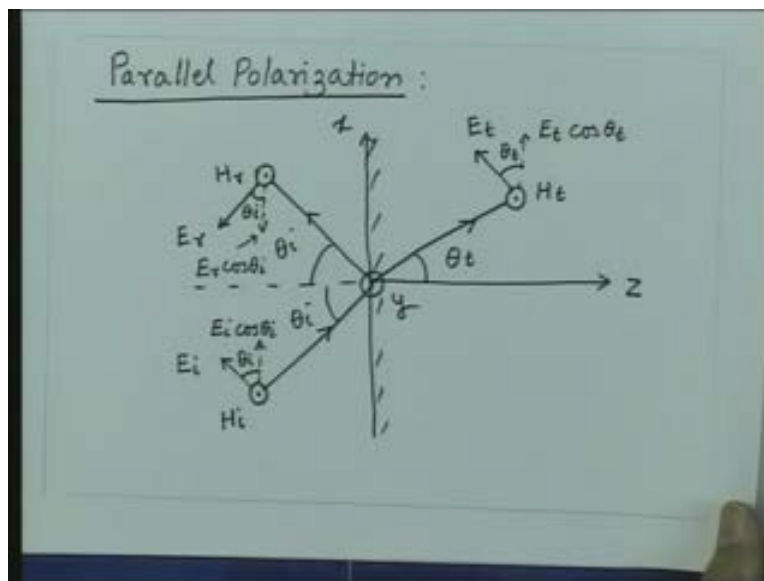
So this quantity is mod gamma put it mod of that magnitude of this quantity less than or equal to one whereas this quantity which is the transmission coefficient let could be less than 1 it could greater than 1. As we recall this quantity here the gamma is the reflection coefficient and the pointing vector is proportional to mod E square. So if gamma is less than 1 that means the pointing vector magnitude for the reflected wave is always less than 1; that is very straight forward. So what essentially we are saying is if you are some wave which was coming to the media then it had got some power density along with it when the wave got reflected the power density in this wave is always going to be smaller 1 power density coming from this wave. So the reflection coefficient is always less than 1 is quite obvious.

What is the meaning of then the transmission coefficient is greater than 1 because that maybe situation here when this quantity  $\eta_2 \cos \theta_2$  is greater than  $\eta_1 \cos \theta_1$  that Time: the amplitude of this would be greater than 1. That means the electric field in medium 2 could be larger compare to the incident electric field. Thus, that the mean that the pointing vector in that for that transmitted wave is greater than pointing vector of this, the answer is no why we got although the electric field is larger here. The intrinsic impedance which is related to this quantity  $\epsilon_1$  and  $\mu_1$  the magnetic field will be reduce in a proportion. So this pointing vector for this will be always less than the pointing vector of this. After all that has to be conservation of the path. So if you got certain power density coming from here, the power density which is going for this wave and this wave they must be some of these 2 must be equal to the power which was carried by the incident wave.

So though the electric field could be larger in the second medium the pointing vector for both the waves the reflected and transmitted will always be less than the pointing vector of the incident wave. When the wave is reflected from the boundary, that Time: there may be a phase reversal for the wave or they may not be a phase reversal from the wave. This quantity is always positive. So when the wave is incident on the dielectric medium the transmitted wave is always in the phase with the incident wave. But the reflected wave can be in phase, can be out of phase 180 degree out of phase that means the electric field if it is like this for the incident wave.

The electric field for the transmitted wave will be always coming out of the plane this if this was coming out of the plane whereas for the reflected wave it might come out of the plane or it might go inside the plane. Both possibilities might exist depending upon the angle of incidence at the medium parameters. So these are some broad conclusions I can draw for this wave which is perpendicularly polarization. The next case which is the parallel polarization the analysis of this wave is identical to this that means you again write down the wave vectors we write down the fields again you write a boundary condition and then you get the expressions for the reflection and transmission coefficient.

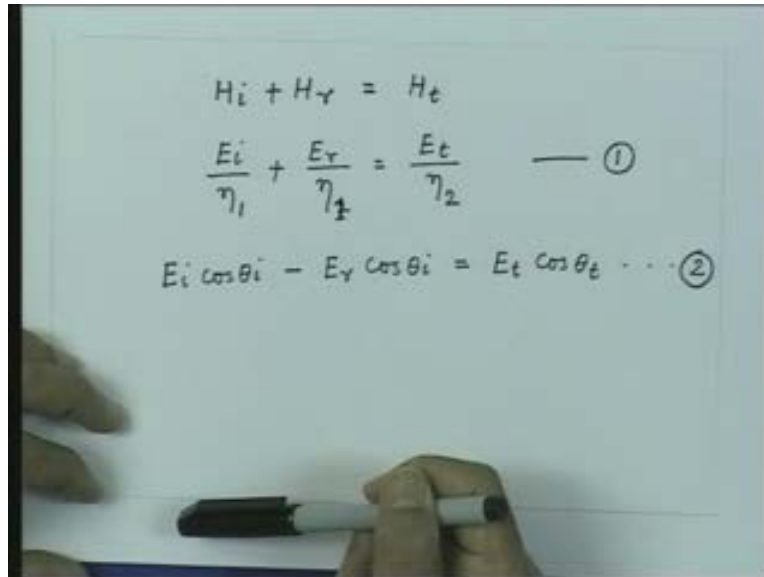
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So very quickly we can get what is called the parallel polarization. That means in this case now the electric field is lying in the plane of incidence. So this this direction is  $x$  this is  $z$  this is  $y$  again the wave is incident at an angle which is  $\theta_i$  it is reflected angle  $\theta_i$ , this is  $\theta_t$  and now the electric field is lying in plane of incidence that means it is lying in this plane of plane of the paper. So if the electric field is lies in the plane of paper the magnetic field must lie perpendicular to the plane of the paper because these 2 must be perpendicular to each other. So in this case without using generality what we assume is that all the magnetic field vectors are  $y$  oriented they are coming out of the plane of the paper and then appropriately choose the electric fields direction so that you get the correct pointing vectors.

So let us say the magnetic field for this was coming out is given by  $H_i$  this is  $H_t$  and this is  $H_r$  and I must get the electric field so that the pointing vector in this direction. So since  $H$  is like this your  $E$  must be going this wave, so  $E \times H$  gives me the correct pointing vector. So I get this is the direction for multiplication  $E_i$  this is the direction for  $E_t$  and for this wave which is going in opposite direction again I must get  $E \times H$  which is this way. So  $E$  will be pointing so this is my  $E_i$ , again I can take the component of these vectors tangential component the magnetic field now is tangential to the interface. So this is if this angle is  $\theta_i$  again this is  $\theta_i$  this is  $\theta_i$  this is  $\theta_t$ . So this component is  $E_t \cos \theta_t$ . This is this component is  $E_i \cos \theta_i$  and this is this is  $E_r \cos \theta_i$  this component. So again by matching the boundary conditions essentially we can get now. So the magnetic field is fully tangential and recalling that the phase function for all of them is same as we saw in the previous case.

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The image shows a whiteboard with three equations written in black marker. The first equation is  $H_i + H_r = H_t$ . The second equation is  $\frac{E_i}{\eta_1} + \frac{E_r}{\eta_2} = \frac{E_t}{\eta_2}$  followed by a horizontal line and a circled 1. The third equation is  $E_i \cos \theta_i - E_r \cos \theta_i = E_t \cos \theta_t$  followed by three dots and a circled 2. A hand holding a black marker is visible at the bottom of the frame.

$$H_i + H_r = H_t$$
$$\frac{E_i}{\eta_1} + \frac{E_r}{\eta_2} = \frac{E_t}{\eta_2} \quad \text{--- (1)}$$
$$E_i \cos \theta_i - E_r \cos \theta_i = E_t \cos \theta_t \quad \dots (2)$$

We get now 2 conditions  $H_i$  plus  $H_r$  is equal to  $H_t$  and substituting for  $H_i$  and  $H_r$  that is  $E_i$  upon  $\eta_1$  plus  $E_r$  upon  $\eta_2$  that is equal to  $E_t$  upon  $\eta_2$  for this is  $\eta_1$  its  $\eta_2$ . So this is your 1 equation and then I have for the electric field which is  $E_i \cos \theta_i$  minus  $E_r \cos \theta_i$  that is equal to  $E_t \cos \theta_t$ . For second equation I have this  $E_i \cos \theta_i$  minus  $E_r \cos \theta_i$  that is equal to  $E_t \cos \theta_t$  that is your second equation. And again by solving these 2 equations we can get a ratio of  $E_r$  by  $E_i$  which is the reflection coefficient and  $E_t$  by  $E_i$  which is the transmission coefficient.

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Ref. Coeff:

$$\Gamma_{||} = \frac{\eta_1 \cos \theta_i - \eta_2 \cos \theta_t}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}$$

Trans. Coeff:

$$\tau_{||} = \frac{2 \eta_2 \cos \theta_i}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}$$
$$1 + \Gamma_{||} = \frac{\eta_1}{\eta_2} \tau_{||}$$

So from here if I do that j get for the parallel polarization the reflection coefficient which is gamma. And since we are talking about parallel polarization that is gamma parallel that will be equal to eta 1 cos theta i minus eta 2 cos theta t divided by eta 1 cos theta i plus eta 2 cos theta t. And the transmission coefficient tau parallel that will be equal to 2 times eta 2 cos theta i divided by eta 1 cos theta i plus eta 2 cos theta t. So the expression which you get for reflection coefficient is similar in the 2 cases except that this eta 1 and eta 2 are interchange. So in the perpendicular case we have eta 2 cos theta i and eta 1 cos theta t whereas in parallel we have eta 1 cos theta i and eta 2 cos theta t. Also note here that in this parallel case 1 plus gamma is not equal to tau because we have from this equation the relation will be 1 plus gamma. If you take eta 1 in the other side will be eta 1 upon eta 2 into tau. So for this case we have 1 plus gamma parallel that will be eta 1 upon eta 2 into tau parallel. But otherwise all the discussion on arguments we had for the parallel polarization will be identical to the perpendicular case also.

So we have already seen in detail the piece of perpendicular polarization. So all the argument that the reflection coefficient magnitude will be always less than 1 whereas transmission coefficient could be greater than or less than 1. All the arguments are valid

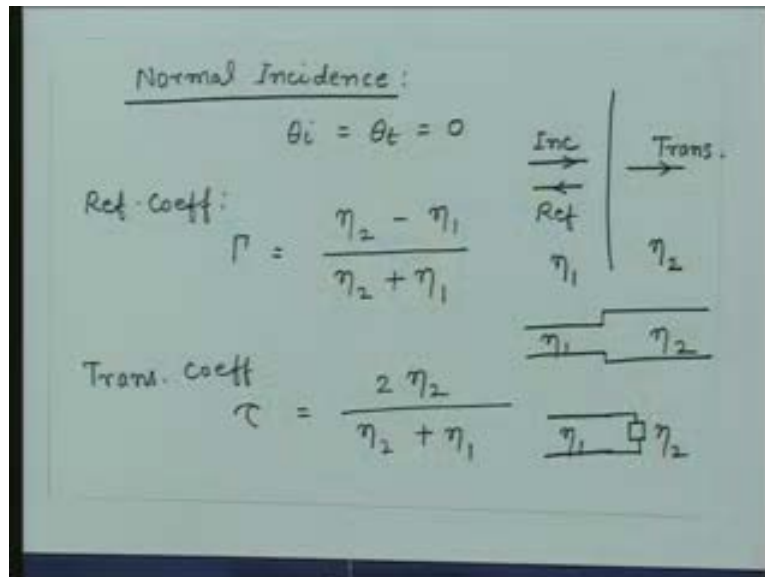


for this polarization also. So once we have now the reflection in transmission coefficient for these 2 cases, then now we can combine the reflected and transmitted fields and then we got the resultant field for the reflected or transmitted wave for any arbitrary polarization. So let me just summarize and what we have done we say if you have any arbitrary polarization of the wave as we have seen the arbitrary state of polarization can be represented by a combination of orthogonal polarization. And in this case we are taking 2 orthogonal polarizations which as linear 1 is perpendicular to plane of incidence other 1 is parallel to the plane of incidence.

Solve the problem separately for these 2 states of polarization. That means you find out the reflection and transmission coefficients for these 2 cases and we get the quantities what is called the gamma perpendicular and tau perpendicular and the gamma parallel and tau parallel. And then, you combine the reflected and the transmitted fields for these 2 polarizations to get the resultant electric field; that means the resultant polarization. So this will discuss little later how the polarization might get for change. But by using this expression now we can find out how much field is induced in the second medium when the uniform plane wave is incident on a dielectric boundary and then from there we can also calculate how much power gets transferred to the second medium and how much power is reflected from medium. Now one of the special cases of this could be that if I take a perpendicular polarization case this case this case.

And if I make this angle 0 we got the special case which will be the normal incidence case. So one way is that okay it is not with the normal incidents than the reflected wave also will be normal because these 2 angles will be equal and Snell's law again this wave also will be moving along the normal. So essentially we have a case wave moves in this direction wave of travels back in this direction which is reflected wave and a transmitted wave also moves in this direction satisfy the boundary conditions. And you can get a reflection and transmission coefficient for that case, other possibilities since we have consider this cases of perpendicular of parallel polarization either of the case if I put theta i equal to 0 then I get the case of the normal incidence. So what we can do?

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The normal incidence case and the polarization which we take whether parallel or perpendicular does not matter because when this angle becomes equal to 0 the electric field will be either this or lying in the plane of the paper. But they will be a perpendicular to this direction. So I can change I can take either perpendicular polarization or parallel polarization I must get the same result. So let us say, take the expression which we got for the perpendicular polarization and from there if we substitute for theta i equal to 0. This we get from this expression if I put theta equal to 0 so theta t also will be 0. So for normal incidence we have theta i is equal to theta t is equal to 0 and then the reflection coefficient gamma as we see that it should be same whether its parallel perpendicular.

So we just simply say if the reflection coefficient gamma for this that will be equal to your eta 2 minus eta 1 divided by eta 2 plus eta 1 and tau will be 2 times eta 2 divided by eta 2 plus eta 1. So that will give me eta 2 minus eta 1 divided by eta 2 plus eta 1 and transmission coefficient that is tau will be equal to 2 times eta 2 divided by eta 2 plus eta 1. And the case of normal incidence is this is your dielectric interface this is your incidence wave these the reflected wave and this is a transmitted. So this is incident this is reflected and this is transmitted. So this case now essentially as become like a 1

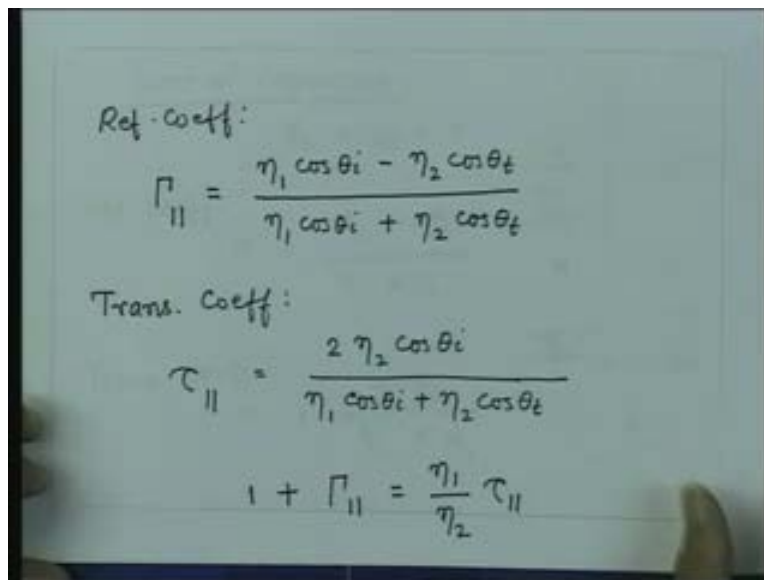
dimensional propagation the electric and magnetic fields the waves are like that. So amplitude of electric and magnetic fields are not varying in this plane anymore it has only variation which is in  $z$  direction. So the fields are this could simply correspond to one dimensional propagation in  $z$  direction.

That means this case is very identical to a transmission line case which was 1 dimensional case the wave you should travel along the transmission line. And you are not worried about what is the variation of the fields' perpendicular to the transmission line precisely same thing we are talking about here that perpendicular to the direction in which this wave is traveling there is no field variation and the problem essentially is a 1 dimensional problem which is identical to the transmission line problem. If I say this medium which is semi infinite and which is characterize by an impedance  $\eta_1$  this medium which is again semi-infinite. See if I see in this direction I will see an impedance is characteristic impedance which is  $\eta_2$ . So if I look rightwards beyond the boundary I see an infinite medium ahead and therefore the impedance seen will be equal to the intrinsic impedance which is  $\eta_2$ . If I see in this direction again I see infinite medium I see an impedance which is equal to the intrinsic impedance of the medium which is  $\eta_1$ .

So that means this case is similar to as if I have 2 transmission lines of characteristic impedance  $\eta_1$  and  $\eta_2$ . And when the wave incident from here this is equivalent to that you have a line which is terminated in an impedance which is  $\eta_2$ . Because this is the infinite medium, so you see the impedance equal to characteristic impedance you get the reflection coefficient on this line which is  $\eta_2 - \eta_1$  divided by  $\eta_2 + \eta_1$ . That is what essentially this quantity. So the normal incidence case which is the special case for any oblique incidence in the dielectric interface is a identical case to the transmission line. So when we are dealing with the transmission line in fact we were handling 1 of the special cases of this reflection and refraction at the interface, also theirs in the transmission line was terminated at this point beyond the load the line was not existing the power was getting lost into this impedance which are located at this location.

But now when we are having a media like this then either we can say that the power this is equivalent to having impedance at the interface the power is lost there. But the power is actually not lost at that location the power is actually gone into the second medium. So equivalently I can say this is like a transmission line but the transmission coefficient in this case we save the power which is gone into beyond this point which was beyond the load point which was not their on transmission line. But the 2 are equivalent so if I calculate what is the power loss at this boundary. This will be exactly same as what I would have got a power loss into the impedance which is terminating the transmission line. So this case the normal incidence case is the 1 of the special situations of the oblique incidence and then 1 can find out the reflection and the transmission coefficient for this. A quick look at as I mention if I take 2 polarizations as a parallel or perpendicular I must get the same reflection coefficient.

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Ref. Coeff:

$$\Gamma_{||} = \frac{\eta_1 \cos \theta_i - \eta_2 \cos \theta_t}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}$$

Trans. Coeff:

$$\tau_{||} = \frac{2 \eta_2 \cos \theta_i}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}$$

$$1 + \Gamma_{||} = \frac{\eta_1}{\eta_2} \tau_{||}$$

But if I go here which is for the parallel polarization and if I substitute theta i equal to 0 and theta t equal to 0. I get a reflection coefficient gamma parallel which is eta 1 minus eta 2 upon eta 1 plus eta 2. One would than wonder if I said by when theta goes to 0 2 cases should become identical why the reflection coefficients are opposite of each other.

Here is  $\eta_2$  minus  $\eta_1$ , this is  $\eta_1$  minus  $\eta_2$  and the reason for this is the convention which you are taken for the electric field when we define the direction for the electric field. For perpendicular case these 2 electric field were in the same direction we taken plus y oriented. So we had a reflection coefficient which is  $\eta_2$  minus  $\eta_1$ . Whereas if you go to parallel polarization the electric fields are in this direction when  $\theta$  goes to 0 the  $E_i$  and  $E_r$  they are opposite direction.

So basically the sign the negative sign which appears in the expression for when you use for parallel polarization is due to the fact that we have already taken the electric field which is oriented in the opposite direction and  $\theta_i$  goes to 0. So the negative sign essentially accounts for it. What that means is that when you are having a normal incident, the reflection coefficient in general essentially can be written like that with the assumption the initial electric field are oriented in the same direction. So if  $\eta_1$  is greater than  $\eta_2$  you will have a phase reversal in the reflected field compare to the incident field. And if  $\eta_2$  is greater than  $\eta_1$  then there will not any phase reversal of the reflected wave when the wave is incident on a dielectric boundary. So what we see something important here.

Now that independent of what are the medium parameters and the angle of incidence the reflection in transmission coefficient all real quantities. That means there could be a direction reversal for the electric and magnetic fields. But there is no arbitrary phase change either in a transmitted wave or in the reflected wave. These cases are the simple reflection and refraction cases when you meet next Time: you will consider the special case where there is a possibility of getting a phase change which is not 0 or  $\pi$ . And that will be 1 of the special cases of these oblique incidences. So will continue with this and we will discuss the oblique incidence case in the next lecture where there could be a possibility of arbitrary phase change at the reflection at the differences.