

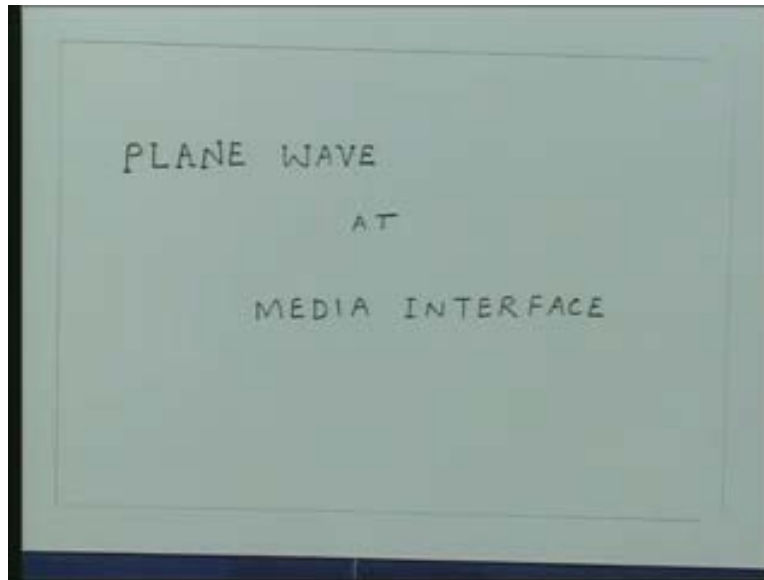
**Transmission Lines and E.M. Waves**  
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**Lecture-30**

Welcome, in the last lecture we investigated the wave propagation in some arbitrary direction with respect to the coordinate axis. Our main objective is to find out the wave propagation in a bound medium and it has been mentioned earlier that when you are having a bound medium the freedom of choosing the coordinate axis is not really there because choosing the coordinate axis in a particular direction may simplify the problem at least algebraically. So normally when we are having the medium boundaries we choose the coordinate axis so that it aligns along the boundaries and then the wave propagates in arbitrary direction with respect to the coordinate axis because the wave is now incident on the boundary or some arbitrary angle.

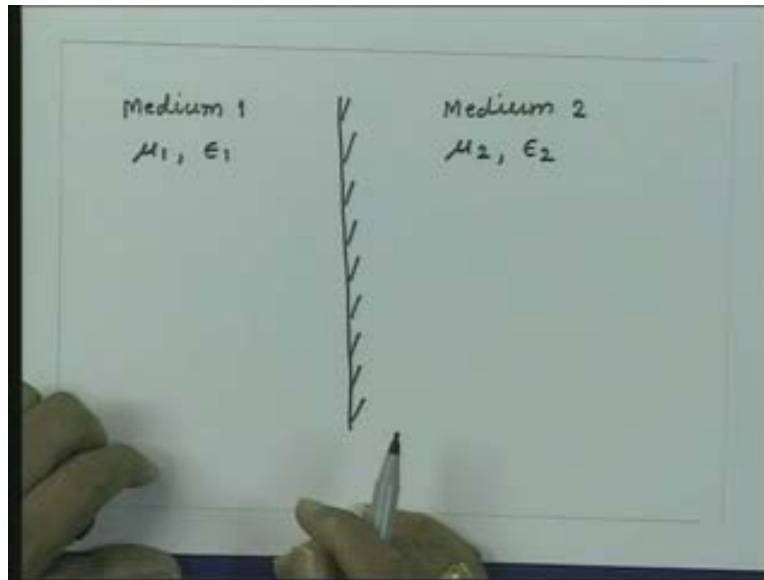
In today's lecture we will investigate the propagation of a plane wave at media interface.

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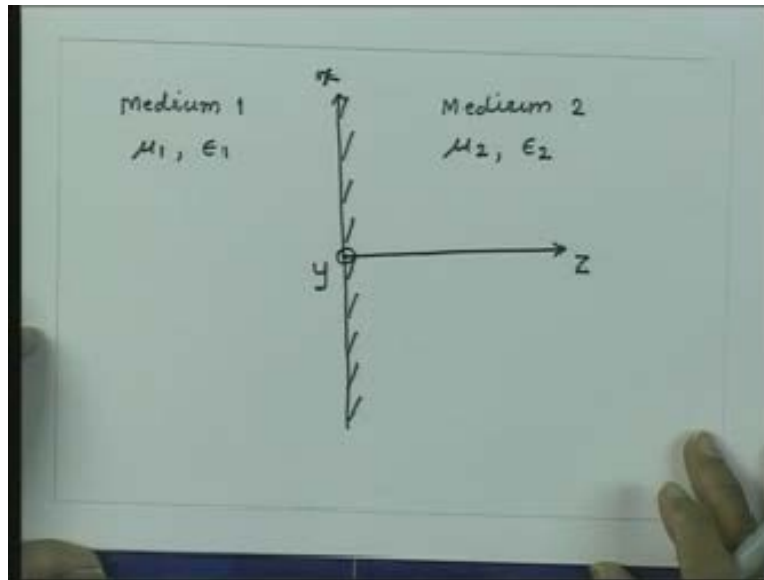
What you are now saying is instead of having an infinite medium if I divide the medium into two semi infinite media so I have a media interface, on the left side of this line we are having a medium which is infinite, on the right side we have a medium which is again infinite and the medium properties abruptly change at this location called a media interface. So let us say we have Medium 1 on the left side of this and we have Medium 2 on the right side and let us assume that the conductivity for both the media is still zero that means the media is still lossless but the permeability and permittivity are different for these two media. So let us say the permeability for this Medium 1 is given by  $\mu_1$ , permittivity is given by  $\epsilon_1$  and for Medium 2 the permeability is given by  $\mu_2$  and the permittivity is given by  $\epsilon_2$ .

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Let us say now we orient the coordinate system such that the media interface is in the  $xy$ -plane. So along this direction we have coordinate axis  $x$  perpendicular to this axis  $z$  and perpendicular to the plane of the paper that is the arrow coming outwards normal to the plane of the paper that is the  $y$  direction. We can again verify that we must have the right handed coordinate system so if my fingers go from  $x$  to  $y$  where  $y$  is coming outwards from the paper then my thumb must point in the direction of  $z$  so this is the correct right handed coordinate system.

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Now let us say we have a wave which is incident on this dielectric interface at some arbitrary angle with respect to the coordinate axis. So I have a wave which is incident at some arbitrary angle like that, this is the direction of the wave so essentially this is the wave vector for the wave which is incident on the dielectric.

Now specifically we are asking questions like when this uniform plane wave is incident on this dielectric interface then what will happen to this wave. Intuitively it appears that part of the energy will get transferred to the second medium so that will again constitute some kind of wave propagation but also what is not obvious at this moment is that part of the energy will get from the interface, this is what essentially we will argue that if the wave is incident on this interface we essentially require two kinds of fields, one

which is in the second medium and also the fields in the first medium all have to be modified to satisfy the boundary conditions.

So essentially when the wave is incident on the dielectric interface, part of the energy will be transferred to the second medium but also the part of the energy will come back to the first medium and that is what essentially we will investigate.

When the energy comes back into the first medium or goes to the second medium what happens to plane wave nature, which direction the energy will be going are questions essentially we will have to ask, also we have to ask for what is the magnitude of the field which goes to second medium, how much power is going to be transferred to second medium.

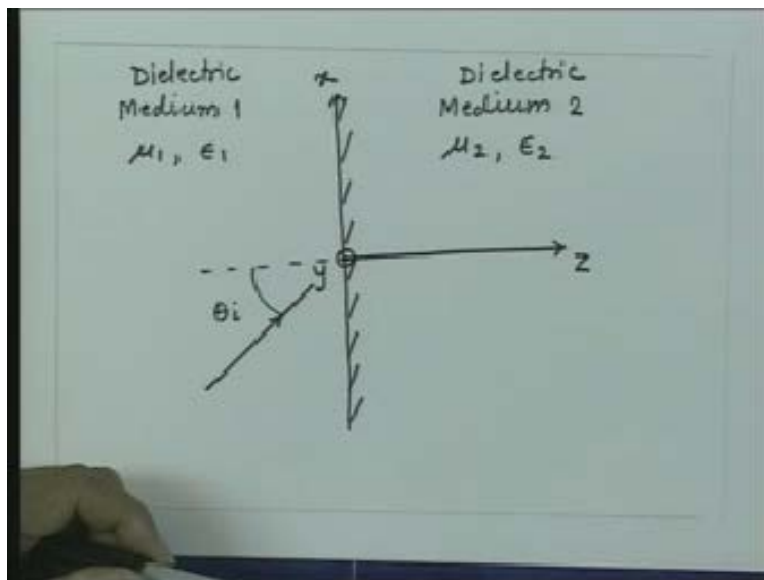
So the analysis of the plane wave at interface essentially includes finding out the direction in which the waves will be moving in the two media, how much power gets transferred from one medium to another medium, how much power comes back from the interface to the first medium itself, what happens to the direction of the electric field that means what happens to the polarization of the electromagnetic wave and so on.

So in this lecture and in the following lectures we will essentially discuss these issues related to the propagation of uniform plane wave across a media interface. And since you have taken conductivity zero for both these media we can at the moment call this as Dielectric Media so this is the Dielectric Medium 1, this is the Dielectric Medium 2. So essentially at the moment we

are investigating propagation of the uniform plane wave at a dielectric media interface, on both sides we are having media which have dielectrics.

Now let us say this is making an angle with respect to this direction that this angle is given by some  $\theta_i$ .

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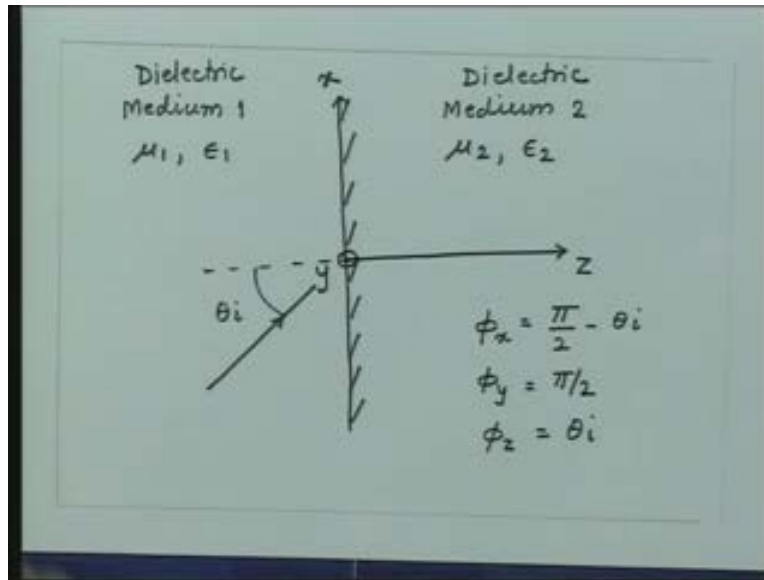
Now you find a certain quantities here that is if I say that  $xy$ -plane is the media interface that means in that plane suddenly the medium properties change the medium is uniform in the  $xy$  direction only suddenly it changes this point along this  $z$  direction and without losing generality we can say this quantity  $z = 0$ . So let us say the origin of this coordinate axis at the interface and the wave vector is making an angle  $\theta_i$  with respect to this direction. Now this  $z$  direction is perpendicular to the media interface so we call this as the

normal to the media interface. So this line the z axis is the normal to the dielectric media interface.

We are now having a wave traveling at an angle  $\theta_i$  with respect to the normal to the media interface that is the problem essentially we are having and now you want to ask when this wave is incident at this angle, what will happen to this wave. So first of all let us represent this wave in the form that is the phase function which has amplitude, let us say without losing generality without specifically saying we are talking about electric field or magnetic field you are having some field vector which is associated with this wave but it has a definite phase function because it is traveling at an angle  $\theta_i$  with respect to coordinate axis.

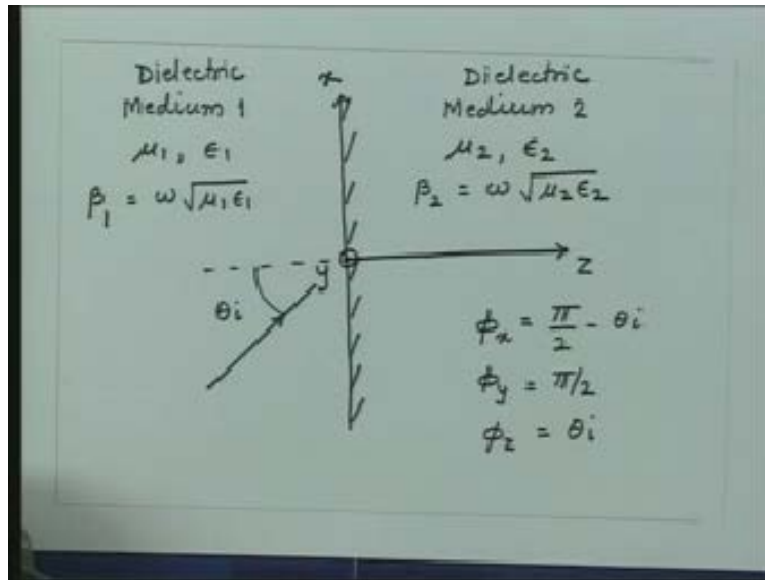
So in this case if I write down as we discussed in the last lecture the wave vector makes three angles  $\phi_x$ ,  $\phi_y$  and  $\phi_z$  with three coordinate axis so these three angles we can write down in that terminology so  $\phi_x$  is the angle which the wave vector makes with the x axis if I extend this such that this angle will be  $\theta_i$  so this angle which the wave vector makes with the x axis will be  $\pi/2 - \theta_i$ , the angle which the wave vector makes with the y axis which is perpendicular to the plane of paper will be ninety degrees so in this case  $\phi_y$  is  $\pi/2$  and the angle which this makes with the z axis is  $\phi_z = \theta_i$ .

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Once I know these three angles which the wave vector makes with the three coordinate axes then I can write down the direction cosines and I can write down the phase function. Since the medium properties for this Medium 1 are  $\mu_1$  and  $\epsilon_1$  we have the phase constant of this first medium is  $\beta_1$  that is equal to  $\omega$  square root  $\mu_1 \epsilon_1$  and in the second medium we have phase constant  $\beta_2$  that is equal to  $\omega$  square root  $\mu_2 \epsilon_2$ .

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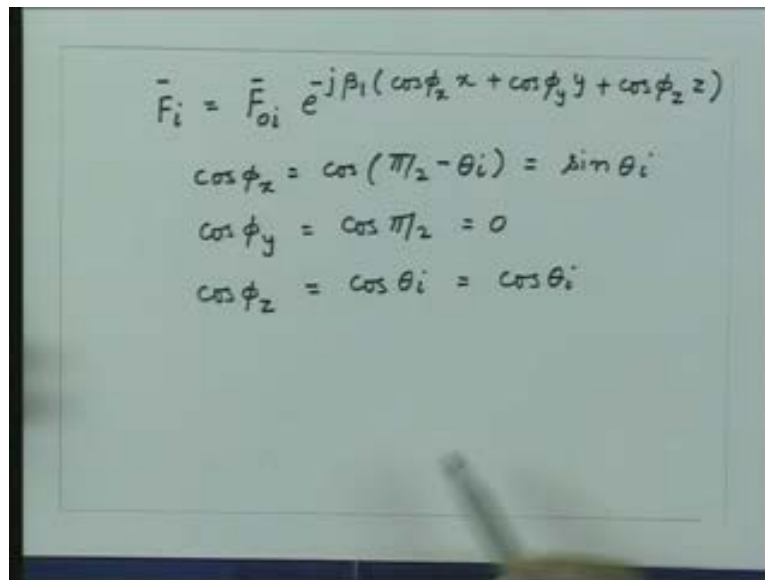
So I know in both media the phase constant for a plane wave and we know the angle which the wave vector makes with the three coordinate axes. So let us say I have some field represented by this wave which is having a vector and the phase function will be given as we saw last time as follows.

Let us say I have some field which is incident and let me call that as some  $F_i$  bar where  $F$  could represent the electric field or magnetic field which is having a magnitude term so let us say  $F_{0i}$  bar which is a vector and then you are having a phase function which is  $e$  to the power  $-j$  since the wave is incident in Medium 1 so this phase constant is  $\beta_1$  so this is  $\beta_1$  into  $(\cos \phi_x x + \cos \phi_y y + \cos \phi_z z)$ .

Now since  $\phi_x$  is  $\pi/2 - \theta_i$  so  $\cos \phi_x$  is  $\sin \theta_i$ ,  $\cos \phi_y$  which is  $\cos \pi/2$  is zero and  $\cos \phi_z$  is  $\cos \theta_i$ . So from here we get  $\cos \phi_x$  is  $\cos(\pi/2 - \theta_i)$  which is again

equal to  $\sin \theta_i$ ,  $\cos \phi_y = \cos \pi/2$  that is zero and  $\cos \phi_z = \cos \theta_i$  that will be  $\cos \theta_i$ .

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$$\begin{aligned}\vec{F}_i &= \vec{F}_{oi} e^{-j\beta_1(\cos \phi_x x + \cos \phi_y y + \cos \phi_z z)} \\ \cos \phi_x &= \cos(\pi/2 - \theta_i) = \sin \theta_i \\ \cos \phi_y &= \cos \pi/2 = 0 \\ \cos \phi_z &= \cos \theta_i = \cos \theta_i\end{aligned}$$

If I substitute this in the expression for the fields in general the incident field we call this wave as the incident wave this is incident on the dielectric media and this is what the suffix denotes this i essentially gives you the incident field so let us call this as incident wave so we have the field for the incident wave which is this magnitude amplitude term  $e$  to the power  $-j\beta_1 (x \sin \theta_i + z \cos \theta_i)$  where this quantity is zero because of  $\phi_y$  is zero

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$$\begin{aligned}\bar{F}_i &= \bar{F}_{oi} e^{-j\beta_1(\cos\phi_x x + \cos\phi_y y + \cos\phi_z z)} \\ \cos\phi_x &= \cos(\pi/2 - \theta_i) = \sin\theta_i \\ \cos\phi_y &= \cos\pi/2 = 0 \\ \cos\phi_z &= \cos\theta_i = \cos\theta_i \\ \bar{F}_i &= \bar{F}_{oi} e^{-j\beta_1(x \sin\theta_i + z \cos\theta_i)}\end{aligned}$$

So the wave which is traveling at an angle  $\theta_i$  with respect to the normal to the interface can be represented by field which is like this so if I look at some instant of time if I look what is the variation of the field amplitude and the function of  $x$  and  $z$  and also  $y$  we can obtain the field variation by taking the real part of this quantity so if I take the field amplitude at some instant of time the amplitude per magnitude will be the real part of this quantity so real part of  $\bar{F}_i$  bar will be  $\bar{F}_{oi}$  bar cosine of this angle which is  $\beta_1$  times this quantity.

Let us say I am interested in finding out what is the variation of the phase in the media interface so I am not worried about how the phase is varying in  $z$  direction just it has a phase constant which is  $\beta_1$  times  $F$  times  $\theta_i$  but at the moment let us say we want to find out what is the phase variation which I am going to get at the interface when the wave is incident at this angle since

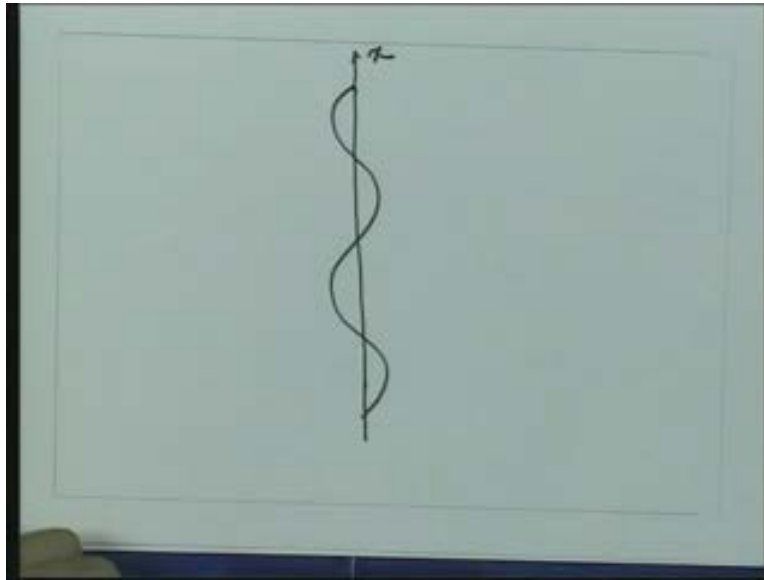
I have taken the origin  $z = 0$  here then I can substitute  $z = 0$  in this and I get the magnitude of the field in xy-plane so that will be your  $F_{0i} \cos(\beta_1 x \sin \theta_i)$

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$$\begin{aligned}\bar{F}_i &= \bar{F}_{0i} e^{-j\beta_1(\cos\phi_x x + \cos\phi_y y + \cos\phi_z z)} \\ \cos\phi_x &= \cos(\pi/2 - \theta_i) = \sin\theta_i \\ \cos\phi_y &= \cos\pi/2 = 0 \\ \cos\phi_z &= \cos\theta_i = \cos\theta_i \\ \bar{F}_i &= \bar{F}_{0i} e^{-j\beta_1(x \sin\theta_i + z \cos\theta_i)} \\ \text{Magnitude of Field in } xy\text{-plane} &= \text{Re}\{\bar{F}_i\} = \bar{F}_{0i} \cos(\beta_1 x \sin\theta_i)\end{aligned}$$

So if I look at the field variation in this xy-plane the field is having a variation which is cosine variation in the x direction and it does not have any variation in the y direction. So essentially this is my media interface if I plot the field amplitude the field amplitude will vary like in the x direction and it is constant in the y direction. So essentially it has created some kind of a corrugated surface if I want to visualize what this field variation is, this will be more like a corrugated surface in this plane where the corrugations are oriented in the y direction because the direction perpendicular to the plane of the paper is the y direction. So I got something like an asbestos sheets which have nice corrugated surfaces.

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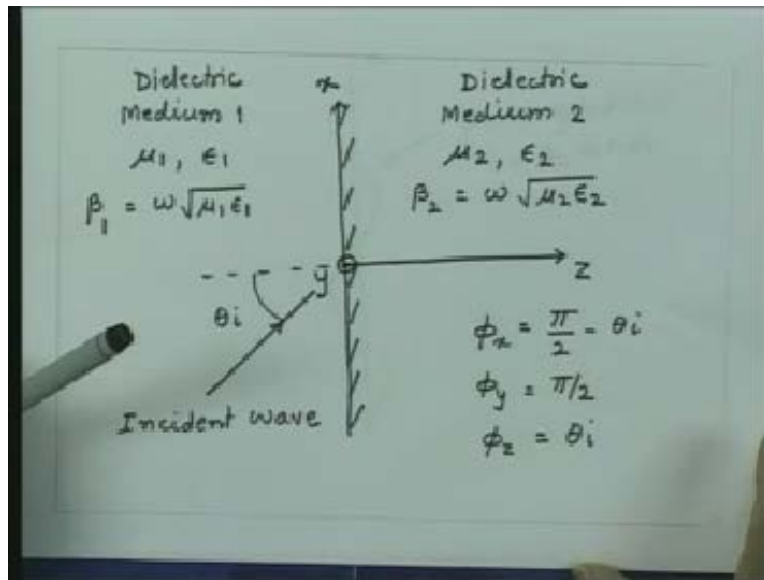


One thing we note here is if the wave is incident in this direction the amplitude variation will appear like a corrugated surface in this xy-plane on the interface or it is equivalent to having a phase gradient in the x direction which is  $\beta_1 \sin\theta_i$ . So I can visualize this field either as a amplitude variation which is like this in the x direction and constant in the y direction or equivalently I can say that it is having a phase variation which is  $\beta_1 x \sin\theta_i$  or the special gradient of the phase will be  $\beta_1 \sin\theta_i$ .

So, this one has a phase gradient which is the phase change per unit length in the x direction will be equal to  $-\beta_1 \sin\theta_i$ . If I make  $\theta_i = 0$  then the phase gradient will become zero so the phase will become constant and that is what will be the case if the wave is moving perpendicular to this plane since the wave was coming perpendicular like this then this  $\theta_i$  will be zero and then

this will be constant phase plane so I will not have any phase gradient in this plane and wave will be traveling normal to it.

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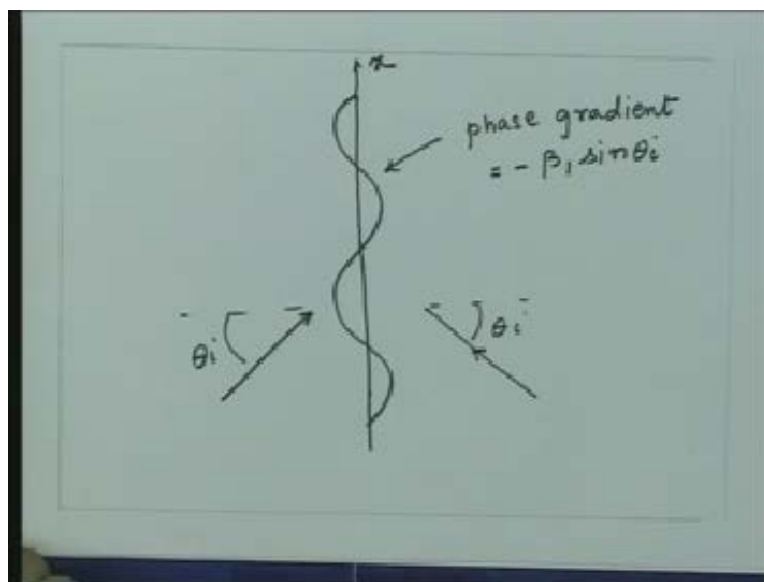
So direction of the wave as it changes introduces a phase gradient on this plane. So creating a phase gradient and tilting the direction of the wave are equivalent that means if the wave direction is changed I get a phase gradient equivalently if I create a phase gradient I will get a wave which will be oriented at some other direction which will satisfy the phase gradient. So from here if the phase gradient is given to you if I equate the phase gradient with  $-\beta_1 \sin \theta_i$  then I will get some quantity  $\theta_i$  that is the effective direction at which the wave is traveling.

Now here what we did is we started from the wave direction and we say this is the phase gradient was created. But now we can go reverse and say if the

phase gradient was created by some mechanism then equivalently the wave is traveling at an angle which will satisfy the phase gradient to be equal to this.

Now in this case the wave was traveling in this direction as we saw was making an angle  $\theta_i$  with respect to this so if I take a direction like that this was the angle  $\theta_i$ . So this wave which is moving at an angle  $\theta_i$  with respect to the normal has created this phase gradient, alternatively I can say if I had created this phase gradient it would be equivalent of having a phase which is coming like this. Now, if I consider this dielectric interface it will be immediately clear to you that even if the wave was incident from this direction exactly at same angle  $\theta_i$  then it also created the same phase gradient but they are making exactly the same angles with respect to the normal.

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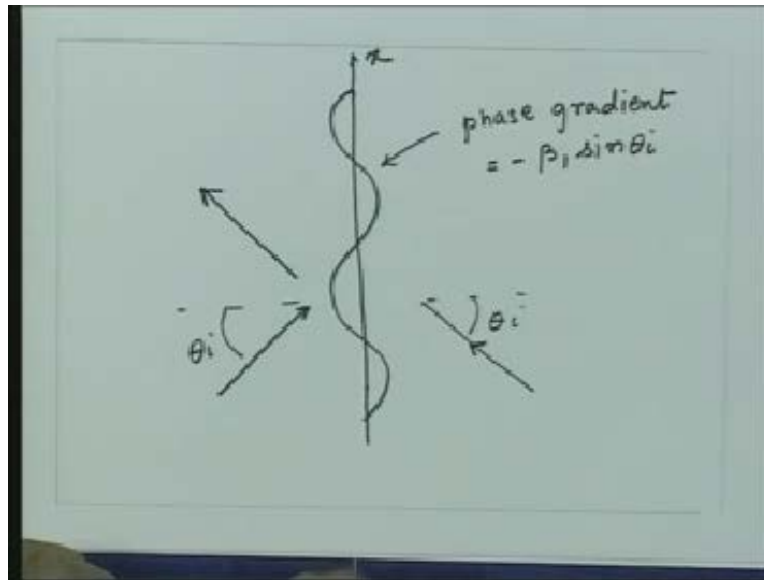


So whether the wave is coming this way on the interface or the wave is coming this way from the interface both are going to create identically same phase gradient. So the wave which was traveling through interface like this will correspond to this phase gradient, if the wave was passing through the interface like this it will create the wave gradient which is like this.

Now this side we have Medium 1, this side we have Medium 2. So firstly if I take this wave and take this on this side essentially the same phase gradient corresponds to the wave which travels away from the interface at same angle because this is the phase gradient which corresponds to this plane in the same direction. So the wave which is approaching to the interface and wave which is going away from the interface at the same angle intersect the normal create the same phase gradient and that is a very important thing.

But we understand this then we can now ask a question when the wave was incident on this dielectric interface what would this wave do? This wave would try to excite certain fields in this second medium because of the continuity of the fields.

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Now if I want to maintain the continuity of the electric and the magnetic fields both but once the electric and magnetic fields are induced into second medium they see infinite medium ahead of it that means again they constitute a wave phenomenon and since the medium seen ahead of them is infinite medium they again see a transverse electromagnetic behavior for the wave.

Now the wave is in the second medium so the ratio of the electric field and magnetic field again is decided by the medium parameter the characteristic impedance or the intrinsic impedance of the second medium whereas the electric and magnetic field ratio was decided by the intrinsic impedance of Medium 1 in this region. So if I say that the fields are continuous across the boundary on one side of the ratio of E and H are satisfied by the intrinsic impedance of this medium and the other side the E and H are related to the

intrinsic impedance of the second medium. It is immediately clear to me that just by having the fields on the other side and the original field which are on this side I cannot satisfy them all together for both electric and magnetic fields. So what that means is I have to induce certain fields in the first medium itself or in other words, we have to modify the fields in the first medium so that the boundary conditions can be satisfied for both electric and magnetic fields at the interface.

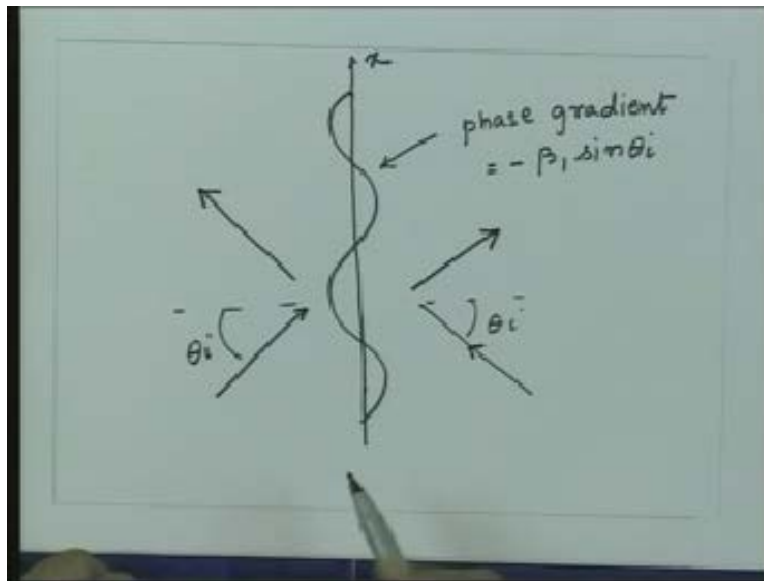
So essentially there will be some fields which will be induced in this medium, there will be some fields which will be induced in this medium and since the induced phenomena is due to this phase variation because this is the one which is created by the incident field both these fields are induced on two sides of the interface will have the same phase variation because that is the origin the induced fields are because of this field which is incident which is having the phase variation.

So essentially what we note now that the field which goes into the second medium, the field which get induced into the first medium also should have a phase variation which is same as the phase variation because they are caused because of this and since this phase variation is uniform in this direction these fields must also constitute a phenomenon for which the field is constant in this direction and I mentioned since this fields are time varying fields, again they will constitute a wave phenomena so the induced field in this medium will constitute a wave phenomena, fields in this medium again will constitute a wave phenomena and these fields will travel away from this boundary this fields which are here will again go away from this boundary. So we will have two types of waves which will be going away from the

interface one is in this direction another is in this direction and one will be incident field.

So we have something which is going like that, there is another way which goes in like this and all these three waves have the same phase variation that is what the important thing to note.

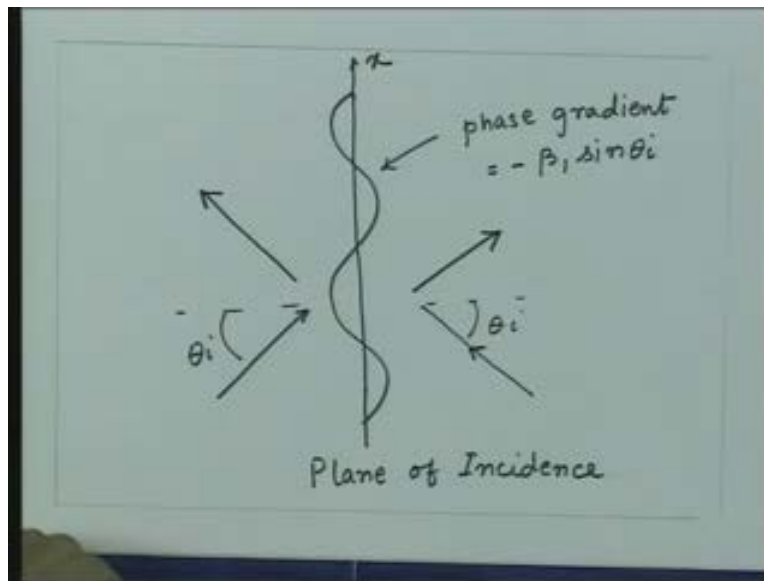
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Now the wave vector for this wave lies in the  $xz$ -plane which is the plane of the paper. So if the amplitude is constant in this plane there is no phase gradient in this plane then the wave normal lies in the plane which is the  $xz$  plane. If we define the plane of the paper as the plane of incidence which is the plane that contains the wave vector and the normal that is the plane which is the plane of the paper we say this is the plane of incidence. So the wave vector for the incident wave lies in the plane of incidence and this plane of incidence is

perpendicular to the direction in which this amplitude is remaining constant which is this y direction but the same is true for two waves which are created which are induced because of the dielectric medium interface.

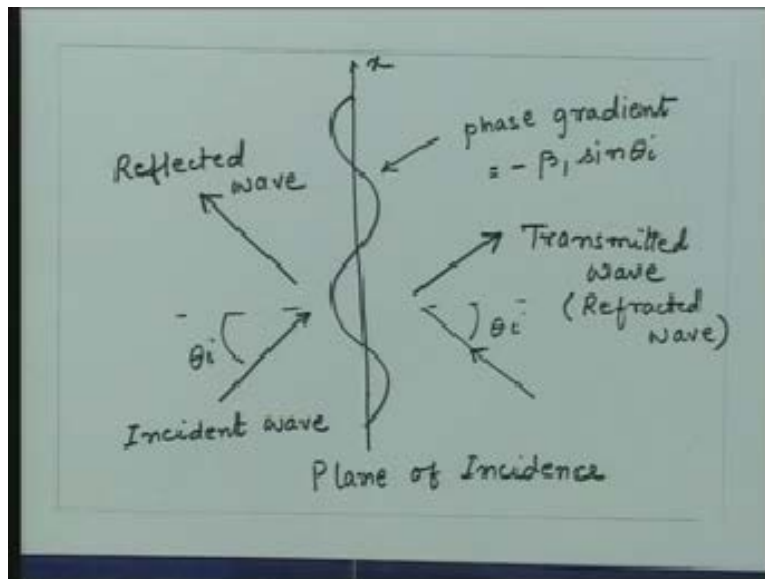
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So you are having a uniform variation for this field also that means the wave vector corresponding to these two waves also must lie in this plane which is the plane of incidence. So what we conclude is make a very important conclusion that the wave vector for the incident wave for this wave induces which is in second medium and the wave vector which is going to first medium all three lie in the same plane then that plane is the plane of incidence. As we mentioned this wave incidence is going away from this interface this will carry some energy so we will say what ever energy came to the interface part of the energy was reflected from the interface and part of the energy was transmitted.

So in general when the wave is incident on a dielectric interface there will be a reflected wave which will carry away some energy which was brought by the incident wave and there will be a transmitted wave which will take the energy to the second medium. So we call this wave as the Transmitted wave and we call this as the Reflected wave. The Transmitted wave is also referred as Refracted wave and this wave which the energy towards the dielectric interface again is the Incident wave.

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So now we have something important conclusions, when the uniform plane wave is incident on a dielectric interface at an angle we induce two waves in which one is called Reflected wave which is going away from the interface Medium 1, we induce another wave which is Transmitted wave which is going away from the interface Medium 2.

All these three waves will have the same phase variation which is  $-\beta_1 \sin \theta_i$  into  $x$  which is the phase gradient and all these wave vectors for these three waves lie in the same plane that is the plane of incidence.

Now if you recall the light is a transverse electromagnetic wave and in the high school we had studied the laws of reflection for light and the first law of reflection was the incident ray, the reflected ray and the refracted ray lie in the same plane which is the plane of incidence. So essentially the statement the first law of reflection comes from matching this phase condition on the interface and that says that these three vectors lie in the same plane.

Now the wave vector in optics is referred to as the direction of the ray so this is the ray in which the light moving. However, we call the direction as the wave vector as we go to general Electromagnetics. So we are having a general statement called the first law of reflection that is the wave vector for incident, transmitted and reflected wave lie in the same plane of incidence and the plane of incidence is the plane which contains the wave vector and the normal to the interface. So we got the first law of reflection established by this condition.

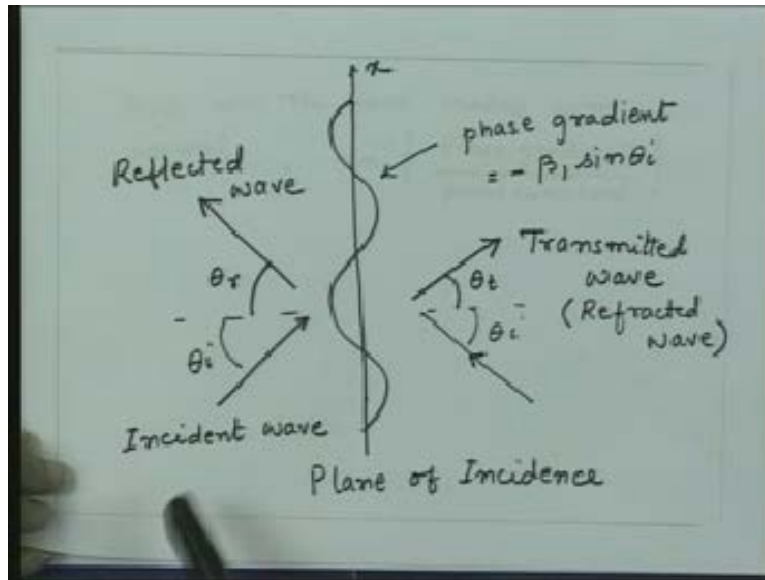
Now the next thing is that when the waves are moving in this direction, in what direction the wave would be moving with respect to normal? Since this angle is  $\theta_i$  and this medium is same the phase gradient what ever I have if I divide that quantity by  $\beta_1$  and take sin inverse of that then that would give me the direction of the wave propagation in that particular medium. So the direction at which the wave moves is if I know the phase gradient the first

phase gradient what ever is created if I divided that by the phase constant of that medium then sine of the angle at which the wave travels is given by that quantity.

So from here I can write the phase gradient divided by the phase constant, the angle which the wave makes with normal is  $\theta$  that is nothing but sine inverse of the phase gradient divided by the phase constant. So somebody has given me the phase gradient then I can take the phase gradient divided by the phase constant in that medium take the sine inverse of that quantity and that essentially gives me the direction of the wave motion in that medium.

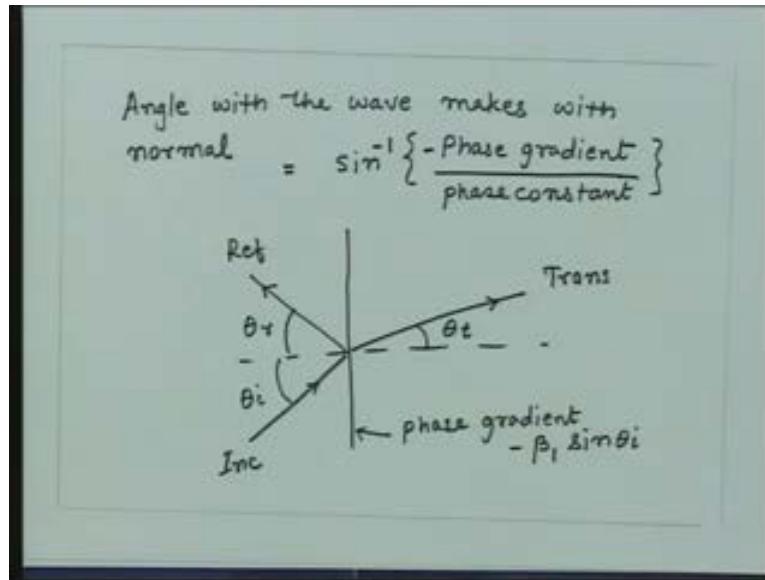
So in this case in Medium 1, let us say this angle at which the reflected wave goes is denoted by  $\theta_r$  the angle of reflection and let us say the wave goes is given by  $\theta_t$  which we call as theta transmission angle so we have a situation where the incident wave is at a angle  $\theta_i$  with respect to normal the reflected angle is at a angle  $\theta_r$  with respect to normal and the transmitted wave is the angle  $\theta_t$  with respect to normal.

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Now we are having a situation with one wave is like that let us say this is my normal this is the angle  $\theta_i$  something goes at some other angle this is my reflected wave let us say this angle is  $\theta_r$  at some other angle I call this as transmitted wave let us say this angle is  $\theta_t$ . This is incident wave, this is reflected wave and this is transmitted wave. And I am having a phase gradient on this surface which is  $-\beta_1 \sin \theta_i$ . I put a minus sign here.

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So what I can do is if I see in Medium 1 I can substitute this phase gradient value and sine inverse of that quantity or sine of this angle will be equal to the minus of phase gradient divided by the phase constant.

Now the phase constant is  $\beta_1$  in this medium and the phase constant is  $\beta_2$  in this medium. So  $\sin \theta_r$  should be minus of phase gradient divided by the phase constant, from here I get  $\sin \theta_r$  that is equal to minus times the phase gradient which is  $\beta_1 \sin \theta_i$  divided by phase constant which is  $\beta_1$ . So this is minus times minus  $\beta_1 \sin \theta_i$  divided by the phase constant in the Medium 1 which is  $\beta_1$  so this is equal to  $\sin \theta_i$ .

So this implies  $\theta_r$  is equal to  $\theta_i$ . This is your second law of reflection that the angle of incidence is equal to angle of reflection. So what ever angle the wave is incident at the same angle with respect to normal the ray will be

reflected or the wave vectors will be making same angle with respect to the normal. This will be going towards the interface this will be going away from the interface but these two angles are equal. So we see another important conclusion from matching of the phase angle that these two angles are equal. This is in medium one which was for the reflected wave.

The same thing we can do for the second medium and that is if I go to Medium 2 then the angle which this thing makes is  $\theta_t$  so  $\sin\theta_t$  should be equal to minus of phase gradient divided by the phase constant but the phase constant in this medium is  $\beta_2$  so from here we get  $\sin\theta_t$  is equal to minus of  $\beta_1 \sin\theta_i$  divided by  $\beta_2$  because now we are having Medium 2. So from here essentially I get,  $\beta_1 \sin\theta_i$  is equal to  $\beta_2 \sin\theta_t$ .

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med. (1)

$$\sin \theta_r = \left\{ \frac{-(-\beta_1 \sin \theta_i)}{\beta_1} \right\} = \sin \theta_i$$

$$\Rightarrow \boxed{\theta_r = \theta_i}$$

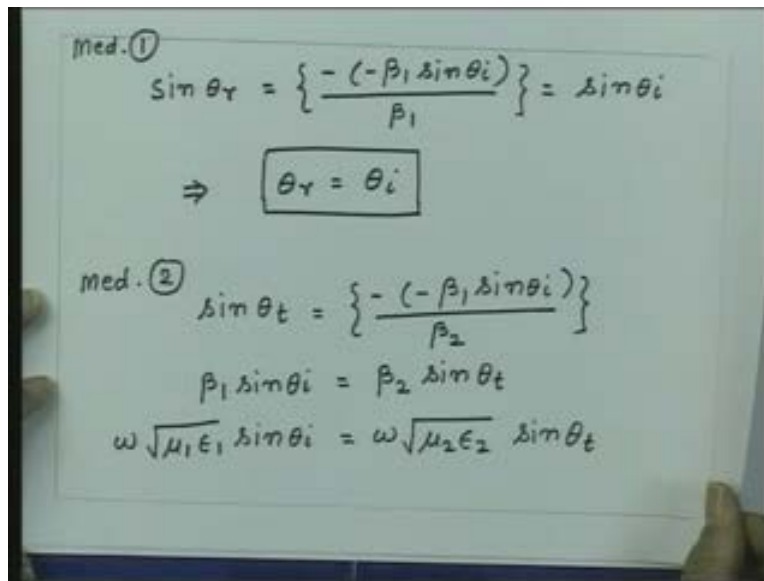
med. (2)

$$\sin \theta_t = \left\{ \frac{-(-\beta_1 \sin \theta_i)}{\beta_2} \right\}$$

$$\beta_1 \sin \theta_i = \beta_2 \sin \theta_t$$

And we know  $\beta_1$  and  $\beta_2$  in medium in which  $\beta_1$  is nothing but  $\omega$  square root  $\mu_1\epsilon_1$  and  $\beta_2$  is  $\omega$  square root  $\mu_2\epsilon_2$  so if I substitute in this, this will be  $\omega$  square root of  $\mu_1\epsilon_1 \sin\theta_i$  that is equal to  $\omega$  square root of  $\mu_2\epsilon_2 \sin\theta_t$ .

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The image shows a handwritten derivation on a piece of paper. It starts with 'med. (1)' and the equation  $\sin \theta_r = \left\{ \frac{-(-\beta_1 \sin \theta_i)}{\beta_1} \right\} = \sin \theta_i$ . Below this, it says  $\Rightarrow \boxed{\theta_r = \theta_i}$ . Then it moves to 'med. (2)' and the equation  $\sin \theta_t = \left\{ \frac{-(-\beta_1 \sin \theta_i)}{\beta_2} \right\}$ . This is followed by  $\beta_1 \sin \theta_i = \beta_2 \sin \theta_t$  and finally  $\omega \sqrt{\mu_1 \epsilon_1} \sin \theta_i = \omega \sqrt{\mu_2 \epsilon_2} \sin \theta_t$ .

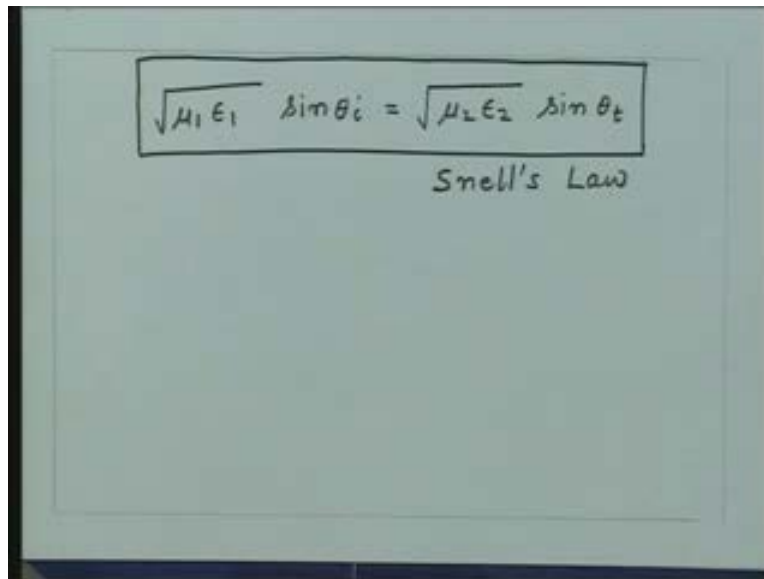
Here both sides cancel so this relation is independent of what is the frequency of the wave and that is square root of  $\mu_1\epsilon_1 \sin\theta_i$  is equal to square root of  $\mu_2\epsilon_2 \sin\theta_t$ . This relation is nothing but the Snell's law which we have already studied in our high school.

Of course, when we are studying the Snell's law there we talk in terms of the refractive indices of the medium and we dealt with only the dielectric media so if we did not have the permeability problem because the materials were not magnetic. So we talked only in terms of refractive indices and we can see from here how do we get the special case that when the media are only

dielectrics and the permeability of the medium is same on both sides. The relation essentially reduces to the relation in terms of the refractive indices.

But this is the important law which tells you the direction of the transmitted wave with respect to the direction of the incident wave for the given media parameter. So we can call this as the generalized Snell's law.

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$$\sqrt{\mu_1 \epsilon_1} \sin \theta_i = \sqrt{\mu_2 \epsilon_2} \sin \theta_t$$

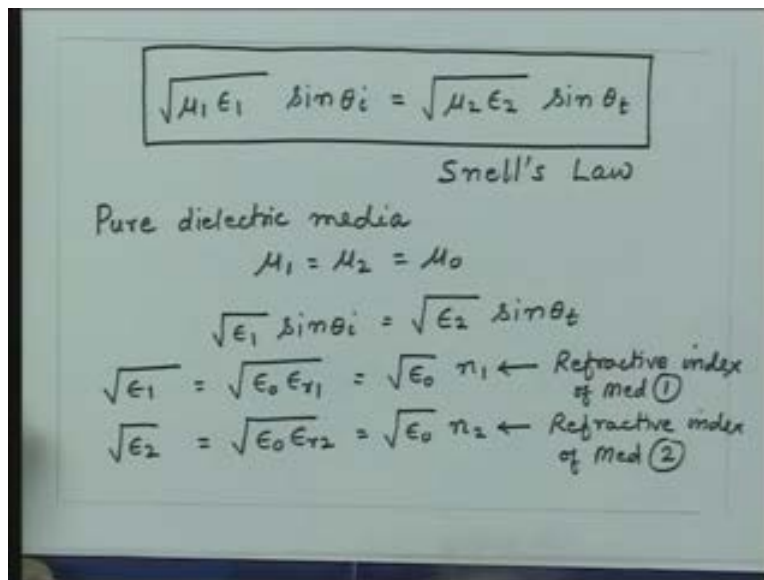
Snell's Law

Now, from here we can calculate the dielectric medium for which the permeability is same so if I take pure dielectric media then for this media  $\mu_1 = \mu_2 = \mu_0$  and then the Snell's law essentially would be square root of  $\epsilon_1 \sin \theta_i$  = square root of  $\epsilon_2 \sin \theta_t$  and we can write down the  $\epsilon_1$  as  $\epsilon_0$  times  $\epsilon_{r1}$  the relative permittivity so the square root of  $\epsilon_1$  is equal to square root of  $\epsilon_0$  into  $\epsilon_{r1}$  that is equal to square root of  $\epsilon_0$  and as we have seen square root of  $\epsilon_{r1}$  is

nothing but the refractive index of the medium so we can call that as  $n_1$  where  $n_1$  is refractive index of Medium 1.

Similarly we can have square root of  $\epsilon_2$  is equal to square root  $\epsilon_0 \epsilon_{r2}$  that is equal to square root of  $\epsilon_0$  into  $n_2$  where  $n_2$  is refractive index of medium two.

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The image shows a handwritten derivation of Snell's Law. At the top, the generalized Snell's Law is boxed:  $\sqrt{\mu_1 \epsilon_1} \sin \theta_i = \sqrt{\mu_2 \epsilon_2} \sin \theta_t$ . Below this, it is labeled "Snell's Law". Then, it specifies "Pure dielectric media" where  $\mu_1 = \mu_2 = \mu_0$ . This simplifies the equation to  $\sqrt{\epsilon_1} \sin \theta_i = \sqrt{\epsilon_2} \sin \theta_t$ . Next, it defines the square roots of the permittivities in terms of the vacuum permittivity  $\epsilon_0$  and relative permittivity:  $\sqrt{\epsilon_1} = \sqrt{\epsilon_0 \epsilon_{r1}} = \sqrt{\epsilon_0} n_1$  and  $\sqrt{\epsilon_2} = \sqrt{\epsilon_0 \epsilon_{r2}} = \sqrt{\epsilon_0} n_2$ . Arrows point from these definitions to the terms in the simplified equation, with labels "Refractive index of med (1)" and "Refractive index of med (2)".

So if I substitute this in your generalized Snell's law then this square root of epsilon cancel on both sides so I will get finally a relation which will be  $n_1 \sin \theta_i = n_2 \sin \theta_t$  or the way we write the Snell's law  $\sin \theta_t$  will be  $n_1/n_2 \sin \theta_i$ .

This is the law you are familiar with because we have already studied this law in optics in our high school physics that if a light ray is incident at an angle  $\theta_i$  and the refractive indices of the media are given by  $n_1$  and  $n_2$  then

the transmitted or the refracted angle which is  $\theta_t$  that will be related by this. So this was a special case of the Snell's law for dielectrics.

In fact when we studied this law in high school physics they appear like these laws came from no where. However, this law essentially comes from the wave nature of the light and since light is a transverse electromagnetic wave when it is incident on a dielectric interface it has to satisfy certain phase conditions and from the phase condition we establish these laws of reflection and refraction.

So let me summarize what we did we started with a wave propagation in some arbitrary direction then we said if the wave is propagating in some arbitrary direction it creates a phase gradient on the media interface this phase gradient is same for the wave which is induced on two sides of the media which we call the transmitted wave and reflected wave and then we found that changing the direction of the wave and creating the phase gradient are essentially equivalent so if I change the direction of the wave I get a phase gradient conversely if I create a phase gradient I get a change the direction of the wave.

So what we find here is that the fields which are induced have the same phase gradients which are created on the interface and from there essentially we can find out the directions in which the waves will be traveling. So we found this relation that the direction of the wave propagation is we can find out from here that is the angle of the wave which it makes with the normal will be minus times the phase gradient divided by the phase constant in that medium sine inverse of that gives me the angle with which finally the wave

will be traveling at that medium. Using this nature essentially we establish the laws of reflection and also the Snell's law which gives the relationship between the incident and the transmitted wave.

Once you understand this then the analysis of wave propagation across the dielectric media is quite straight forward. The idea here is that you write down the phase function for both the media for all the three waves match the boundary conditions at the interface and then find out the amplitude of electric and magnetic fields in this media for these two wave with respect to the incident wave.

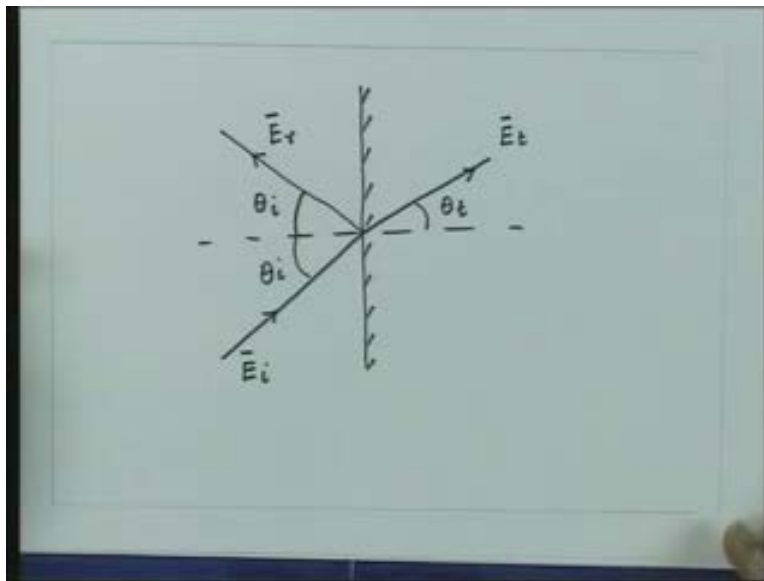
So let us see if I have a incident wave having an amplitude which is  $F_i$  of a electric field with is  $E_i$  or magnetic field  $H_i$ . Then we want to find out what are the relative amplitudes of the electric and magnetic fields for the reflected wave and for the transmitted wave.

So let us say we have a wave which will be having electric and magnetic fields. now the problem essentially we want to investigate is, there is a wave which is incident on this we know this is going to be reflected at same angle part of the thing this angle now is  $\theta_i$  and this angle also is now theta  $\theta_i$  because  $\theta_r = \theta_i$  and this angle is theta  $\theta_r$ . This is the incident wave so let us say this one had a electric field which is given by  $E_i$  bar, say I take electric field for this is given by  $E_r$  bar and I take the electric field which is given by  $E_t$  bar.

So we have a magnitude for the electric field. These are not directions for electric fields these are the wave vectors electric field will be oriented in

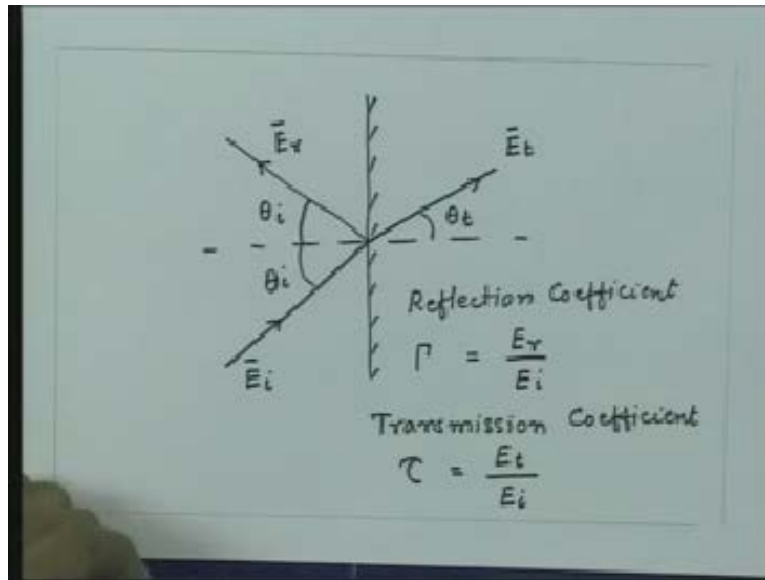
some direction which will be perpendicular to the wave vector. But if you have the amplitude of these three fields which are given by  $E_i$ ,  $E_r$  and  $E_t$  we want to find out what is the relationship between these two so what is the ratio of this and this and that quantity we call as a transmission coefficient for this interface and the ratio of  $E_0$  and  $E_i$  we call as the reflection coefficient for this interface.

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So we again have the quantities which are we are familiar in some sense because we have seen the reflection coefficient in case of transmission line that is when ever we are having impedance mismatch then the part of the energy is reflected.

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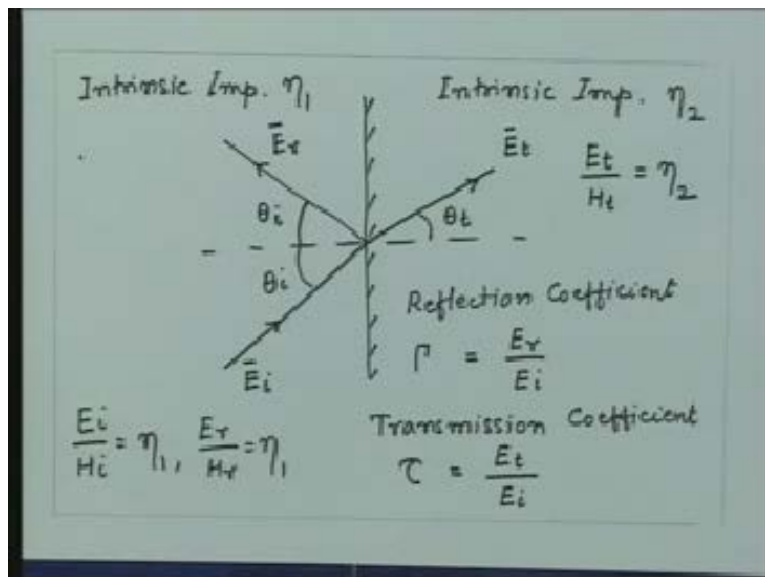
We are having same situation here that when the wave travels in this medium it suddenly sees a change in the intrinsic impedance and because of that impedance discontinuity the part of the energy gets reflected and we want to know what is the reflection coefficient which is ratio of  $E_r$  and  $E_i$ .

So for this the two quantities of interest one is the reflection coefficient and let us denote that as  $\Gamma$  and that is equal to  $E_r/E_i$  and then we can define transmission coefficient generally this is denoted by  $\tau$  and that is equal to  $E_t/E_i$ . Since we are defining this coefficient for the electric field we can specifically call them as the electric field reflection coefficient and electric field transmission coefficient the similar quantities you can define for magnetic field also but we know since the electric and magnetic fields are related to each other by the intrinsic impedance of the medium, if we know

this quantity is electric field in the three media then we can find out appropriately the magnetic fields also.

The important thing to note here is that the wave which was incident on this was the transverse electromagnetic wave so the ratio of E and H was equal to intrinsic impedance in this medium, the ratio of the electric and magnetic field for this wave is also intrinsic impedance of this medium for this again a transverse electromagnetic wave and this wave is again a transverse electromagnetic wave so the ratio for the electric and magnetic field for this medium is also intrinsic impedance. So if I say the intrinsic impedance for this medium is let us say  $\eta_1$  and intrinsic impedance for this medium is  $\eta_2$  we know for this medium your  $E_i/H_i = \eta_1$ , also  $E_r/H_r = \eta_1$  and  $E_t/H_t = \eta_2$ .

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So using these relations and the boundary conditions at the interface now we will investigate what will be the reflection and the transmission coefficient. Keep in mind that since we are now dealing with the vector quantities the electric and magnetic fields will in general make an arbitrary angle with respect to this plane so you may have electric field which might be oriented at some angle like this. So normally what we do we decompose the problem into taking the component of the electric field in the direction perpendicular to the plane and a direction which is in the plane.

So essentially what we do is any electric field we decompose into two cases or any general case we decompose into two what we call as the parallel polarization where electric field lies in the plane of incidence and a perpendicular polarization where electric field lies perpendicular to the plane of incidence. So we have to find out these quantities for these two cases for parallel polarization and perpendicular polarization and once we get that then we can get the total characteristic of the wave which is polarized at an arbitrary angle with respect to the plane of incidence.

So, in the next lecture essentially we will take these two cases specifically the parallel and perpendicular polarization and then investigate their reflection and transmission properties across the dielectric media.

Thank you.