Transmission Lines and E.M. Waves
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Lecture-3

Welcome, in the last lecture we studied the effect of Transit time on the circuit analysis. We also introduced the concept of distributed elements. Then we derived the relationship between voltage and current which was the differential equation. And then we found the solutions of the differential equation for voltage and current and also we found that these solutions essentially represent the wave phenomena on Transmission Line.

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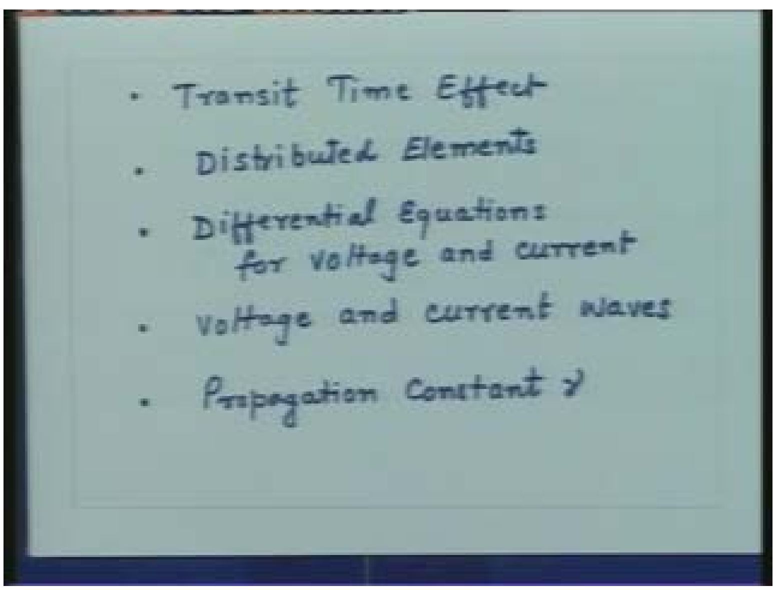

So, voltages and currents exist in the form of traveling waves on a Transmission Line. The characteristics of these traveling waves are governed by a parameter called a propagation constant γ.

In this lecture we will understand the physical significance of this quantity γ which is complex in nature and also we will solve a problem so that we can get a feel how to apply this parameter γ for the Transmission Line analysis problems. Later on, we will go to the complete solution of the differential equation then get the final expression for the voltage and current on the Transmission Line.

As we have seen the propagation constant γ is related to the elementary constants of Transmission Line. So γ is defined as the $\sqrt{(R + j\omega L)(G + j\omega C)}$ where R, L, G and C are the primary constants of the Transmission Line and ω is the angular frequency of the signal which is applied to Transmission Line.

As we mention last time this quantity γ in general can be written in the complex form with the real part α and an imaginary part β so $\gamma = \alpha + j\beta$.

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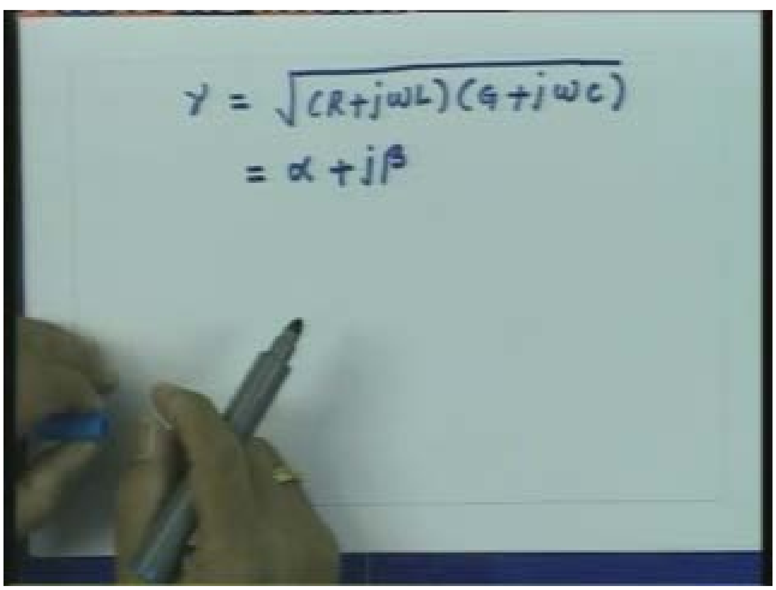

Today we will try to understand the physical significance of these two parameters α and β and what is their effect of signal propagation on Transmission Line.

Let us concentrate on the traveling wave in the forward direction. As we have seen the traveling wave in the forward direction is given as $V^+ e^{-\gamma x}$ which we can we write as $V^+ e^{-(\alpha + j\beta)x}$. Without loosing generality if I assume that V^+ is the real quantity this thing I can write as mod of $|V^+| e^{-\alpha x} e^{-j\beta x}$. So the voltage on the Transmission Line for a traveling wave in the positive x direction has an amplitude which is given by $|V^+|$ multiplied by $e^{-\alpha x}$ and a phase which is equal to $e^{-j\beta x}$.

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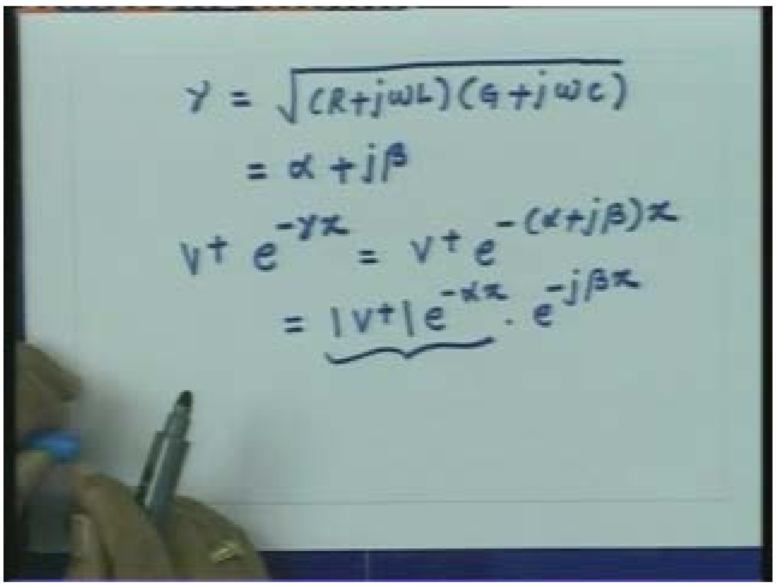

So this quantity γ essentially has two components for the wave propagation thus real part α essentially controls the amplitude of the wave and imaginary part β controls the phase variation of the traveling wave along the Transmission Line. So the phase as a function of distance on Transmission Line is equal to $-\beta x$ so what we call as the space phase is equal to minus $-\beta x$. That means as the wave travels in the positive x direction the phase lags more and more and the phase linearly varies as the function of x for a given value of β. Since this quantity $-\beta x$ is the phase where β essentially represent the phase change per unit length so this quantity β is the phase change per unit length. Hence the dimensions for β are the radian per meter or per unit distance so we have the dimension for this which is Rad/met.

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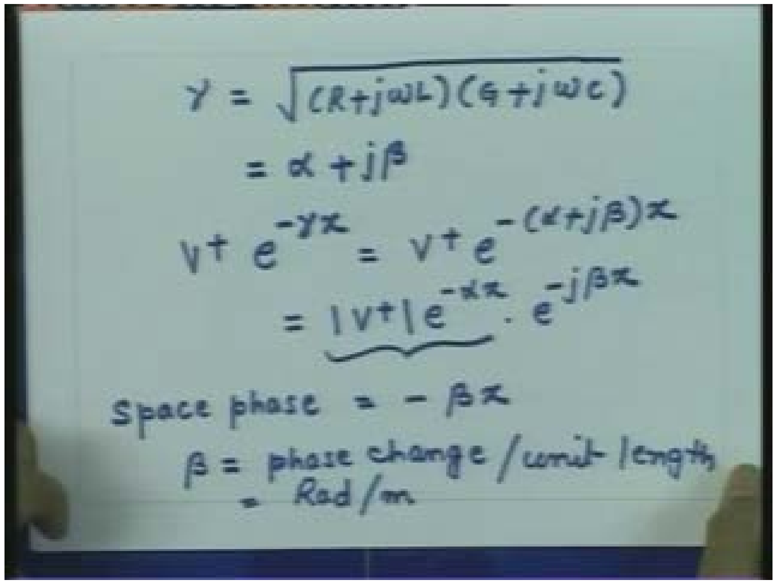

As you know from our basic Physics quotes that whenever there is a phase change of 2π in a wave motion then that distance is called the wavelength which means this phase constant β which is the phase change per unit length also called as the phase constant which is again related to the wavelength of the wave on the Transmission Line.

So by definition, we can say that the phase change is equal to 2π for a distance of wavelength λ so the phase change per unit length is $\frac{2\pi}{\lambda}$. So by definition, we have this quantity β which is equal to $\frac{2\pi}{\lambda}$.

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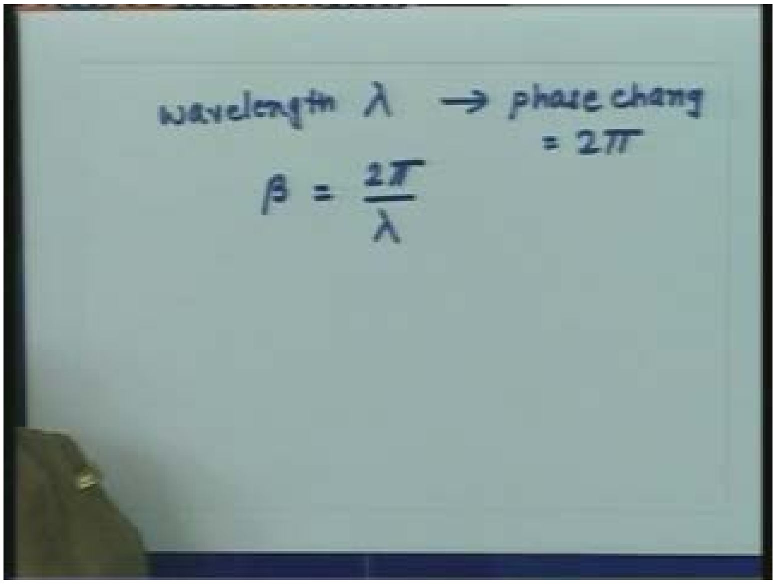

In fact in the wave motion λ is not the one which is first determined. In a complex geometry of wave propagation first you analyze the parameter β. And then by using this relation $\frac{2\pi}{\beta}$ you get a quantity which called as the wavelength.

For a simplified problem we know the velocity and from the velocity divided by the frequency we get the wavelength. However, when you are having a complex structure on which the wave is propagating the velocity is a parameter which is unknown in other words this quantity β is unknown. So normally when we have a wave analysis problem first we solve this parameter β then 2π divided by that quantity β gives you a number which is having a dimension of length and that quantity is called the wavelength of that particular wave.

As we have already seen that since γ depends upon the primary constant R, L, G and C, β is also a function of R, L, G and C and also it could be in general a function of ω. Therefore we conclude that this quantity β which is called the phase constant is a function of the primary parameters of the line R, L, G and C and also is the function of frequency.

Or in other words we conclude that the wavelength of a traveling wave on Transmission Line is a function of line parameters and the frequency. So in general the wavelength changes and the phase constant changes as the line parameters change or the physical structure over which the wave propagation is changed.

The other parameter which we have in the propagation constant is this quantity α, this is representing a wave traveling in the x direction whose amplitude varies as $e^{-\alpha x}$. What it means is that the amplitude of this wave will be modulus of $|V^+|$ as x = 0 and as the wave travels in the x direction its amplitude decreases exponentially with a loss which is $e^{-\alpha x}$.

So if I plot the amplitude of this wave as a function at x = 0, I have a magnitude of the wave which is V^+ and the amplitude of the wave goes on exponentially decreasing with $e^{-\alpha x}$. So α is a parameter which essentially tells you how the amplitude reduction of the wave takes place when it travels on the Transmission Line. Therefore this quantity α is called the attenuation constant because it represents how the wave attenuate in amplitude as it travels along this structure. And it has a special unit that is called the Nepers/meter.

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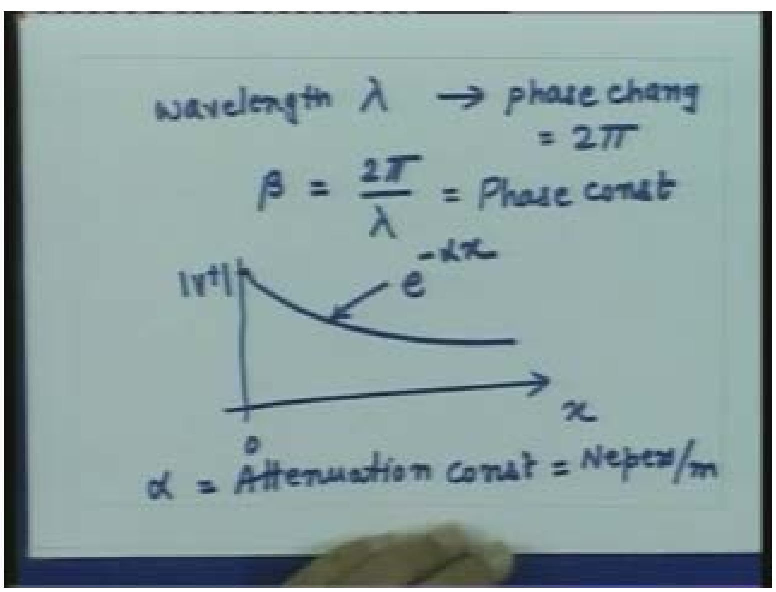

So we can say that if α is equal to one Nepers/meter then the voltage amplitude will reduce from its initial value to one over e of that value over a distance of one meter. If I substitute α = 1 Neper/meter if I put x = 1meter then this part will be e^{-1} times V^+. So the amplitude would reduce to e^{-1} of its initial value which is $|V^+|$. So α is related to a distance over which the wave attenuates to one over e of its initial value. Therefore this length is some kind of characteristic length over which the wave travels effectively on the Transmission Line.

Then one can say that a distance of $\frac{1}{\alpha}$ tells you over what distance the wave effectively is going to travel because the wave amplitude is going to reduce to one over e of its initial value over that distance so we can have something called a Effective travel distance on the line which is of the order of $\frac{1}{\alpha}$.

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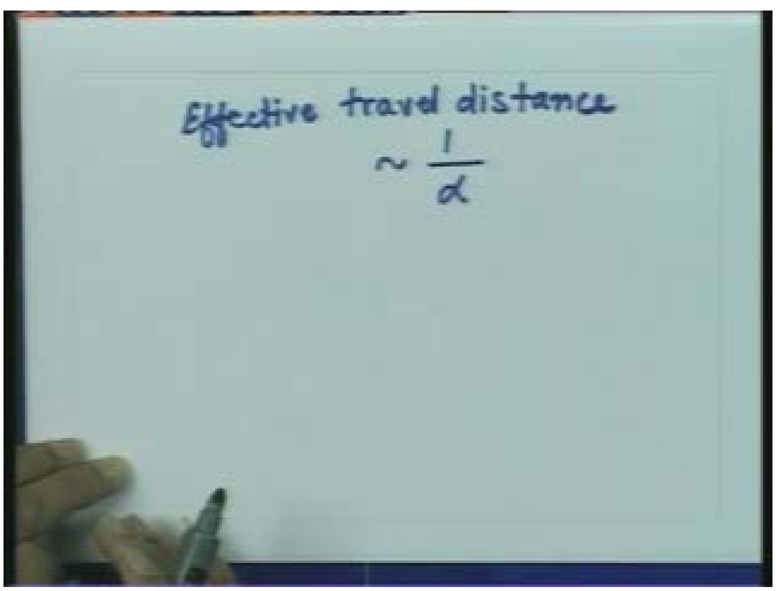

Since the wave amplitude is reducing to one over e of its initial value the power of the wave also reduces so I can represent this quantity in terms of dBs.

If I take the ratio of the two voltages the initial value which is V^+ and the final value after one meter distance which is $V^+ \ e^{-\alpha x}$. You can define the ratio of these two and data quantity i can call as dB so I have dB which is decibel which is given as ratio of these two so $-20 \log(e^{-\alpha x})$.

If I take $\alpha = 1$ Neper/meter and $x = 1$m then this quantity will become e^{-1} so a dB will be $-20 \log(e^{-1})$ which will be 8.68 dB. So that means now we have an important relationship for α in two units one is Neper/meter and other is in dB/meter so we have 1 Neper/meter = 8.68 dB/meter.

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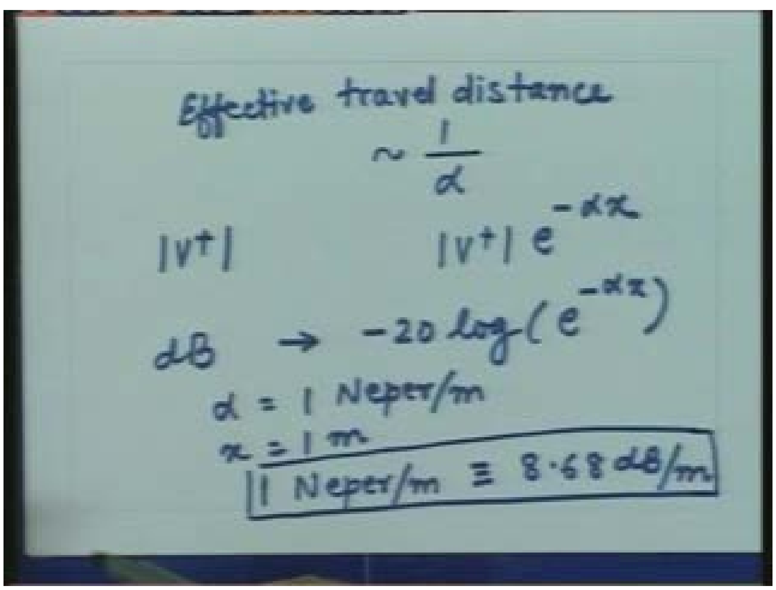

Many times when we have the data sheet the attenuation constant of the Transmission Line is given in terms of dBs. However it should be kept in mind that when you are applying this attenuation constant for solving the Transmission Line problems its value should be always converted to Nepers per meter. So make sure that you never make a mistake of substituting the α in terms of dBs in the expression which is the voltage or current expression, always the value should be converted from dBs to Nepers and that value should be used in the expression for the voltage and current.

Again as you have seen in terms of the phase constants the attenuation constant is also a function of the primary constant or the line parameters and also it depends upon the frequency. So now in general the propagation constant which is a combination of phase and attenuation is a function of line parameters and is a function of frequency.

In variably as the frequency is increased the attenuation constant goes on increasing and that is the reason the same structure which works very satisfactorily at low frequencies becomes more and more lossy as you go to high frequencies.

This effect will be understood in more detail as we progress in this course but at this point it is enough to know that as the frequencies increase the attenuation constant increases and the same structure becomes more and more lossy at high frequencies.

let us take a simple problem so that we can get a feel for this parameters so let us say I have a transmission line which are having primary constants as R = 0.5 Ω/m, L = 0.2 μH/m, the capacitance per unit length C = 100 pF/m and the conductance G = 0.1℧ /m. And let us say the frequency of the operation is 1 GHz.

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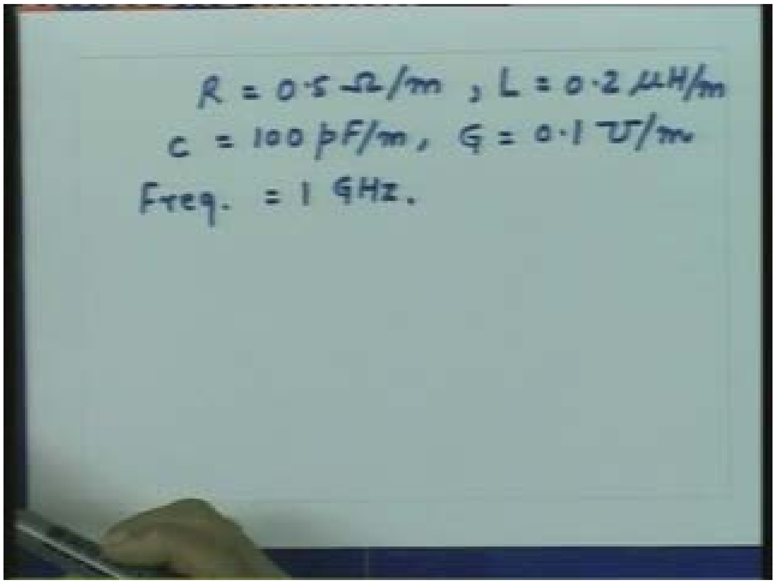

One may ask you to find out what is the propagation constant or explicitly we can calculate what is the attenuation and the phase constant for this line.

First converting the frequency into the angular frequency we have ω which is equal to 1GHz which is again equal to 10^9 hertz into 2π radians/second.

Then the propagation constant γ which is $\sqrt{(R + j\omega L)(G + j\omega C)}$ which we can write it here as $\sqrt{(0.5 + 0.2\, j\omega \times 10^{-6})(0.1 + j\omega \times 100 \times 10^{-12})}$. By separating the real and imaginary parts, we get γ = 2.23 + j28.2.

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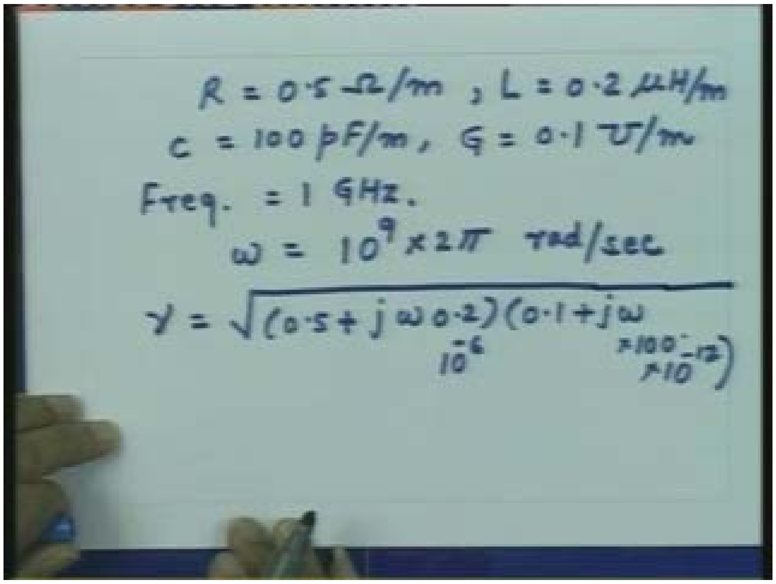

So the real part here which is 2.23 represents the attenuation constant which is α so we have α = 2.23Nepers/m and we have the phase constant β which is equal to 28.2 rad/m.

Once we know the primary constants of the line which are R, L, G and C one can calculate the frequency of operation from this expression the complex propagation constant then separate out the real and imaginary parts of γ then we get the attenuation constant α and the phase constant β.

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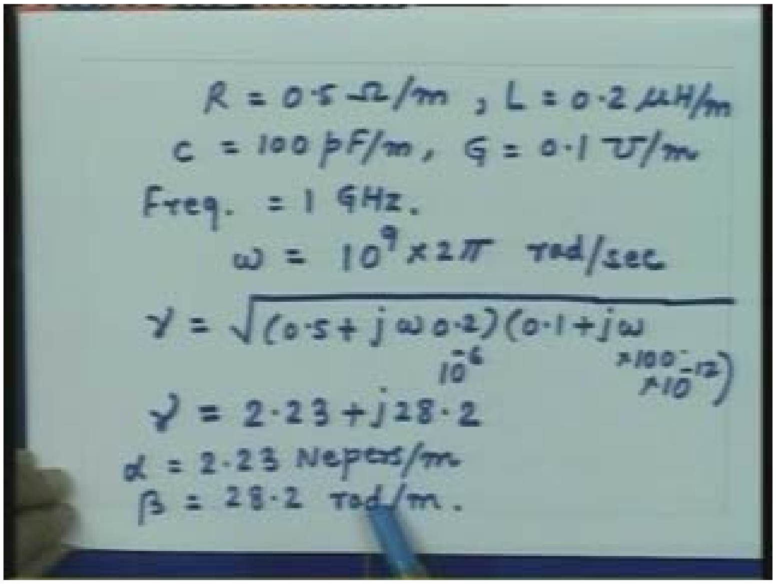

Now let us apply this to the voltage expression on the Transmission Line. Let us say I have a traveling wave on a Transmission Line the α and β for that Transmission Line are given by this because the primary constants of the line are given by this parameters. So I know this quantity α and β for Transmission Line. Let us say at some instant of time and at some location of the line which we call as x = 0, t = 0 the voltage measured was some value and we want to find out what the voltage would be at some other point on the line at some other time.

So let us say I have a line here I have some location here which I call x = 0 and at some instant of time I measure the voltage between the terminals of the line I want to find out what will be the voltage at some other point on the Transmission Line at some other instant of time. Let us say the voltage measured at x = 0, t = 0 8.66Volts and let us say I want to find out the voltage at some other distance x = 1m and t = 100nanosecond. Also we want to find out let us say what is the peak voltage which will reach at this location. As we can notice here since we are having a phase difference between these two points the voltage may not be at the peak value at t = 100nanosecond.

So we have to find out two quantities what is the voltage at this instant of time at this location x = 1m and what could be the highest value of the voltage which can reach at this location. Also we can pose this problem for two cases. One is if the voltage wave was traveling from left to right what would be the voltage at this location and second is if the voltage wave was traveling from right to left what will be the voltage value at this location.

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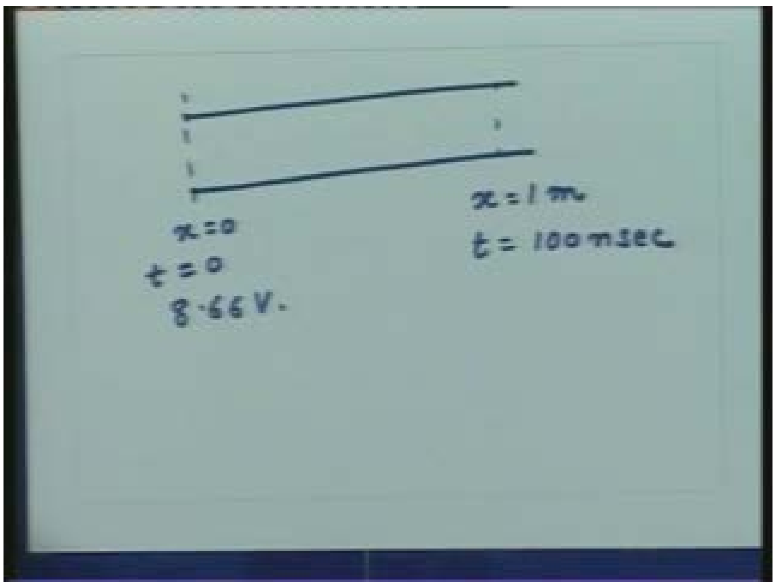

Let us first consider the voltage wave which is traveling from left to right. As we have seen the voltage as a function of time is given as the real part of $V^+ e^{-\alpha x} e^{-j\beta x+\omega t}$. This V^+ in general is a complex quantity because the voltage which is given here 8.66 may not be the maximum value at this location. So in general this quantity can have a phase and amplitude so this is equal to $|V^+| \cos(\phi + \omega t - \beta x)\, e^{-\alpha x}$.

So here this quantity $V^+ e^{-\alpha x}$ gives you the amplitude variation as the function of distance and this gives you the phase including the initial phase which the signal might be having at x = 0.

Now substituting the condition which is given to you that x = 0, t = 0 the voltage is 8.66 if I substitute into that I get 8.66 that is equal to for about x = 0, t = 0 this quantity will go so I will get $|V^+| \cos\phi$ without knowing the initial phase at this location I cannot solve the problem. One possibility is that I may assume is this quantity has zero phase however in general this phase might not be zero so let us say this voltage which I measure here is 8.66 volts and the phase of this time signal was 30˚.

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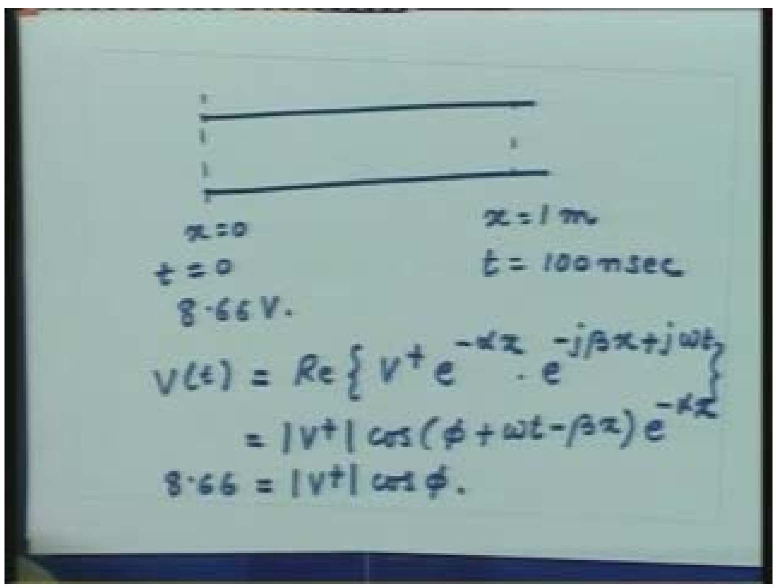

So the initial phase ϕ is 30˚ at this location x = 0. Once I substitute that I can get from here the $|V^+| \cos 30˚$. By inverting this I get the amplitude of the wave at x = 0 which is $|V^+|$ that will be equal to 10volts.

Now substituting this V^+ in this expression I can now calculate what will be the value of voltage at x = 1m, t = 100nanoseconds.

When I substitute here t = 100 nanosecond, we know the frequency which is 1GHz, we have calculated β, the ϕ is given 30˚ and V^+ is known which is equal to 10Volts so I can find out what is the instantaneous value of the voltage v(t) which will be equal to -0.88V.

The peak voltage corresponding to this at this location would be the value of V^+ multiplied by $e^{-\alpha x}$ so since the α is given as 2.23 Nepers/meter I can substitute α there.

We can substitute x = 1meter and I can get the peak value of the voltage at this location that is equal to $10 \times e^{2.23}$ which again will be equal to 1.75Volts.

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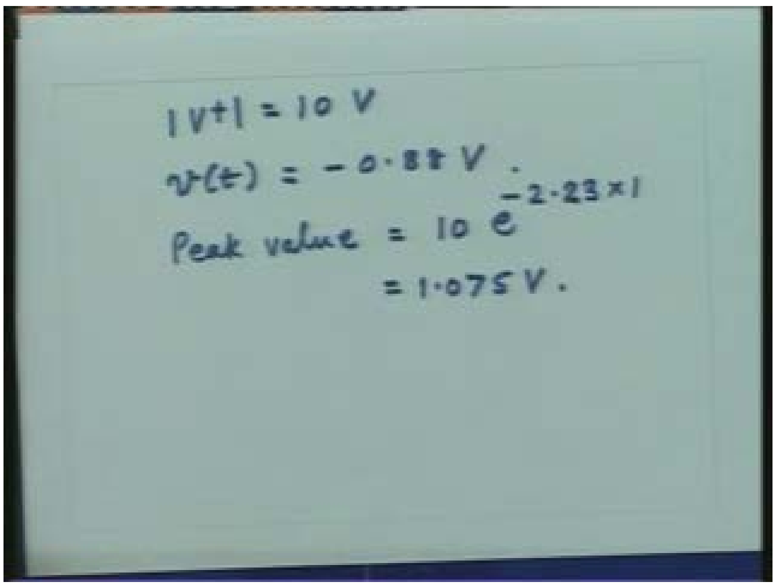

So in this problem we understand that if the value of the voltage was given at some particular location on the line at some instant of time then by using this relation we can find out the voltage at any other instant of time at any other location on line. Also we can find out what maximum voltage can reach at that location on the Transmission Line so we can find out the peak value of the voltage, also I can find out the instantaneous value of the voltage.

As we can see from here since α is positive the wave is traveling from left to right the amplitude of the wave here is 10Volts, by the time the wave reaches to one meter its amplitude has reduced to 1.07Volts. So the wave attenuates as it travels from left to right. If I take other situation where the wave travels from right to left then since the wave is traveling from right to left its attenuation is from right to left that means now if I work backwards I know the wave amplitude at this location if I work out backwards its amplitude at this location will be more compared to the amplitude at this location. So the two waves when they are traveling in different directions since α is positive, their amplitude behavior are not same the wave always attenuates in the direction of propagation. So if I take a wave which is moving in the forward direction its amplitude will reduce from left to right, if I take a wave which is traveling from right to left then its amplitude will increase as a function of x because the wave is traveling in the backward direction and attenuation is in this direction. Since I know the value of the voltage at this location if I move in the positive x direction the amplitude of the voltage is increased.

So by using the same expression and changing this sign of β because the backward wave is given as $|V^{+}|\ e^{+\alpha x}\ e^{+j\beta x+j\omega t}\ e^{j\phi}$.

Now the amplitude grows for this wave as we move in the positive x direction and if I substitute the values of α, β, distance and time and the initial phase in this expression then I will get the voltage at the same time and distance that is x = 1m, t = 100 nanoseconds will be equal to -83.77Volts.

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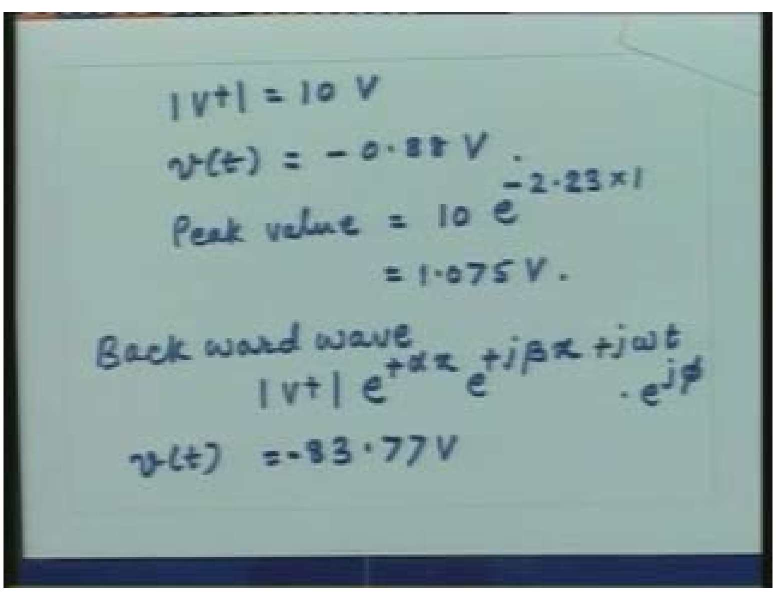

So its amplitude at this location would be 1.075Volts when the wave was traveling from left to right. If the wave was traveling right to left then this amplitude would be -83.77 Volts at this location. Though at x = 0, t = 0 both the waves we have the same amplitude which is 10 Volts.

Now we can go to getting the complete solution of the differential equation with this understanding of the propagation constant γ. Till now we have simply solved the second order differential equation got a general expression for voltage and current and just tried to see what this two terms of the solution represent and they were the traveling waves traveling in the opposite directions.

Now without getting into the boundary conditions I can still do some manipulations to reduce certain arbitrary constants so that I can make a step towards the complete solution of the voltage and current. So first thing I should notice is the voltage and the current expression which I have written is completely general in nature, which means they must satisfy the original differential equation at every point on Transmission Line. So the first thing I can do is I can take this voltage and current and substitute into the original differential equation. So if you recall that we had the differential equation that is $\frac{dV}{dx} = -(R + j\omega L)I$ and $\frac{dI}{dx} = -(G + j\omega C)V$, now I know the expressions for V and I so I can take any of these two equations and substitute the general expressions for V and I in this and then try to establish a relationship between the arbitrary constants.

So substituting for V and I expressions for the general voltage solution I get $\frac{d}{dx}\left\{V^+e^{-\gamma x}+V^-e^{\gamma x}\right\}$ that is again equal to $-(R + j\omega L)\left\{I^+e^{-\gamma x}+I^-e^{\gamma x}\right\}$.

By differentiating these two terms this I can get $-\gamma V^+e^{-\gamma x}+\gamma V^-e^{\gamma x} = -(R + j\omega L)\left\{I^+e^{-\gamma x}+I^-e^{\gamma x}\right\}$.

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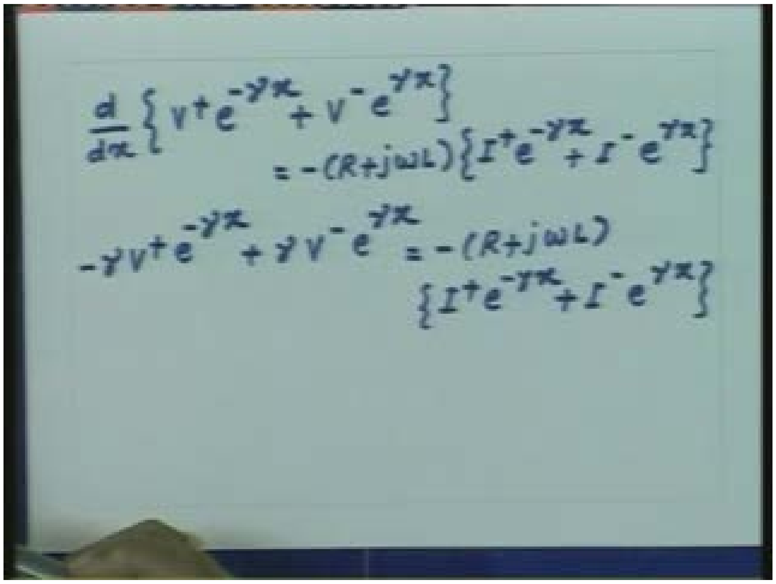

In first look it appears that there is one equation which is relating the voltages and currents. However if you look at it little deeply what will you notice is here these two terms the V^+ and the V^- are representing two waves which are traveling in different directions. Similarly I^+ and I^- terms are also representing the current waves which are traveling in two different directions. This general relation between the voltage and current has to be satisfied at every location on the Transmission Line.

Since the two waves which are traveling in the opposite direction have different phase histories you cannot satisfy this equation for every value of x unless these individual waves satisfy the conditions on the line of these equations at every point on the Transmission Line. What that means is within this one equation there are two embedded equations the forward traveling wave which is having V^+ amplitude should be related to the forward traveling wave for current, similarly the backward traveling wave for voltage should be related to the backward traveling wave for current. So in fact from this one equation you can get two equations because both the waves have to satisfy the differential equations individually because this voltage expression and current expression should satisfy this condition or differential equations at every point on Transmission Line, I get, $-\gamma V^+ e^{-\gamma x} = -(R + j\omega L)I^+ e^{-\gamma x}$ and $\gamma V^- e^{\gamma x} = -(R + j\omega L)I^- e^{\gamma x}$.

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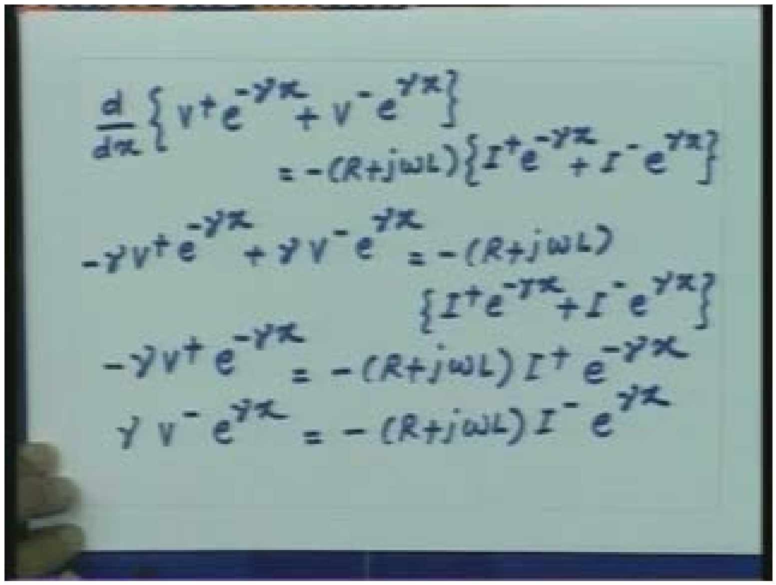

Now, from this I can get the relationship between the amplitudes of the voltages and currents for the two waves in this expression $e^{-\gamma x}$ will cancel. So I will get a relation between V^+ and I^+ and from this equation $e^{-\gamma x}$ will cancel so I will get a relation between V^- and I^-.

Therefore I get, $\frac{V^+}{I^+} = \frac{(R + j\omega L)}{\gamma}$ and $\frac{V^-}{I^-} = \frac{-(R + j\omega L)}{\gamma}$.

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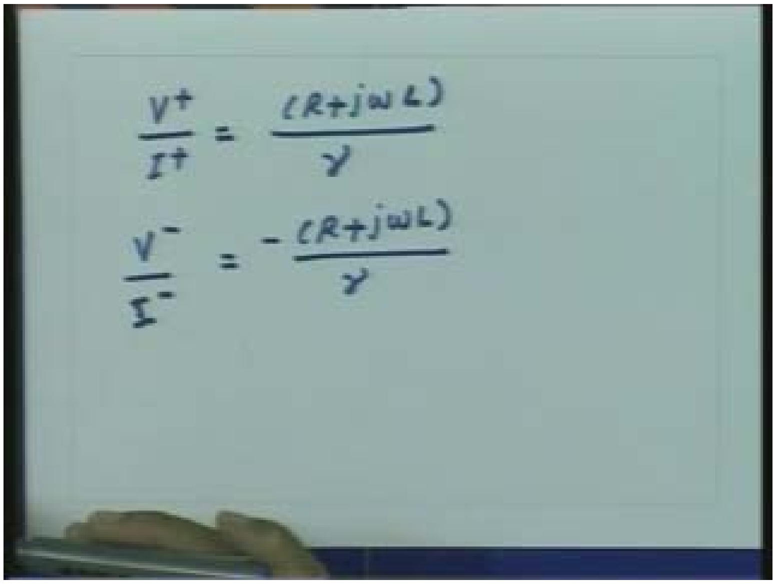

Substituting for $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$, so this will be equal to $\sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$ and this is equal to $-\sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$.

Now this quantity again looks like a characteristic quantity of the line because this quantity is related to only the primary constants of the line which are R, L, G and C and also frequency so as we had a parameter γ which was the propagation constant which was a combination of the line parameters and the frequency it looks like another parameter which looks like a characteristic parameter of the line. Also, this parameter is a ratio of voltage and current so this quantity has a dimension of impedance. That is the reason why we define this quantity as the characteristic impedance of the line and normally this quantity is denoted by Z_0 of the Transmission Line.

So we define a quantity called the characteristic impedance which is denoted by Z_0 that is equal to $\sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$.

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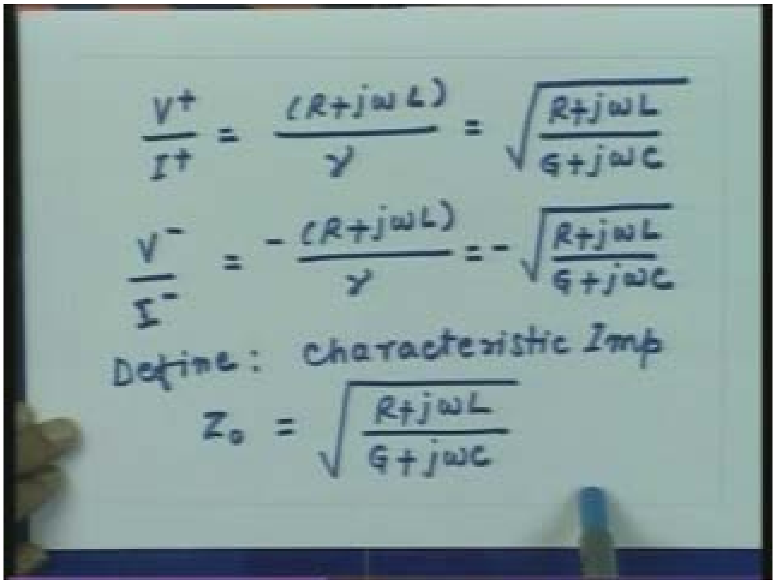

Later on you will see that this quantity Z_0 or the characteristic impedance of the line governs the energy flow on the Transmission Line. So the two parameters the propagation constant γ and the characteristic impedance Z_0 are completely characterize the propagation of a Transmission Line. The R, L, G and C though they are the primary parameters hardly we go to these parameters in the Transmission Line calculations. most of the time the Transmission Line characteristics are defined in terms of the propagation constant γ and the characteristic impedance Z_0 and these two parameters are adequate to analyze any of the Transmission Line problems.

So until and unless one specifically wants to calculate the attenuation and phase constants from the primary constants of the line one can use the data sheet where the γ and Z_0 are directly given and thereby it can be used of for solving the problem of Transmission Line.

Now, what does this parameter Z_0 tells you? You will notice from this equation that the ratio of the V^+ to I^+ is equal to Z_0 where V^+ is the amplitude of a voltage traveling wave in the forward direction I^+ is the amplitude of the current traveling wave in the forward direction that means is at any location on the Transmission Line if I take a ratio of the voltage and current for the forward traveling wave that is always equal to the characteristic impedance irrespective of what other boundary conditions are there on Transmission Line.

Recall we have not applied any boundary conditions on transmission lines. Till now we have found out the general solution of the differential equation for the voltage and current and substituting this general voltage and current solutions into a differential equation, we have established a relation between the voltage and current amplitudes of the traveling waves. So we can make a general statement that the forward traveling wave has a ratio of voltage and current which is always equal to the characteristic impedance of Transmission Line.

Similarly we can get the ratio of the voltage and current at any point on transmission line for a backward traveling wave which is always equal to $-Z_0$. In other way the forward traveling wave always sees an impedance which is equal to Z_0 where as the backward traveling wave always sees an impedance which is equal to $-Z_0$.

So as long as we are having the traveling waves on Transmission Line their voltage and current relationship is fixed by this parameter characteristic parameter of Transmission Line called the characteristic impedance. If I look at this quantity which is $\sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$ you will get the real part of this quantity which will be positive. What that means is the forwards traveling wave sees impedance whose real part is positive means it sees a resistance as it travels on the Transmission Line.

Since the Transmission Line has a passive structure with resistance, inductance, capacitance and conductance which makes a sense that a voltage or current source sees a resistance ahead of it. However, if I look at the backward wave since this quantity Z_0 is same for this two if the real part of this is positive that it looks like a resistance it will be negative for the backward traveling wave or it will appear that the backward traveling wave sees a negative resistance as it travels in the positive direction. The wave is not actually traveling in the positive direction the wave is traveling actually in the backward direction but if I calculate the ratio of this its impedance is equal to negative and what negative resistance represents is the energy is not supplied but rather energy is received. So this essentially telling you is that if I look in the positive x direction I will receive the energy which make sense in this case because now the wave is traveling backwards which essentially is carrying the energy backwards.

Since we are having a energy source at the left it is equivalent to saying that there is something on the right side which is supplying energy to the to the energy source on the left side. So the backward wave essentially is like an energy source supplying energy to the generator that is equivalent to negative resistance. Here negative resistance is not in the conventional sense it essentially the manifestation of the direction of the energy flow on the Transmission Line. So again concluding that irrespective of the boundary conditions on the Transmission Line we establish the forward traveling wave always sees ahead of it the impedance which is equal to characteristic impedance and a backward traveling wave sees characteristic impedance if I look backwards but if I see forward then the backward traveling wave will see an impedance which will be negative of the characteristic impedance.

So as I mentioned these two quantities γ and Z_0 are adequate to solve the Transmission Line problems once I get this then I can reduce the arbitrary constants in a voltage and current expressions because I can substitute for I^+ which is a $\frac{V^+}{Z_0}$ and I^- can be substituted by $\frac{V^-}{Z_0}$.

So now the voltage and current expressions on the Transmission Line can be written as $V = V^+ e^{-\gamma x} + V^- e^{\gamma x}$ and I which is $I^+ e^{-\gamma x}$ but I^+ is $\frac{V^+}{Z_0}$ so I can substitute that, similarly I^- is $\frac{V^-}{Z_0}$.

So, I^+ will be $\frac{V^+}{Z_0} e^{-\gamma x} - \frac{V^-}{Z_0} e^{\gamma x}$.

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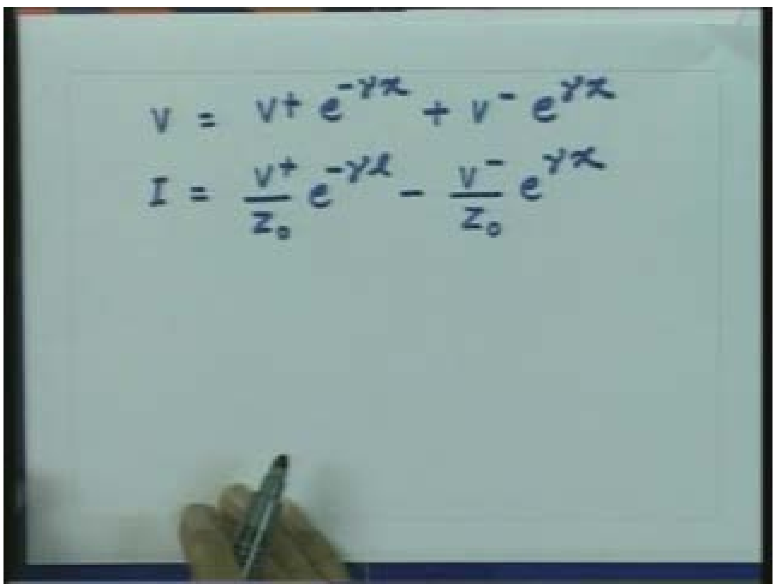

So essentially by substituting the voltage and current expressions in the differential equation we eliminated two arbitrary constants I^+ and I^- and now we are left with only two arbitrary constants which have to be evaluated are V^+ and V^-.

Till now we have not even defined the origin on the Transmission Line. Before I apply the boundary conditions let us define the origin of Transmission Line. Normally we have two special locations on Transmission Line, in which one is at this end where the generator is connected and the other end where some impedance will be connected called the load impedance. So when I define the boundary conditions for this problem either I can define the origin for the boundary condition at this location which is at the load end or I can define the origin at this location which is at the generator end.

Generally it is preferred to define the origin at load end. So we define this point as zero location and then all distances essentially are traveled towards the generator. So now let me define instead of x which was moving in this direction let us say I have a parameter l which moves in this direction so I have a distance which is now measured towards the generator from the load with l = 0 at the load point. So now I am defining every value of x. So now l is negative of x because x was moving from left to right and l moves from right to left and l = 0 is at the location of the load.

Substituting for x which negative of l I can write down the voltage and current in terms of this l which is the distance measured from the load towards the generator.

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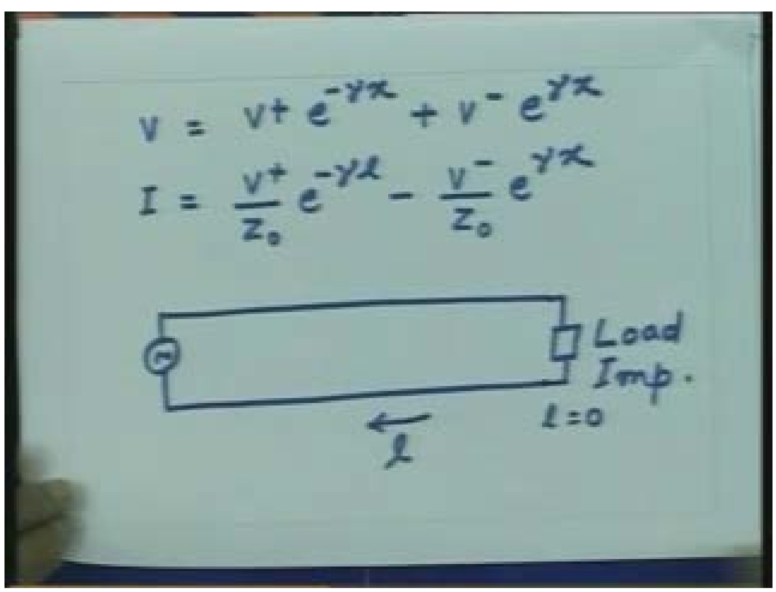

So I get the voltage which is $V^+ e^{\gamma l} + V^- e^{-\gamma l}$ where l = -x and $I = \frac{V^+}{Z_0} e^{\gamma x} - \frac{V^-}{Z_0} e^{-\gamma x}$.

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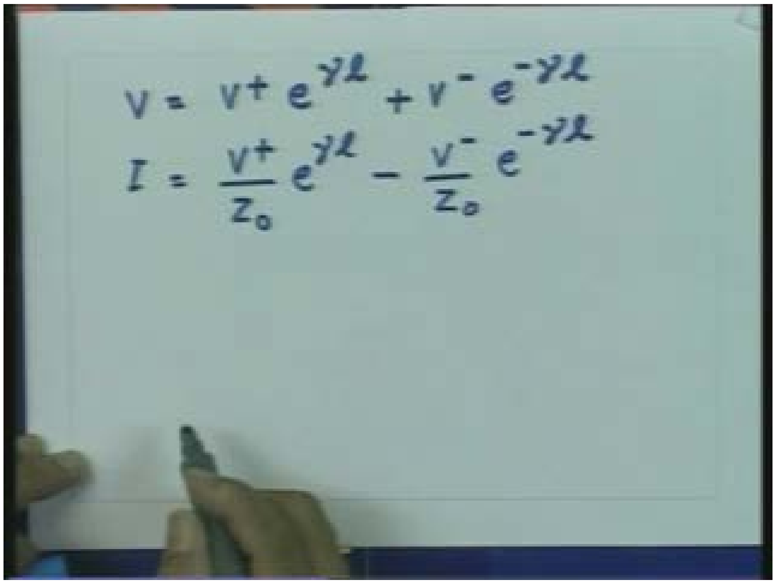

And l = 0 corresponds to the location of the load or the other end of the Transmission Line. Sometime that end is also referred as the receiving end of the Transmission Line and the generator end is also called as the transmitting end of the Transmission Line.

Once I get this then I have a boundary condition which I can apply at l = 0 because the line is terminated into a load impedance which is a known load impedance. So I have a boundary condition that is at l = 0 at the load point I have terminated a line in some given impedance so the impedance Z is equal to some impedance which is equal to Z_L.

So I can take the ratio of the voltage and current at l = 0 and from there I can apply the boundary condition that this ratio V over I should be equal to the load impedance.

So taking the ratio of these two I will get $Z_L = \left.\frac{V}{I}\right|_{l=0} = Z_0 \frac{V^+ + V^-}{V^+ - V^-}$.

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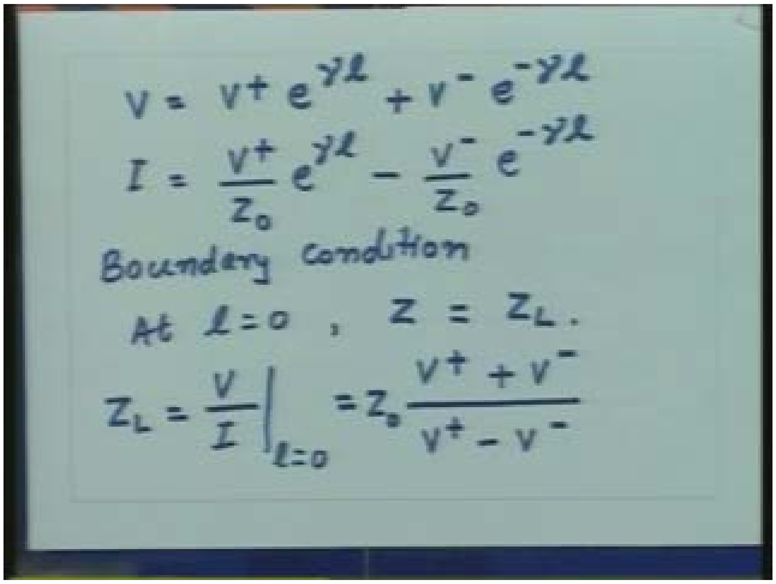

So one thing is clear that the load impedance is related to the characteristic impedance, also it is related to the amplitude of the forward wave and amplitude of the backward wave. One can also notice from this expression that I can take this quantity V^+ common and this can be written as $Z_0 \frac{(1+\frac{V^-}{V^+})}{(1-\frac{V^-}{V^+})}$.

What that means is that the absolute values of V^+ and V^- do not matter as long as we are applying the boundary condition for impedance it is the relative value of $\frac{V^-}{V^+}$, they will decide what is the load impedance.

Therefore the ratio of these two quantities $\frac{V^-}{V^+}$ is the meaningful quantity as far as impedance boundary condition on the Transmission Line is concerned.

So now we define a new parameter which is the measure of the backward and the forward wave which is $\frac{V^-}{V^+}$ and we call that quantity as the reflection coefficient.

Let us say I define a parameter Reflection coefficient denoted by Γ and this is defined as ratio of the backward wave to the forward wave. So at any location if I take the voltage for the backward wave which will be $V^- e^{-\gamma l}$ and if I take the voltage for the forward wave wgich is $V^+ e^{\gamma l}$ the ratio of these two quantities is called the reflection coefficient.

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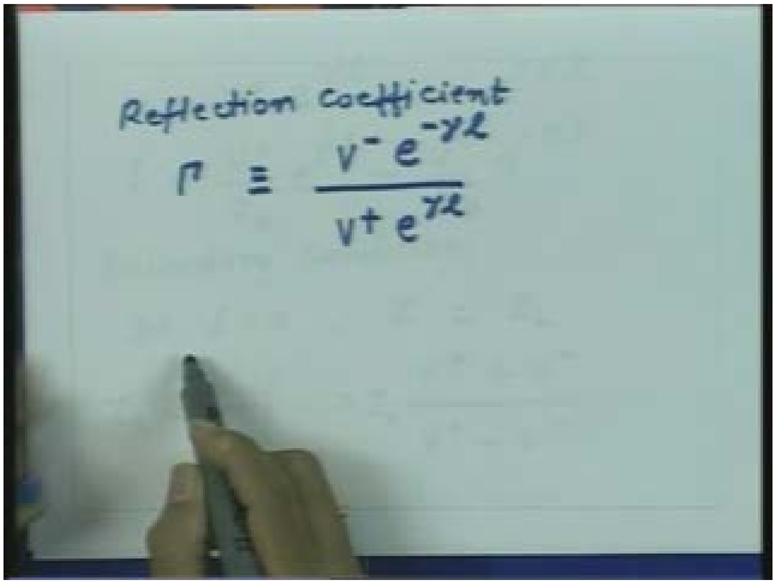

Since we are taking the ratio of the backward and forward voltage waves we call this Reflection coefficient as the Voltage Reflection coefficient. So we have parameter for this Voltage Reflection coefficient which is ratio of the backward traveling wave to the forward traveling wave and at l = 0 the value of the reflection coefficient will be $\frac{V^-}{V^+}$.

So in general at location l the Reflection coefficient is defined like this but at l = 0, Γ(0) the reflection coefficient at zero distance which means at the load point will be equal to $\frac{V^-}{V^+}$.

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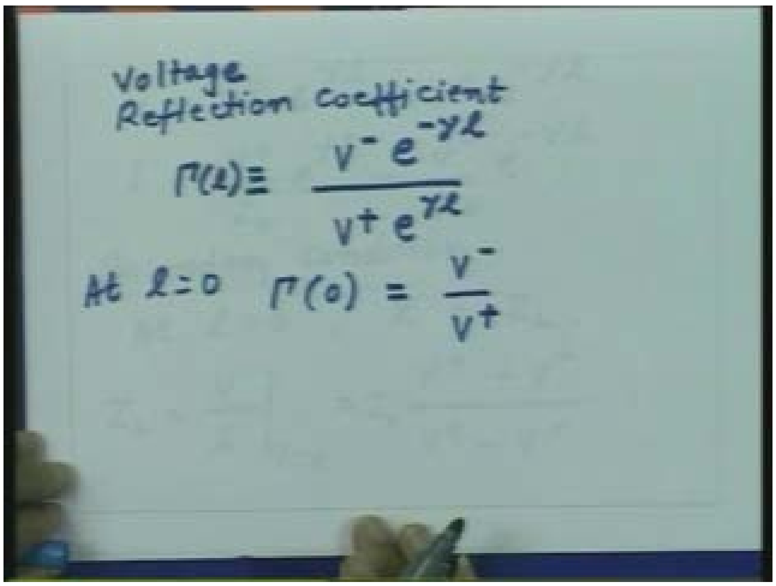

Now substituting this in impedance relation here, I can get $Z_0 \frac{(1+\frac{V^-}{V^+})}{(1-\frac{V^-}{V^+})}$.

So here I have the load impedance Z_L = $Z_0 \left\{ \frac{1+\Gamma(0)}{1-\Gamma(0)} \right\}$.

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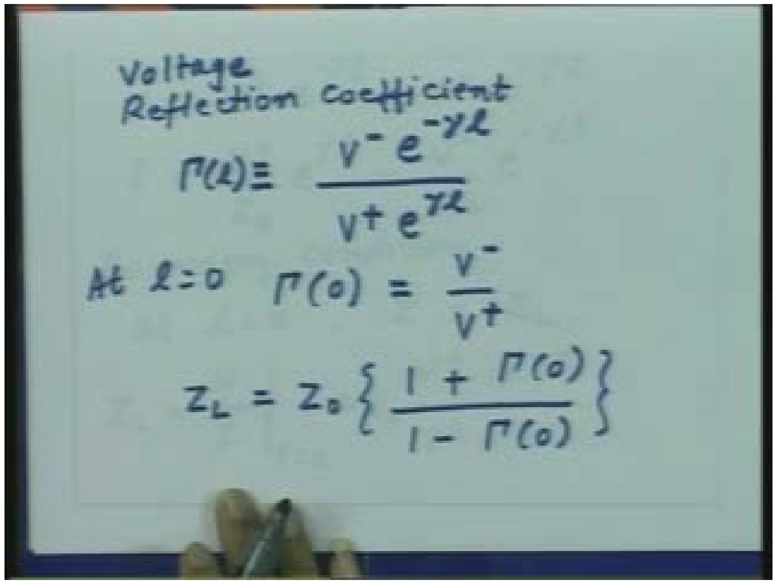

Inverting this relation essentially now we get the quantity Reflection coefficient which will define like this in terms of the impedance which is connected to the line. What does reflection coefficient tell you? It tells you the ratio between the reflected voltage and the incident voltage that means it is a measure of how much energy is reflected from the load end of the Transmission Line compared to what was incident on that. Ideally if my intention was to deliver the whole energy to the load this quantity should be as small as possible.

What we will do in the next lecture is we will establish the conditions under which you will have the full transfer of the power to the load or there will no reflection from the line. Also, we will briefly discuss why there is reflection on the line, what the reflection coefficient essentially indicates and what is the role of the load impedance Z_L and what is the role of the characteristic impedance in reflecting the energy from the load and the Transmission Line.

Thank you.