Transmission Lines and E.M. Waves
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Lecture-29

Welcome, till now we have discussed the propagation of plane wave in an unbound medium we oriented our coordinate system such that the wave propagated in direction of one of the axis that was z axis and we could do this because unbound medium essentially is symmetric in all directions so no matter in what direction we looked the medium appears same and that is the reason we had the flexibility in choosing the direction of the coordinate axis.

However if you have a bound medium then the medium does not appear same in all directions and then the choice of coordinate axis might affect the analysis essentially the algebraic manipulation which we require in different ways. So essentially we choose the coordinate system so that the analysis of the problem become little simpler and when we do this the choice of coordinate axis gets restricted.

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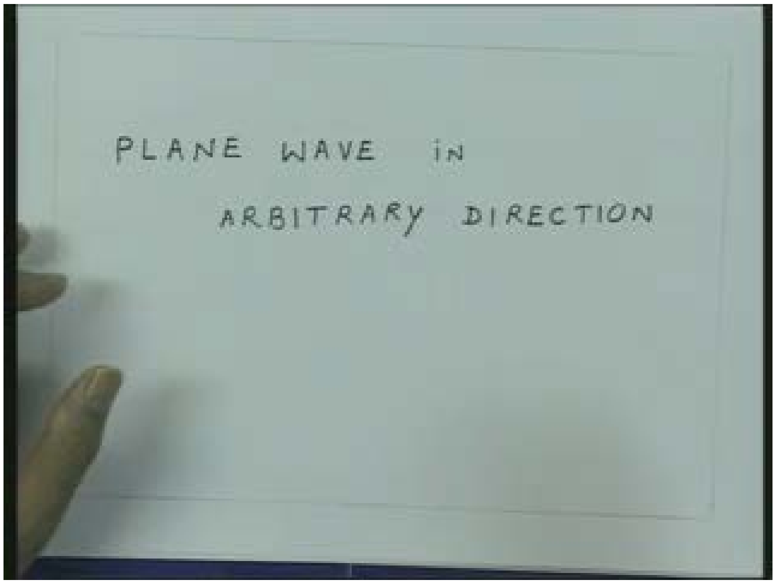

So first we will try to put a medium which is the semi infinite medium that means you will divide the unbound into two halves then we will investigate the property of Electromagnetic Wave in this semi infinite medium and then slowly as we proceed in this course we will try to capture the Electromagnetic Wave in more and more bound medium.

However, before we do this analysis of the propagation of the Electromagnetic Wave in a bound medium we require a representation of an Electromagnetic Wave which is traveling in some arbitrary direction with respect to the coordinate axis.

So, today we investigate the plane wave propagation in an arbitrary direction with respect to the coordinate system. If you recall when you had a coordinate system such that the wave was propagating in the z direction the phase variation for this Electromagnetic Wave was in the z direction and the electric and the magnetic fields lied in a plane which was perpendicular to the z direction that means they lied in the xy plane. We still have a medium which is unbound but the only thing is we are orienting the coordinate system in such a way that the wave is moving in some arbitrary direction with respect to the coordinate axis.

So let us say if you have a coordinate axis like this and if this is the x direction, this is y direction then by right hand rule when we put my fingers like this I will get this direction at the z direction.

Now let us say the wave is propagating in some arbitrary direction which is moving in this direction so that is the direction of the wave propagation so let us say this is wave direction and the constant phase planes that means the plane in which the electric and magnetic fields lies they are perpendicular to this direction so we have some planes which are perpendicular to this direction. So we call these planes as the constant phase planes so where the wave propagates essentially is the phase which are moving and that essentially gives you the phenomena of wave propagation.

Now if you want to represent this wave the wave is characterized by the wave function that means we have to write down the wave function for this wave which is traveling in some arbitrary direction.

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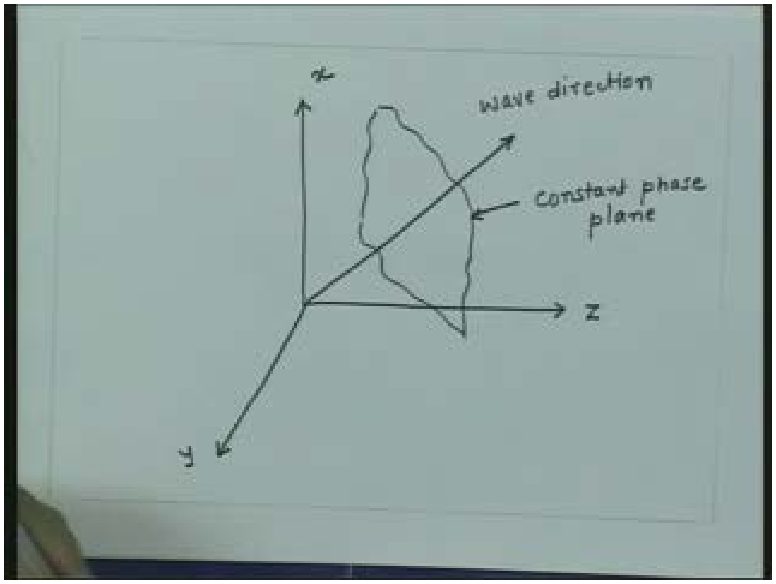

Let us say the directions which it makes the x, y and z axis are given by the angles $\o_x$, $\o_y$, $\o_z$. Then we can write down the direction cosines of this line which will be cos $\o_x$, cos $\o_y$, cos $\o_z$. So essentially we can write down the unit vector in the direction of the wave propagation let us denote that by n cap which can be written as we have the unit vector n cap = cos $\o_x$ x cap + cos $\o_y$ y cap + cos $\o_z$ z cap where x, y and z are the unit vectors in the direction x, y and z respectively and these quantities cos $\o_x$, cos $\o_y$ and cos $\o_z$ are called direction cosines of this line. So these quantities are the direction cosines.

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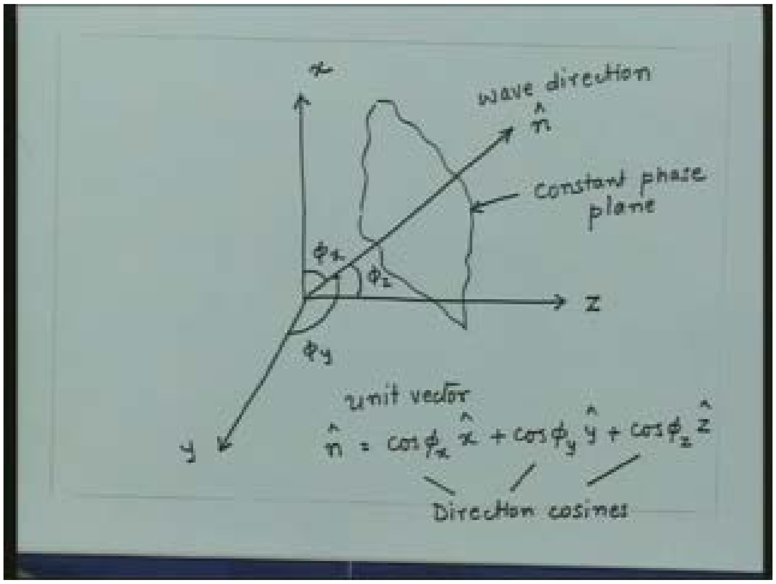

So the wave motion which is in this direction is given by the unit vector n is essentially characterized by this direction cosines which are the cosines of the angles where this line makes with the three axis x, y and z. Let us say this vector intersect this wave front at some point here let us say this point is O and this point is A.

Now if I take some arbitrary point on this phase front lets say this point is P. I can write down the vector for OP if I know the coordinate of this point and let us say the coordinate of this point are (x, y, z). So I can write down the vector OP which x x cap + y y cap + z z cap so I know this vector which is the unit vector in the direction of the wave motion. So we can write this vector OP bar = x x cap + y y cap + z z cap.

Now if I look at this thing carefully the normal distance you take any point on this plane which is the constant phase plane the normal distance of this is given by OA. If I take any point on this plane and if I find out this projection of that direction in the direction of the normal is OA and that is fixed irrespective of what point I take from the phase.

So, essentially the equation of this phase front is the dot product of this vector and the unit vector is equal to this which is constant. So we have equation of the constant phase plane from here and that is unit vector n cap dot product OP bar, this dot product will always be equal to this distance OA where this point is the normal point on the wave front then this is equal to |OA| which is equal to constant.

Now if I substitute for n cap and OP from here I will get x cos $ø_x$ + y cos $ø_y$ + z cos $ø_z$ is equal to constant. And this constant is nothing but magnitude of OA that means it is telling me the distance of this plane from the origin.

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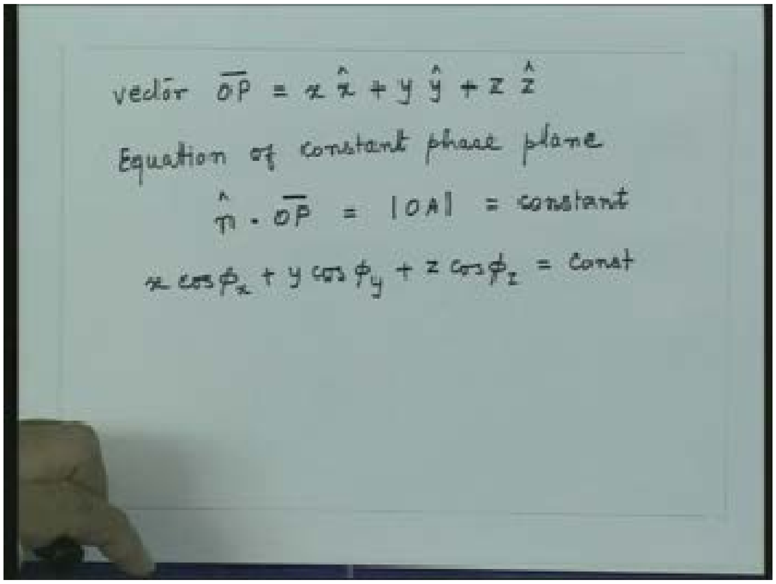

Now, if the wave is having a phase constant β and if this distance OA the phase of this plane is nothing but β times the distance traveled to OA. So if I assume that the wave was having zero phase when it was passing through the origin then the phase of this plane is distance OA multiplied by the phase constant which is β. So I have the phase of the plane equal to β into OA which is nothing but β into n cap dot vector r bar because this OP vector is the position vector of this point which is nothing but denoted vectorially the r bar vector.

So I have a point P which is denoted by this position vector r then the phase of the plane is β times the dot product of the unit vector in the direction of the wave propagation and the position vector of any point on a constant phase plane. Once you get the phase of this plane then writing the expression for the electric or magnetic field which is corresponding to this wave propagation is very straight forward. Essentially what we have is some electric field vector which is lying in this plane and the magnetic field is the line perpendicular to the electric field vector and the direction of wave propagation and the phase variation for electric and magnetic fields both is given by this one which is β into n dot r

So what we can do is we can write down the electric field for this wave say E has a magnitude which is a vector quantity so this E_0 tells me the magnitude of the vector and this vector tells me the direction which is lying in the plane of this constant phase ø in this plane and the phase variation of this is e to the power -jβ(n.r) and since electric field vector has to be perpendicular to the direction of the wave propagation we have the electric field E_0 dot n should be equal to zero because E_0 is perpendicular to n

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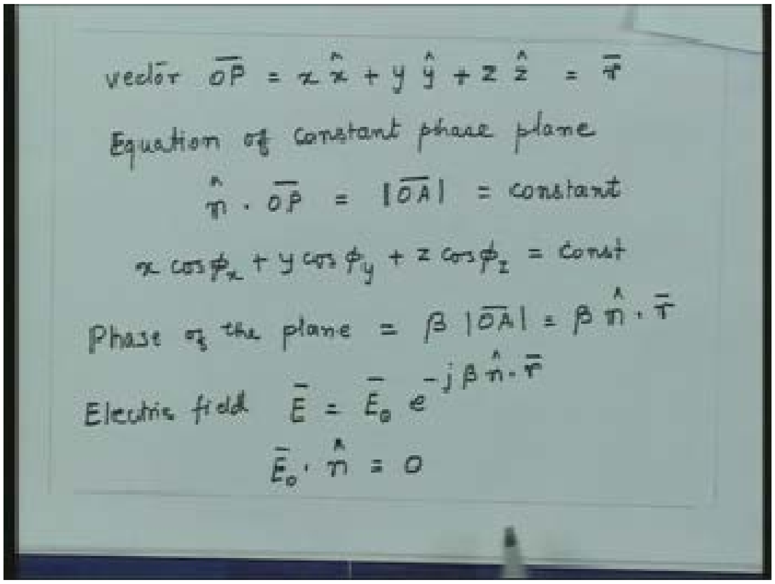

So a transverse electromagnetic wave traveling in some arbitrary direction is given by the unit vector n cap represented essentially by this.

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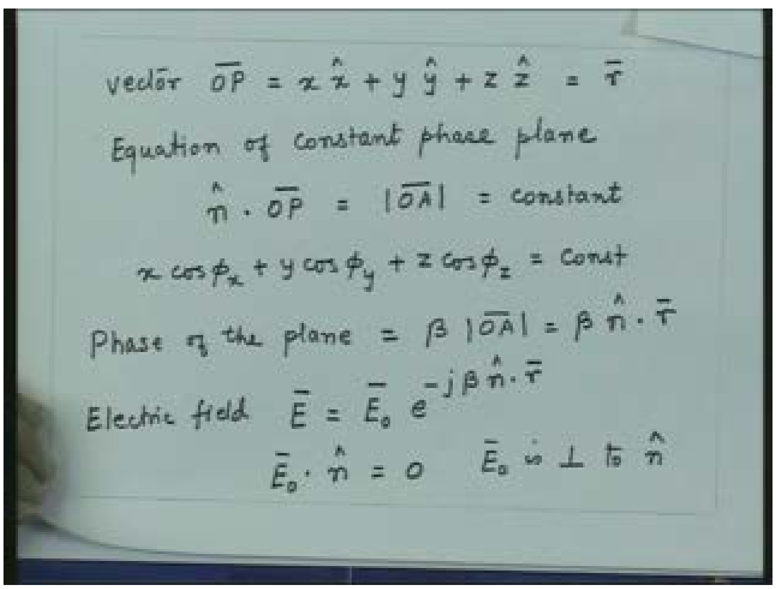

What we can do is we can combine this β to this n and we can define another vector which is β times n cap and we can call that vector as the wave vector.

So essentially we define a parameter for this wave which is traveling in some arbitrary direction as the wave vector and is denoted by small K bar which is nothing but β into n cap. Expanding for n cap this is equal to β into {cos $ø_x$ x cap + cos $ø_y$ y cap + cos $ø_z$ z cap}. Taking β inside you get this wave vector K bar which is equal to β cos $ø_x$ x cap + β cos $ø_y$ y cap + β cos $ø_z$ z cap.

Then this quantity essentially tells you the component of this wave vector in the x direction, this component tells me the component of the K vector in y direction and this quantity tells me the component of the wave vector in the z direction.

So we can denote these quantities as K_x, K_y and K_z so we can call this quantity as K_x, we can call this quantity as K_y and we can call this quantity as K_z. So this vector K bar is K_x x cap + K_y y cap + K_z z cap.

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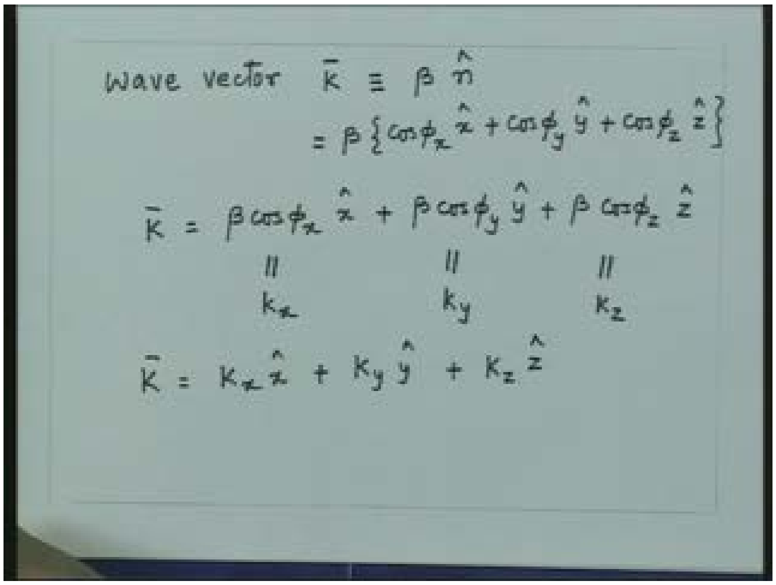

So if I know the direction of wave propagation if I know the phase constant of β which depends upon the medium parameters then I can find out this wave vector which completely characterizes the wave propagation in the arbitrary direction.

And then as we wrote here that E_0 dot n should be equal to zero. The same thing we can write down for the K vector that the direction of the electric field and the K vector are perpendicular to each other that means the dot product of these two quantities should be equal to zero.

Now we can verify when you have said the wave was traveling in the z direction if I go to this then my direction cosines if the wave is traveling in z direction the $ø_z$ is zero, $ø_y$ is ninety degrees and $ø_x$ is ninety degrees so this is zero, this is zero and this is one. So I get this K which is equal to β into z cap and when I take the dot product of that with this vector r then I get essentially β into (multiplied by) z. So the phase variation would be e to the power -jβz which is the same expression we had got for the wave was propagating in the z direction.

So this is the representation of the electric field for a uniform plane wave which is traveling in some arbitrary direction making angles $ø_x$ $ø_y$ and $ø_z$ with respect to the three coordinate axes.

The next step which we will need is to finding out the magnetic field corresponding to this electric field and for this we can again go back to the original Maxwell's equation we are still dealing with the media which now do not have conductivity so let us say we have a medium which is the source medium there are no currents no finite conductivity so the conductivity is zero and then the Maxwell's equation is del cross E that is equal to -jω μ into H bar.

Since now E is known it is given by this expression I can substitute in this and I can find out corresponding magnetic field so I get from here H bar equal to -1 upon jωμ into del cross E which I write in the determinant form which will be -1 upon jωμ determinant x cap y cap z cap d/dx d/dy d/dz E_x E_y E_z.

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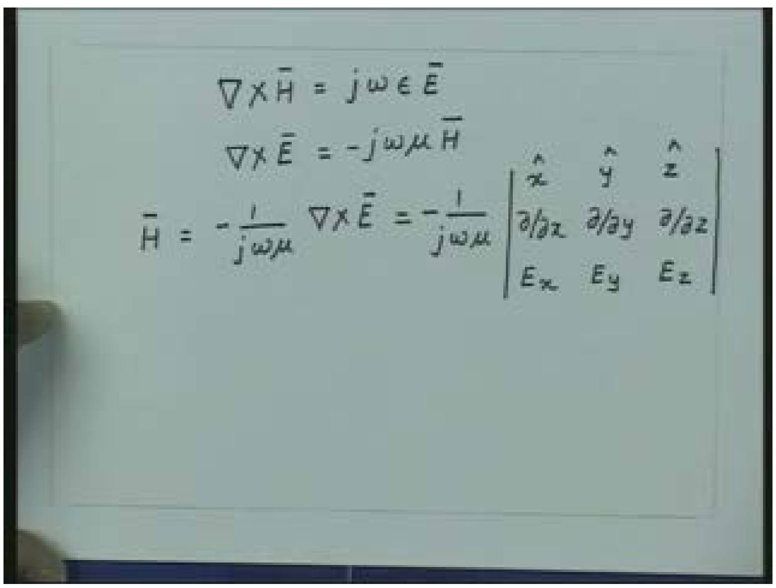

Now this electric field vector which we have got here also wrote down explicitly in its components. So essentially we have the electric field E in general if I expand this E_0 in its components this will be E_{0x} x cap plus E_{0y} y cap plus E_{0z} z cap and the phase function which is e to the power -jk bar dot r bar.

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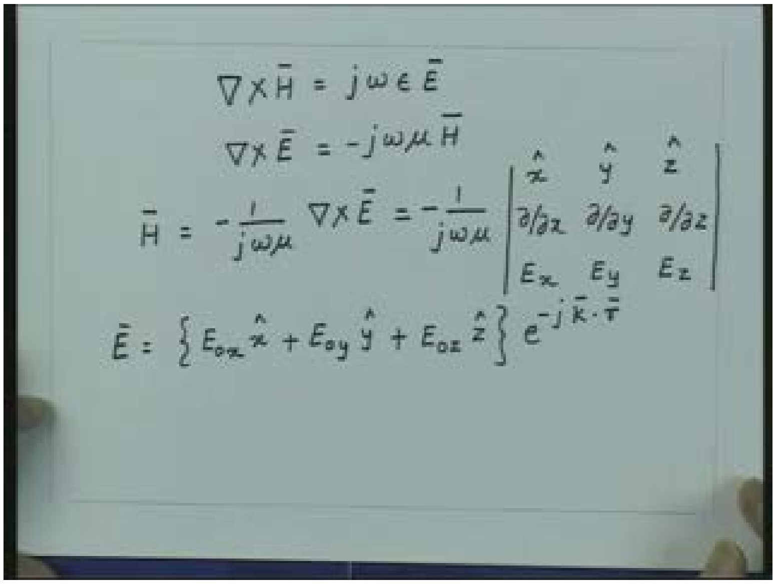

So the E_x component is E_{0x} e to the power -jk dot r and E_y component is E_{0y} multiplied by the same phase function and the E_z component E_{0z} multiplied by e to the power -j k dot r. So I have three components for the electric field which I can substitute here. Now note here this quantity E_{0x} E_{0y} and E_{0z} are not functions of the space this electric field is constant. As we saw for uniform plane wave the electric and magnetic fields vary only in the direction of the wave propagation so they are constant everywhere in the space and we have already taken out the variation of the phase variation which is in direction of the wave propagation out so this vector essentially is the constant vector. So E_{0x} E_{0y} and E_{0z} are not functions of space these are constant quantities. So, only phase variation which you have is only in this term which is e to the power -j k dot r.

So let us see if I take any of the component when I take its derivative with respect to x will be derivation of this with respect to x quantity so we have d/dx of any of these components E_x component or E_y component or E_z component that is equal to -j k_x which will get from here multiplied by the same quantity which is either E_x or E_y or E_z.

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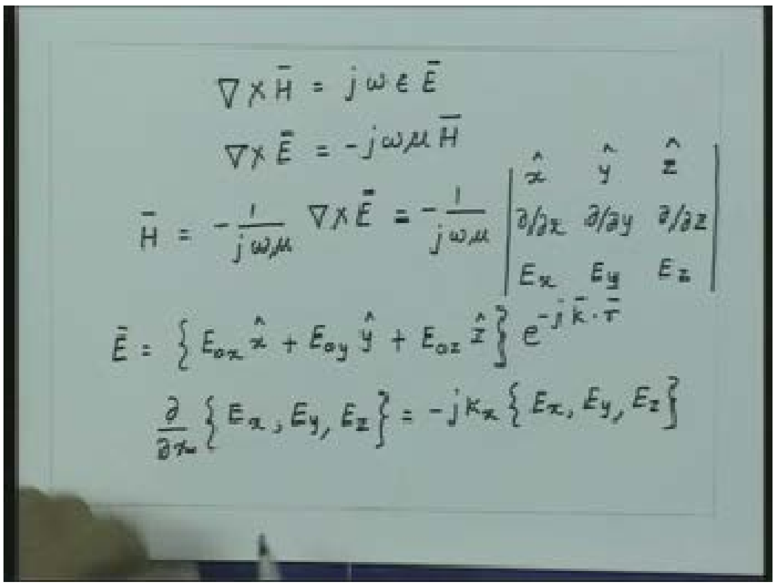

So what that means is this operator d over dx is equivalent to multiplying the quantity by –j k_x. So this implies that this operator d/dx is equivalent to multiplying this quantity -j k_x.

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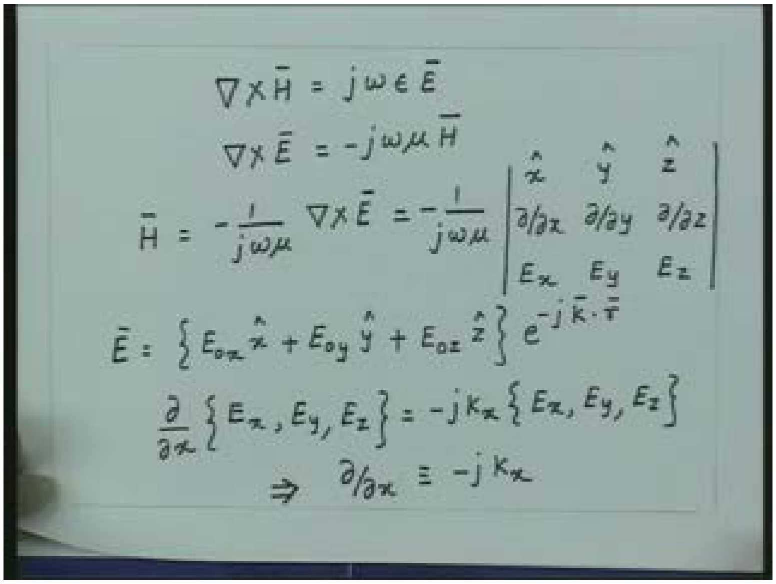

Similarly we can also do for the other two derivatives. So we have as a same token d/dy = -j k_y and d/dz = -j k_z.

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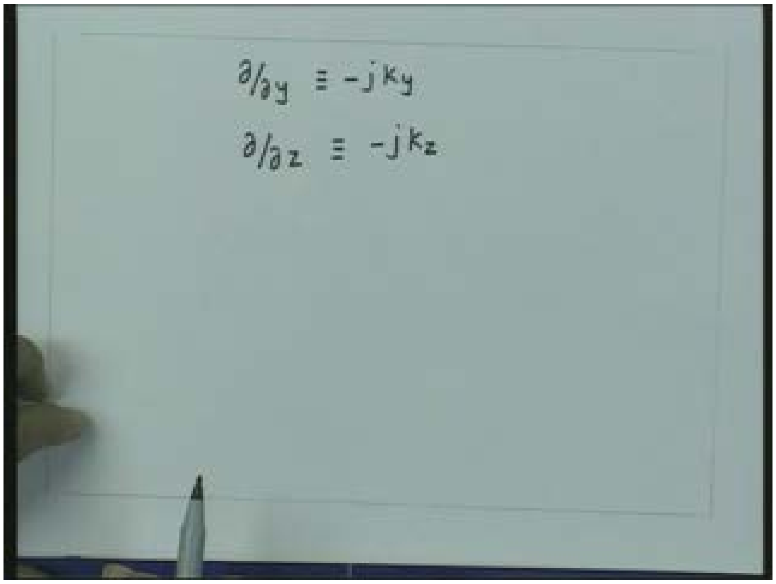

So this quantity here this determinant in this d/dx can be replaced by -j k_x, d/dy can be replaced by -j k_y and d/dz can be replaced by -j k_z. By substituting from here we can get the magnetic field H bar equal to -1 upon jωμ x cap y cap z cap -jk_x -jk_y -jk_z E_x E_y E_z but this quantity is nothing but the cross product of this vector which is nothing but -j times k cross E.

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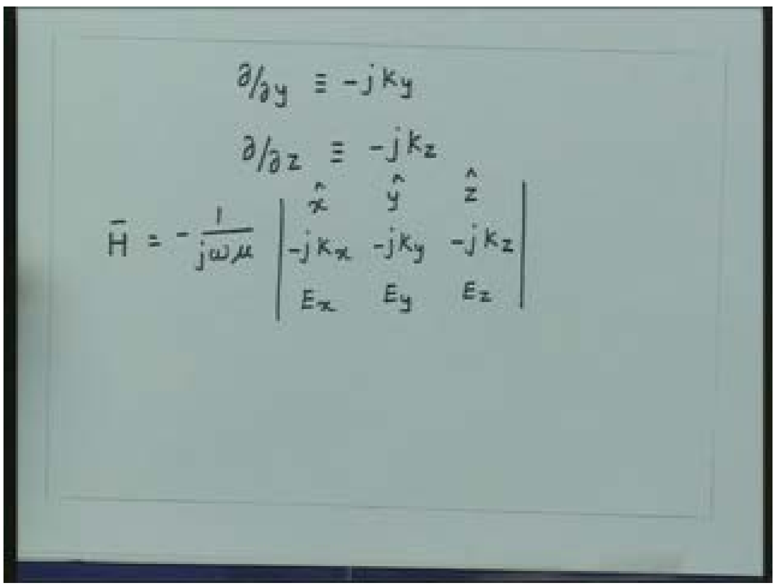

So we can write down this quantity as -1 upon jωμ -j k bar cross E bar where this -j will gets cancelled so this is equal to 1 upon ωμ k bar cross E bar.

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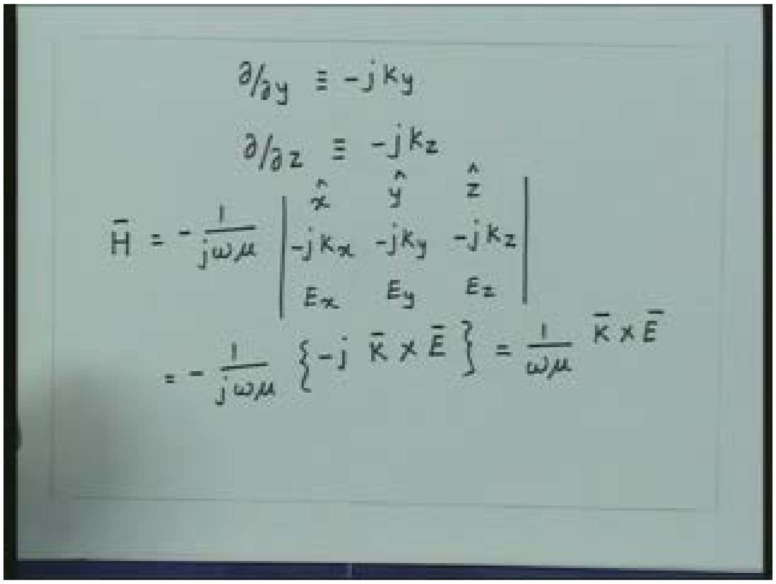

We can immediately note here that k is the direction of the wave propagation E is the direction of the electric field and magnetic field is the cross product of these two that means the magnetic field lies perpendicular to the direction of the wave propagation and also the direction of the electric field that is what we have for a traverse electro magnetic wave that the electric and magnetic fields are perpendicular to each other and they are also perpendicular to the direction of the wave propagation. So from here essentially we can calculate the vector magnetic field if the electric field is known.

Now we started with some coordinate axis and we took the wave which is propagating in some arbitrary direction which is making angles with respect to the coordinate axis and then we found out the expression for the electric field in the arbitrary direction which was this. Then substituting this electric field in one of the Maxwell’s equations essentially we got the expression for the magnetic field and that expression essentially is this.

So now one can find out electric and magnetic fields for a traverse electromagnetic wave traveling in some arbitrary direction. Let us try to see how this behavior of the wave in arbitrary direction with respect to the coordinate axis affects our understanding of quantities like velocity of the wave and so on. Let us say I have a wave which is traveling in some arbitrary direction and for that electric field is given as E_0 e to the power -j k bar dot r bar. If I expand it this is E_0 e to the power -j and if I write down the k as β times cos $ø_x$ x + β cos $ø_x$ y + β cos $ø_x$ z.

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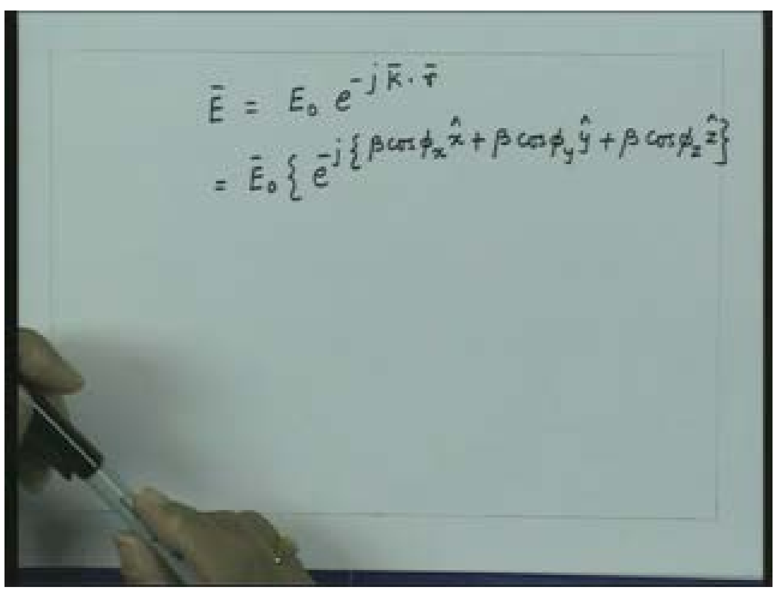

Now let us look at the phase variation which is in z direction so what I can do is I can just take this portion separately and the phase variation which is in z direction separately. So I can write this as E_0 bar take this term separate so e to the power -j and let me take β common here so this is β cos $ø_x$ x + cos $ø_x$ y multiplied by the phase term which is e to the power -jβ cos $ø_z$ z.

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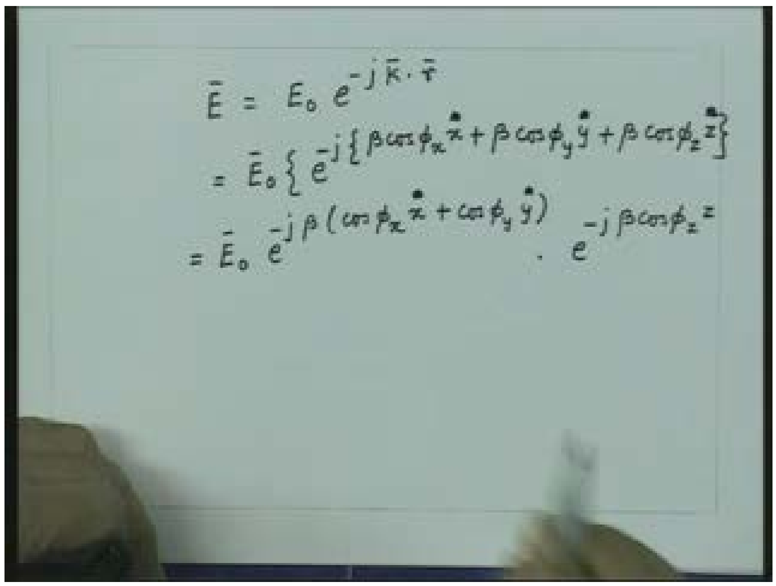

There are two things must be noted from this expression that this electric field first of all if I move in the xy plane then I have a phase variation in x and y direction that means the xy plane is not a constant phase plane and that is very obvious from this very first picture we have taken that constant phase plane is this plane now which is inclined with respect to the axis. So if I take this xy plane then this plane does not represent the constant phase so the phase is not constant in this plane.

So essentially you have a variation of the phase in the xy plane which is varying like cos $ø_x$ and cos $ø_y$ but you are having a variation in the z direction which is given by β times cos $ø_z$.

Now if I look at this way and then I ask the question that the coordinate axis was given and if the wave was traveling at some arbitrary direction then what is the velocity with which the phase point moves in the z direction or what is the phase velocity of the wave in the z direction? Essentially what we do is we take the phase in the direction of z which is -β times cos $ø_z$ so the effective phase constant which this wave sees is the direction z is β into cos z.

So if I write down this we have the wave phase constant in z direction that is let us call the quantity $β_z$ and that is equal to β into cos $ø_z$. So this wave for which the xy plane is not a constant phase plane but if I ask what is the effective phase constant with the wave in the direction z. Then we have the phase constant $β_z$ which is nothing but β into cos $ø_z$.

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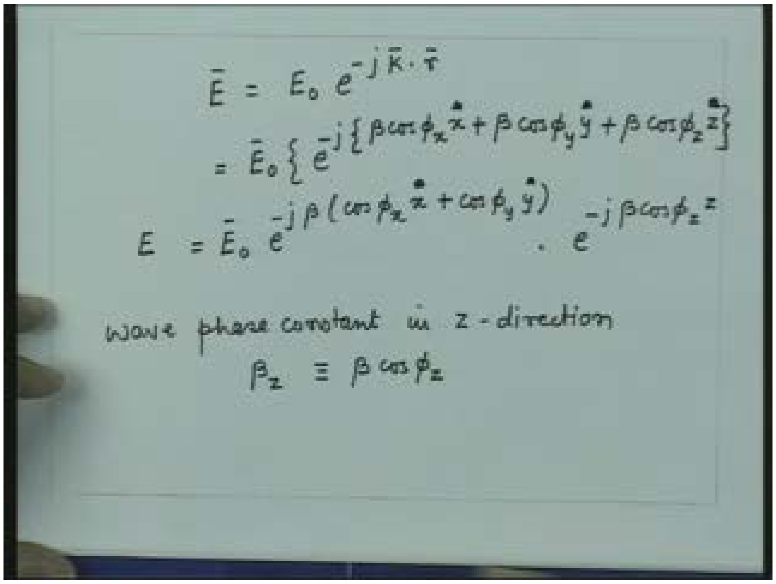

Also the same thing will happen in other two directions, the phase constant which will have in the x direction the β_x will be β into cos $ø_x$ and the phase constant which is in the y direction will be β_y again will be β into cos $ø_y$.

Once we have this then we go to our expression for phase velocity and phase velocity is nothing but ω divided by the phase constant in z direction so ω/β_z gives me the phase velocity of the wave in the direction z. So we get from here the phase velocity in z direction, let us call that as v_{pz} which is nothing but ω/β_z which is again is ω divided by β into cos $ø_z$.

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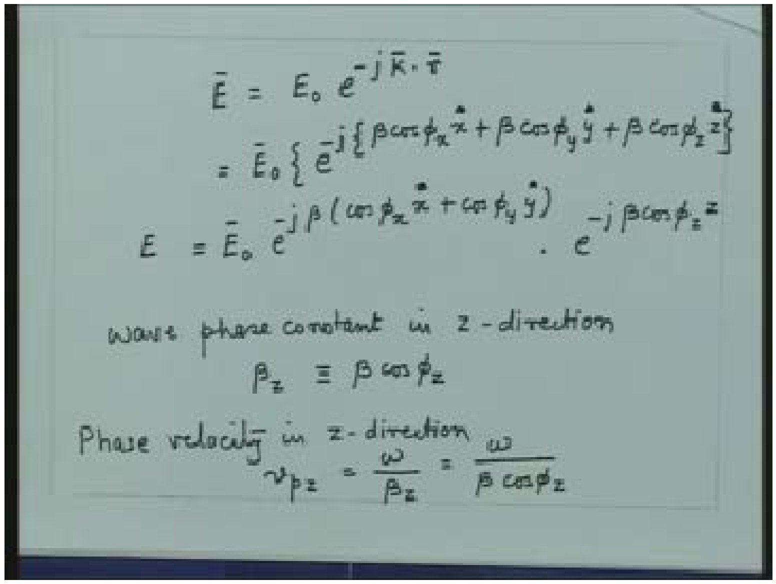

But ω by β is the velocity of the wave in the direction of the wave motion. So if I say that I do not have coordinate axis in this direction the phase fronts are moving if I ask what is the velocity of this wave front in this direction that quantity is nothing but ω by β. So this essentially gives me the velocity of the wave in that unbound medium let us denote that quantity omega by beta as some v_0 divided by cos $ø_z$. So the v_0 is the velocity of the wave in the direction the perpendicular to the constant phase front that is the velocity of the wave in an unbound medium. But the phase velocity of the wave in the z direction is that the velocity v_0 divided by cos $ø_z$.

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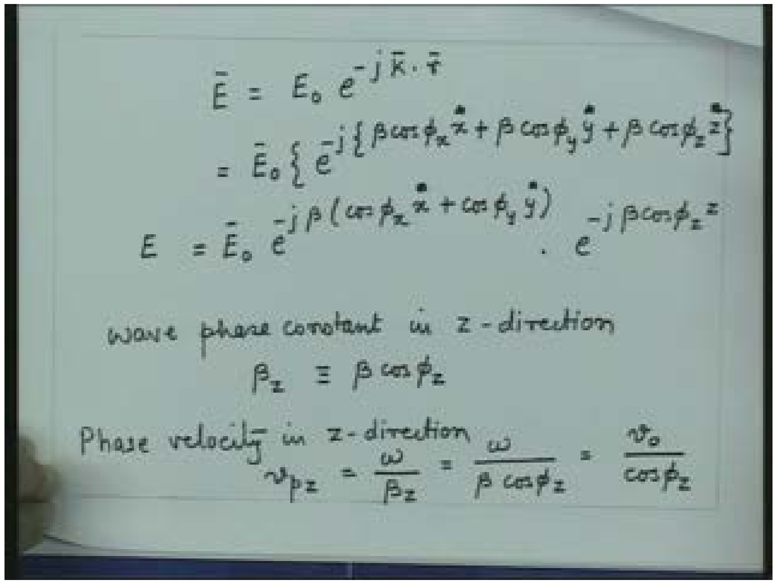

By the same token we can have the phase velocities in the other direction because your $\beta_x = \beta \cos \o_x$, $\beta_y = \beta \cos \o_y$ so this gives your phase velocities v_p in x direction which is ω divided by β_x is equal to ω divided by $\beta \cos \o_x$ is equal to v_0 upon $\cos \o_x$. And similarly we get v_{py} is ω divided by β_y is equal to ω divided by $\beta \cos \o_y$ which is again equal to v_0 upon $\cos \o_y$.

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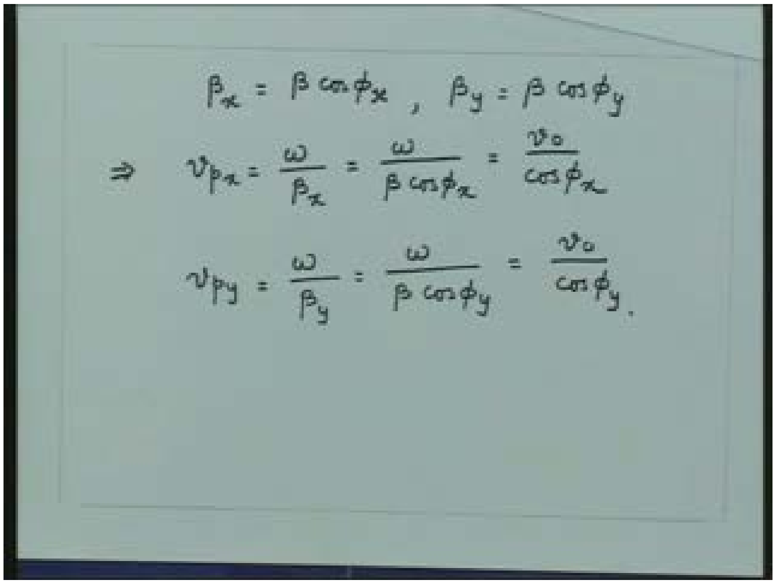

So the phase velocities of this wave in free coordinate axis are essentially given by the intrinsic velocity of the wave in that medium which is v_0 divided by the direction cosine which is cos $ø_x$, cos $ø_y$ and cos $ø_z$.

Now since the cos $ø_x$, cos $ø_y$ and cos $ø_z$ are always less than 1 this quantity the phase velocity is always greater than the phase velocity intrinsic phase velocity of the wave v_0. So one important thing which we see from here is that these quantities v_{px} or v_{py} or v_{pz} is always greater than or equal to the intrinsic velocity of the wave in that medium which is v_0 when the direction cosine is equal to 1.

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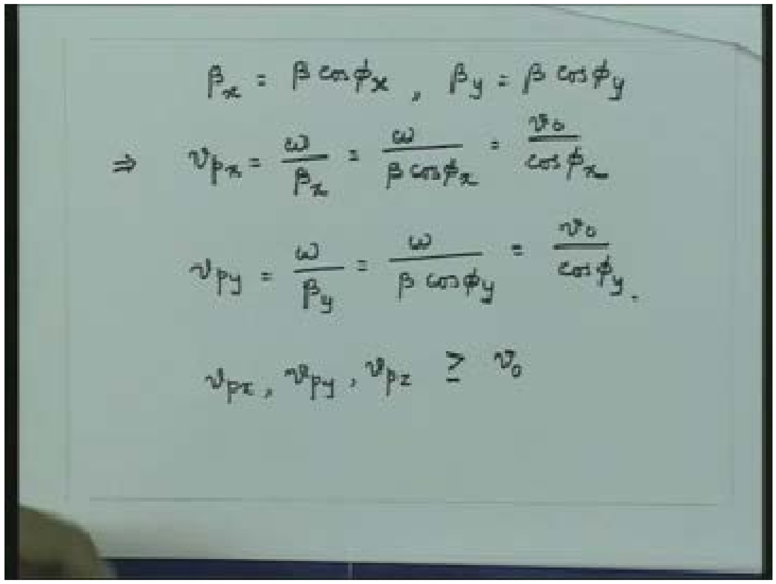

Otherwise this quantity is always greater than the intrinsic velocity of the wave. Another important thing which we note here is we can always have a wave which is traveling perpendicular to x axis so that cos $ø_x$ will be ninety degrees and cos $ø_x$ will be zero so the phase velocity v_{px} will be equal to infinity that means we may have a situation in which the phase velocity may go to infinity.

So essentially the phase velocity always is greater than the intrinsic velocity of the wave in that medium and it can be as high as infinity. So there is no bound on the phase velocity on upper side it can go as high as infinity the lower bound on the phase velocity is the intrinsic velocity of the wave in that medium. That is something interesting because now we are talking about the velocity which is greater than that of the intrinsic velocity of the wave.

We know that the light is a transverse electro magnetic wave so the intrinsic velocity for light will be nothing but the velocity of light in that medium, so this quantity would be C. Then the phase velocity will be always greater than the velocity of the light in the medium and it can go as high as infinity. We know from our basics of physics that the velocity for any physical system cannot be greater than velocity of light.

Then what is happening here is we are having these parameters called the phase velocities which are always greater than velocity of light and they can go even as high as infinity. Does that mean that you have found a mechanism of sending information with speed as high as infinity? No, this velocity is not the velocity of any energy packet or physical point in space, this is because the wave we have defined the phase velocities which are based on the constant phase front they essentially give you this condition that the phase velocities will be greater than the velocity of the wave of the intrinsic velocity in that medium.

Let us see this little carefully. So let us say to make the case simpler let us say I have this is x and z and let us say the wave is traveling in this plane in this direction so this direction of the wave and these are the constant phase fronts which we have.

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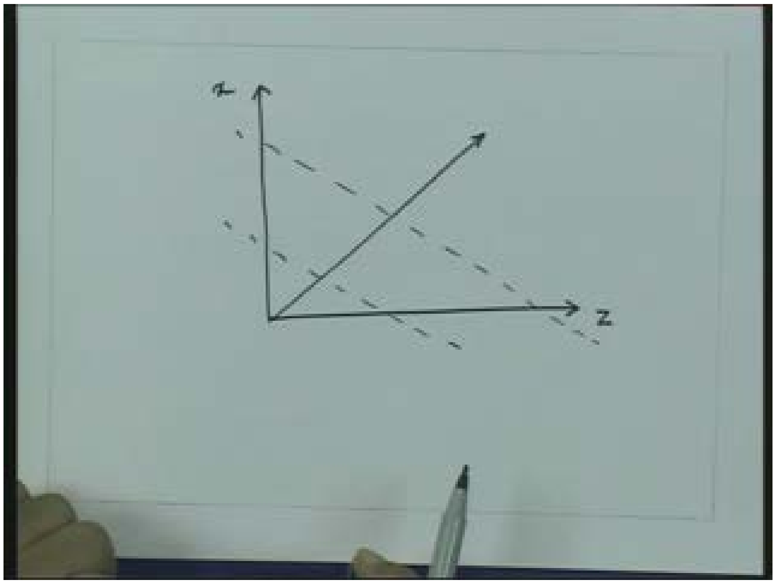

Now by the time this wave have traveled from this point to this point the constant phase point which is given by the entire phase front this constant phase point moves from this point to this point. So unless the wave is parallel to this, this distance is the phase point moves is always greater than the actual movement of this wave front. Why this is happening is in fact this point has not moved to here this point has actually moved from here to here say if I take some point A the A point will move to A prime whereas the point which has come here along the z axis was originally not A point it was this point B and that point has come here which is B prime.

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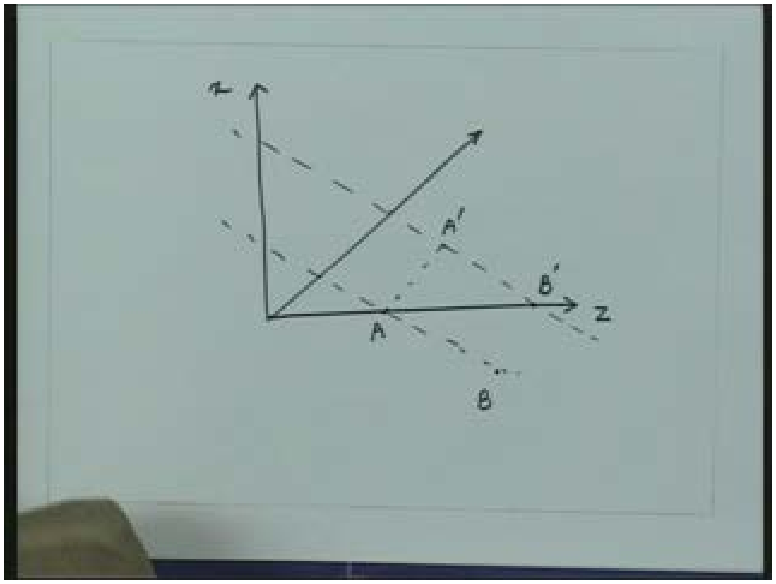

So when the wave is moving if I just define a particular point on the wave front this point is only moving by a distance which is A A prime. However when we are measuring the phase velocity what we do is we simply measure those separation the points on the z axis which are separated by a phase and I ask a question how much time it has taken to change the phase from this value to this value. So essentially we measure this distance find out how much phase it has undergone and from that we essentially get the phase velocity.

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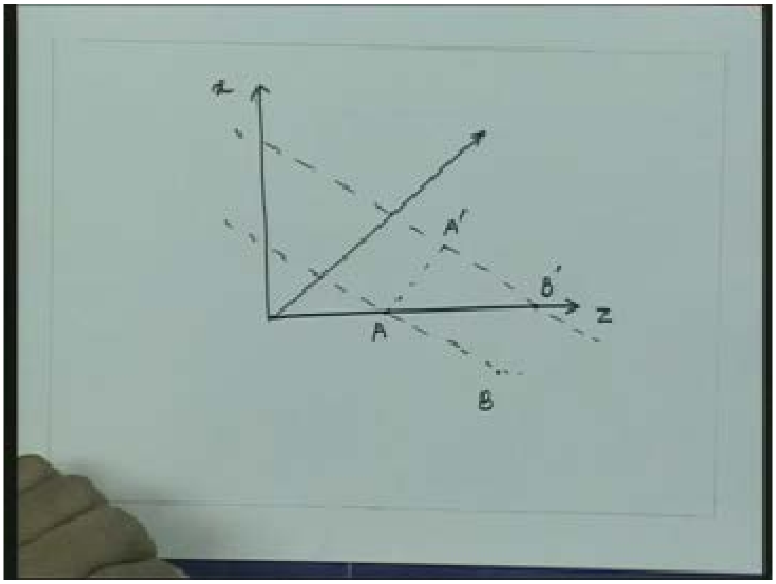

So the phase velocity is not actually giving you the velocity of the particular point on the phase front in fact when we define the phase velocity the entire constant phase plane itself is behaving like a point so here on this phase front we take this point on this phase front we take this point but both these points represent the same phase. So if you find the same phase point here we say that the distance with the constant phase point and moved from this point to this point and that is the reason we get the velocity which is greater than the intrinsic velocity of the wave in the medium because this is not representing the velocity of a particular point on the phase front.

So this velocity what ever phase velocity we get this is not simply the resolution of the velocity vector in the three directions because if you resolve the velocity in three directions the components of the velocities would be always less than the actual vector. But in this case we see that the components of the phase velocities in three directions v_{px} v_{py} v_{pz} are always greater than the velocity vector v_0. So this is not a simple vector resolution of the velocity vector of the wave in fact the phase velocities are due to calculate from the distance traveled by the constant phase point along the z axis and that gives you this intrinsic velocity divided by the direction cosines.

So as the wave becomes more and more perpendicular to the z axis that means moving in x direction parallel to x axis perpendicular to z direction the phase fronts will become parallel to z axis and this point by small tilt you will see that this point will be moving very rapidly you want to move a small moment of this point would have moved by a large distance along z direction whereas the point has moved very little in the x direction when this phase front is almost parallel to the z axis. So what we find from here is that if the wave was moving along x direction if the phase fronts were parallel to z axis then by small moment of the wave the point moves by a very large distance and if the wave was perfectly parallel to z axis the point essentially moves from minus infinity to infinity even for a infinitesimal moment of the phase front in the x direction.

So when the phase velocity approaches infinity that time the phase fronts are moving in the x direction that means the wave is moving in the x direction that means the wave is moving in x direction so the pointing vector for this wave is in the x direction there is no power which is moving in z direction the power is moving in x direction.

When the phase velocity approaches infinity we can ask with what velocity the energy is moving in the z direction. In general we can ask a question that if the wave is moving with a phase velocity which is some v_{pz} in this direction with what velocity the energy will be moving in this direction or what is the velocity of the power flow in the z direction and that we can say very easily that now the velocity which is in this direction if I take this velocity v_0 then the velocity in this direction will be v_0 multiplied by the cos $ø_z$. So the velocity with which this point A will be moving in z direction will be v_0 multiplied by cos $ø_z$ and it will be moving in x direction.

This velocity with which actually a given point in the phase front moves in the z direction is called the Group velocity that is the velocity with which a particular point on the phase front moves or that is the velocity with which the energy or power will move in the z direction.

So from here the velocity of this point movement in this direction will be called the Group velocity and that is denoted by v_g and since we are talking of group velocity in z direction so we can put suffix z that is equal to the intrinsic velocity v_0 zero multiplied by cos $ø_z$.

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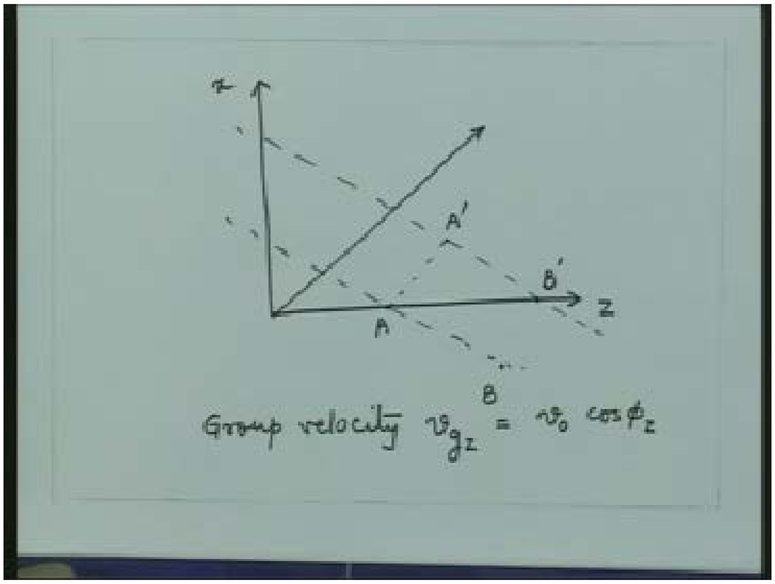

Since cos $ø_z$ is always less than 1 the Group velocity which is the velocity of the energy packet or a particular point on the phase front that is always less than or equal to the intrinsic velocity of the wave in that medium. In fact the Group velocity is a resolution of the velocity vector in three directions the phase velocity is not the resolution of the velocity vector in three directions.

If the wave was moving in some arbitrary direction we essentially write down the group velocities in the three directions the similar lines so we have Group velocity in the x direction which is $v_{gx} = v_0 \cos ø_x$, $v_{gy} = \cos ø_y$ and $v_{gz} = \cos ø_z$.

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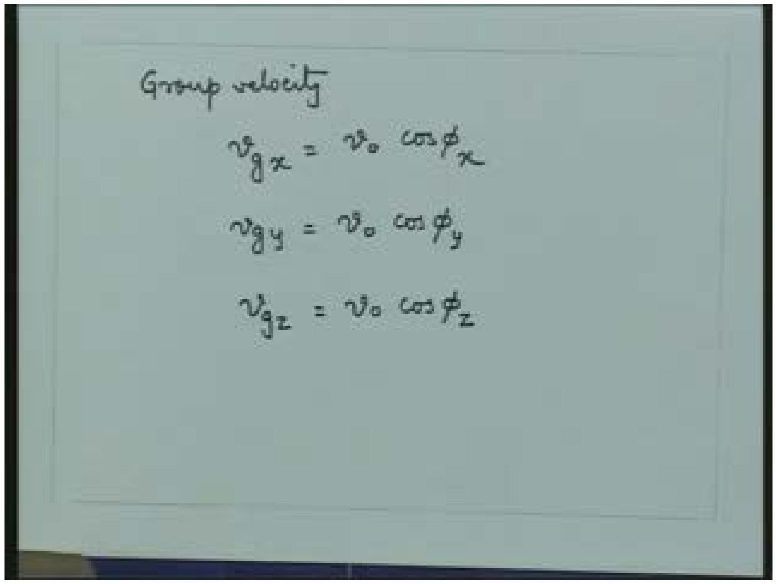

So these Group velocities are bound between 0 and v_0, if the angle is ninety degrees this quantity will become zero so these three here is v_{gx} v_{gy} v_{gz} lies between 0 and v_0.

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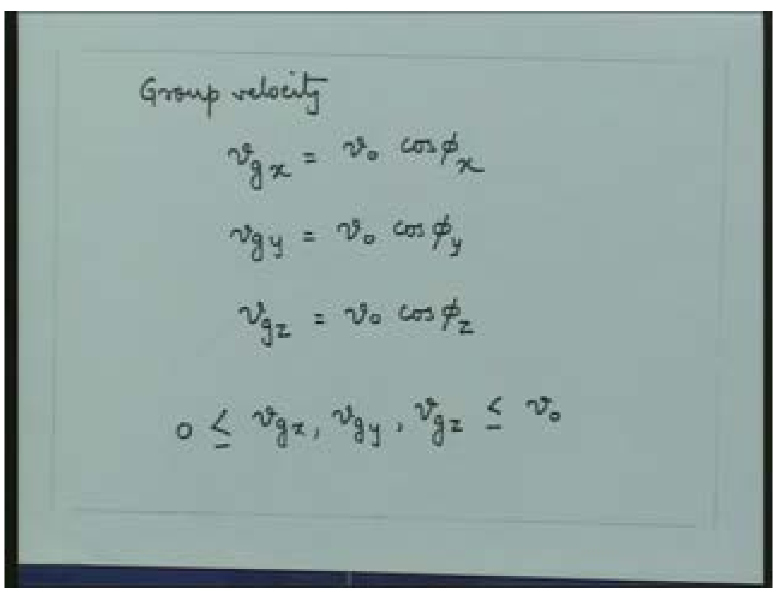

Now we are having a very interesting situation for the phase velocity we have bound which is infinity and v_0 so the phase velocity never comes below v_0 and the group velocity never goes above v_0 in fact the domains for the group velocity and the phase velocity are quite exclusive.

So we have some kind of dividing line the intrinsic velocity of the wave in that medium v_0, on this side you have the phase velocity v_p and this side you have a group velocity v_g the phase velocity always lies from v_0 to infinity and this will lie from v_0 to 0.

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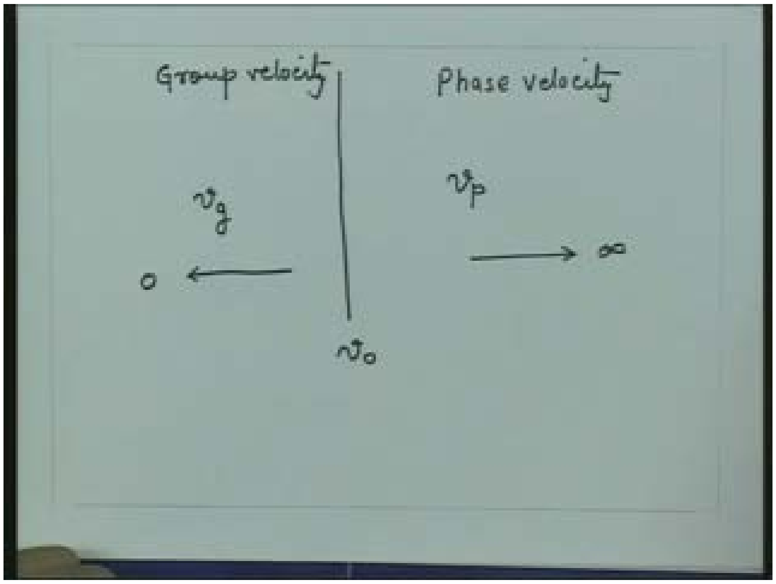

Only in a situation when the wave is traveling in the direction either x y or z or if I find the phase velocity and group velocity of the wave in the direction of the wave motion then both the quantities v_p and v_g will be equal and they will be equal to v_0. Otherwise what we see from here since the group velocity is v_0 into $ø_x$ and the phase velocity v_{px} equal to v_0 upon $ø_x$.

The product of the group velocity and the phase velocity is equal to v square. So you take any direction you like and we have a very important thing that is v_p multiplied by v_g that is equal to v_0 square

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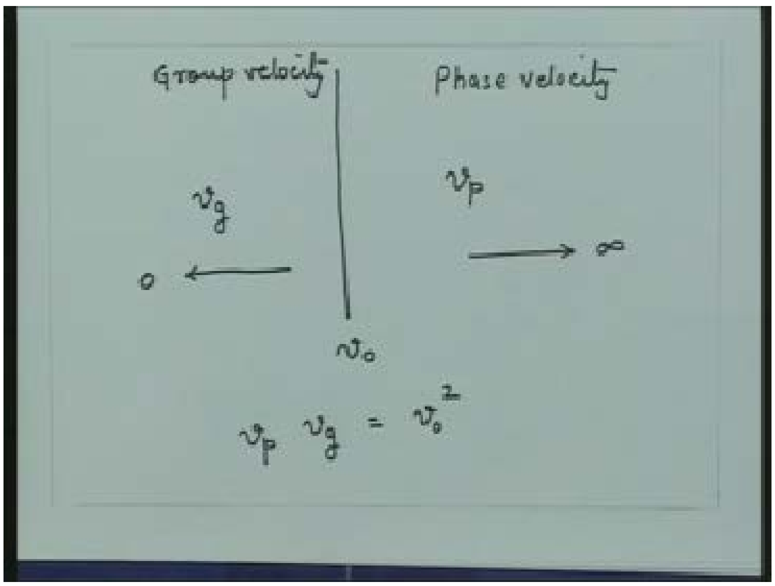

So you take the phase velocity in any arbitrary direction, you take the group velocity in the same direction and the product of these two quantities is always equal to the square of the intrinsic velocity of the wave in that medium. What that now means is that the phase velocity approaches infinity the product of these two is constant which is v_0 square. So this quantity v_g must approach to zero so that the product is still a finite quantity.

If you want energy flow in the medium that means the group velocity should not be zero and the phase velocity should not be infinity. So as the phase velocity becomes higher and higher the velocity of the energy in that direction becomes smaller and smaller, when we see in the direction of the wave motion both the velocities will become equal so if it travels with the phase velocity v_p equal to v_0 then v_g will also become equal to v_0 and then essentially the phenomenon is that the phase and the energy they all are moving in the same direction with the same speed in the direction of the wave motion.

Now we have the important conclusion that when ever we talk about the energy flow or the velocity of the energy in the medium we have to find out this quantity called the group velocity. However when we simply talk about the moment of the phase in the medium then the velocity is given by the phase velocity and which is always greater than the intrinsic velocity of the wave in that medium. Once you know this velocity then the phase velocity divided by the frequency gives you the parameter called the wave length. So from here you can get the wavelength of the wave which is now different in different directions because in next direction the velocity will be different so I can take λ in x direction let us say that is equal to v_{px} divided by frequency f, which is again equal to v_0 upon cos $ø_x$ divided by frequency and v_0 by frequency is nothing but the wavelength in the medium let us call that quantity as λ_0 divided by cos $ø_x$.

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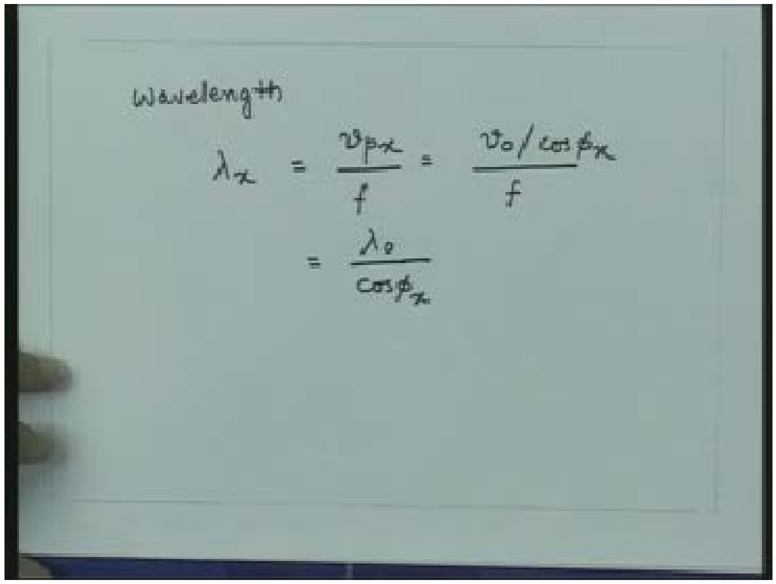

The same thing we can have for other directions also that means the wavelength which we measure in some arbitrary direction will be always longer than the intrinsic wavelength of the wave. If I measure the wavelength in the direction of the wave motion that is λ_0 but if I measure the wavelength in some arbitrary direction that wavelength will always be greater than the intrinsic wavelength of the wave.

In this lecture we saw some important things about a wave traveling in some arbitrary direction that is the phase velocity is always greater than the intrinsic velocity of the wave in that medium. We introduce a new velocity called Group velocity with which the energy travels and that velocity is always less than the intrinsic velocity of the wave in that medium and the product of group and phase velocities is always equal to a square of the intrinsic velocity of the wave in that medium and then we also find the wavelength in that medium which is always longer compared to the intrinsic wavelength of the wave in that medium.

So using this concept, now we can go to the propagation of the wave in a semi infinite medium where we can Orion the coordinate system which will suit the boundaries and the wave will be traveling in some arbitrary directions.

Thank you.