Transmission Lines and E.M. Waves
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Lecture-28

Welcome, in the last lecture we studied a very important concept called a Poynting Vector of an Electromagnetic Wave. The Poynting Vector tells you the density of the power flow at any location in the space. So if you are having complex electric and magnetic field distribution then one can ask that how much power is flowing at every location because of the electric and magnetic fields essentially we can find Poynting Vector the magnitude of the Poynting Vector tells me the density of the power that is watts per meter square and the direction of the Poynting Vector give the direction in which the power is flowing.

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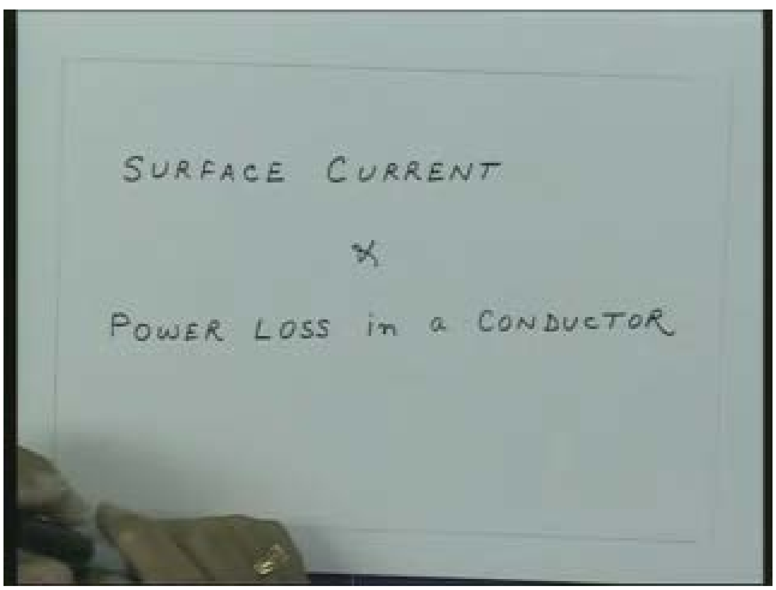

Now using this concept then we can ask a question that if the electromagnetic wave is incident on a conducting surface how much is the power loss in the conducting surface. We also discussed today an important concept the surface current we already discussed when we talked about boundary conditions about the surface current. However still it is not very clear that what the origin of the surface current is, do we really have surface current in practice. So surface current essentially is the phenomenon which is lying on the surface of any medium and essentially this is the phenomenon which is ideal for a conducting surface.

Now in practice we do not have ideal conductors we have conductors which are having very high conductivity but still the conductivity is finite. Then one can ask the question that does this surface current have any meaning in the practical system or it is just only a concept of abstractness. So what we will do is essentially we start from the volume current density inside a conductor we know the concept of skin depth in a good conductor and then from there we will essentially find out a current which is equivalent to the surface current and then we will say even in practical system the concept of surface current is very useful in finding out how much loss takes place in the conducting surface. So first we discuss about the surface current and then we will go to the power loss in a conducting surface.

One can ask a very simple question that if you have a conducting surface we already said that if the normal to this conducting surface is given by $\hat{n}$ and there is a magnetic field $\overline{H}$ which is this on the conducting surface then $\hat{n} \times \overline{H}$ essentially gives me the direction of the linear current density called the surface current. So we have the $\hat{n} \times \overline{H}$ if we take like that so our fingers should go from $\hat{n}$ to $\overline{H}$ the direction of the current will be inverse so essentially there will be linear current which will be going inside the plane of the paper, this is a linear surface current density that is what we talked when we talked about boundary conditions.

Now, one can ask the question that if the surface current is flowing here what the driving mechanism for this surface current is. So we already said this phenomena is phenomena for infinite conductivity so if I take a ideal conductor then may be at some point of time instantaneously there is some electric field at this state in this medium and since it was a very short lived phenomena we have the electric and magnetic fields together.

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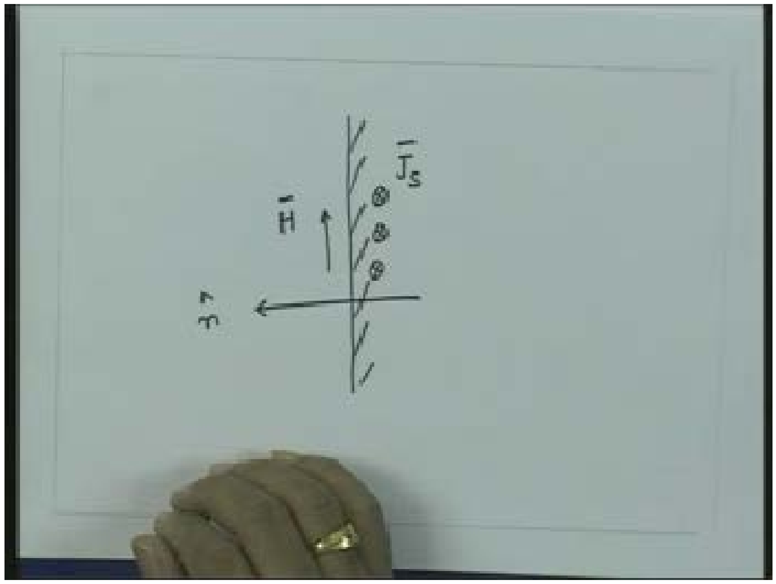

Now for that momentary existence of the electric field would put the charges into motion which will constitute this current and since the conductivity is infinite even if the electric field does not exist anymore when the charges are put in motion this charges will keep moving for infinite times that means they will have current. So when the conductivity is infinite one may visualize this that at some instant of time some electric field induce the motion to these charges these charges now are set in motion and then they keep moving which is essentially is the surface current and then this is balanced by the magnetic field and $\hat{n}\times\overline{H}$ essentially gives you the surface current density.

So basically there are two situations now that you have a tangential electric component of electric field on the conduction surface which is zero but there are no surface currents and the tangential component of electric field is zero but there is surface current. So in both the situations we have the tangential component of electric field is zero for ideal conductor but in one case we have surface current, in other case we may not have surface current but if you have a surface current then it must be balanced by the magnetic field. So if you have magnetic field tangential to the conducting boundary then and then only you will have surface current otherwise we will not have surface current.

Now this was a hypothetical situation when the surface current was truly throwing on the surface. What we now do is we just try to see if we take a good conductor then we still make use the concept of surface current. It is very clear that if you have conductivity which is not infinite then because of the electric field there will be always a finite conduction current density inside the conductor also the skin depth is of finite width that means this phenomena is no more really truly surface phenomena but when the electric field exist on the surface essentially if I say this is the conductor from this side we have the conductor. So let us say this is direction say x, let us say this direction is y which is coming out of the plane of the paper if I put my fingers next to y like that then this is direction is the z direction. So let us say this is my conducting medium which is for z greater than zero so this side I have dielectric, this side I have conductance so I have here conductor for z greater than zero.

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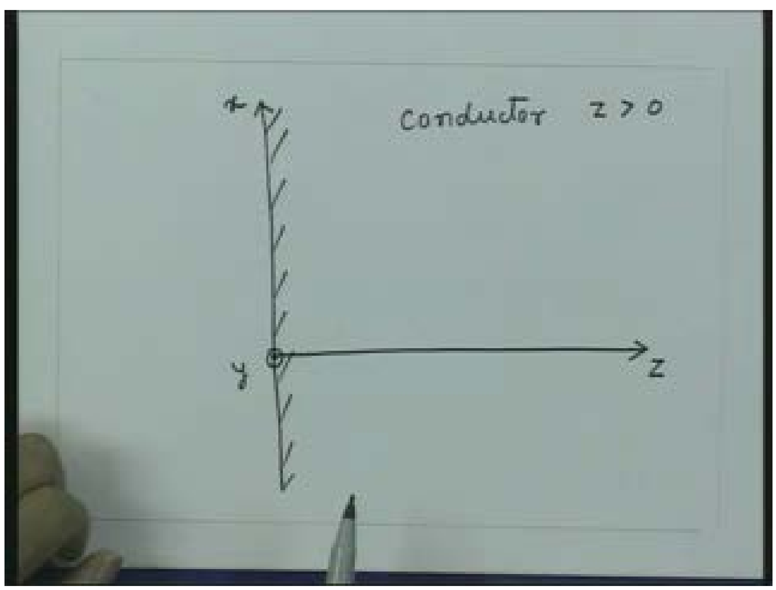

Now if I have some electric field at the surface of the conductor since the conductivity is not infinite which is not an ideal conductor I have a conductivity for this conductor which is given by σ so the tangential component of electric field is not zero now because we are having the conductivity which is not infinite. So let us say I have a certain value of electric field for this in this direction and that electric field exponentially will die down if I say the value of electric field in this direction was some E_0 and the magnetic field which will be oriented in y direction will be H_0. So we will have electric field which is oriented in this direction and then we have magnetic field oriented in y direction, so this is magnetic field.

Then the electric field amplitude will decrease exponentially as the wave travels inside the conductor. So we are having an exponential variation inside this given as $\overline{E}_0$ which is the value of the electric field at the surface of the conductor $e^{-\gamma z}$ where γ is the propagation constant of this wave inside the conductor.

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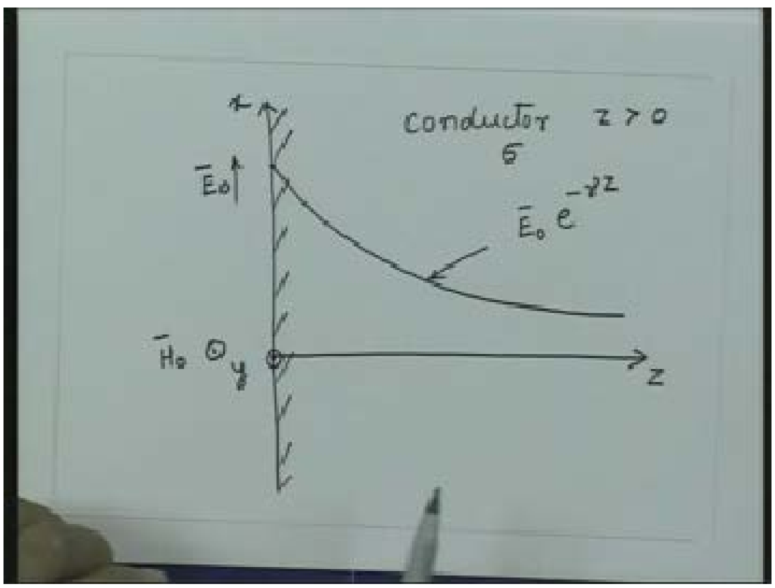

So once we know the electric field value at the surface of the conductor then this law is very well defined this law is exponential law and for once you know the conductivity of the medium and the frequency this γ is the parameter of this medium, this quantity γ which is the propagation constant of this medium as you have seen earlier is $\sqrt{j\omega\mu\,\sigma}$ if the conductivity is very large that means this is a very good conductor so let me call this as a good conductor.

By separating out the real and the imaginary part we know it gives you the attenuation constant and the phase constant. So this thing we have done earlier it is also $\sqrt{\frac{\omega\mu\sigma}{2}}+j\sqrt{\frac{\omega\mu\sigma}{2}}$.

Now since this law is very well defined and the current is now throwing this way in the direction of the electric field so we have a conduction current density which is σ times $\overline{E}_0$ which is very exponential as we go very deeper inside the conductor. So what we can do is we just take a thin sheet which is having a depth of dz and the conduction current the density is going to be in this will be σ $\overline{E}_0$ $e^{-\gamma z}$.

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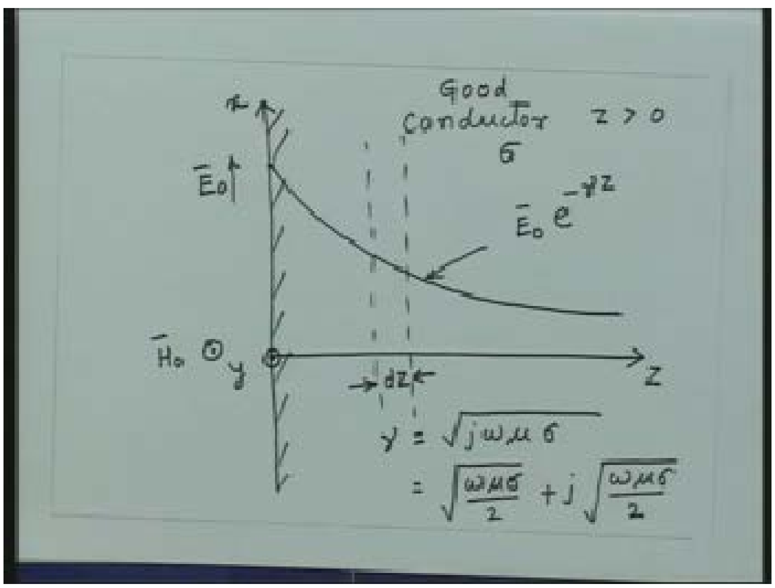

So now if I ask what is the current which is flowing in the sheet per unit area in the xy-plane? So if I take the area in this direction the current is flowing this way if I ask what is the current flowing in an area in this xy-plane and I can find out the total current which is flowing under this surface of unit area in the x direction.

So essentially if I take the conduction current density and integrate over this depth I essentially get the total current which is flowing under this surface xy and if I take the surface as the unit surface then I will get the current which is flowing under unit area. So essentially we can write down in this region the conduction current density we have conduction current density j which will be a function of z and that will be equal to σ times $\overline{E}_0$ at that location z which is nothing but equal to $\sigma \overline{E}_0 e^{-\gamma z}$ and since I am assuming that the electric field is oriented in x direction so I can take the direction of this conduction current density as the x direction I can separate out the real and imaginary part of that to write α and β.

So this I can also write as $\sigma \overline{E}_0 e^{-\alpha z} e^{-\beta z}$ and unit vector x. So I know the conduction current density at this location if I want to find out what is the current which is flowing of depth dz which will be equal to the conduction current density multiplied by this because I am taking the depth in this y direction which is equal to unity. So the area of cross section if I take that will be dz into unity in that direction so the total current for unity depth in the y direction will be one into dz into the conduction current density.

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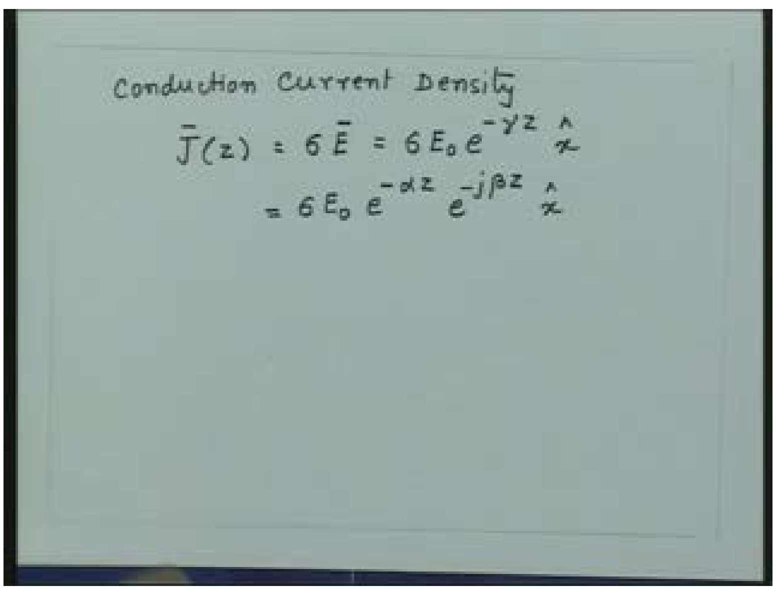

So now I have here the current flowing in this small region so the current I(z) in this slab will be conduction current density $\overline{J}(z)dz$. So substituting in this, this will be $\sigma \overline{E}_0 e^{-\gamma z} dz$ $\hat{x}$.

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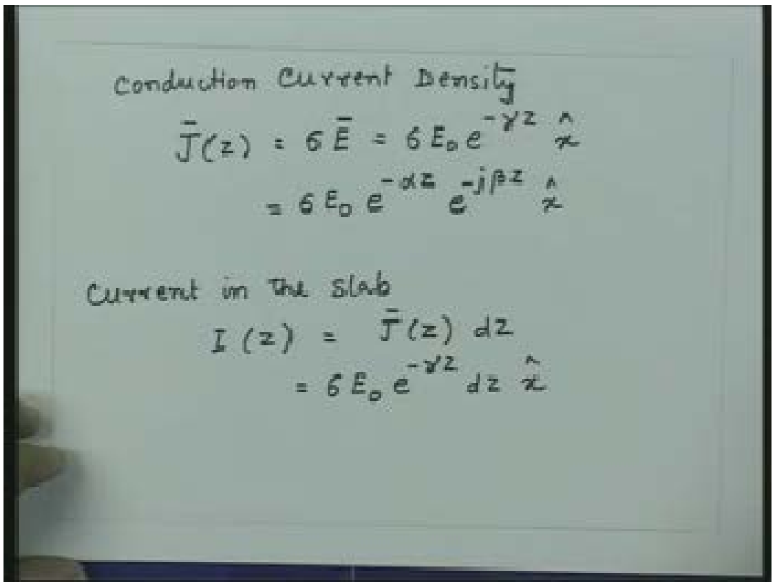

Then as we mentioned we want to find out what is the current flowing under the unit area in this that means per unit length in the y direction how much is the total current flowing under this surface so we integrate this in the z direction. Then we get the total current and we call that current as the surface current. Now note here there is no two surface current the current is actually flowing in the depth and actually it is theoretically extending up to infinity. However since the conductivity is very large we know the skin depth is very small that means this function is a very rapidly decreasing function so effectively this current will be very confined to very close to the surface but is not truly confined to the surface.

So what ever we say that if we integrate this quantity in the z direction we get that total current and let us call that surface current to start with we will justify why we can still call this surface current let us say if I integrate this I get a quantity which we call as the surface current say $\overline{J}_s$ and that will be from integrated over the entire depth that means z going from zero to infinity this total current which is confined in the slab so I substitute here which is $\int_0^\infty \sigma \overline{E}_0 e^{-\gamma z}\, dz\, \hat{x}$.

This integral is very simple essentially this is equal to $\int_0^\infty \frac{\sigma E_0}{-\gamma}\left[e^{-\gamma z}\, dz\right]_0^\infty\ \hat{x}$ when z = 0 this quantity is one when z = ∞ since γ has a positive real part which is attenuation constant which is non zero in this case that quantity will be zero. So essentially this surface current $\overline{J}_s$ will be $\frac{\sigma E_0}{\gamma}\hat{x}$.

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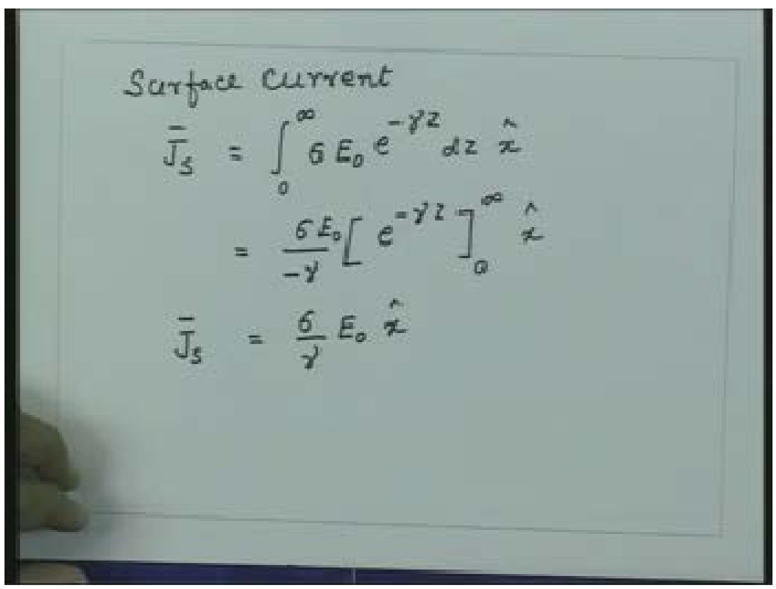

Now I have mentioned we are calling this quantity as the surface current density however we have to justify completely that though this is not truly a surface phenomena it has all the properties which the surface current has and one of the quantities we know that the surface current is related to the magnetic field which is tangential component of the magnetic filed to the surface, also it should satisfy the relationship that n x H should give you the magnetic field.

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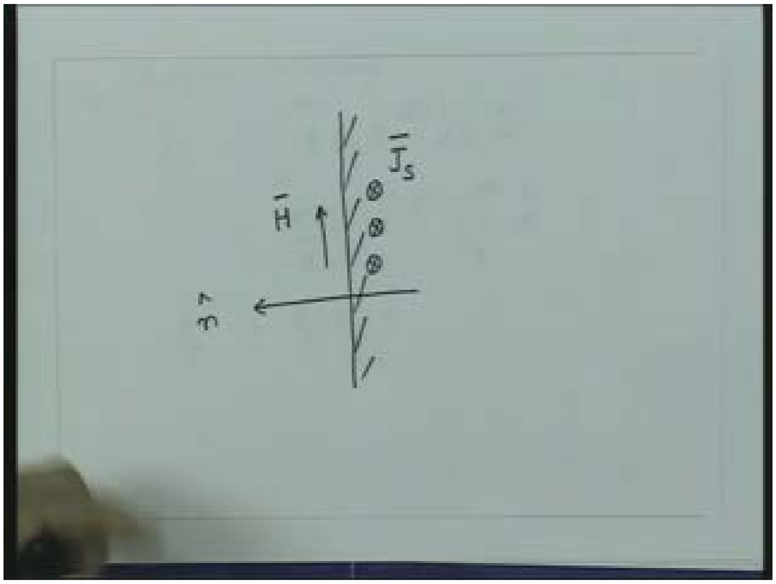

So in this case we had seen that $\hat{n} \times \bar{H}$ should be equal to the surface current density so if this quantity which ever we are getting this $\bar{J}_s$ should serve the purpose of surface current it must be related to the magnetic field and also it should be satisfying this condition that $\hat{n} \times \bar{H}$ should be equal to the surface current. So let us see since I have the electric field on the surface $\bar{E}_0$ and the magnetic field $\bar{H}_0$ on the surface here.

If I go inside this, this is giving me a wave which is traveling inwards in the z direction and the second is the traverse electromagnetic wave because this medium is unbound in this direction. So your $\bar{E}_0$ and $\bar{H}_0$ must satisfy the relation that the ratio of the magnitude of these E and H should be equal to the intrinsic impedance of the medium.

So if I calculate the Intrinsic Impedance of the medium for a good conductor $\eta_c = \sqrt{\frac{j\omega\mu}{\sigma}}$ and the electric field and the magnetic field are related that magnetic field $H_0 = \frac{E_0}{\eta}$ where $\frac{E_0}{H_0}$ is η so this quantity will be $\frac{E_0}{\eta_c}$.

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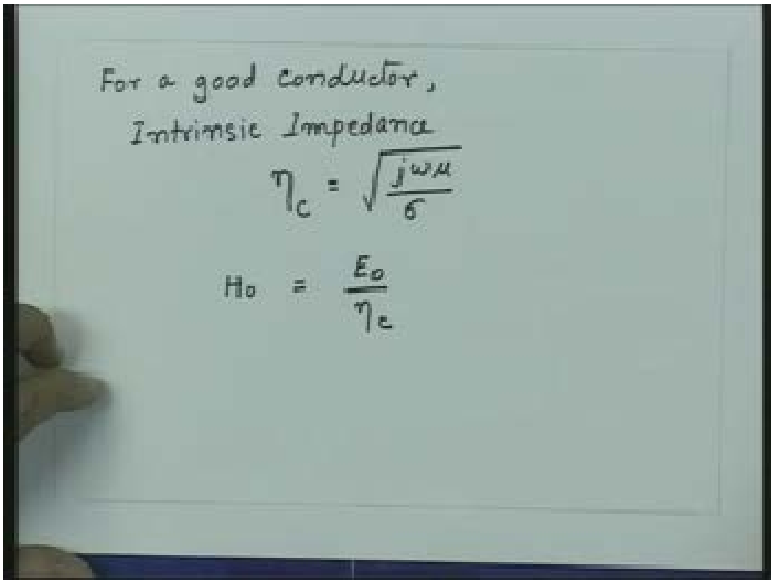

Now recall the γ which we got is this $\sqrt{j\omega\mu\ \sigma}$ so I can write the η_c as $\sqrt{\frac{j\omega\mu\ \sigma}{\sigma^2}}$ but this quantity $\sqrt{j\omega\mu\ \sigma}$ is nothing but γ so this is equal to $\frac{\gamma}{\sigma}$.

So I can write this quantity η_c as $\frac{\gamma}{\sigma}$ so H_0 will be equal to $\frac{\sigma E_0}{\gamma}$.

If I take magnitude of magnetic field that is what this quantity is I will get the magnitude of the magnetic field that is magnitude of electric field divided by the intrinsic impedance of the medium which is η_c which I can manipulate to get in this form but this quantity is same as we have derived for the surface current $\frac{\sigma E_0}{\gamma}$ that means magnitude of H_0 is equal to the magnitude of the surface current.

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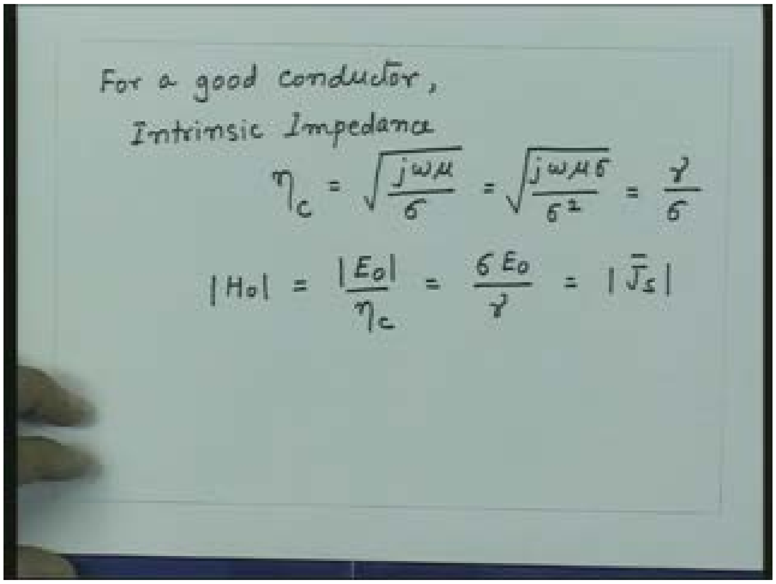

One relation that we wanted is if this quantity is the surface current the way which we visualize then it must be related to the magnetic field and precisely that is what we see here that the magnitude of this surface current is related to the magnetic field. The second thing which we want is the $\hat{n}\times\bar{H}$ must be giving you the surface current direction.

Now again from this case since the electric field was in this direction the conducting current density J was in that direction we have integrated this J over depth to get the surface current so the direction of surface current is also same as this which is x direction so we have direction of surface current which is x oriented we have magnetic field which is y oriented.

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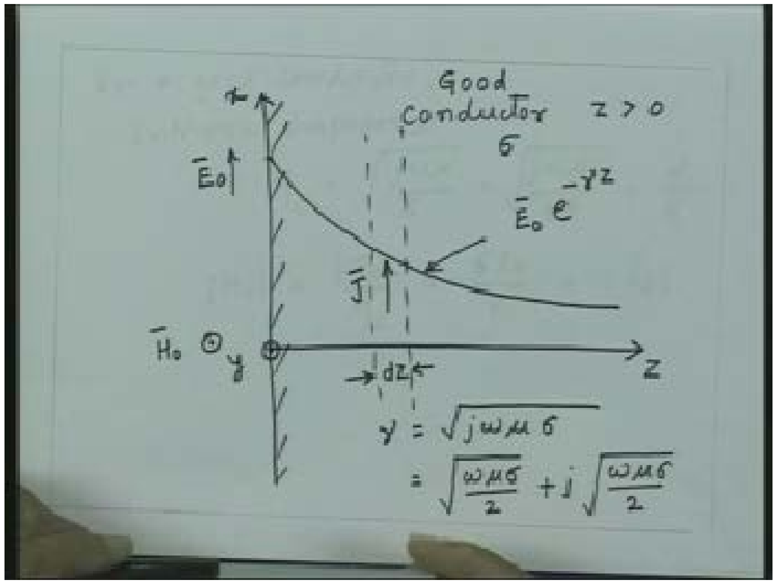

And the unit normal to the conducting surface that means the direction of the normal to the conducting surface $\hat{n}$ is $-\hat{z}$. So if I calculate $\hat{n}\times\bar{H}$ where H is oriented in y direction I will get the orientation of H from here so $\hat{n}\times\bar{H}$ will be $-\hat{z}\times\bar{H}$ the magnitude of the magnetic field oriented in y direction so this is $\hat{y}$ so that is equal to H_0 $\hat{x}$.

So the $\hat{n}\times\bar{H}$ gives me the direction which is x direction, we see that the surface current we have got is related to the magnetic field that means this quantity which we have got here is called the surface current. In fact have all the properties which the surface current should have. So $\hat{n}\times\bar{H}$ gives the surface current and the magnitude of the surface current is same as the tangential component of magnetic field at the conducting surface. That is the reason in practice though this current is not truly surface current we use this as the surface current, also the conductivity is very large this current effectively will be confined to very thin layer that is the skin depth.

And as we have seen earlier if we go to the frequency like few hundred mega hertz the skin depth typically lies in the range of about few tons of microns. So essentially this is the current which is flowing in a very thin sheet close to the conductor, also it has the characteristic of the true surface current that is it is related to the magnetic field and $\hat{n} \times \bar{H}$ should give me the direction of the surface current. Now this quantity can be used as a surface current for a good conductor although in true sense there is no surface current for good conductor. So that is the reason in practice although we do not have true surface current because the conductivity is always finite we can make use of this quantity as surface current and then we can do the further calculations by using this concept of surface current.

Now since we get the surface current then we define an important parameter for this boundary called the Surface Impedance. This is very useful when ever we do our calculations on the conducting surface.

Let us define a parameter called the Surface Impedance and this quantity is defined as $Z_S = E_{tangential}$ to the surface divided by the linear surface current density. Now let me remind you here that the conduction current density have units of amperes per $(meter)^2$ we have integrated over one length which is dz so this has units of ampere per meter which is the dimension of the linear surface current density.

So here we have the unit for E which is volts per meter, we have the linear surface current density which is amperes per meter. So that gives me essentially volt divided by ampere which is ohms. So, this quantity Z_S is some impedance and which is a surface phenomena because it is related to a surface current. So if I know the tangential component of electric field then the surface current can be obtained from a Surface Impedance or vice versa.

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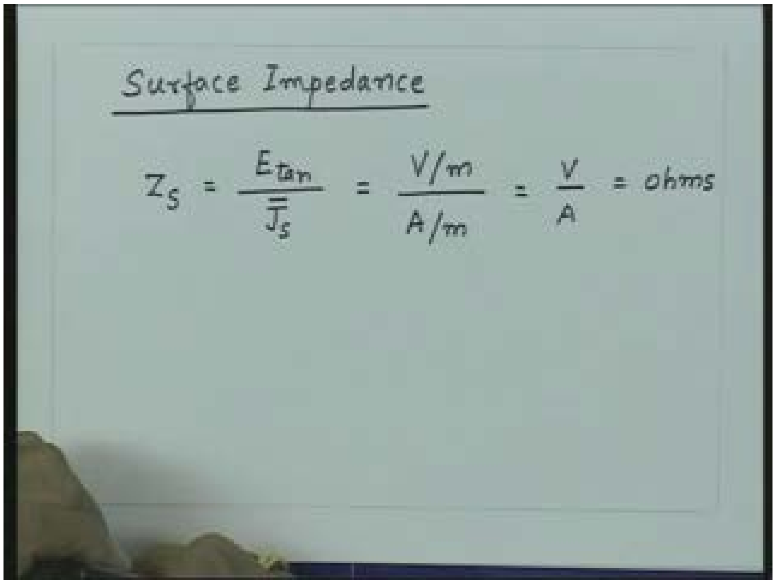

Now from this expression if I substitute for J_s a tangential component of electric field which is E_0 essentially we get this quantity Z_S that is equal to the tangential component of E_0 and surface current which we got here is $\frac{\sigma E_0}{\gamma}$. So this $\frac{\sigma E_0}{\gamma}$ the Surface Impedance is equal to $\frac{\gamma}{\sigma}$. If I substitute for $\gamma = \sqrt{j\omega\mu\ \sigma}$ then I can take this σ inside this square root sign so that will be equal to $\sqrt{\frac{j\omega\mu}{\sigma}}$ and this quantity is nothing but the intrinsic impedance of the conductor so this quantity is η_c. So for a good conductor the Intrinsic Impedance of the medium is same as the Surface Impedance

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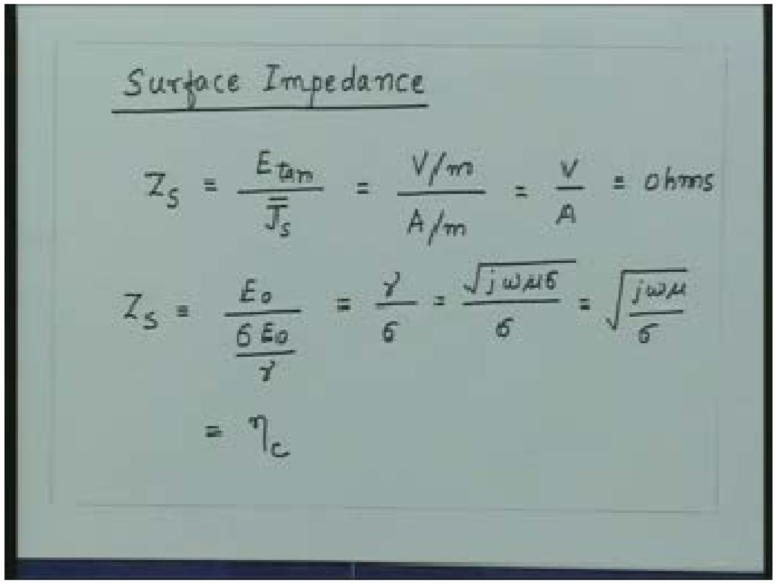

However, we will see later the Surface Impedance concept essentially is used to calculate the power loss. So if I know the tangential component of the electric field if I know the conductivity I can calculate the intrinsic impedance of the medium and that quantity will be same as the Surface Impedance.

Once we have this surface current density then we can ask how much is the power loss because of this current which is flowing in the surface of this conductor. So what we now try to do is we just take a surface and ask if I have a unit area on this surface then how much is the total power loss in this unit area of the conducting surface. As we have already seen since the current is flowing deep inside the conductor essentially the power loss is not taking place only on the surface the power loss is taking place all along the depth of the conductor. However we will just ask what is the total loss which is taking if I take the total loss inside the depth taken inside the conductor.

So the idea here is to find out if I take unit area on the conducting surface so if I take a area which has the depth dz and this is the unit area so this thing is unity, this thing is unity So on the surface of the conductor I am asking if I take a thin layer which is parallel to the surface of the conductor and since there is a finite conduction current density there we can ask how much is the power loss in the thin sheet and then if we integrate over the entire depth from the surface of the conductor we will get the total power loss taken place in the unit area of that conductor

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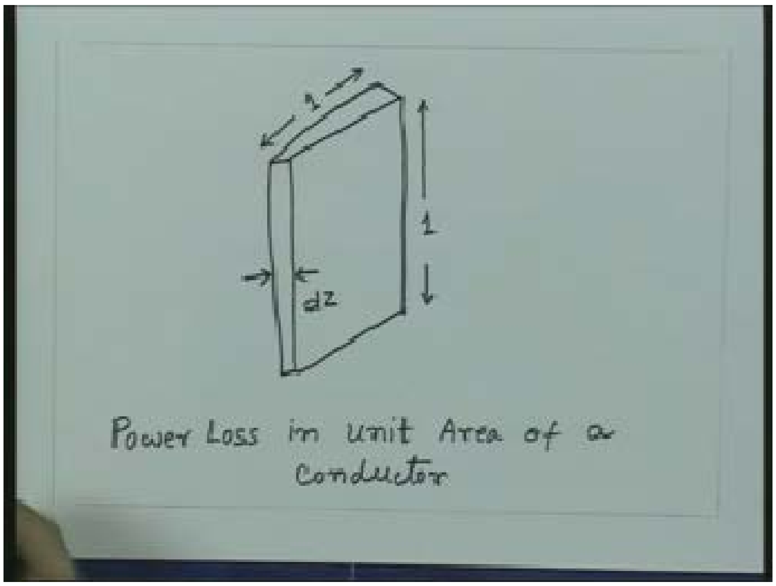

Now let us find out what is the power loss in unit area of a conductor. The idea is very simple essentially you find out what is the resistance of this thin slab of thickness dz and area is 1 and the current in this is flowing in this direction J. So from here I can find out what is the current which is flowing in this slab and from there I can find out what is the power loss which is the I^2 r loss inside this slab and then I integrate over the depth to get what is the total power lost in the conductor.

So since the conduction current density is $\overline{J}$ here and the current is flowing in this area it is the area of Cross-Section which is 1 into dz so the area of Cross-Section A for this conductor is 1 into dz that is dz and the length of this conductor is unity so that is equal to 1. The conductivity of this medium is σ so it has a resistivity equal to $\frac{1}{\sigma}$. So the resistance of this slab if you calculate we have a resistance which is equal to the resistivity multiplied by length divided by area of cross section so it is $\frac{1}{\sigma} \cdot \frac{1}{\mathrm{dz}} = \frac{1}{\sigma \mathrm{dz}}$.

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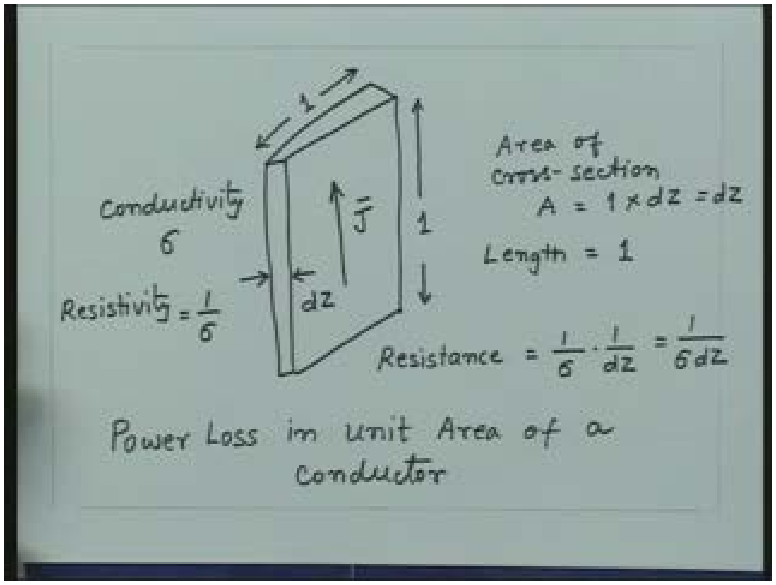

How much is the current flowing in this is $\overline{\mathrm{J}}$ times the area of the Cross-Section which is $\overline{\mathrm{J}}$ times dz that is the current flowing in this slab.

So I have the current equal to $\overline{\mathrm{J}}$ times dz so the resistance of this slab is $\frac{1}{\sigma \mathrm{dz}}$ and the current which is flowing in this is $\overline{\mathrm{J}}$ times dz. Then I have to calculate what the power loss in this I^2 square R loss is.

So if I calculate what is the Ohmic loss in the slab let us say this thing is dw I am using the dw because I am taking a thin sheet here I am saying this incremental slab how much is the power loss so let us say that this quantity is dw so that is equal to the current square multiplied by the resistance and let us say this current is I so let us say ½ $|I|^2$ into the resistance R, I can substitute in this $\overline{J}$ as σ $\overline{E}_0 e^{-\gamma z}$, I can substitute this into this so this will be ½ | σ $\overline{E}_0 e^{-\alpha z} e^{-j\beta z} dz|^2$ and resistance is $\frac{1}{\sigma dz}$.

I can simplify this so this gives me half sigma square one sigma will get cancelled so this is σ, this is $|\overline{E}_0|^2 e^{-2\alpha z}$ dz.

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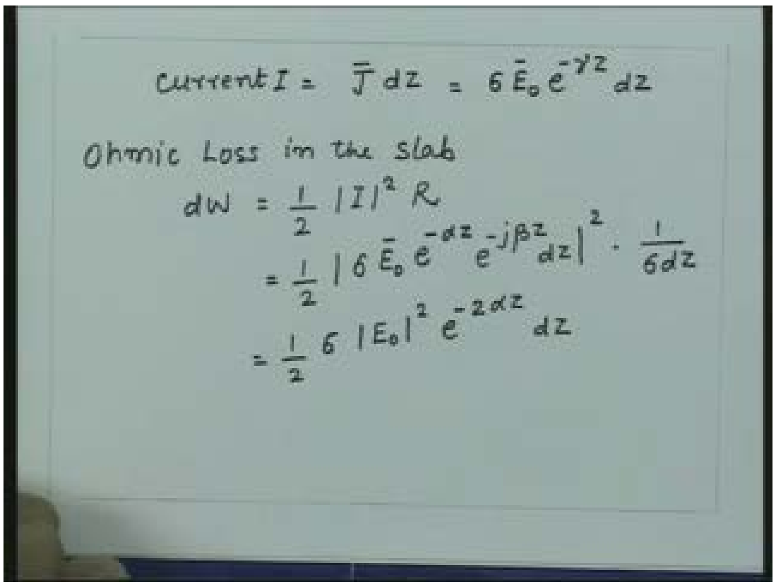

So the power loss in this thin slab of depth dz is essentially by this. Once we get that then we can find out what is the total power loss under this unit area so now we get the total power loss under unit area on the surface w which is nothing but integral of this from 0 to ∞ so this is $\int_0^\infty \frac{1}{2} \sigma |\overline{E}_0| e^{-2\alpha z} dz$.

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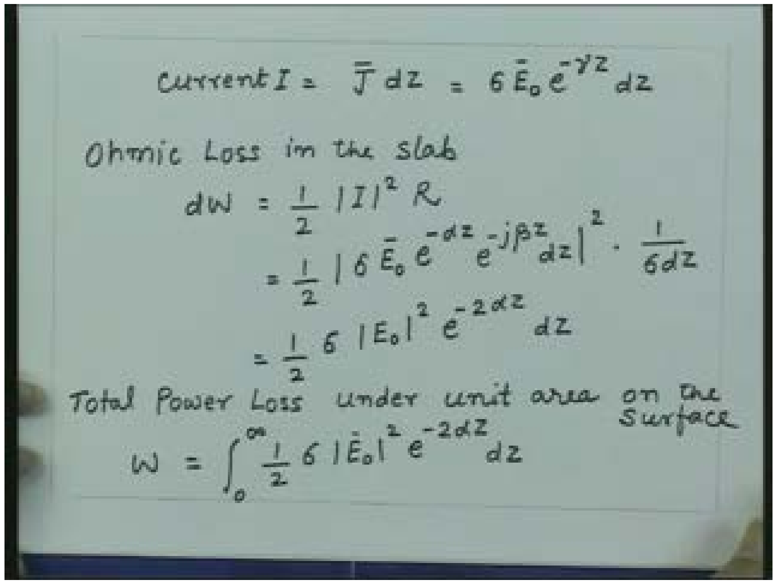

This integral again is very simple the α is a positive quantity so this total power loss which we get is $\frac{1}{2}\,\sigma\,|\bar{E}_0|^2 \left[\frac{e^{-2\alpha z}}{-2\alpha}\right]_0^\infty$.

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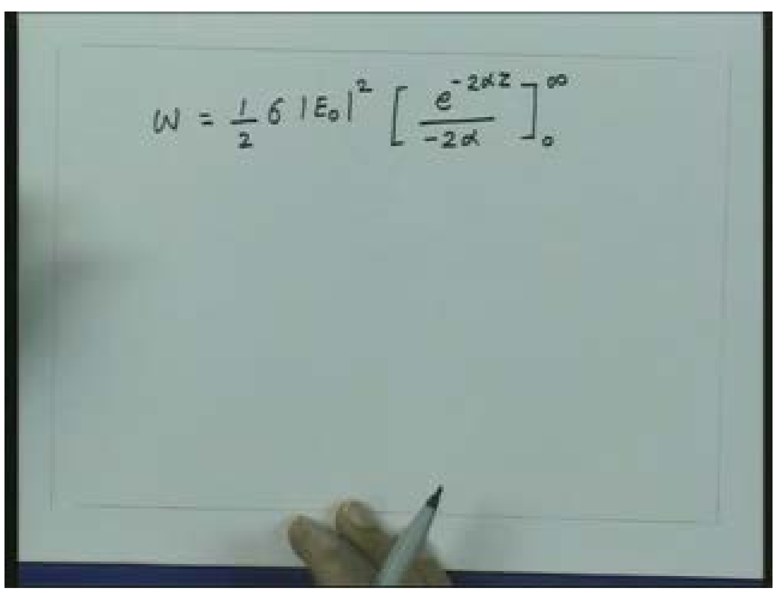

And since α is a positive quantity when z is infinity, when z is zero this quantity is 1 so this essentially gives you $\frac{1}{2}\frac{\sigma}{2\alpha}|\bar{E}_0|^2$.

Now we got the power loss for unit area of the conducting surface and now it is the matter of only doing some algebraic manipulation to simplify for this term which is $\frac{\sigma}{\alpha}$. So if I substitute for α as we have seen earlier the propagation constant γ is given by this. This quantity is α and this quantity is β like I substitute in terms of α here in this and I will get that is equal to $\frac{1}{2}\frac{\sigma}{2\sqrt{\frac{\omega\mu\sigma}{2}}}|E_0|^2$ which I can combine this.

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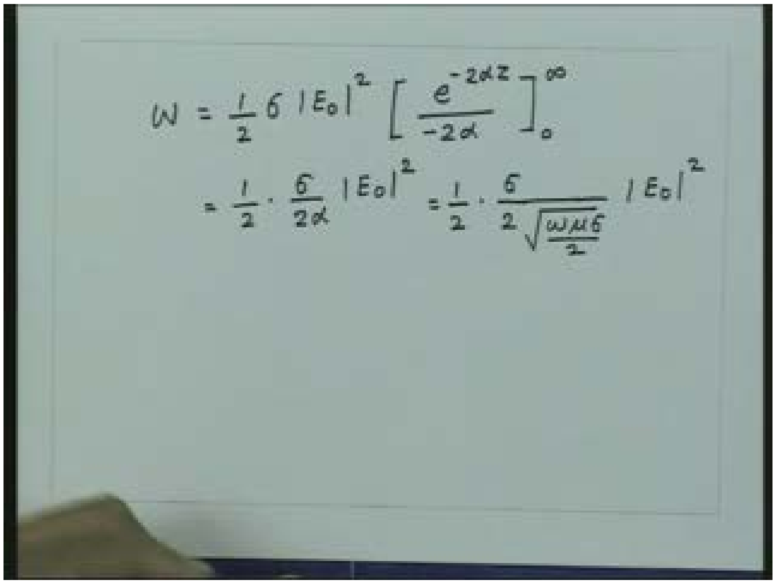

Note here that α is this quantity but |γ| is $\sqrt{\omega\mu\sigma}$. So essentially this thing can be written as I can take this 2 inside so that becomes equal to ½$\frac{\sigma}{\sqrt{2\omega\mu\sigma}}$ but $\sqrt{\omega\mu\sigma}$ is nothing but |γ| so this $|E_0|^2$ so this is equal to $\frac{1}{2}\frac{\sigma}{\sqrt{2}|\gamma|}|E_0|^2$.

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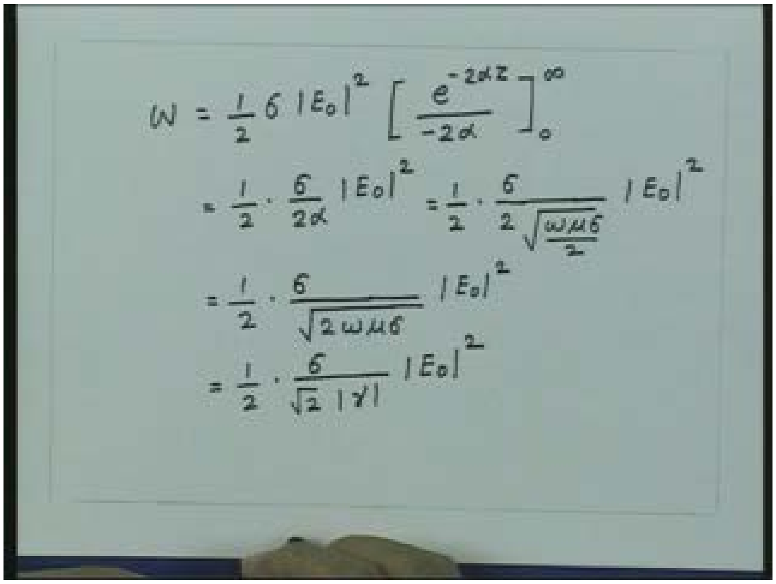

And if I go back to my surface current density this quantity $\frac{\sigma}{\gamma}$ is nothing but the surface current that is what we have to write. So this quantity here this $\frac{\sigma}{\gamma}$ times E_0 is nothing but surface current.

So what we can get from here is this quantity if I write down appropriately in terms of the surface current density I will get the power loss which can be written as from here power loss w = $\frac{1}{2}\frac{\sigma}{2\alpha}\frac{|\gamma|^2}{\sigma^2}|J_S|^2$.

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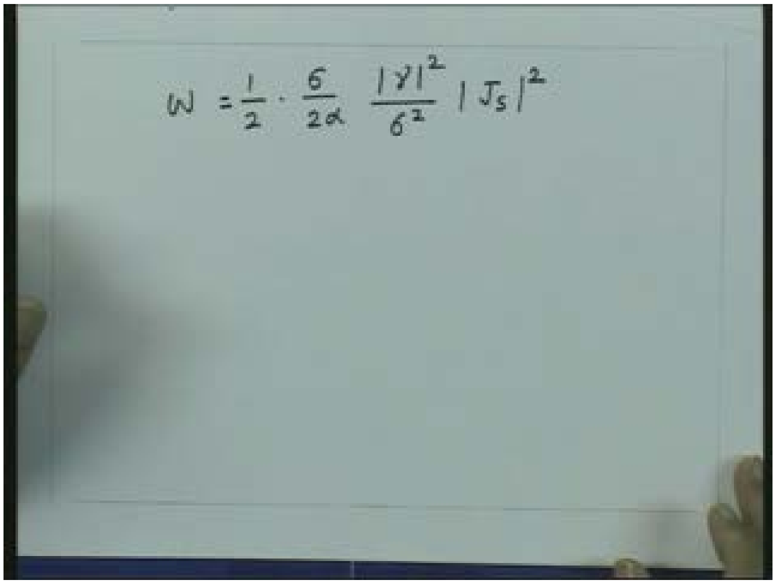

Now if I substitute for γ appropriately as we did in terms of the Intrinsic Impedance this thing can also be written as $\frac{1}{2}|J_S|^2 \sqrt{\frac{\omega\mu}{2\sigma}}$. And if I go back to the characteristic impedance which I have defined then I get this quantity here the characteristic impedance of the intrinsic impedance of the medium of a surface impedance which we have got is $\sqrt{\frac{j\omega\mu}{\sigma}}$ which is same as $\sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$. So I can say the surface impedance has a resistive part which I call the surface resistance and a reactive which i call the surface reactance. So this one let us say we write as surface resistance plus some reactance which is the surface reactance.

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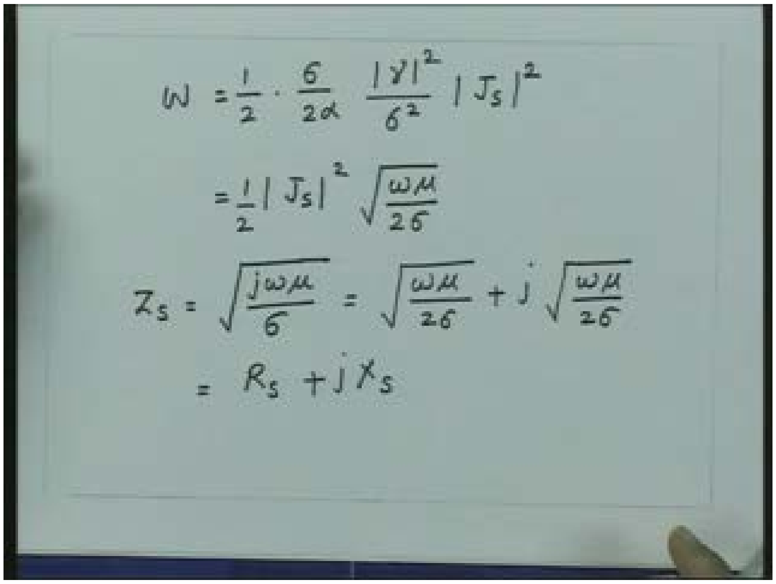

Now this quantity which we have got here is nothing but $\frac{\omega\mu}{2\sigma}$ which is same as this so we can write this as equal $\frac{1}{2} | J_S |^2$ into the surface resistance.

Now this is a very important result that the power loss per unit area of the conducting surface can be obtained by this expression that means if I know the surface current density for this conducting boundary and if I know the surface resistance then the power loss can be calculated as half linear current density square multiplied by the surface resistance.

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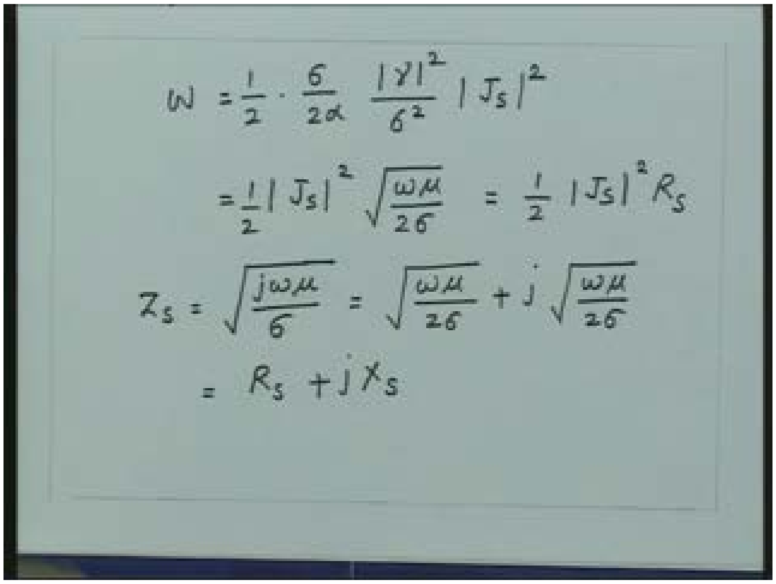

So later on we will see when we go to the structures like wave guides or something like that then the conductor loss is calculated from the surface current density because surface current density we can obtain from the magnetic fields so if I know the tangential component of the magnetic field on the surface of the conductor I can find out using the relation n x H the linear surface current density and once I know the surface current density and if I know the surface impedance then from there I can find out what is the power loss per unit area of the conductor.

So this concept or this calculation of power loss using the surface impedance or surface register and the linear current density is extremely useful in calculating the power losses in the conducting medium. Of course here we have calculated the power loss by using essentially the circuit concept what I mean by that is I found out the current, I found out the resistance in this slab then we find out the I^2 R loss and then we got the total power loss which was under the surface of that conductor.

We can find the same thing by an alternative approach and that is by the wave approach. What we argue is that if there was an electric and magnetic field which was on the surface of a conductor

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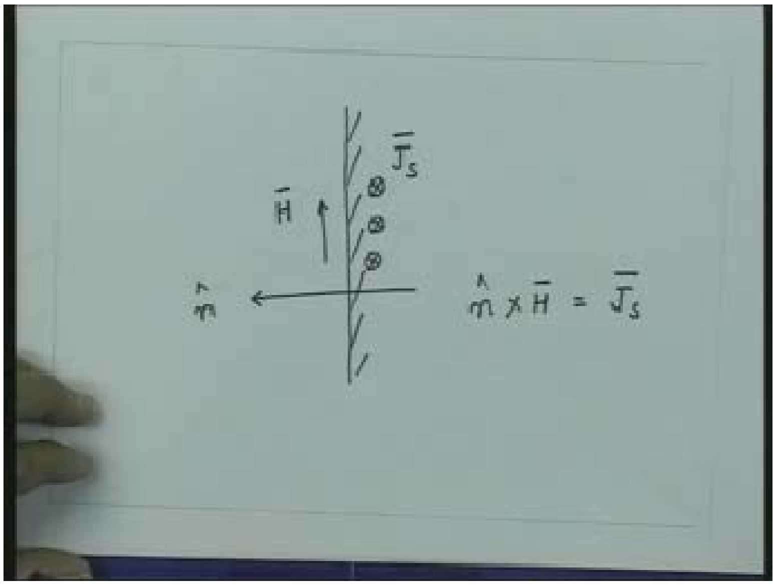

so we have a electric field and a magnetic field and once we have electric and magnetic fields they are essentially having a wave phenomena which is going in z direction. So the power going with this wave inside the conductor is the one which is essentially in these losses of the Ohmic conductor because the medium is infinite and this power is decreasing as the wave travels no power is coming back.

So I can say that equivalently what ever the power flow inside the conductor is the measure of what is the loss inside the conductor, So instead of doing the calculation of the power loss from this electrical point of view like finding out the current and the resistance I can use the wave concept and find what the power loss inside the conductor is. So if this is your interface and if the electric field in this direction is E_0 and the magnetic field is H_0 and the wave is traveling in this direction z direction. I can find out what is the power flow at this location since I am asking for a power flow per unit area of the surface. Essentially now I want to know what the Poynting Vector on the surface of the conductor is and that is the power which is essentially going inside the conductor so it will be lost in the heating of the conductor which is Ohmic loss.

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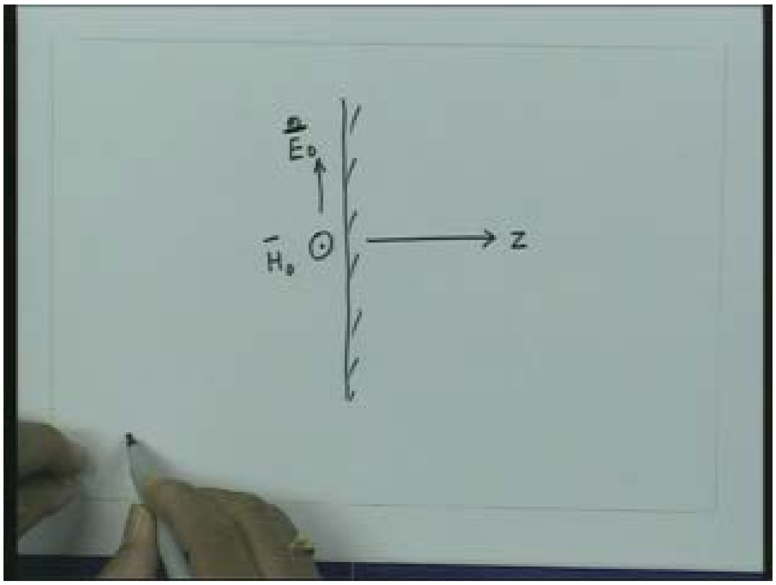

So if I calculate, the Poynting Vector $\overline{p}$ will be E x H where E and H are perpendicular to each other and this is in x direction and this is in y direction. So essentially we get $E_0\ H_0^*$ in the z direction. I can substitute for $\overline{H}_0$ for this medium that is $\frac{E_0}{\eta_c}$ so this is equal to $\frac{|E_0|^2}{\eta_c^*}$.

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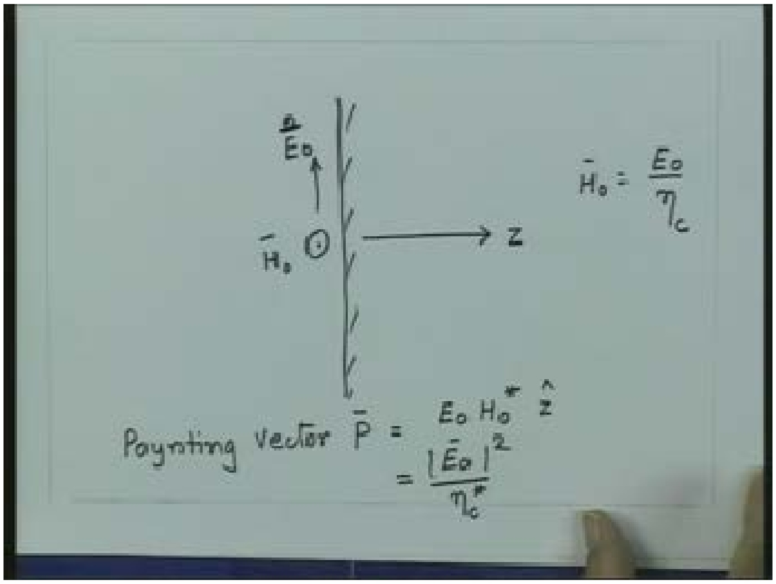

If I substitute for η_c and rationalize it I essentially get the power flow p that is equal to or we use rms value so we have average power which is ½ E x H. So here the p will be half and if I rationalize this quantity η_c which is $\sqrt{\frac{j\omega\mu}{\sigma}}$ then I will get this ½ $|E_0|^2$ $\sqrt{\frac{2}{\omega\mu\sigma}}$.

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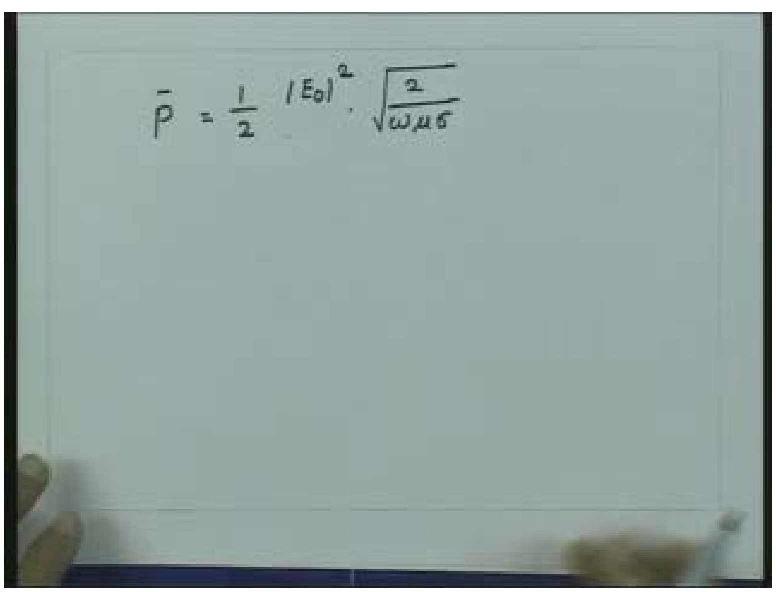

So this expression is exactly same as what we have got the power loss from our surface impedance calculation. So essentially we can calculate the power loss either by using the circuit concept like find the conduction current density then you find out Ohmic loss which is I^2 R loss or you can use the wave concept to find out what is the power going inside the surface of the conductor and that power essentially should get lost inside the conductor because the field essentially goes to zero at z tends to infinity that means there is no power flow at $z = \infty$.

So what ever power has gone inside the conductor must have been lost in the heating of the conductor. So either by using the wave concept or by using the electrical circuit concept we can calculate the power loss per unit area of the conductor.

However, the interesting thing to notice is that the power which is getting lost inside the conductor is inversely proportional to the conductivity that means as the conductivity becomes larger and the same thing is true for frequency as the frequency become larger the power flow inside the conductor reduces and when you go to very higher frequency or you go to ideal conductor where $\sigma = \infty$ then the power will grow identically to zero. That means for an ideal conductor there is no power going inside the conductor. Similarly when the frequency becomes larger there is no power which is going inside the conductor. What that means is for higher conductivity or high frequency the wave essentially finds the resistance penetrating the conductor the power does not go inside so the fields are dying exponentially very rapidly inside the conductor and we have taken analogy that this case is very similar to Lossy Transmission Line. However now there is a difference in a Lossy Transmission Line the power was getting lost in the hitting of the line but in this case the field starts dying down very rapidly but the power is not lost in the Ohmic loss.

In fact the power is not able to penetrate this layer. So as the conductivity becomes equal to infinity what ever power you try to put on the conductor no power penetrates the conductor in fact the entire power will be reflected from this boundary which is the conducting boundary.

So we will use this concept when we go the media interfaces and the behavior of the wave in media interfaces that when ever we have the difficulty in penetration of the boundaries the energy will be reflected and then in a medium from where the energy is coming you have the wave which is incident on the boundary and the wave which gets reflected because it finds difficulty in the penetration of the boundary.

So the circuit concept or the wave concept both can be used for calculating the power loss in the conductor and we conclude a very important conclusion that when the conductivity is infinite the wave just does not penetrate the medium there is no power loss and the entire energy which is incident on the interface is reflected from the boundary.

Thank you.