Transmission Lines and E.M. Waves

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Lecture-27

Welcome, in the earlier lectures we have seen that the time varying electric and magnetic field constitute a wave phenomena. Obviously then this wave requires some power or energy to flow with it. In this lecture essentially we investigate the power flow associated with an electromagnetic wave. We will do some derivation starting from the basic Maxwell's equation and then ultimately point out how much will be the power flow associated with electric and magnetic fields.

Till now we have investigated a uniform plane wave which is a wave propagation in a unbound medium. However, when we are developing this power flow calculations associated with electromagnetic waves we will do the general analysis and not restricted to the uniform plane waves. Of course at the end of the discussion we will find out how much is the power flow associated with the uniform plane waves which we have discussed in the last two lectures.

So when ever we have beginning of analysis in electromagnetic essentially we go back to the Maxwell's equations and find the answers which is consistent with the Maxwell's equations. The same thing we do here again where we ask the question if you go back to the Maxwell's equations what answer I get for the power flow associated with electromagnetic fields. So in this lecture we are going to do the power flow associated with an electromagnetic wave.

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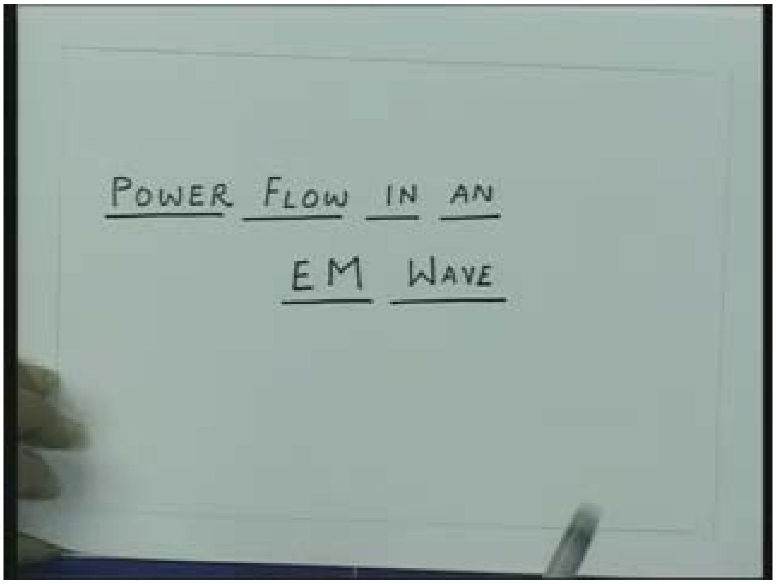

Now going to the Maxwell's equations and essentially the curl equations we have in general the $\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$ that is if I assume the permeability of the medium is not a function of time I can take μ out and this can be written as -μ $\frac{\partial \overline{H}}{\partial t}$ and the second equation which will be $\nabla \times \overline{H} = J + \frac{\partial \overline{D}}{\partial t}$ where they are writing D in terms of the electric field and again assuming that the permeability of the medium is not a function of time so this can be written as $\overline{J} + \varepsilon \frac{\partial \overline{E}}{\partial t}$.

So again we start with these two basic equations and then try to investigate the power flow associated with electric and magnetic fields.

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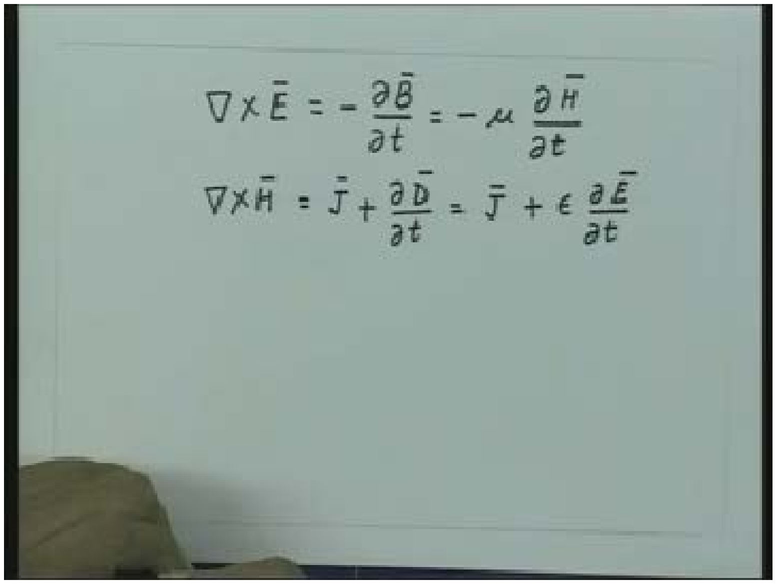

Here essentially we make use of the vector identities and then try to find out meaning to some of the terms which I am going to get in the expansion of the vector identity. So let me just take the vector identity which is $\nabla \cdot \left(\overline{\mathrm{A}} \times \overline{\mathrm{C}}\right) = \mathrm{C} \cdot \left(\nabla \times \overline{\mathrm{A}}\right) - \mathrm{A} \cdot \left(\nabla \times \overline{\mathrm{C}}\right)$ where A and C are some two arbitrary vectors. So if I have any two arbitrary vectors A and C we have this vector identity for these two vectors. Now what we can do is let us say this vector $\overline{\mathrm{A}}$ is the electric field and this vector $\overline{\mathrm{C}}$ is the magnetic field so substituting for A as E and C as H essentially the vector identity for these two vectors the electric and the magnetic field can be written as $\nabla \cdot \left(\overline{\mathrm{E}} \times \overline{\mathrm{H}}\right) = \mathrm{H} \cdot \left(\nabla \times \overline{\mathrm{E}}\right) - \mathrm{E} \cdot \left(\nabla \times \overline{\mathrm{H}}\right)$ and we can substitute for $\nabla \times \overline{\mathrm{E}}$ from this equation and for $\nabla \times \overline{\mathrm{H}}$ from this equation.

So from here I can get this quantity as $\mathrm{H} \cdot \left(-\mu \frac{\partial \overline{\mathrm{H}}}{\partial t}\right) - \mathrm{E} \cdot \left(\overline{\mathrm{J}} + \varepsilon \frac{\partial \overline{\mathrm{E}}}{\partial t}\right)$.

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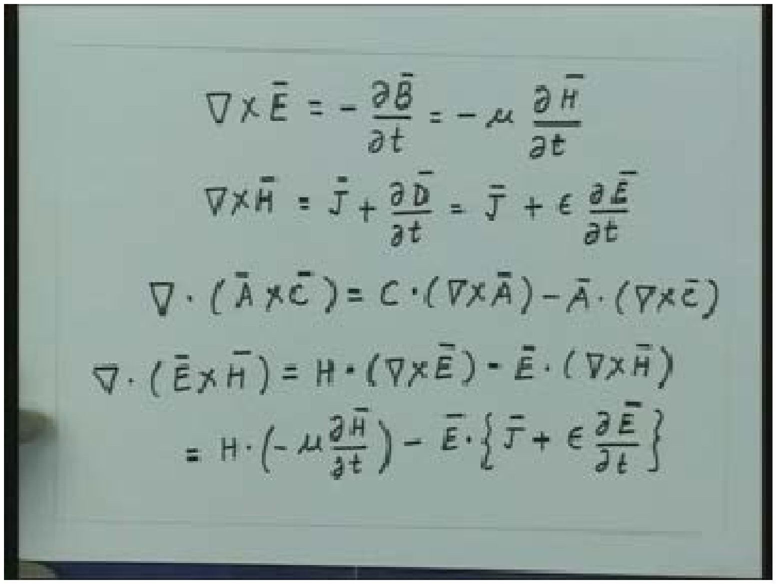

I can rewrite for the two arbitrary vectors A and C, again if I take a time derivative of $\frac{\partial}{\partial t}\left(\bar{A} \cdot \bar{C}\right)$ where $\bar{A}$ and $\bar{C}$ are some two vectors. This is nothing but $\bar{A}\frac{\partial \bar{C}}{\partial t} + \bar{C}\frac{\partial \bar{A}}{\partial t}$ so this is true for any two arbitrary vectors A and C.

If I take both the vectors A vectors then I can get $\frac{\partial}{\partial t}\left(\bar{A} \cdot \bar{A}\right) = \bar{A}\frac{\partial \bar{A}}{\partial t} + \bar{A}\frac{\partial \bar{A}}{\partial t}$ so that quantity is nothing but $2\,\bar{A}\frac{\partial \bar{A}}{\partial t}$.

So from here essentially we get $\bar{A}\frac{\partial \bar{A}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}\left(\bar{A} \cdot \bar{A}\right)$ which is nothing but mod of A^2 so this is $\frac{1}{2}\frac{\partial |A|^2}{\partial t}$.

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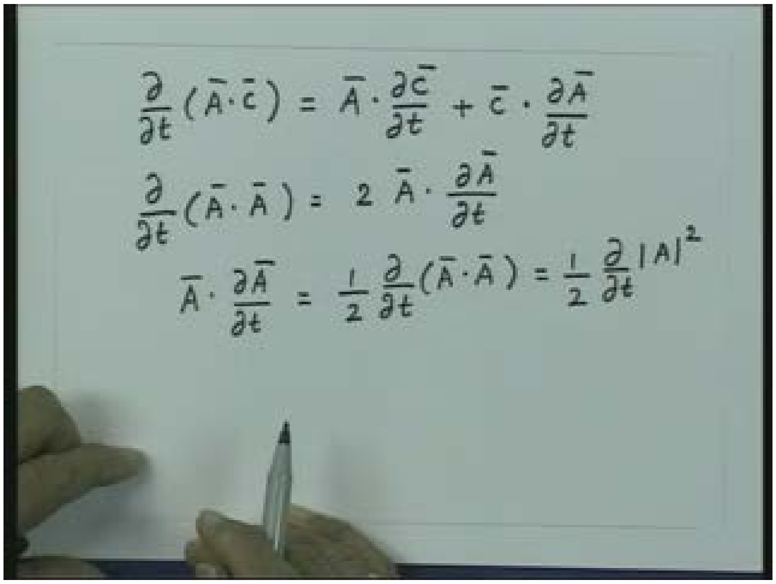

So we can make use of this relation for simplifying this. Essentially if I take this μ out this is $H\cdot\left(\frac{\partial \bar{H}}{\partial t}\right)$ just a quantity which is similar to this, similarly if I take this then this will be $E\cdot\left(\frac{\partial \bar{E}}{\partial t}\right)$ so I can substitute from this into this equation and I get the equation as $\nabla\cdot\left(\bar{E}\times\bar{H}\right)$ that is equal to from here I am substituting $\bar{A}\frac{\partial \bar{A}}{\partial t}$ which is $\frac{1}{2}\frac{\partial |A|^2}{\partial t}$ so this will become $\frac{1}{2}\frac{\partial |H|^2}{\partial t}$ so this will become $-\frac{\mu}{2}\frac{\partial |H|^2}{\partial t}$ and we can have the second term as this which is $-E\cdot\left(\frac{\partial \bar{E}}{\partial t}\right)$ so that will become $-\frac{\varepsilon}{2}\frac{\partial |E|^2}{\partial t}$ and then finally we can have this term $-\bar{E}\cdot\bar{J}$.

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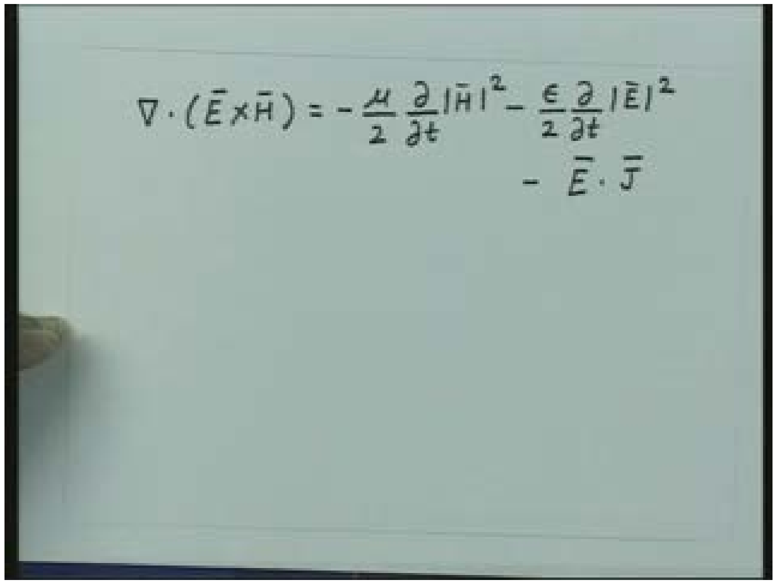

Up till now we started with Maxwell’s equations which we have point relations that means these equations are valid at every point in space. This relationship which we have got here is essentially a point relationship so any point in the space essentially this condition is satisfied. In general, if I am having a medium which is having a finite conductivity that gives the conduction current density J and if I am having medium which is not varying at a function of time then in general the electric and magnetic fields satisfies these equations.

Now what we can do is we can integrate this quantity over a closed surface or a volume and then we will have some meaning associated with these quantities. So let us say if I integrate this over a volume I can get this because this is a triple integral integrated over a volume $\int_v \nabla\cdot\left(\bar{E}\times\bar{H}\right) dv = \int_v -\frac{\mu}{2}\frac{\partial |H|^2}{\partial t}dv - \int \frac{\varepsilon}{2}\frac{\partial |E|^2}{\partial t}dv - \int_v -\bar{E}\cdot\bar{J}$.

Again assuming that this volume is not varying as a function of time that means the fields are only time varying but the space is not varying as a function of time we can interchange this $\frac{\partial}{\partial t}$ with the integration so we can take this $\frac{\partial}{\partial t}$ out of the integration sign and the same thing I can do here also. And I can apply divergence theorem on this left side to change the volume integral to the surface integral.

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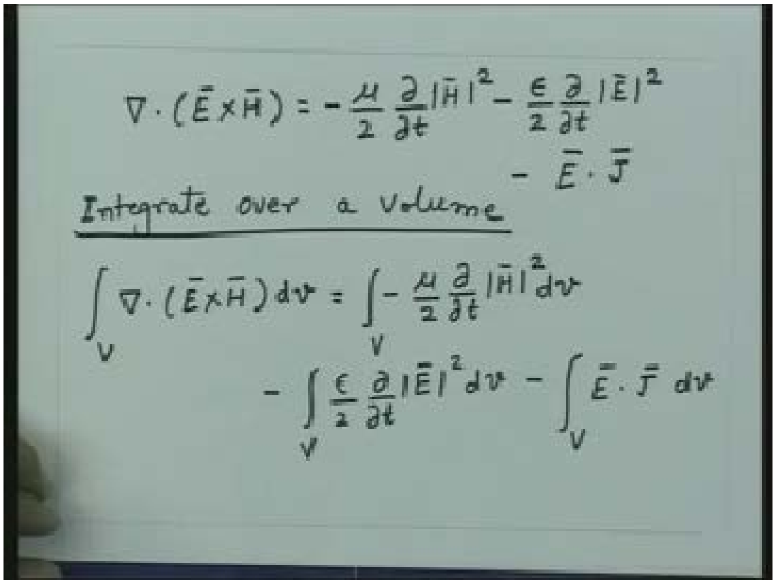

So this gives the left side then becomes the surface integral over a closed surface that will be $\int (\bar{E} \times \bar{H}) \cdot d\bar{a}$ that is the application of divergence theorem so this thing now the volume integral is converted to the surface integral by using divergence theorem and interchanging the sign for the time derivative and the integral sign we get - $\frac{\partial}{\partial t}\int_v \frac{\mu}{2}|H|^2 dv$

this is on closed surface again $-\frac{\partial}{\partial t}\int_v \frac{\varepsilon}{2}|E|^2 dv - \int_v -\bar{E} \cdot \bar{J} dv$.

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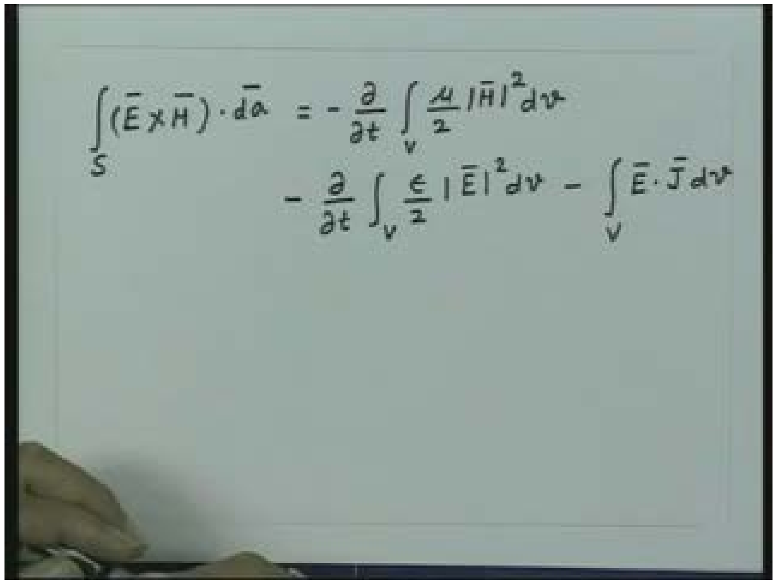

I can substitute for $J = \sigma \bar{E}$ so this term here if I look at and if I substitute for $J = \sigma$ then this term will become $\sigma\, |E|^2$.

Now if I look at this quantity and here this quantity essentially gives me the density of the magnetic energy stored in this volume, this quantity tells me the electric energy stored in the volume. So this is basically the electric energy density, this is the magnetic energy density integrated over the volume gives me the total energy stored in this volume v due to magnetic field, this is the total energy stored in this volume due to electric field so $\frac{\partial}{\partial t}$ of this quantity essentially gives me the rate of change of the magnetic energy stored in that volume, similarly this term gives me the rate of change of the electric energy stored in that volume. Also the negative sign shows that the rate of change is negative that means there is a decrease in the energy as a function of time. So this quantity essentially tells me the rate of decrease of the magnetic energy stored in that volume v, this quantity tells me the rate of decrease of electric energy stored in that volume v.

By substituting $J = \sigma \bar{E}$ this quantity essentially tells you the Ohmic laws into the medium. So what we find is here the first term tells me the rate of decrease of magnetic energy in that volume, this quantity tells me the power loss taking place in that volume because of the finite conductivity of the medium.

So if you have a total energy includes in a surface this power loss total must be equal to the energy which is equal to essentially leaving in that box. So if I take this volume v which is having a corresponding surface area s since there is no other mechanism of consuming energy from the conservation of energy essentially we get that this quantity must represent the flow of energy coming from the surface or rate of flow of energy coming from the surface.

So from here what essentially we see is this quantity the surface integral for $\bar{E} \times \bar{H}$ tells you the net power flow from a closed surface. Then from here this quantity has represents $\int_s (\bar{E} \times \bar{H}) \cdot d\bar{a}$ is net power flow from a closed surface.

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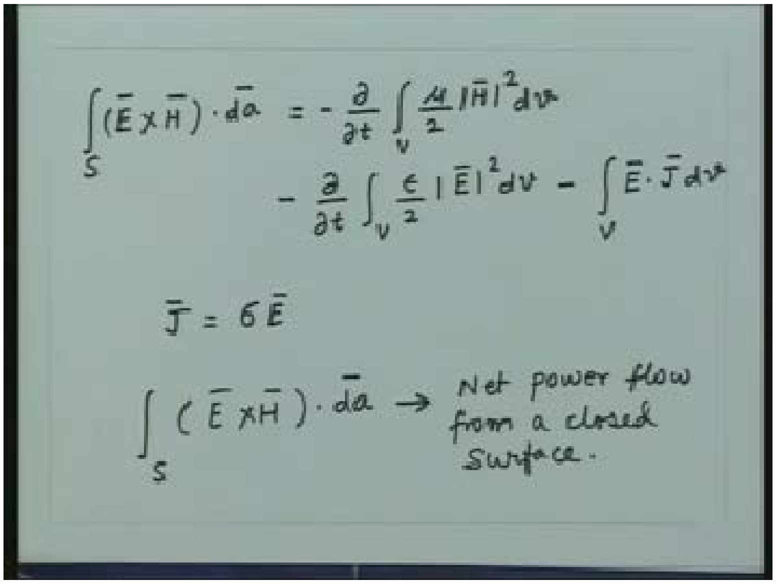

So essentially we find a very important thing that by doing simple vector manipulations as we have done started with vector identity substituted the Maxwell's equations in the vector identity and from there we find something interesting that this surface integral of $\overline{E}\times\overline{H}$ over a closed surface gives me the net power flow associated with this electric and magnetic field. This statement is called the Poynting Theorem.

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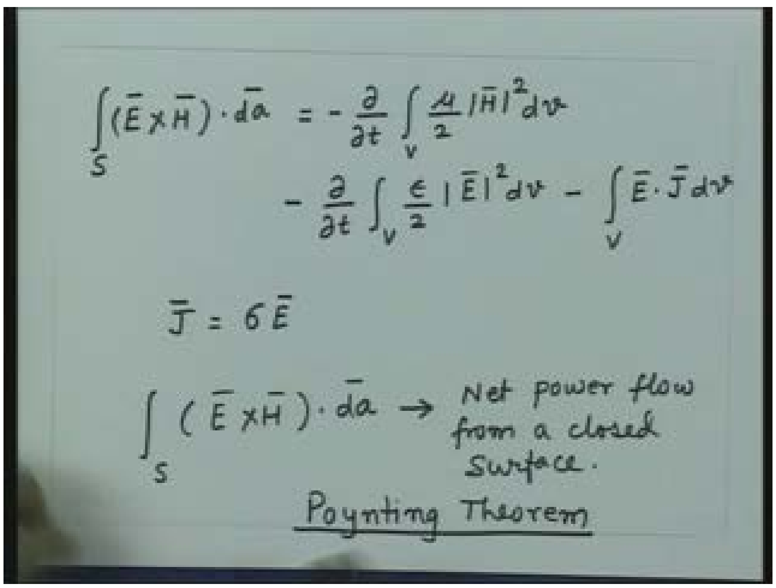

So the Poynting theorem says that for the electric and magnetic fields if I take the cross product of that and integrate over a closed surface that gives me the total power flow from that closed surface.

Now if this is the quantity which is representing the total power flow then we can say this quantity $\overline{E}\times\overline{H}$ is essentially the power density or power flow density on the surface of this closed surface. So when we integrate this power density over the surface area then that gives me the total power flow from the surface. However we should keep in mind just saying that this whole integral is giving me net power flow that is why this quantity should gives me the power density at every point on the surface of the sphere is a arbitrary definition.

The Poynting Theorem does not say that this quantity is representing the power density or the power flow per unit area at every point on the surface of this volume. what I am telling you is that the total power coming out of this is equal to this quantity. So this quantity $\overline{E}\times\overline{H}$ is a power density and that is true at every point on the surface of the sphere is a arbitrary definition of the power density which we take from here. It so happens in most of the practical situations this arbitrary definition gives you the power density correctly. However if you ask rigorously whether knowing this quantity should be said as this is representing power density at every point on the surface of this volume, this statement is not correct.

In fact there may be special methodological phases where this argument will fail that if you have $\overline{E}\times\overline{H}$ at some particular point it may give you a power flow where there is actually no power flow. So while using this quantity as the power density one has to be little careful. However, in most of the practical situations as I mentioned this arbitrary definition that $\overline{E}\times\overline{H}$ gives me the power flow density at particular location that normally is valid.

Now essentially what the important thing that we get is a Power Flow Density and let me call that quantity as some $\overline{p}$ which is a vector and that is equal to $\overline{E}\times\overline{H}$. Then we call this quantity $\overline{p}$ as the Poynting vector for these fields or for this so this quantity is called the Poynting vector.

So Poynting vector is a very important concept in the electromagnetic waves because it tells you what the density of the power flow at a particular point in the space is and also it tells you in which direction the power is flowing. We know this quantity is a vector quantity so first thing we note here is if you have electric and magnetic fields then the Poynting vector is in a direction perpendicular to both of electric and magnetic fields because we have this cross product that means this vector $\overline{p}$ is perpendicular to both of these vectors.

So firstly if this quantity has to be non zero if there is a power flow. Now first thing we note here is that $\overline{E} \times \overline{H}$ should not be parallel to each other. If you have $\overline{E} \times \overline{H}$ electric and magnetic fields parallel to each other then the cross product will be identically zero and there will not be any power flow associated with this. So only the component of electric and magnetic field which are perpendicular to each other they contribute to the power flow and the direction of power flow is perpendicular to both the electric and magnetic fields or in other words that will have a power flow in the due to the fields the E and H must cross each other. When ever there is a crossing of E and H there is a possibility of power flow and this is the word possibility here because this quantity essentially is telling you the so called the instantaneous power if I know the value of E and H at some instant of time at some point in space I can always find this cross product at that instant of time and I will get a number this quantity $\overline{p}$ which will give me the Poynting vector at that instant of time. This is possible that even if there is a $\overline{p}$ which is finite at some instant of time there may not be any net power flow over long time periods that means if I say in time average sense there may not be any power flow associated with the system.

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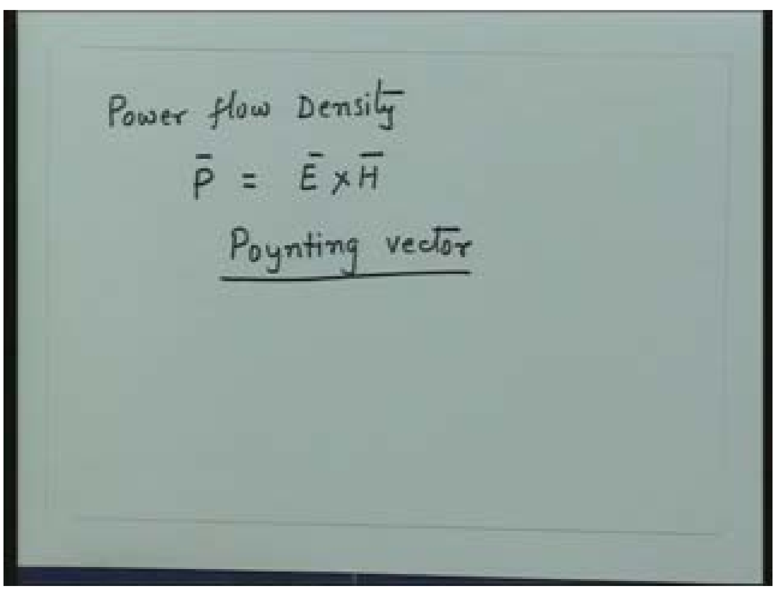

So Poynting vector which we define as $\overline{E} \times \overline{H}$ serve the purpose of defining the power flow but if I seen a practical system probably more useful quantity will be time average value of this Poynting vector because if I take some instant of time first of all this quantity one by one negative so if I say this is telling you power it may even give me the power which is negative. Of course when we are dealing with the space we can say negative power means the direction of the power flow which is changed but all those complications will come if I use simply the $\overline{E} \times \overline{H}$ and get the value of $\overline{p}$ because $\overline{p}$ can go positive negative also depending upon the time phases between E and H even this quantity can go as the complex point.

So what we do is we essentially try to get the time average value of the Poynting vector and that is what more meaningful quantity for finding out whether there is a net flow of power associated with the electric and magnetic fields. As we have seen earlier in our analysis essentially we are interested only in time harmonic fields. So we will do the analysis for time harmonic fields here. So again we assume that the electric and magnetic fields are varying sinusoidally as a function of time, only thing they can have is phase difference between them the temporal phase difference. And then we can ask the general question what would be the average power flow or the Poynting Vector associated with those electric and magnetic fields.

Now let us define the general time varying fields for electric and magnetic fields which could be varying as a function of space and time. Let us say at some point in space I have the electric field $\overline{E}$ which is having some magnitude $\overline{E}_0$ and is having a variation $e^{j\omega t}$ and let us say it has some phase which is given as e so this quantity is having some phase j times ϕ_e.

Similarly I can have a magnetic field $\overline{H}$ which is oriented in some direction so it having a magnitude $\overline{H}$ in some arbitrary direction but it is having the same frequency so it is $e^{j\omega t}$ but it may have a time phase which could be different so this is ϕ_h.

What we can do is we can just take out this vector associated with this as a unit vector and just write down this quantity only as the magnitude of the electric field, the magnitude of the magnetic field. If I take the instantaneous values of the electric and the magnetic field then I can get the instantaneous values of the fields as E at some instant of time which will be the real part of this quantity and as I have mentioned I can take the unit vector out of this I can keep only the magnitude so this will be the magnitude which is E_0 $e^{j\omega t}$ $e^{j\phi_e}$ multiplied by the unit vector which is $\hat{e}$ where $\hat{e}$ gives me the unit vector in the direction of this electric field.

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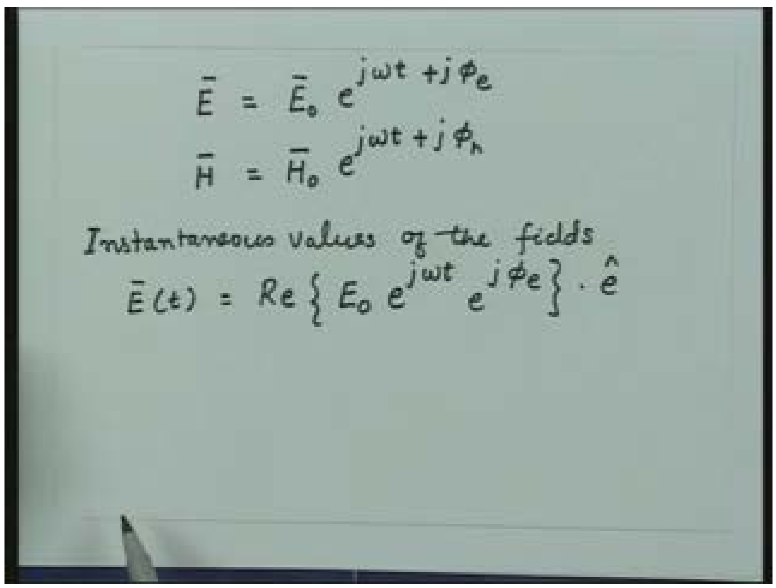

Similarly I can get the instantaneous value of the magnetic field at some time t which will be the real part of again I will do the same thing I will take the magnitude of this magnetic field H_0 $e^{j\omega t}$ $e^{j\phi_h}$ phase of the magnetic field multiplied by the unit vector which is the $\hat{h}$ direction.

So $\hat{e}$ and $\hat{h}$ essentially gives me the vectors in the direction of electric and magnetic fields and ϕ_e and ϕ_h will give the phase of electric and the magnetic fields respectively.

And E_0 and H_0 are the amplitudes of the electric field at peak amplitudes associated with this. So if I take real part of this quantity that gives me the instantaneous value of the electric field and the instantaneous value of the magnetic field.

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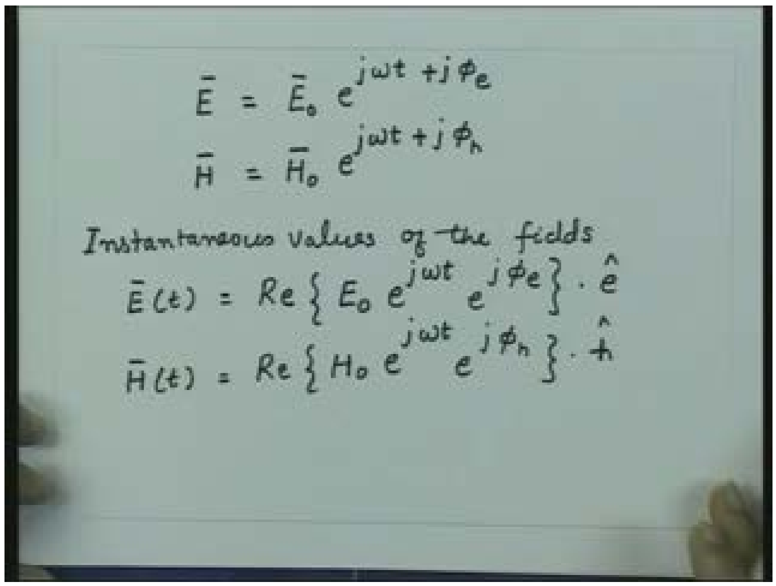

Once I know this quantity then I can find out at that instant of time the Poynting vector which essentially is taking $\bar{E} \times \bar{H}$. So before that if I just separate out the real part of this from here we get the instantaneous value $\bar{E}(t)$ which will be real part of this quantity so that is equal to $E_0 \cos(\omega t + \phi_e)$ multiplied by the unit vector $\hat{e}$. And $\bar{H}(t)$ will be $H_0 \cos(\omega t + \phi_h)$ multiplied by the unit vector $\hat{\hbar}$.

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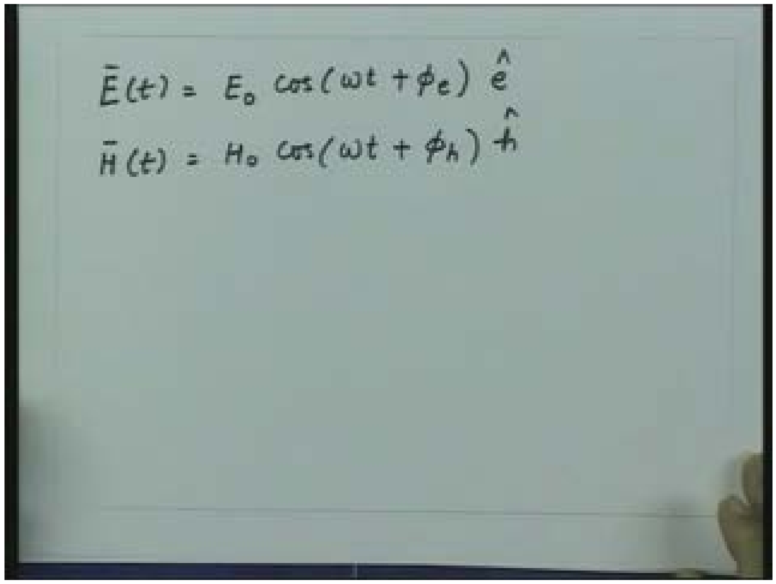

Once we know these vector quantities at that instant time of t then we can calculate now the Poynting vector and that gives me the Power Flow Density at that instant of time t. So from here we can get the Poynting vector $\overline{p}$ which is $\overline{E} \times \overline{H}$, I just take product of these so that is equal to E_0 H_0 cos(ωt + ϕ_e) cos(ωt + ϕ_h)$\left\{\hat{e} \times \hat{\hbar}\right\}$.

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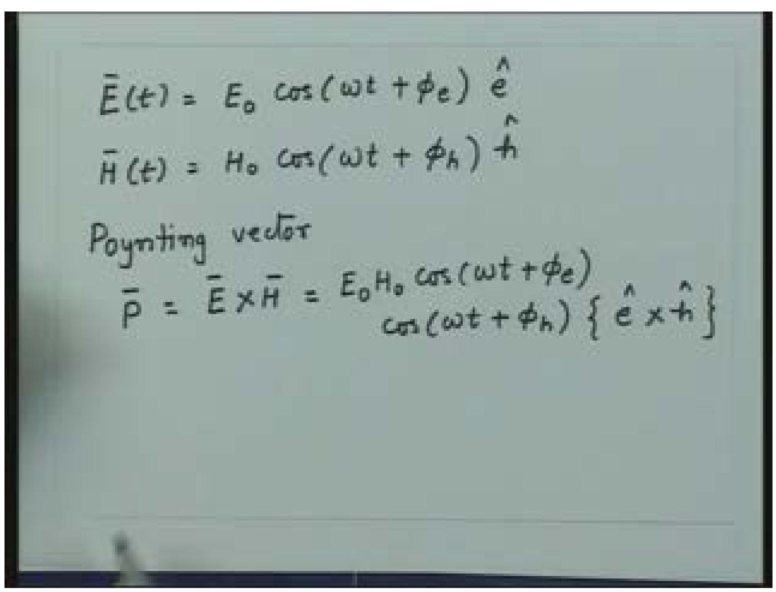

So this quantity is the scalar quantity and you have the cross product which essentially is the cross product of the unit vectors, I can simplify this so this gives me essentially $\frac{E_0H_0}{2}\left\{\cos(\phi_e - \phi_h) + \cos(2\omega t + \phi_e + \phi_h)\right\}\left(\hat{e}\times\hat{\hbar}\right)$.

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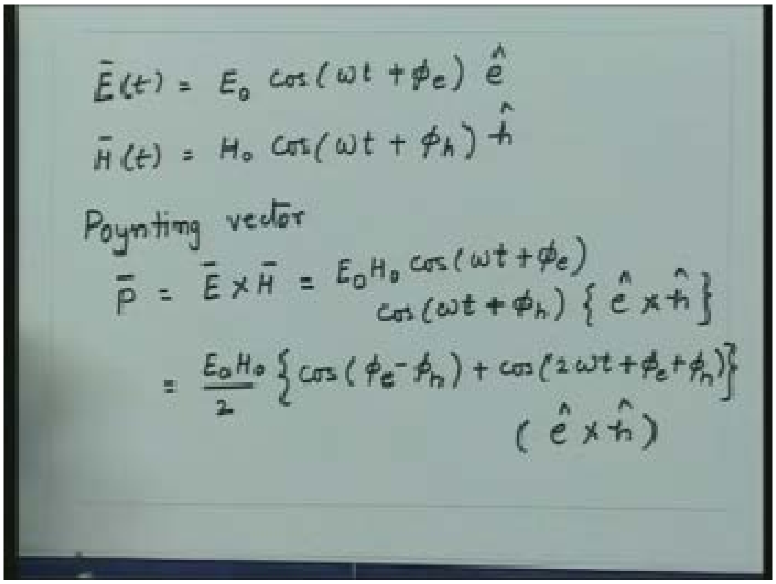

Now this is the instantaneous value of the Poynting vector and as we mentioned we are now interested in finding out what is the average value of the density or what is the average value of this Poynting vector. So we can take a time average of this over a period of this signal, if I integrate this power density over one period or one cycle so essentially we get the average power density associated with this but this quantity will go to zero over that one period.

So, essentially this is corresponding to a waveform which is having a frequency of 2f or angular frequency 2ω. So over a period corresponding to ω this quantity will identically go to zero so if I take the time average of this quantity which is $\bar{P}_{average} = \frac{1}{T}\int_0^T \bar{P}dt$ where T is the time period associated with this angular frequency ω so T = $\frac{2\pi}{\omega}$ then I get the average Poynting vector and in that this quantity will essentially goes to zero. So I get the average value of the Poynting Vector $\overline{P}_{average} = \frac{1}{T}\int_0^T \frac{E_0 H_0}{2} \cos(\phi_e - \phi_h)\, dt\, \left(\hat{e} \times \hat{\hbar}\right)$.

So now the average Poynting vector is the average value of this quantity and you have a cross product of the unit vectors of the electric and the magnetic fields. Now this quantity is not a function of time this is constant so this can be taken out. So the integral will be $\frac{1}{T}$ integral zero to T dt which is nothing but equal to one.

So now we have $\overline{P}_{average} = \frac{E_0 H_0}{2} \cos(\phi_e - \phi_h)$ and this cross product of unit vectors $\hat{e}$ and $\hat{\hbar}$. we can do little more algebraic manipulation to write again back the electric and the magnetic fields in the vector form. So what we can do is this quantity now can be written as $\frac{1}{2}$ Re{[E_0 $e^{j\omega t + j\phi_e}\hat{e}$][$H_0$ $e^{-j\omega t - j\phi_h}\hat{\hbar}$]}.

So what we have done is we have added this quantity $e^{j\omega t}$ and $e^{-j\omega t}$ in the expression if you see here this will be E0 H_0 these are scalar quantities, we will have cross product of E and H which is this and if I take the real part of this quantity $e^{j\phi_e}$ multiplied by $e^{-j\phi_h}$ will give me the $\cos(\phi_e - \phi_h)$.

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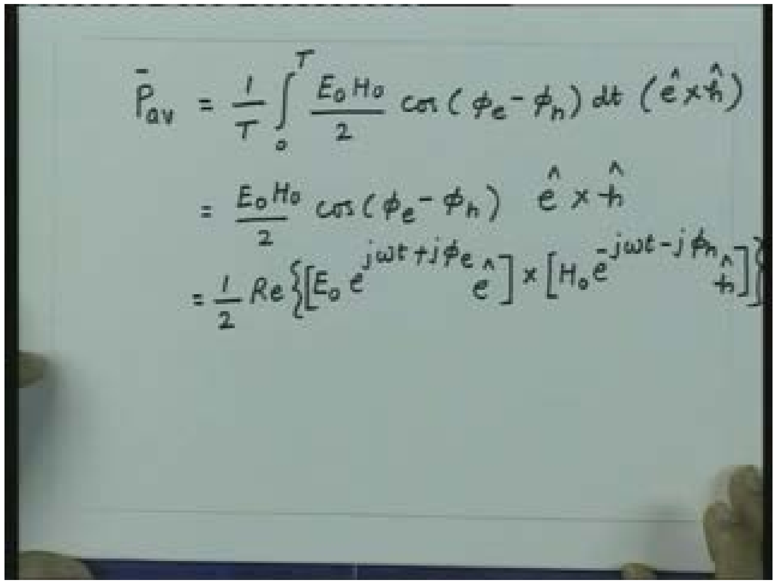

Now if I take this negative sign here this quantity is the scalar quantity so you can write this also like the real part of this quantity which is $[E_0 e^{j\omega t+j\phi_e} \hat{e}] \times [H_0\ e^{j\omega t+j\phi_h} \hat{h}]$. So the conjugate of this quantity will be a scalar quantity real quantity so complex quantity is only this $e^{j\omega t+j\phi_h}$ so if I take the conjugate of this essentially this represents this quantity. But this quantity is the original magnetic field which we have defined in the vector form, similarly this is the quantity is the original electric field which we define in the vector form. So essentially we have this quantity here the electric field is zero $e^{j\omega t+j\phi_e}$ multiplied by unit vector, same is true for magnetic field here.

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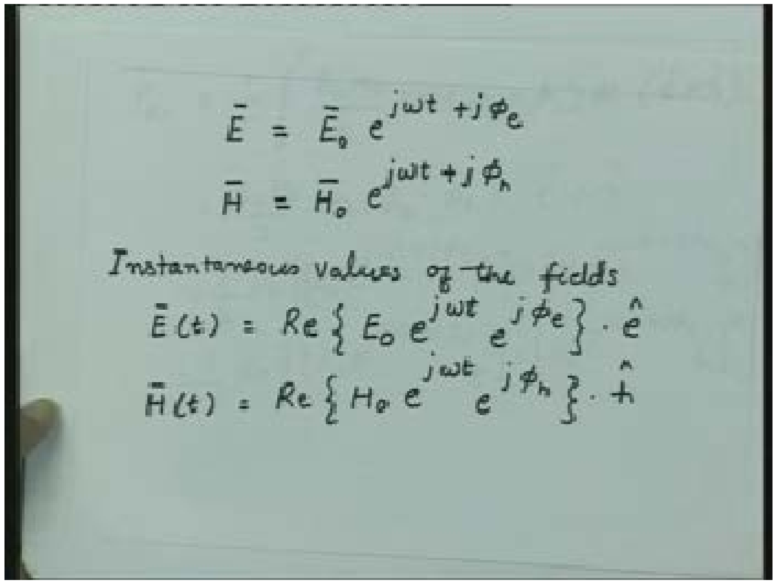

So this quantity is nothing but electric field and this is the magnetic field so this is half real part of $E \times H^*$ which is the average Poynting vector. So if I know the electric and magnetic fields in the complex form that means the electric and the magnetic field may not be in time phase all the time then in general we can just calculate this cross product of $E \times H^*$ and real part of that the half vector is essentially because of the rms value we get in the signals since the signals are time varying sinusoidally essentially this is the rms factor so now this gives me the average power flow which will be associated with that electromagnetic wave.

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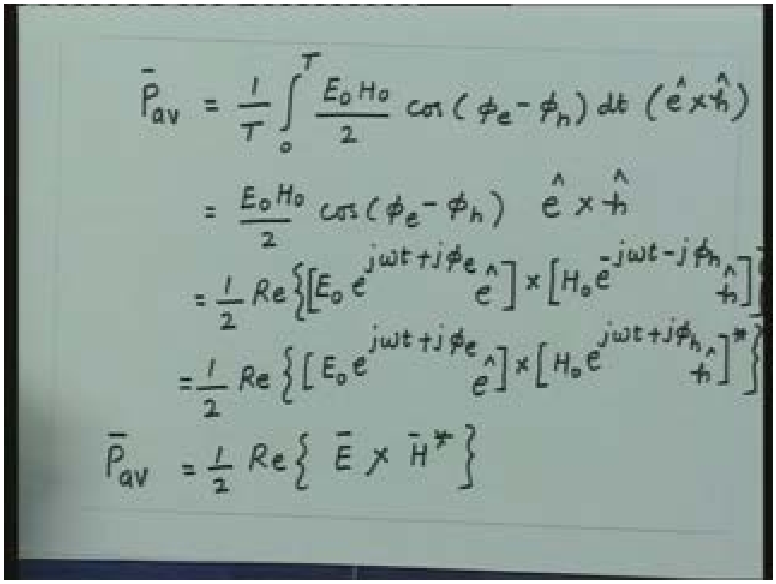

Now this quantity is the real quantity as we are taking a real function of this so all those problems which we had with the instantaneous power flow could either become complex depending upon the phase difference between them and all those have been taken care of and also it tells me the overall power flow which is associated with this fields at a particular location. So it is possible at a particular location the instantaneous Poynting vector might be negative or positive but when you calculate the average Poynting vector then that will be always positive and that will give me the net power flow which will be associated with those electric and magnetic fields. So this is the concept which is very regularly used in finding out the average power flow associated with an electromagnetic wave.

Again now there are two things are essential to have the average power flow, one is the electric and the magnetic fields must have a component perpendicular to each other then only you will have a cross product which is non zero and at the same time the electric and magnetic fields should not be in time quadrature that means the phase difference between the electric field and magnetic fields should not be 90˚ because if it is 90˚ then the real part of this quantity will be zero and then you will not have any real power flow associated with that one.

So in general it is possible if you take the electric and magnetic fields you will have the complex power the real part of that quantity gives me the net power flow at that location but the imaginary part of that quantity $E \times H^*$ gives me the power which is oscillating around that point so some instant of time the power might be going in certain direction if you see after some time the power will be essentially coming back in the same direction. So the imaginary part of $E \times H^*$ gives me some kind of a oscillating power which you call as a reactive power whereas the real part of $E \times H^*$ gives me the net power flow or the resistive power flow at a particular location.

This concept of Poynting vector and the average Poynting Vector is the very important concept because by using this concept we can calculate the net power flow at a particular location. One can then apply this concept to the case of the uniform plane wave. So we can ask if you are having a uniform plane wave then how much power density the uniform plane wave carries when it travels in the medium. We have seen for uniform plane waves first of all the electric and magnetic fields are perpendicular to each other so if I take a uniform plane wave then the electric field the magnetic fields are perpendicular to each other and then let us say this is electric field $\overline{E}$, this is the magnetic field which is $\overline{H}$ and the power will be flowing in this direction which is the cross product of these two. So this is the direction of the Poynting Vector $\overline{P}_{av}$.

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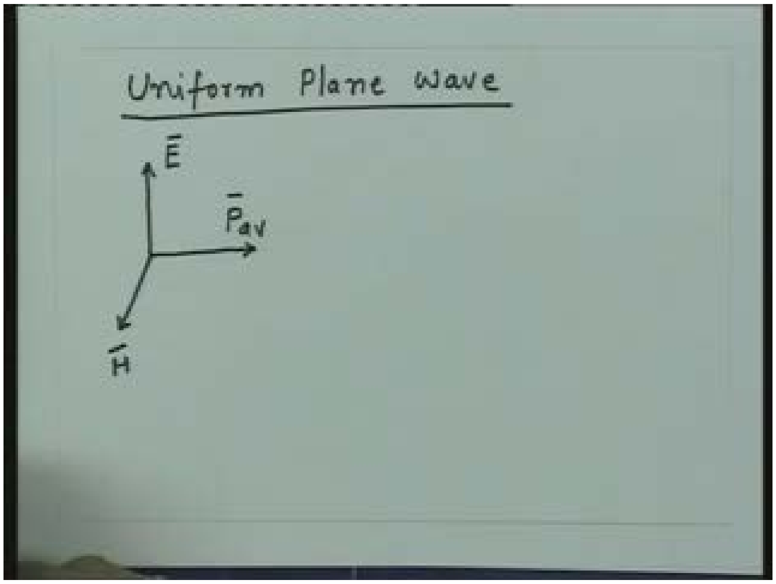

Again we can apply the right hand rule to find out whether the power will be flowing in this direction or will be flowing in this direction. Since we are taking the cross product of E and H again we point the fingers in the direction of E to H and the thumb should be in the direction of the cross product which is the direction of this. This direction is same as the direction of the wave propagation also because we have talked about uniform plane wave E and H and the direction of the wave propagation essentially form the three coordinate axis in the same sequence that means if I go from E to H my fingers point from E to H the thumb should go to the direction of the wave propagation. So we correctly get the direction of the average Poynting vector which is the same as the wave propagation.

Secondly, now if I say the electric field is some magnitude E_0 and is having a phase variation which is $e^{-j\beta}$ let us say the wave is traveling in z direction so this is the z direction $e^{-j\beta z}$ and E can be oriented let us say x direction then H will be oriented in y direction so I can say this is oriented in x. Then the magnetic field H will be oriented in y direction since it has a magnitude H_0 $e^{-j\beta z}$ orientation is the y direction.

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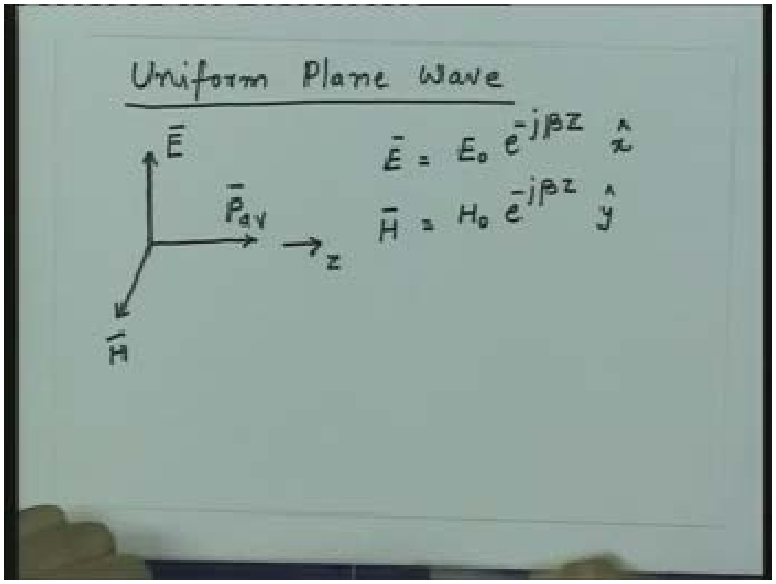

So the average Poynting vector associated with this $\overline{P}_{av}$ is equal to the half real part of $E \times H^*$, now the cross product of x and y will give my direction z so this average Poynting vector is in z direction so this is half of real part of this conjugate so essentially that will become E_0H_0 so this will give me E_0 H_0 in the direction $\hat{z}$.

In general if I assume this quantity could be complex quantity I can put still even the conjugate sign at this. So for a uniform plane wave the average Poynting vector will be half of $E_0 \times H_0^*$ and the direction of this will be z, if you say the electric field was oriented in the x direction and the magnetic field was oriented in y direction.

Now we can take specific cases for the unbound medium as the uniform plane wave is propagating we can take first the medium which is dielectric medium. Now for a dielectric medium or in general if I take a unbound medium first of all we know there is a relationship between these two quantities E and H that is the magnitude of the electric and magnetic fields are related to what is called the intrinsic impedance of the medium. So I also have a relation for a uniform plane wave having electric and magnetic field and that is equal to η which is Intrinsic Impedance.

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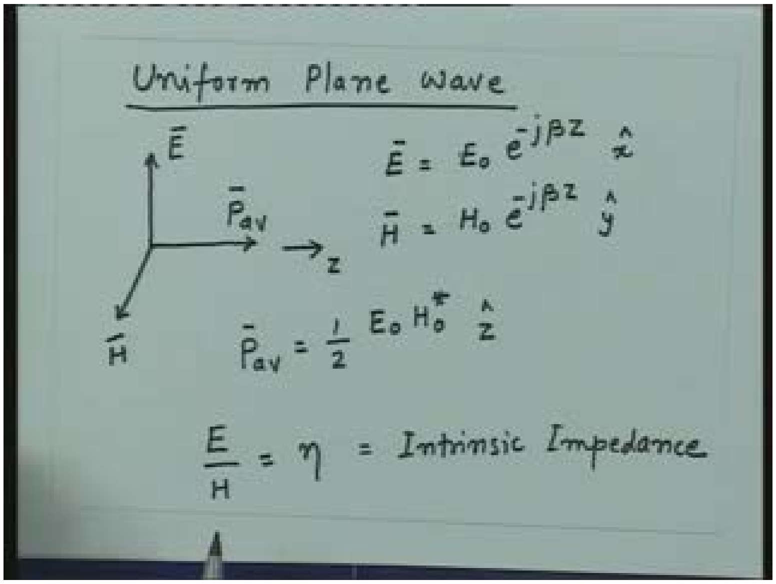

So I can substitute for the magnetic field from here that will be E upon η or I can substitute for electric field which is H times η. So I get the average Poynting vector $\overline{P}_{av}$ = $\frac{1}{2} E_0 \left(\frac{E_0}{\eta}\right)^* \hat{z}$ or if I write in terms of the magnetic field this will also be $\frac{1}{2}\ H_0 H_0^* \hat{z}$.

Now E_0 E_0^* is mod $|E_0|^2$ so this is equal to $\frac{1}{2}\frac{|E_0|^2}{\eta^*}\hat{z}$ or of course if I am putting the conjugate here I must put the real part of that so this is real part of this quantity, the same thing you have to put here this is the real part so this is again $\frac{1}{2}\mathrm{Re}\left\{\eta |H_0|^2\right\}\hat{z}$.

So from here essentially we can find out what is the average power flow associated with the uniform electromagnetic wave in an unbound medium.

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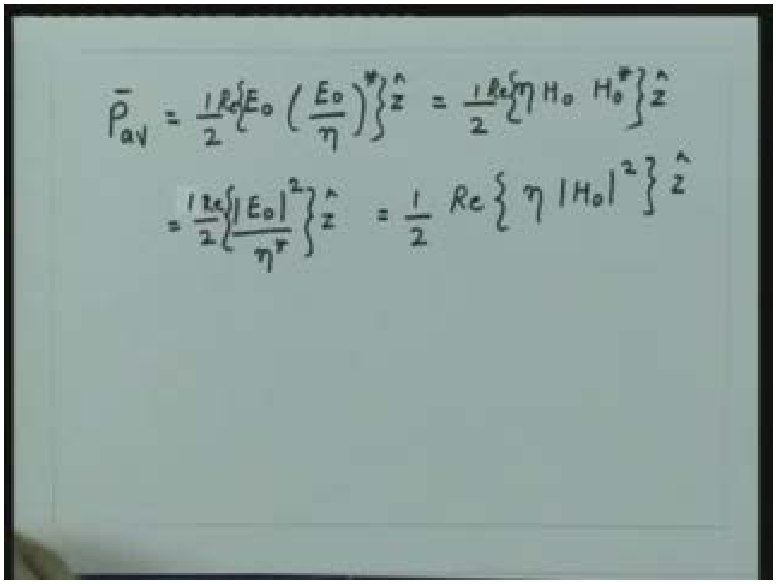

Now if I take a dielectric medium an ideal dielectric medium that means there is no conductivity in this medium for which we know that $\eta = \sqrt{\frac{\mu}{\varepsilon}}$.

So this quantity is a real quantity for an ideal dielectric medium so this quantity essentially the η^* since this is a real quantity the same is η so in this case the average power density $\overline{P}_{av}$ will be equal to $\frac{1}{2}\frac{|E_0|^2}{\eta}$ and that will also be equal to $\frac{1}{2}\eta|H_0|^2$.

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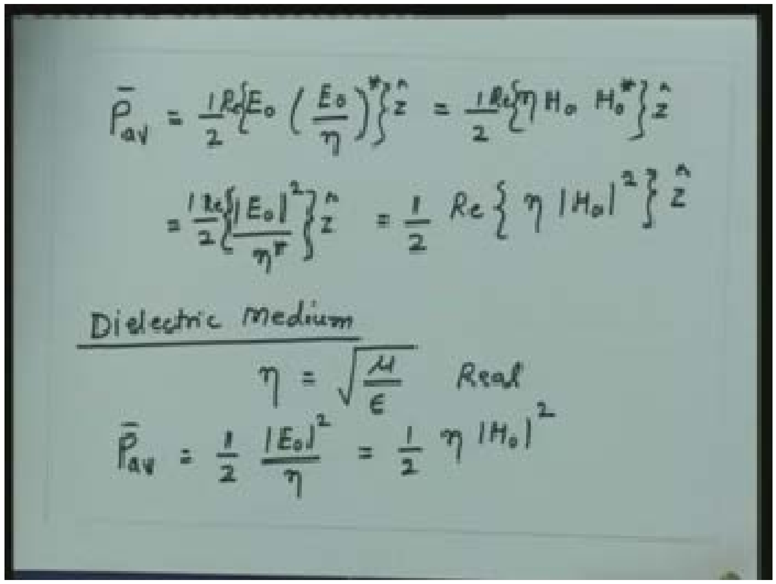

So in a dielectric medium if I know the magnitude of the electric field or this is the peak amplitude of the electric field and I know the permittivity and the permeability of the medium then I can find out the Intrinsic Impedance of the medium, this quantity is real. So just by knowing the amplitude of the electric field I can get the power flow density associated with this uniform plane wave.

In general if this medium is having a conductivity which is neither zero nor very large which is like a conductor then we have to really go through this expression to find out what is the net power flow associated with it. However we can take an extreme case that is if you have a good conductor then we know that the Intrinsic Impedance of this medium is approximately equal to $\sqrt{\frac{\omega\mu}{2\sigma}} + j^2\sqrt{\frac{\omega\mu}{2\sigma}}$ which we have already seen.

So if I take this Intrinsic Impedance and substitute in this expression here then I can get the average power flow density $\overline{P}_{av}$ will be equal to $\frac{1}{2}\eta|E_0|^2\sqrt{\frac{\sigma}{2\omega\mu}}$.

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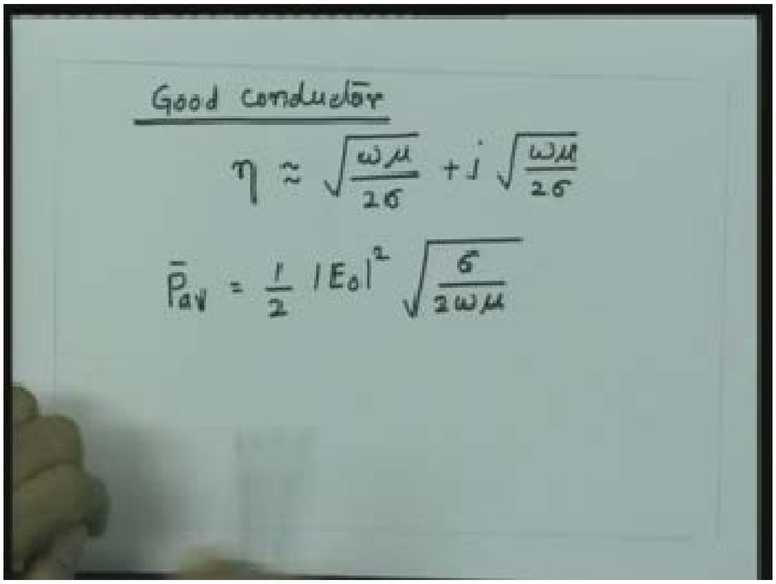

So essentially by using the concept of average Poynting Vector we can find out the power flow in any medium and at any particular location in space. In case of the dielectric the calculation is very straight forward because the intrinsic impedance of the medium is real whereas when we go to the medium which is like a good conductor or in general medium where conductivity is finite then one has to go to the more general expression of finding out the average power flow associated with electric and the magnetic fields.

Thank you.