

**Transmission Lines and E.M. Waves**  
**Prof R.K. Shevgaonkar**  
**Department of Electrical Engineering**  
**Indian Institute of Technology Bombay**

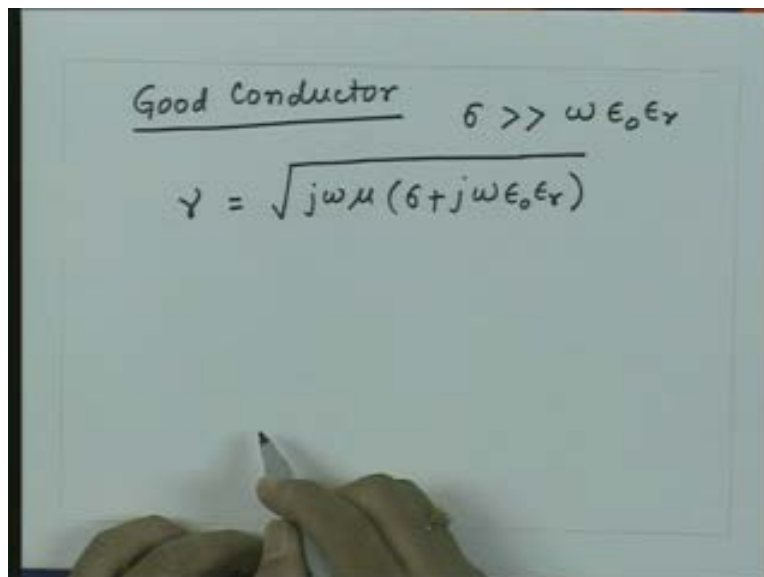
**Lecture-26**

Welcome, in the last lecture we saw the wave propagation in a medium which is not ideal but a good dielectric, now we investigate the wave propagation in a medium which is a good conductor that means in this medium the conduction current is much larger compared to the displacement current. So let us take a medium which we call as the good conductor as we characterize earlier a good conductor is characterized as  $\sigma$  is much larger compared to  $\omega \epsilon_0 \epsilon_r$  that means the conduction current density is much larger compared to the displacement current density then we call that medium as the good conductor.

As we did in the previous case again we find out what is the propagation constant for this medium, we separate the real and imaginary parts and from there we get the attenuation and phase constants. As we have seen the propagation constant  $\gamma$  for this medium will be

$$\sqrt{j\omega\mu(\sigma + j\omega\epsilon_0\epsilon_r)}.$$

(Refer Slide Time: 02:24 min)



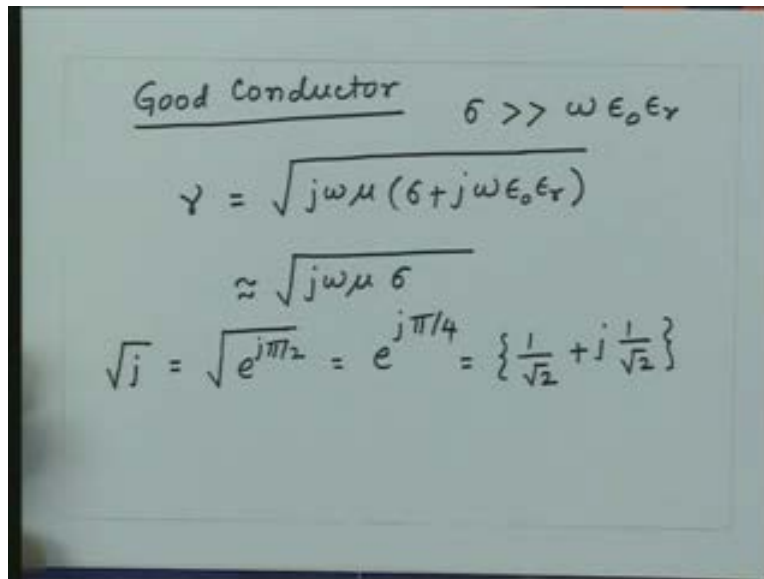
Good Conductor  $\sigma \gg \omega \epsilon_0 \epsilon_r$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon_0\epsilon_r)}$$

Now if I substitute this condition  $\sigma \gg \omega \epsilon_0 \epsilon_r$  then this approximately can be written as  $\sqrt{j\omega\mu\sigma}$  because now this quantity is negligible compared to this quantity. So if I have a conductor essentially the displacement current contribution is neglected now and because of that we have the propagation constant which essentially is given by that.

Now if you separate the real and the imaginary part of this then the  $\sqrt{j\omega\mu\sigma}$  is straight forward we want the  $\sqrt{j}$  so  $\sqrt{j}$  can be written as  $\sqrt{e^{j\pi/2}}$  which again equal  $e^{j\pi/4}$  that is equal to  $\left\{ \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \right\}$ .

(Refer Slide Time: 03:42 min)



Handwritten derivation on a slide:

$$\begin{aligned} \text{Good Conductor} \quad \sigma \gg \omega \epsilon_0 \epsilon_r \\ \gamma &= \sqrt{j\omega\mu(\sigma + j\omega\epsilon_0\epsilon_r)} \\ &\approx \sqrt{j\omega\mu\sigma} \\ \sqrt{j} &= \sqrt{e^{j\pi/2}} = e^{j\pi/4} = \left\{ \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \right\} \end{aligned}$$

So if I substitute this into that I will get this quantity propagation constant  $\gamma$  will be equal to  $\sqrt{\omega\mu\sigma} \left\{ \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \right\}$ .

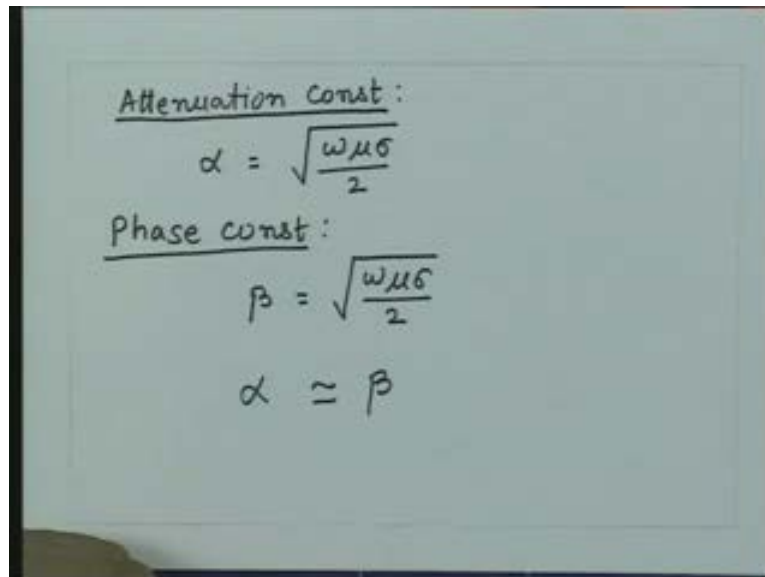
(Refer Slide Time: 04:08 min)

The image shows a handwritten derivation on a piece of paper. At the top, it says 'Good Conductor' followed by the condition  $\sigma \gg \omega \epsilon_0 \epsilon_r$ . Below this, the propagation constant  $\gamma$  is given as  $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon_0\epsilon_r)}$ . This is then approximated to  $\gamma \approx \sqrt{j\omega\mu\sigma}$ . Next, the square root of  $j$  is derived as  $\sqrt{j} = \sqrt{e^{j\pi/2}} = e^{j\pi/4} = \left\{ \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \right\}$ . Finally, the propagation constant is expressed as  $\gamma = \sqrt{\omega\mu\sigma} \left\{ \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \right\}$ .

$$\begin{aligned} \text{Good Conductor} \quad \sigma \gg \omega \epsilon_0 \epsilon_r \\ \gamma &= \sqrt{j\omega\mu(\sigma + j\omega\epsilon_0\epsilon_r)} \\ &\approx \sqrt{j\omega\mu\sigma} \\ \sqrt{j} &= \sqrt{e^{j\pi/2}} = e^{j\pi/4} = \left\{ \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \right\} \\ \gamma &= \sqrt{\omega\mu\sigma} \left\{ \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \right\} \end{aligned}$$

As you have done earlier we call this real part of this quantity as the attenuation constant and we call the imaginary part as the phase constant. For this medium we have the attenuation constant  $\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$  and we have phase constants  $\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$  that is for this medium the attenuation constant  $\alpha$  and the phase constant  $\beta$  are almost equal so we have  $\alpha$  is almost equal to  $\beta$ .

(Refer Slide Time: 05:15 min)



The image shows a handwritten note on a light blue background. It contains three lines of text: the first line is 'Attenuation Const:' followed by the equation  $\alpha = \sqrt{\frac{\omega \mu \sigma}{2}}$ ; the second line is 'Phase const:' followed by the equation  $\beta = \sqrt{\frac{\omega \mu \sigma}{2}}$ ; and the third line is the equation  $\alpha \approx \beta$ .

$$\text{Attenuation Const:}$$
$$\alpha = \sqrt{\frac{\omega \mu \sigma}{2}}$$
$$\text{Phase const:}$$
$$\beta = \sqrt{\frac{\omega \mu \sigma}{2}}$$
$$\alpha \approx \beta$$

Now if you go back to our understanding of Transmission Line we have characterized the Transmission Line as a lossy line whenever we have attenuation constant comparable to the phase constant. In this situation the attenuation constant is equal to the phase constant means this medium if at all we want to visualize similar to the Transmission Line this is an extremely Lossy Transmission Line.

So for a good conductor since the attenuation constant is equal to the phase constant this medium is like a very lossy line case. That means when the wave tries to propagate in this medium its amplitude dies down very rapidly in the direction of propagation. So that means when the wave tries to enter the conducting medium its amplitude reduces very rapidly because it is having the attenuation constant which is equal to the phase constant. So a good medium can be visualized like an extremely Lossy Transmission Line.

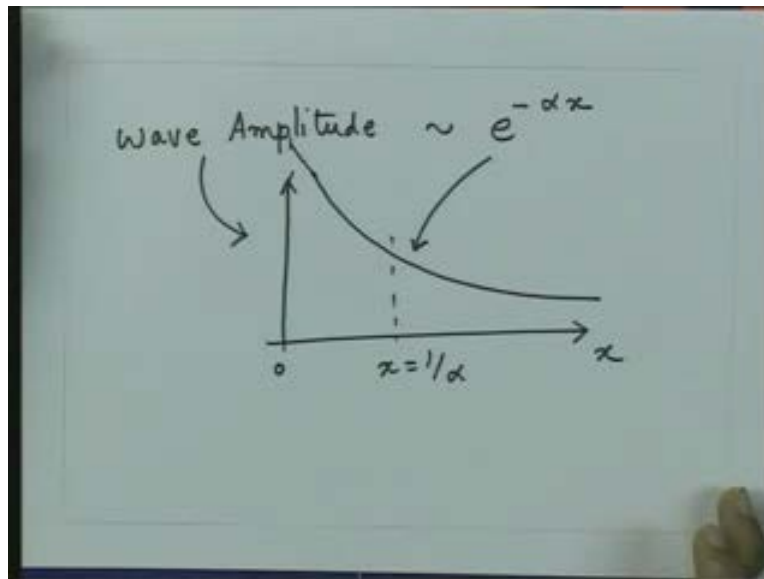
However there is a small difference between a Lossy Transmission Line and the conducting medium. In a Lossy Transmission Line the power was getting lost into the resistance of the line or the conductance of the medium separating the two conductors of the line so there was a loss of power when the wave propagated along the Transmission

Line. However, in this case when the wave is attenuating not necessarily the power is getting loss into the conductor. In fact we will see little later when we have the case  $\sigma = \infty$  so  $\alpha$  becomes equal to infinity, that means over a zero distance the wave will die down as you try to penetrate in this medium but in the small thickness there is no power loss. So we will see that when we talk about the conductors the wave cannot penetrate in this medium because the  $\alpha$  is very large there is a lot of attenuation, that does not mean that the power is getting loss into the heating of the medium something else hammers to the power and that we will investigate it later.

However, at this point we can say that if a good conductor can be characterized like a Lossy Transmission Line and there is the reason we can have this quantity  $\alpha = \beta$  and then the wave amplitude will vary as  $e^{-\alpha x}$  where  $x$  is the direction of the wave propagation.

So if I plot the wave amplitude as a function of  $x$  if I plot this I will get essentially a exponential curve which is this law. So if I travel a distance  $x = \frac{1}{\alpha}$  then the wave amplitude would die down to one over  $e$  of its initial value so if I take this value to be zero and I have a certain wave amplitude at  $x = 0$  here, over a distance of  $\frac{1}{\alpha}$  the wave amplitude will die to one over  $e$  of its initial value.

(Refer Slide Time: 09:12 min)



So now I can say that when the wave tries to propagate in this conducting medium effectively the propagation of the wave is over this distance which is  $x = \frac{1}{\alpha}$  we got beyond this the field would become very small. So what we can do is as the rule of thumb we can say that effectively the wave is going to propagate over this distance in this medium this is the effective length over which the propagation of the wave takes place from the beginning of the medium, what that means is that any electromagnetic wave cannot go really deeper into the conducting medium basically this tries to penetrate little bit from the surface of the conductor or what that means is essentially the energy is in the dielectric medium when this energy in the form of electromagnetic waves tries to go inside the conducting medium it can penetrate only a short distance from the surface of the conducting medium and deeper into the conducting medium the fields are negligibly small.

So, essentially this creates some kind of a surface phenomenon on the conducting surface. If I say this is the effective width over which the wave propagate now I can say that beyond this the medium does not exist because there are no fields beyond this point so no matter what is the thickness or the depth of this conductor only this thickness where

the field die down to one over  $e$  of its initial value they are the one which simply matter in the propagation of electromagnetic wave, beyond that whether the path conducting medium is there it doesn't really affected.

So in practice when ever we are having the conducting boundaries essentially this region which is very close to the surface of the conducting boundary will decide the propagation characteristic of the electromagnetic wave. What is deeper in the medium is it is of no relevance because the fields essentially die down very rapidly as we go inside the conducting medium.

So if I write this  $\alpha$  over this depth what ever this quantity is let us call that quantity as  $\delta$ .

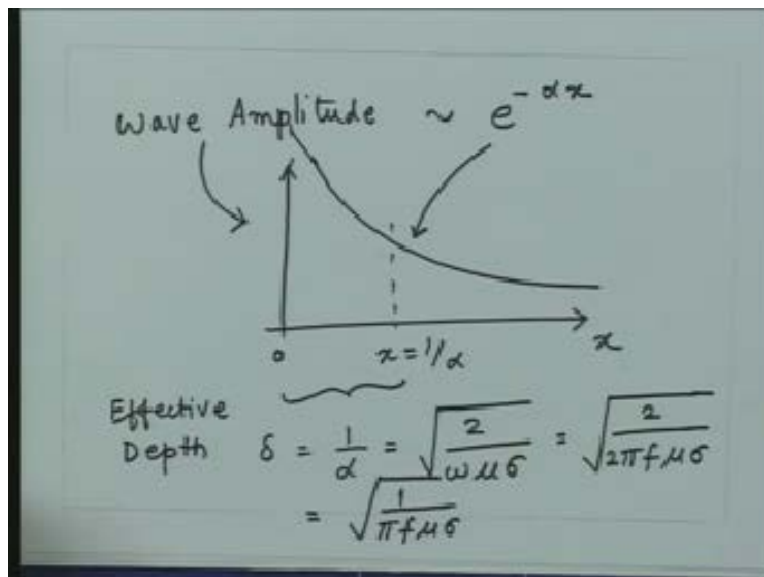
So here the effective depth  $\delta$  is  $\frac{1}{\alpha}$  and if I substitute the value of  $\alpha$  from here then that

will be  $\sqrt{\frac{2}{\omega\mu\sigma}}$ .

If I substitute for  $\omega$  which is two by frequency so that will be equal to  $\sqrt{\frac{2}{2\pi f\mu\sigma}}$  where 2

will cancel so from here essentially I can get  $\sqrt{\frac{1}{\pi f\mu\sigma}}$ .

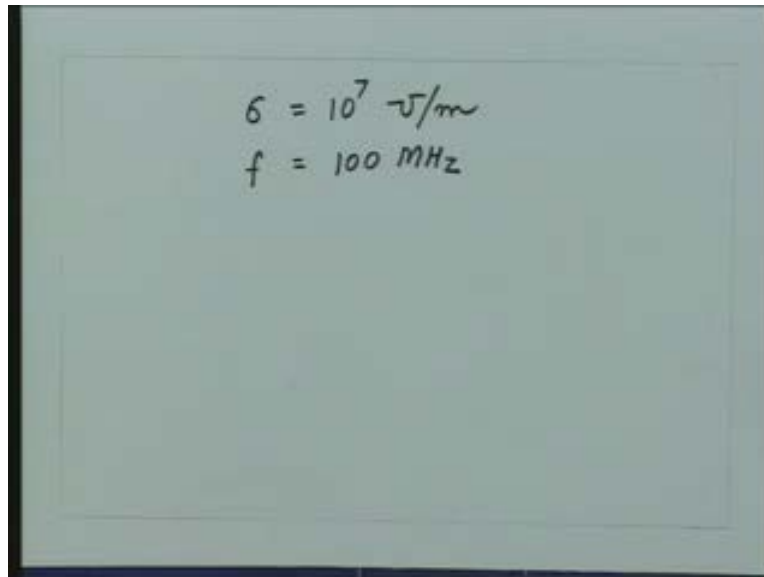
(Refer Slide Time: 12:56 min)



So I have an effective depth of penetration of the electromagnetic wave inside a good conductor that is inversely related to the square root of the frequency it is inversely related to the square root of the conductivity. So higher the conductivity or higher the frequency less will be the depth of penetration, not only that when the conductivity is infinite that means if I take the ideal conductor in that case this depth will be zero that means the wave will not penetrate the medium at all. So if you take a ideal conducting medium there is no penetration of electromagnetic energy in the ideal conductor. Similarly if I go to very arbitrarily high frequencies the depth of penetration becomes extremely small.

Let me take some typical numbers and just to get the feel for what depth we will have if you operate at the radio frequencies. So let us say I take some arbitrary medium for which the conductivity is approximately  $10^7 \text{ } \Omega / m$ , it is a good conductor and let us say I take a frequency  $f = 100 \text{ MHz}$  a typical frequency which is used for the TV transmission or for the Radio propagation.

(Refer Slide Time: 14:31 min)



A photograph of a green chalkboard with two handwritten equations in black chalk. The first equation is  $\sigma = 10^7 \text{ } \Omega / m$  and the second equation is  $f = 100 \text{ MHz}$ .

$$\sigma = 10^7 \text{ } \Omega / m$$
$$f = 100 \text{ MHz}$$



Now from here if I substitute for this  $\delta$  I get this  $\delta$  is equal to square root of one upon  $\pi$  into frequency which is 100 mega hertz which is  $10^8$  Hz so this is into  $10^8$  into  $\mu$  and let us assume the permeability of this medium is the same as the free space which is four power  $10^{-7}$  so let me assume the permeability of the medium  $\mu = \mu_0$  which is equal to  $4\pi \times 10^{-7}$  Henry per meter so I can write here  $4\pi \times 10^{-7}$  multiplied by  $\sigma$  which is  $10^7$ . If I simplify, this will be  $\frac{1}{2\pi \times 10^4}$  meters or I can multiply this by  $10^6$  to make it micro meters so this will be approximately about fifteen micrometers.

(Refer Slide Time: 16:14 min)

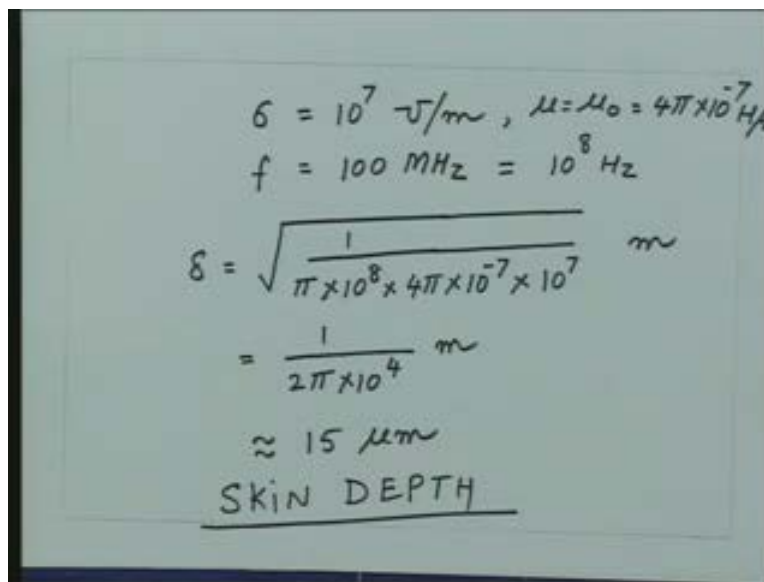
$$\begin{aligned} \sigma &= 10^7 \text{ S/m}, \mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \\ f &= 100 \text{ MHz} = 10^8 \text{ Hz} \\ \delta &= \sqrt{\frac{1}{\pi \times 10^8 \times 4\pi \times 10^{-7} \times 10^7}} \text{ m} \\ &= \frac{1}{2\pi \times 10^4} \text{ m} \\ &\approx 15 \mu\text{m} \end{aligned}$$

If I take a good conductor which is having the conductivity of order of  $10^7$  and let me remind you that a good conductor like copper have a conductivity of  $5.6 \times 10^7$  the conductivity even higher than this. If I take a good conductor material in practice like copper or silver and if I take the frequency let us say hundred mega hertz then the depth of penetration of the electromagnetic wave is only fifteen microns that means the wave propagation is just fifteen microns from the surface of this good material which is having conductivity  $10^7$  and we saw as the frequency increases the skin depth becomes smaller. So, if you go to frequency of Giga hertz the frequency which is used for Satellite

communication or the microwave frequency or the millimeter wave frequencies this number will become even smaller.

So what we find is effectively for a good conductor if you take the frequencies which are typical radio frequencies the depth of penetration typically is of order of few microns or few tens of microns. Basically this phenomenon is taking place only on the skin of the material which is of this order of few microns to few tens of microns. That is the reason why we call this effective depth as the SKIN DEPTH of the material. So we call the delta an effective width as the SKIN DEPTH of the material.

(Refer Slide Time: 18:14 min)



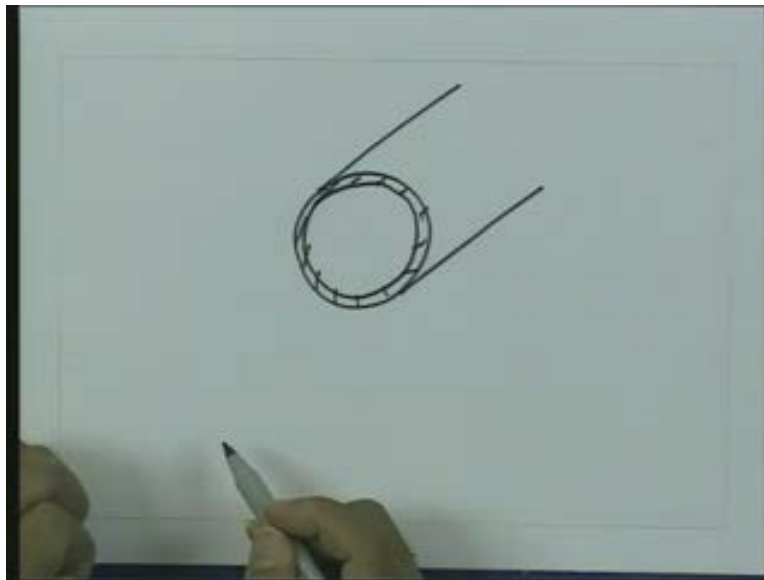
Handwritten calculation of skin depth ( $\delta$ ) on a green chalkboard:

$$\begin{aligned}\sigma &= 10^7 \text{ } \Omega/\text{m}, \mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/A} \\ f &= 100 \text{ MHz} = 10^8 \text{ Hz} \\ \delta &= \sqrt{\frac{1}{\pi \times 10^8 \times 4\pi \times 10^{-7} \times 10^7}} \text{ m} \\ &= \frac{1}{2\pi \times 10^4} \text{ m} \\ &\approx 15 \text{ } \mu\text{m} \\ \text{SKIN DEPTH}\end{aligned}$$

So skin depth is a parameter which tells you over which the electromagnetic energy is going to exist from the surface of the good conductor. Beyond this depth again as I mentioned the properties of the material do not really matter. So if you have a half frequency system a component since the energy is going to lie or the fields are going to lie only within a distance of few tens of microns we have to make that layer very good because any perturbation in the properties of that layer is going to affect the propagation of the electromagnetic wave.

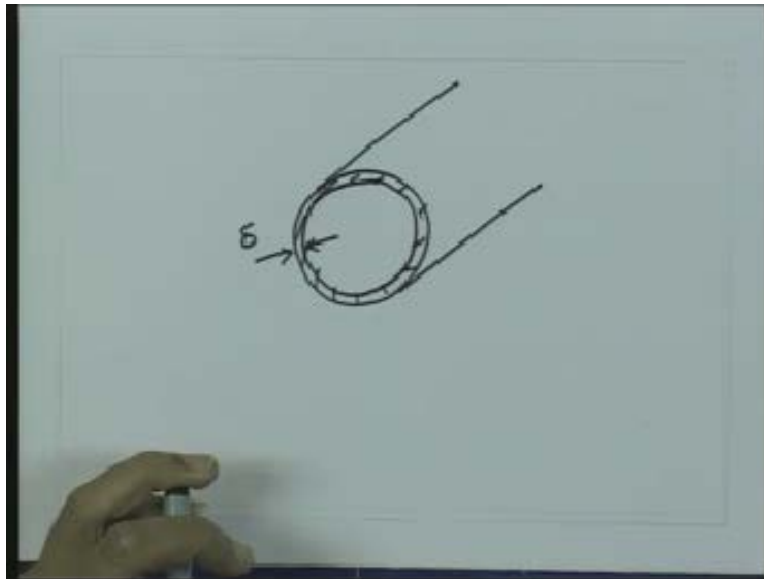
Let us say I have a solid conductor and I want to send from high frequency signal using this conductor the fields are essentially is going to lie only on the surface of that conductor within the skin depth. So if I take a conductor let us say something like that and you have to say the electromagnetic energy is going to propagate along this conductor. The current which are going to be excited because of these fields are not going to lie in the entire cross section of the conductor they essentially be confined to the skin depth of the conductor only this is the region where the fields are going to exist and because of that the currents are going to flow.

(Refer Slide Time: 19:48 min)



So this region though you are having conductor here is not really contributing to the flow of current because the current is just confined to this where this thickness is  $\delta$  which is typically of the order of few tens of microns.

(Refer Slide Time: 20:04 min)



So even if you make this conductor hollow the propagation of the energy along this conductor will not get affected because this layer still remains intact. So two things happen here firstly since the phenomenon is only taking place on the surface this material does not play role in the propagation of electromagnetic energy, secondly if you wanted to have a good propagation the properties of this small thin layer should be very good because any perturbation in the conductivity of this layer would affect the properties of electromagnetic waves significant. So that is the reason when we go to the microwave component where the frequency is very high and the skin depth becomes only a micron or something like that the surface has to be extremely good and perfect because any small perturbation on their surface where most of the energy is confined to will affect the propagation of electromagnetic wave.

Once we understand this then we find something interesting now and that is let us say we have a finite conductivity of this medium through which we were sending the signal around this conductor and we have certain area of cross section or radius for this you have extremely low frequencies the current will be uniformly distributed over this so I can find out the area of the cross section and from the area of cross section I can find out

what is the resistance of this wire per unit length because I know the resistivity of the medium.

However as we increase the frequency and I have the skin depth starts coming into the picture the current is now getting more and more confined to the surface that means the area of cross section over which the current is flowing is reducing now. As a result the resistance of this conductor starts increasing as you go to higher frequency. So because of the skin depth the resistance is now becomes a function of frequency. For a given conductivity as the frequency increases the skin depth reduces, as the skin depth reduces the area of cross section over which the currents are flowing have reduces and because of that the resistance of the conductivity increases.

So now what we have found is the resistance which we treated more like the property of the wire is not so when we go to high frequencies and when the skin depth starts playing a role. As we go to higher and higher frequency the same conductor start showing more and more resistance and the wave propagation essentially is confined to the surface of the conductor.

Another aspect of the high conductivity of the medium is if we look at the Characteristic Impedance of the medium or the Intrinsic Impedance of the medium  $\eta_c$  is we saw earlier

is  $\sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$  and now we are having  $\sigma$  much larger compared to  $\omega\epsilon$  so this is approximately  $\sqrt{\frac{j\omega\mu}{\sigma}}$ .

Again if you separate the real and imaginary parts then the intrinsic impedance of the medium will be  $\sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$  or if you write in terms of the magnitude and phase this quantity will be  $\sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$ .

(Refer Slide Time: 24:38 min)

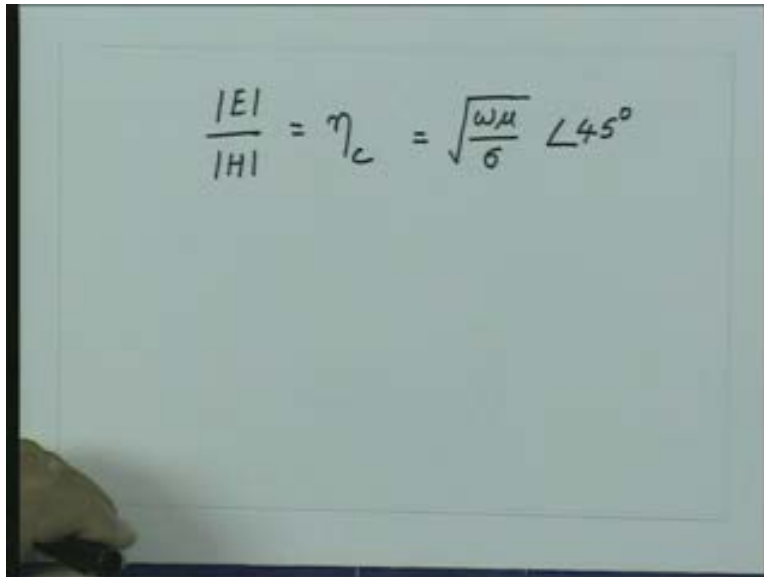
The image shows a handwritten derivation of the intrinsic impedance  $\eta_c$  on a piece of paper. The title 'Intrinsic Impedance' is written at the top. The derivation starts with the formula  $\eta_c = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$ , which is then approximated to  $\approx \sqrt{\frac{j\omega\mu}{\sigma}}$ . This is further simplified to  $= \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$ , and finally expressed in polar form as  $= \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$ .

$$\begin{aligned}\text{Intrinsic Impedance} \\ \eta_c &= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \approx \sqrt{\frac{j\omega\mu}{\sigma}} \\ &= \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}} \\ &= \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ\end{aligned}$$

If you recall the electric and magnetic fields in a medium or related to this quantity called the intrinsic impedance of the medium we have seen for dielectric medium or for a medium where the magnitude of the electric field and the magnitude of the electric field are equal to  $\eta$ . So in this case the ratio of this will be equal to the Characteristic Impedance or the Intrinsic Impedance of the medium and now this quantity is equal to

$$\sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ.$$

(Refer Slide Time: 25:25 min)


$$\frac{|E|}{|H|} = \eta_c = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

So what we find is the electric and magnetic fields are perpendicular to each other the trans electromagnetic wave nature is still there but now there is a time lag between the magnetic field and the electric field. So if you take the ratio of these two the ratio is now dependent on frequency also not only on a medium parameter that is used to be in the case of dielectric medium and also the magnetic field now lags the electric field by  $45^\circ$ .

So when we talk about a good conducting medium important things happen where when the wave tries to penetrate in the conducting medium it penetrates over a very short distance called a skin depth and also the electric and magnetic fields are not in time phase any more but there is a phase difference in time for the electric and the magnetic field and which is for a good conductor is approximately  $45^\circ$ .

So what we find from this discussion is a wave propagation in a dielectric medium and in a conducting medium are completely different. In a dielectric medium the wave can really go deeper into the medium the electric and magnetic fields are in time phase the wave amplitude reduction is very small whereas if I go to the conducting medium the wave

attenuates rapidly it cannot go really deeper into the medium, also the electric and the magnetic fields have a phase difference in time with each other.

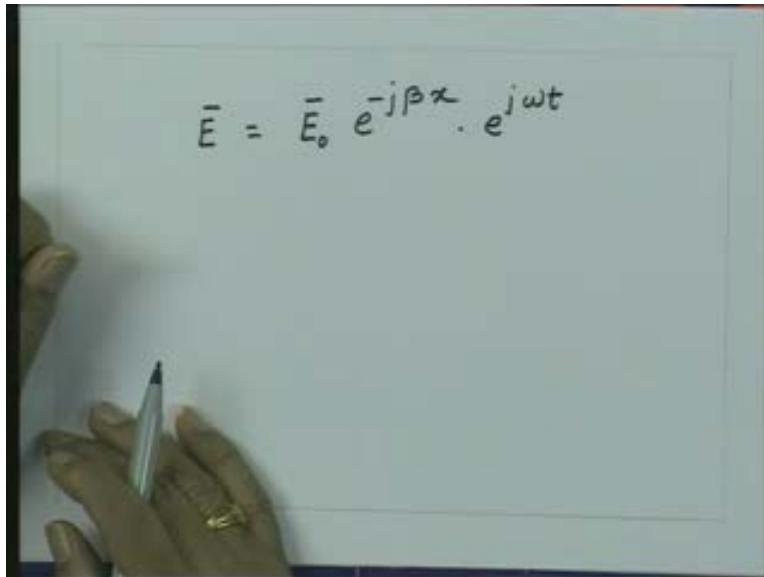
Now if you take a general medium which is a combination of the Dielectric and the Conductor. Then of course all these relationships are going to be complex we cannot make any approximations. So for those media for which the conduction current is comparable to the displacement current but in the two extreme cases for a good dielectric and for a good conductor these approximations can be made easily and one can understand the wave propagation in a more comprehensible manner than in a general media.

With this understanding now we go with another aspect of the wave propagation and that is what the velocity of the wave when it tries to move in this medium.

Now first let us take a medium which is again a good dielectric and without loosing generality let us say the losses in dielectric are very small so I can treat this dielectric almost like a lossless dielectric. Then how the wave is going to propagate in this medium? As we have seen if the medium was lossless then the propagation constant is purely imaginary and then any field  $\bar{E}$  can be written as some  $\bar{E}_0 e^{-j\beta x}$  and if I want to put explicitly the time function  $e^{j\omega t}$ .



(Refer Slide Time: 29:05 min)

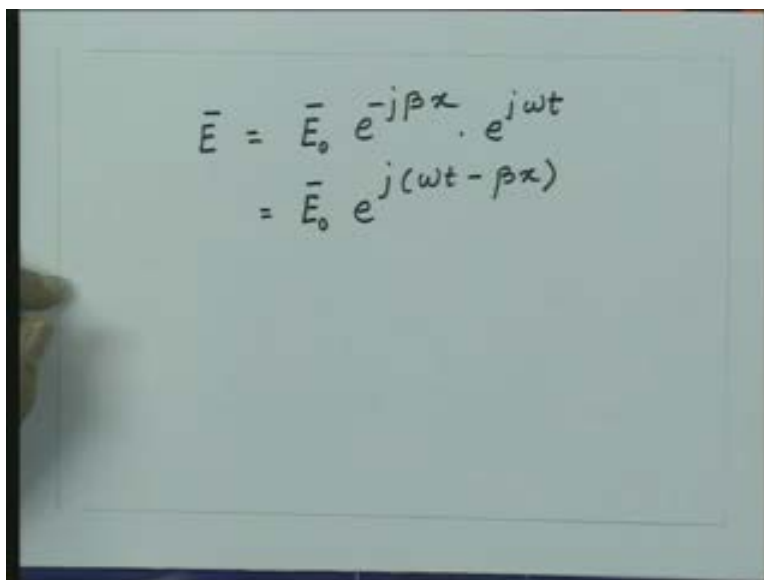


A photograph of a whiteboard with a handwritten equation. The equation is  $\bar{E} = \bar{E}_0 e^{-j\beta x} \cdot e^{j\omega t}$ . A hand holding a pen is visible in the bottom left corner of the frame.

$$\bar{E} = \bar{E}_0 e^{-j\beta x} \cdot e^{j\omega t}$$

I can combine this to get the amplitude vector  $\bar{E}_0 e^{j(\omega t - \beta x)}$ . And as you have seen earlier this phenomenon essentially represents a wave phenomenon the wave is traveling and in this case when we take minus  $-j\beta x$  the wave is traveling in positive  $x$  direction which we already seen.

(Refer Slide Time: 29:35 min)



A photograph of a whiteboard with a handwritten equation. The equation is  $\bar{E} = \bar{E}_0 e^{-j\beta x} \cdot e^{j\omega t}$  followed by  $= \bar{E}_0 e^{j(\omega t - \beta x)}$ . A hand is visible in the bottom left corner of the frame.


$$\begin{aligned}\bar{E} &= \bar{E}_0 e^{-j\beta x} \cdot e^{j\omega t} \\ &= \bar{E}_0 e^{j(\omega t - \beta x)}\end{aligned}$$

Now if I stand on a particular location in the space that means if  $x$  is constant and look at the wave which is moving in the space the phase of this wave will be varying with respect to time as  $\omega t$  that means the phase is linearly increasing as a function of time. If I instantaneously look at the wave in the space again we see the phase is varying as  $-\beta x$  because again it is varying linearly at a function of distance in the space.

So if we become an observer and stand in the space and look at the wave which is moving then at any particular point of the wave if you monitor its phase will vary linearly the function of time.

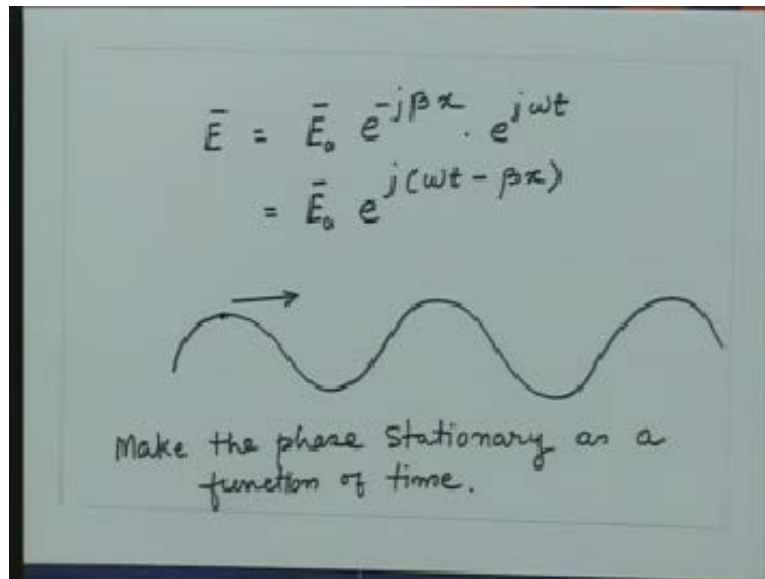
However suppose I hold on to a particular phase point on the wave let us say this is the wave if I take any particular point and I hold on to this point so the observer is standing and watching the wave moving but let us say we are holding on to a particular phase point on the wave so this is the point I want to hold on to. Now since the wave is moving if I have to hold on to a particular phase point I must move along the wave then only I will hold on to this point, otherwise the wave will simply move and the phase will change. So an observer has to move along with this wave with a velocity with which the wave is traveling if he does not want to lose this particular point to which he is holding on to or in other words now if the observer is **not** losing this point if you look at the phase of the wave the phase is not changing anymore for him because he is holding on to his point we are looking at the wave and for him the phase is always same because he is moving with the wave.

(Refer Slide Time: 32:09 min)

$$\begin{aligned}\bar{E} &= \bar{E}_0 e^{-j\beta x} \cdot e^{j\omega t} \\ &= \bar{E}_0 e^{j(\omega t - \beta x)}\end{aligned}$$


So if we consider an observer such that he holds on to a particular point and thus whatever is needed not to lose this point then for that observer the phase appears to be constant as a function of time. So what we do is we take this quantity the total phase of the electromagnetic wave and if I make this quantity constant as a function of time if I make this stationary as a function of time whatever speed the observer has to move to make this stationary is the velocity of this wave. So what we say is 'Make the phase stationary as a function of time to get the velocity of the wave.'

(Refer Slide Time: 33:20 min)



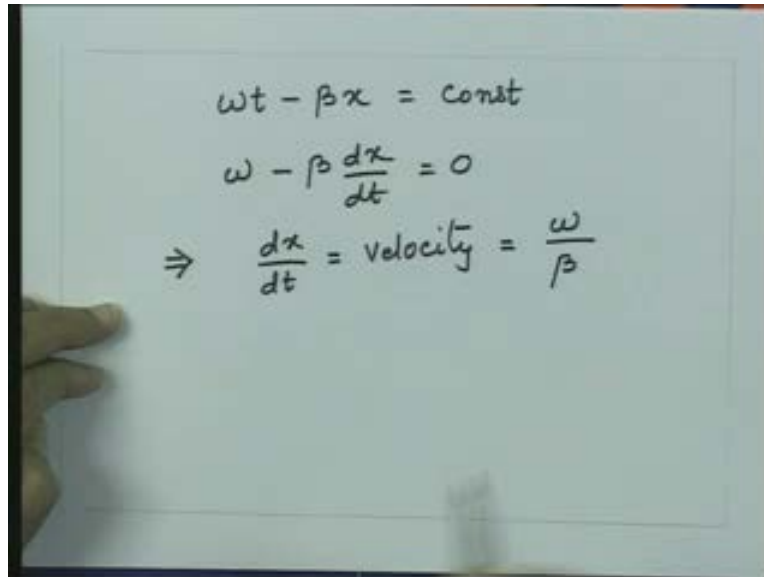
The image shows a handwritten derivation of the complex representation of a wave. At the top, the electric field  $\bar{E}$  is expressed as the product of its amplitude  $\bar{E}_0$ , a spatial phase factor  $e^{-j\beta x}$ , and a temporal phase factor  $e^{j\omega t}$ . This is then simplified to a single exponential term  $\bar{E}_0 e^{j(\omega t - \beta x)}$ . Below the equations is a hand-drawn sinusoidal wave moving to the right, indicated by an arrow above the first peak. At the bottom, a note reads: "Make the phase stationary as a function of time."

$$\begin{aligned}\bar{E} &= \bar{E}_0 e^{-j\beta x} \cdot e^{j\omega t} \\ &= \bar{E}_0 e^{j(\omega t - \beta x)}\end{aligned}$$

Make the phase stationary as a function of time.

So if I say that this quantity has a total phase which is a combination of space and time if I make that constant I say  $\omega t - \beta x$  is constant for that observer which is moving with the velocity of the wave. Then from here if I take a derivative of this with respect to time I will get  $\omega - \beta \frac{dx}{dt} = 0$  will give me this quantity  $\frac{dx}{dt}$  which is nothing but velocity that is equal to  $\frac{\omega}{\beta}$ .

(Refer Slide Time: 34:18 min)



The image shows a whiteboard with handwritten mathematical equations. The first equation is  $\omega t - \beta x = \text{const}$ . The second equation is  $\omega - \beta \frac{dx}{dt} = 0$ . The third equation, preceded by an arrow, is  $\Rightarrow \frac{dx}{dt} = \text{velocity} = \frac{\omega}{\beta}$ .

Now since this velocity we have calculated from the stationarity of the phase we call this velocity as the phase velocity of the wave which means the faster phase point of the wave is moving with this velocity. Later on we will see this concept is essential because there is another velocity with which actually the energy travels. In this case when we are talking about unbound medium you have chosen a coordinate system such that the wave is traveling along x direction, this appears very trivial.

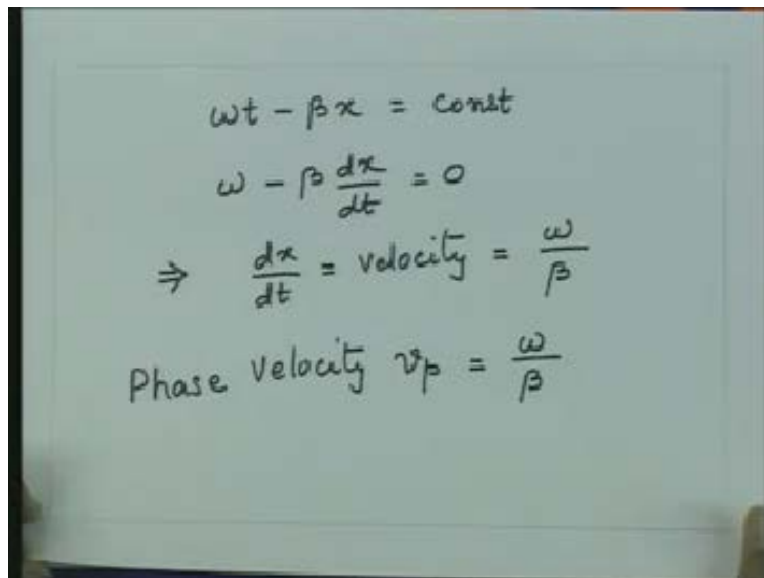
However in general when we talking about a wave moving about an arbitrary direction this concept of saying that the phase is moving with this velocity but the energy might move with some other velocity will become more apparent. In this situation when you said that the phase is moving with this velocity we call this velocity of the wave as the phase velocity. So we have to explicitly saying that this velocity is the phase velocity, normally it is denoted by  $v_p$  and this quantity is nothing but  $\frac{\omega}{\beta}$ .

So the quantity  $v_p$  is not the quantity which people know a priory for any medium first for a given frequency we find out what is the phase constant then  $\frac{\omega}{\beta}$  gives me a velocity

which is velocity and we call that velocity as the phase velocity. So essentially knowledge of propagation constant or phase constant is important because that tells me of what is the velocity of the phase of the electromagnetic wave when it travels into the medium. In fact, for a traveling wave this is very straight forward so I get a relation which is like this.

However the concept of phase velocity can be extended to any arbitrary wave propagation. For example suppose I have two waves which are traveling in opposite direction form the standing wave I can still define the total phase which is a combination of space and time and by making that quantity stationary as the function of time I may get the expression for the phase velocity of the wave. So the concept is very general though for traveling wave it is very simple and then we get a phase velocity of a traveling wave which is equal to  $\frac{\omega}{\beta}$ . So we will see that this relation that we will use very often when we go to the propagation of electromagnetic wave.

(Refer Slide Time: 37:06 min)



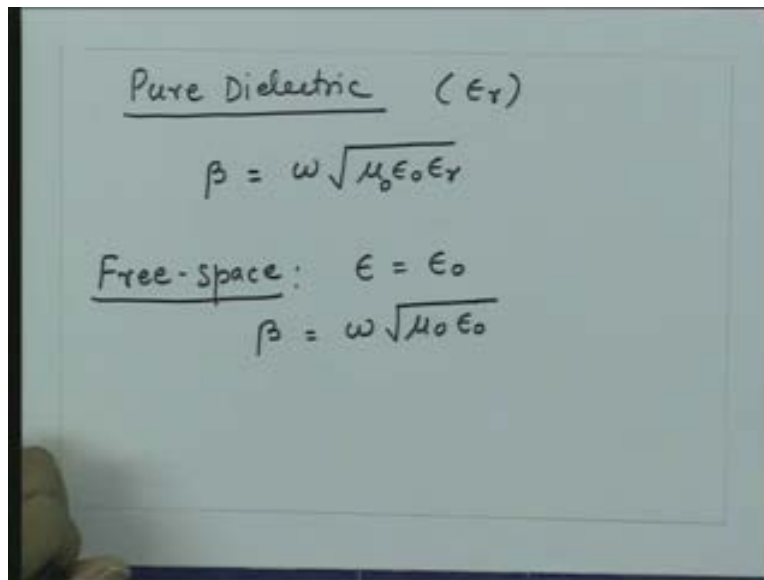
The image shows a handwritten derivation of the phase velocity formula on a piece of paper. The text is written in black ink and follows these steps:

$$\omega t - \beta x = \text{const}$$
$$\omega - \beta \frac{dx}{dt} = 0$$
$$\Rightarrow \frac{dx}{dt} = \text{velocity} = \frac{\omega}{\beta}$$
$$\text{Phase velocity } v_p = \frac{\omega}{\beta}$$

Now as you have seen earlier if I take a Pure Dielectric with dielectric constant  $\epsilon_r$  then for this medium the phase constant  $\beta$  is  $\omega\sqrt{\mu\epsilon_0\epsilon_r}$ . Since the medium is dielectric medium permeability of the medium permeability of the free space so the phase constant is  $\omega\sqrt{\mu_0\epsilon_0\epsilon_r}$ .

Now let us consider a medium which is free space that means for which epsilon  $\epsilon_r$  is 1. So this is for this dielectric for free space where  $\epsilon = \epsilon_0$  and  $\beta = \omega\sqrt{\mu_0\epsilon_0}$ .

(Refer Slide Time: 38:28 min)



Handwritten notes on a whiteboard:

Pure Dielectric ( $\epsilon_r$ )

$$\beta = \omega\sqrt{\mu_0\epsilon_0\epsilon_r}$$

Free-space:  $\epsilon = \epsilon_0$

$$\beta = \omega\sqrt{\mu_0\epsilon_0}$$

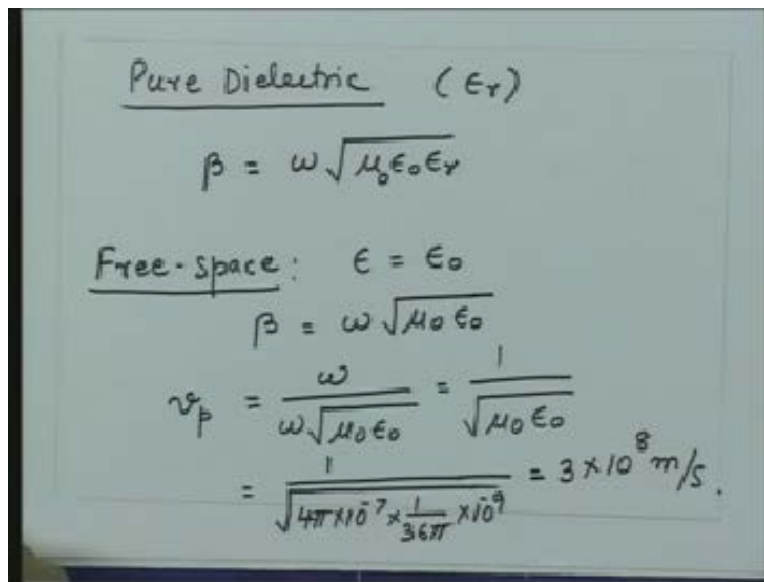
So the velocity for the free space  $v_p = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}}$  which is again equal to  $\frac{1}{\sqrt{\mu_0\epsilon_0}}$ .

If I substitute for  $\mu_0\epsilon_0$  in this then this will be  $\frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{36\pi} \times 10^{-9}}}$  and that will be

equal to  $3 \times 10^8 \text{ m/s}$ .

So this is the velocity of the wave when the wave is traveling in the free space and that is  $3 \times 10^8 \text{ m/s}$ . It will immediately strike to us this is the velocity of the light in free space. So since light is a transverse electromagnetic wave it is traveling with a velocity which is given by this quantity which is the phase velocity and that is the reason why velocity of light is  $3 \times 10^8 \text{ m/s}$ .

(Refer Slide Time: 40:00 min)



Handwritten derivation on a slide:

Pure Dielectric ( $\epsilon_r$ )

$$\beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r}$$

Free-space:  $\epsilon = \epsilon_0$

$$\beta = \omega \sqrt{\mu_0 \epsilon_0}$$

$$v_p = \frac{\omega}{\omega \sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$= \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{36\pi} \times 10^9}} = 3 \times 10^8 \text{ m/s}.$$

So not only light but any wave which is transverse electromagnetic wave and it is traveling in the free space will have a velocity which is equal to  $3 \times 10^8 \text{ m/s}$ .

So what we can do is we can substitute from this one into this. So  $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$  is the velocity of light in vacuum and normally is denoted by the parameter  $c$ .



(Refer Slide Time: 40:30 min)

Handwritten notes on a slide showing the derivation of the phase velocity of a wave in a dielectric medium.

Pure Dielectric ( $\epsilon_r$ )

$$\beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r}$$

Free-space:  $\epsilon = \epsilon_0$

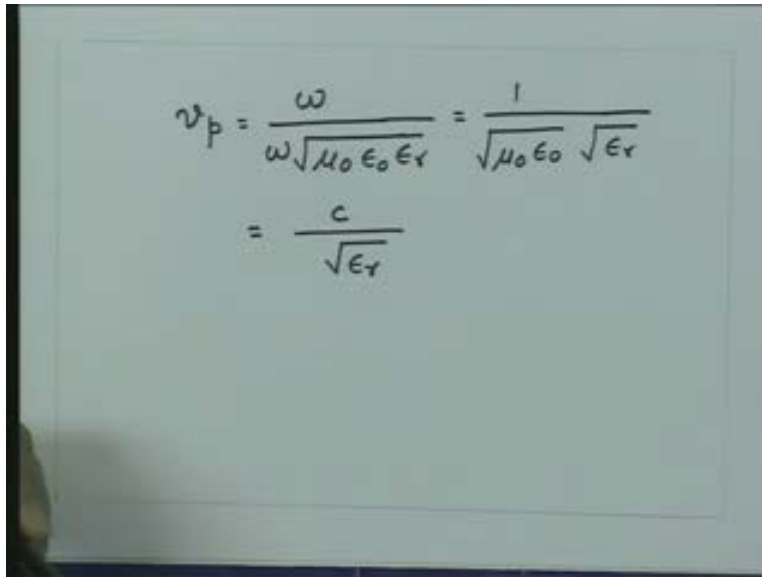
$$\beta = \omega \sqrt{\mu_0 \epsilon_0}$$
$$v_p = \frac{\omega}{\omega \sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$
$$= \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{36\pi} \times 10^9}} = 3 \times 10^8 \text{ m/s} = c$$

So essentially the velocity of wave in a dielectric medium  $v_p$  will be  $\frac{\omega}{\omega \sqrt{\mu_0 \epsilon_0 \epsilon_r}}$  which is

some  $\frac{1}{\sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r}}$ . And this quantity  $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$  is nothing but velocity of light in vacuum

which is  $c$  so that is equal to  $\frac{c}{\sqrt{\epsilon_r}}$ .

(Refer Slide Time: 41:17 min)



A photograph of a whiteboard with handwritten mathematical equations. The equations show the derivation of phase velocity  $v_p$  from angular frequency  $\omega$  and wave number  $k$ . The first line is  $v_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r}}$ . The second line shows this is equal to  $\frac{c}{\sqrt{\epsilon_r}}$ .

$$v_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r}}$$
$$= \frac{c}{\sqrt{\epsilon_r}}$$

So here first thing to note is when the electromagnetic wave travels in a medium it always slows down because if this quantity is greater than 1 then the phase velocity will be always less than the velocity of electromagnetic wave in the free space which is the velocity of light. So velocity of any electromagnetic wave in a dielectric is always less than the velocity in the free space.

We also know that the refractive index of the medium is defined as the ratio of the velocity of light in vacuum divided by the velocity of light in that medium. If I go by that definition we know that we have the Refractive Index of the medium let us say we denote that by small  $n$  which will be equal to velocity of light in vacuum divided by velocity of light in medium. So this quantity is nothing but  $\frac{c}{v_p}$  so that is  $\frac{c}{v_p}$  which is nothing but  $\sqrt{\epsilon_r}$ .

So we have a very important relation that is the Refractive Index of the medium a lossless medium is equal to the square root of the dielectric constant of the medium.

(Refer Slide Time: 43:17 min)

$$\begin{aligned}
 v_p &= \frac{\omega}{\omega \sqrt{\mu_0 \epsilon_0 \epsilon_r}} = \frac{1}{\sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r}} \\
 &= \frac{c}{\sqrt{\epsilon_r}} \\
 \text{Refractive Index } n &= \frac{\text{Vel. of Light in Vacuum}}{\text{Vel. of Light in medium}} \\
 &= \frac{c}{v_p} = \sqrt{\epsilon_r} \\
 \boxed{n} &= \sqrt{\epsilon_r}
 \end{aligned}$$

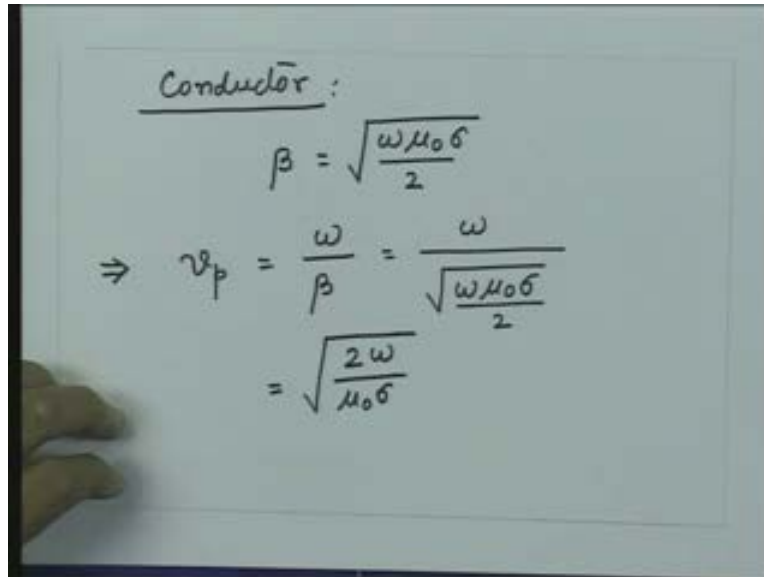
So when ever we have a medium which is pure dielectric or a good dielectric though Refractive Index and the dielectric constant are related to this relation and the electromagnetic wave always slows down compared to the vacuum when it enters any medium which having a dielectric constant greater than 1.

Now also the same thing we can do for the conducting medium we can take a conductor and no matter what is more the penetration the wave has inside the conductor we can still ask what is the velocity of this so again for this conductor we have  $\beta$  which is  $\sqrt{\frac{\omega \mu_0 \sigma}{2}}$  so

we can have the phase velocity  $v_p = \frac{\omega}{\beta}$  which again is equal to  $\frac{\omega}{\sqrt{\frac{\omega \mu_0 \sigma}{2}}}$  so that is equal

to  $\sqrt{\frac{2\omega}{\mu_0 \sigma}}$ .

(Refer Slide Time: 44:44 min)



The image shows a handwritten derivation on a piece of paper. At the top, the word "Conductor" is underlined. Below it, the phase constant  $\beta$  is given as  $\beta = \sqrt{\frac{\omega \mu_0 \sigma}{2}}$ . Then, the phase velocity  $v_p$  is derived as  $v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega \mu_0 \sigma}{2}}} = \sqrt{\frac{2\omega}{\mu_0 \sigma}}$ .

So the velocity which we had for a dielectric medium was  $\frac{c}{\sqrt{\epsilon_r}}$  the velocity was not a function of frequency so if the dielectric constant of the medium is not a function of the frequency the velocity also is not a function of frequency that means all frequencies travel with the same speed in the medium.

However if I take a medium which is a conductor even if the medium properties are not varying as a function of frequency the phase constant to the function of frequency and the velocity also is the function of frequency. Then we call this medium as a dispersive medium because the velocity is varying as a function of frequency. So when  $v_p$  is the function of frequency we call this medium as the dispersive medium.

So what we find is a medium which is conductor the velocity becomes a function of frequency that means all frequency does not travel with the same speed and because of that we have dispersion in the medium. So now we have something important conclusions to draw. First of all the electromagnetic wave when traversed in the medium we can define a parameter what is called the phase velocity that is the parameter which

tells you with what speed the phase or constant phase point on the wave is traveling in the medium. Then we find for dielectric medium the phase velocity depends on the dielectric constant of the medium and for dielectric constant greater than one the phase velocity is always less than the phase velocity in vacuum that means the wave always slows down when it travels in a medium with higher dielectric constant.

However a dielectric medium is not dispersive medium because the wave slows down but all frequencies travel with the same speed. Then we also find that the parameter called a Refractive Index of the medium which is the ratio of velocity of a wave in vacuum to the ratio of the velocity of the wave in the medium is equal to the square root of the dielectric constant of the medium. And then when we go to the conductor we find that the velocity becomes a function of frequency that means the medium becomes a dispersive medium. So a pure dielectric medium is a non dispersive medium but a good conductor is a dispersive medium.

This essentially summarizes the propagation of transverse electromagnetic wave in an unbound medium. Before we proceed further now we investigate first the power flow associated with the electromagnetic waves and then we will go to more complex analysis of this which is propagation of electromagnetic wave in bound medium or in a medium which are having at least the interfaces dielectric or the conductive interfaces.

Thank you.