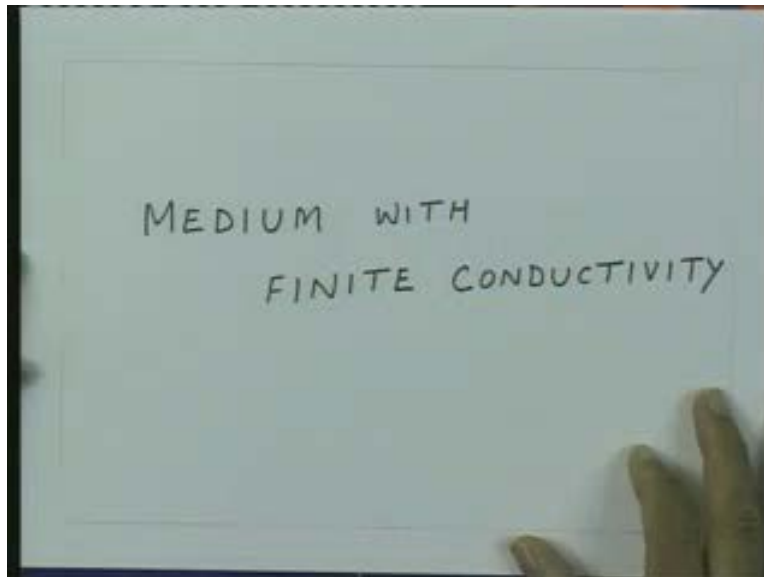


Transmission Lines & E. M. Waves
Prof. R. K. Shevgaonkar
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture – 25

We are discussing the wave propagation in an unbound medium. In last few lectures we investigated the wave propagation in an unbound medium which does not even have conductivity. Now we take a medium which has a finite conductivity and see how the wave propagation gets affected because of the finite conductivity.

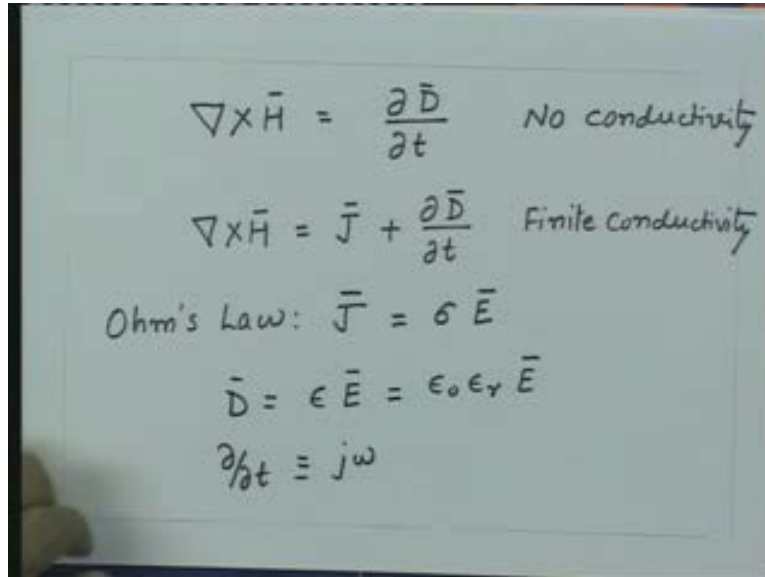
(Refer Slide Time: 00:01:09)



So, we investigate in next few lectures the propagation of electromagnetic wave in a medium with finite conductivity. Whenever we start a new problem in electromagnetics, essentially we have to go back to the Maxwell's equations and start the problem from there. Now if you recall the conductivity of the medium appears in the Maxwell equation which corresponds to the amperes law, so we had that $\nabla \times \mathbf{h}$ is equal to the conduction current density plus the displacement current density and when the conductivity of the medium was 0, the conduction current density was 0, so we had $\nabla \times \mathbf{h}$ which was equal to the displacement current density. So, we have now two cases

first what we investigated when the conduction current density was 0 and now we want to take into account the conduction current density which is related to the conductivity of the medium.

(Refer Slide Time: 00:04:54)



$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \quad \text{No conductivity}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{Finite conductivity}$$

$$\text{Ohm's Law: } \vec{J} = \sigma \vec{E}$$

$$\vec{D} = \epsilon \vec{E} = \epsilon_0 \epsilon_r \vec{E}$$

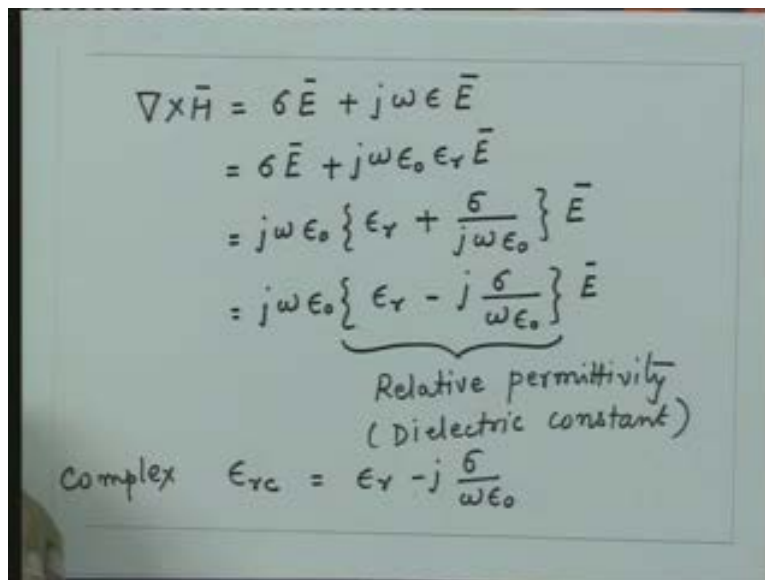
$$\frac{\partial}{\partial t} \equiv j\omega$$

So, we had the equation $\nabla \times \vec{H}$ that is equal to $\frac{d\vec{D}}{dt}$ when no conductivity. However when we are having a finite conductivity we have conduction current, so we have $\nabla \times \vec{H}$ that is equal to in general \vec{J} plus $\frac{d\vec{D}}{dt}$, \vec{D} is a vector when we have finite conductivity. We know from the ohms law \vec{J} is equal to the conductivity of the medium into the electric field and again we are considering the simplest case that is the medium is isotropic, is homogeneous, so the conductivity σ is uniform in the space also it is not depending on the direction.

So, what that means is that if I have a medium which is having a finite conductivity and if we impress the electric field on the medium then there are two types of currents which are going to flow in the medium, one will be corresponding to this σ which we call as a conduction current density and the other one will be this quantity $\frac{d\vec{D}}{dt}$ which will be what is called a displacement current density. So, in general in any medium there will be two types of currents which will be flowing that is the conduction current and the

displacement current. Substituting for D as epsilon times E, so if I take D is equal to epsilon times E which if I write explicitly in terms of the dielectric constant epsilon r, so this we can write as epsilon 0 epsilon r times E. And as we have seen earlier the time derivative is equivalent to multiplying the quantity by j omega. So, for all time harmonic fields d by dt is equivalent to j omega. So, from here essentially dD by dt will be j omega epsilon times E or epsilon_0 epsilon_r times E.

(Refer Slide Time: 00:09:58)



$$\begin{aligned}
 \nabla \times \vec{H} &= \sigma \vec{E} + j\omega \epsilon \vec{E} \\
 &= \sigma \vec{E} + j\omega \epsilon_0 \epsilon_r \vec{E} \\
 &= j\omega \epsilon_0 \left\{ \epsilon_r + \frac{\sigma}{j\omega \epsilon_0} \right\} \vec{E} \\
 &= j\omega \epsilon_0 \underbrace{\left\{ \epsilon_r - j \frac{\sigma}{\omega \epsilon_0} \right\}}_{\text{Relative permittivity (Dielectric constant)}} \vec{E}
 \end{aligned}$$

Complex $\epsilon_{rc} = \epsilon_r - j \frac{\sigma}{\omega \epsilon_0}$

I can now substitute these things into this equation with finite conductivity and I get the curl equation which is del cross h that is equal to sigma times E plus j omega epsilon times E. I can also substitute explicitly for epsilon as we have done earlier, so this is sigma times E plus j omega epsilon_0 epsilon_r times E. We can take this quantity j epsilon_0 common from this equation and then we can write this equation j omega epsilon_0 into epsilon_r. I am taking this term first plus sigma upon j omega epsilon_0 times E.

I can take this j in the numerator, so this will be j omega epsilon_0 epsilon_r minus j sigma upon omega epsilon_0 times E. Now what we can do is we can compare this equation with the Maxwell equation without the conductivity that is del cross h is equal to dD by dt and

d by dt is equal to $j\omega$. So, essentially this quantity is $j\omega$ times the permittivity of the medium times the electric field, we can substitute for D into this.

So, if I have a medium like a dielectric medium, I have $\nabla \times \mathbf{h}$ which is equal to $j\omega$ times permittivity times \mathbf{E} . If I have finite conductivity then I get $\nabla \times \mathbf{h}$ which is like this. Now if I compare this equation with this equation and treat the medium like a dielectric then this quantity whatever is the coefficient of this $j\omega \epsilon_0$ times \mathbf{E} as this equation we call that quantity is the relative permittivity or dielectric constant of the medium.

So, we can say that if there was a finite conductivity in the medium then this quantity I can call as the dielectric constant of the medium. So, we can say this is the relative permittivity or dielectric constant of the medium. What is the, what has happened now is that because of the finite conductivity the dielectric constant has become complex quantity. So, we can treat the medium like a dielectric medium however if we replace the dielectric constant of that medium by the proper complex dielectric constant then the conductivity of the medium can be taken into account and the analysis can be done on the line similarly to what we were doing for the dielectric medium.

Since, we have already investigated the wave propagation in a dielectric medium, we can essentially substitute for dielectric constant for that medium as the complex dielectric constant which we get from here and essentially we get the wave propagation characteristics in a medium with finite conductivity. So, essentially the conductivity of the medium can be accounted for by effectively introducing the concept of what is called the complex dielectric constant.

So, we have this quantity which is complex dielectric constant and we call that as ϵ_{rc} just to denote that this quantity is complex and that is equal to nothing but ϵ_r minus j sigma upon $\omega \epsilon_0$. So, two things should be noted here that when we have a finite conductivity into the medium then the dielectric constant has

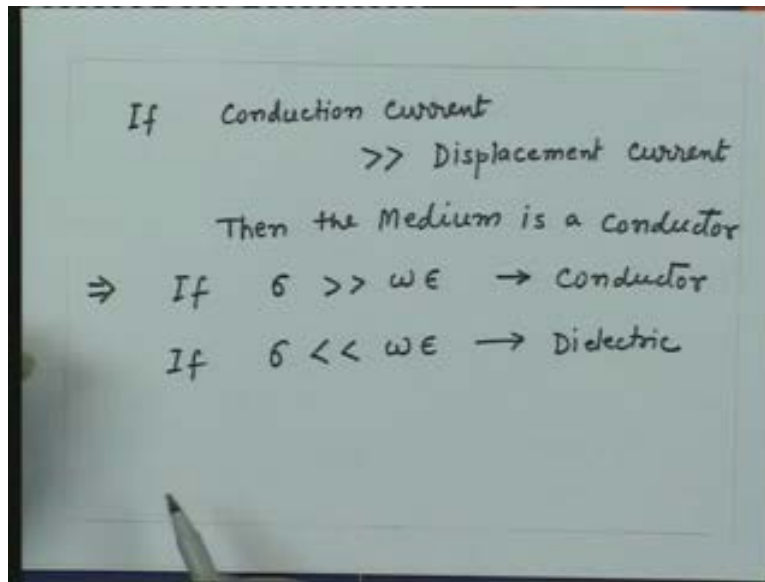
become a complex and also the dielectric constant now has become a function of frequency.

Earlier when we saw the medium properties, the medium properties were not depending upon frequency but if we introduce the concept of complex dielectric constant then the medium property which is the relative permittivity or dielectric constant that quantity now has become a complex quantity. So, by introducing the concept of complex dielectric constant which is the function of frequency, we can analyze the wave propagation problem in a medium with finite conductivity by just replacing ϵ_r by ϵ_{rc} in the medium and that's what precisely we will do when we go to the wave propagation in the medium with finite conductivity.

However, at this point one can ask when I have a medium like this which is having finite conductivity and of course it has dielectric constant, so you will have displacement current in a conduction current. What should we call this medium, should we call this medium a conductor or should we call this medium a dielectric? The answer again lies in these two terms which you are having in this equation. We can ask a question if certain electric field is imposed on the medium which current component dominates in the medium? If the conduction current dominates over the displacement current then we say this medium is like a conductor.

On the other hand if the displacement current dominates over the conduction current then we say that this medium is more like a dielectric. If the two terms are comparable then the medium can neither be called a dielectric nor it can be called a conductor. So, now what we are having is we are having a condition that if conduction current is much much greater than displacement current then the medium is a conductor, is a conductor and if the opposite is true that means if the conduction current density or conduction current is much much smaller than the displacement current then the medium we call as a dielectric medium.

(Refer Slide Time: 00:14:00)



So, substituting for the conduction current and the displacement current this quantity is a conduction current density, this quantity is the displacement current density, so magnitude wise if sigma times E is much much greater than omega epsilon then we will say the medium is a conductor. So, that means this implies that if sigma is much much greater than omega epsilon then we call that as conductor whereas if sigma is much much less than omega epsilon then we call that as dielectric. And since this quantity is frequency dependent for given permittivity and given conductivity of a medium, the medium might behave more like a conductor or dielectric depending upon what frequency range we are operating at. If you take a frequency which is very low then essentially we will have this sigma will be much much larger compared to omega epsilon.

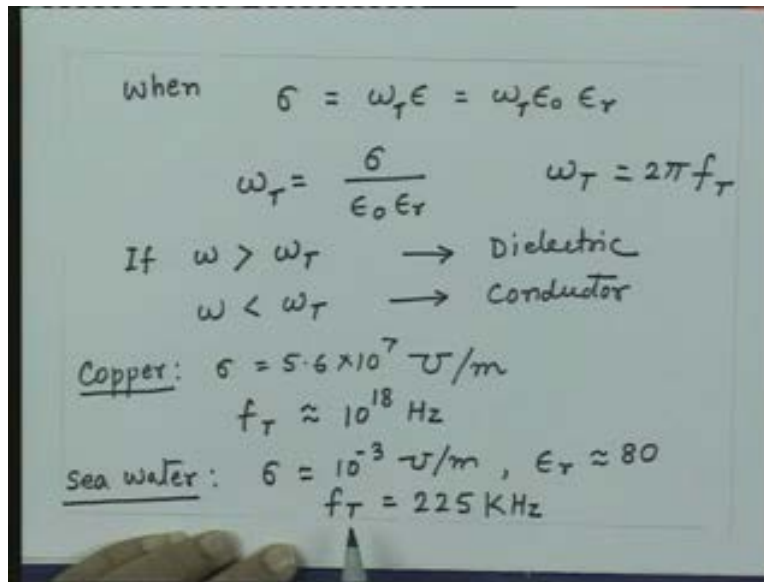
So, let us say if we take some value as a dielectric constant or the permittivity of the medium and if you go to very small frequency you will always find a frequency for with this quantity will become much much smaller than the conductivity expect when the conductivity is 0. So, any medium which is having a finite conductivity will always behave like a conductor if you go towards the lower end of the frequency spectrum.

On the other hand if I go towards the higher end of the frequency spectrum where ω is very large that for any finite value of the conductivity, we will always find a frequency range where this is much larger compared to this and the medium will behave more like a dielectric medium. So, there is nothing very special about a particular medium like a conductor or a dielectric, it depends upon what frequency you are operating at and if you change the frequency the behavior of the medium may change from the conductor to dielectric and vice versa. What that also means is that there is no meaning to the absolute number for the conductivity. We all know that the copper is a good conductor, this conductivity is 5.6×10^6 mhos per meter and we always think for a medium to be conductor the conductivity should be like that but now from here if you look at that will not be the case I may take a medium for which the conductivity maybe as low as 10^{-3} mhos per meter the medium can still behave like a conductor if I go to arbitrarily small frequency.

So, the absolute number which we are used to seeing and saying that looking at the number we say that the medium is conductor or dielectric is not quite justified because now the behavior of the medium depends upon the frequency of operation. So, we can say that for the dividing a line when σ is equal to $\omega \epsilon_0 \epsilon_r$ that is equal to $\omega \epsilon_0 \epsilon_r$ and again write down for ω is $2\pi f$ or I can leave it in ω , so that frequency, angular frequency ω is equal to σ divided by $\epsilon_0 \epsilon_r$. So, if the frequency is more than this quantity than all this as a some transition frequency ω_T , so at this frequency the conduction current density and the displacement current densities become equal.

So, if ω is greater than ω_T then the medium will behave like dielectric if ω is less than ω_T then the medium will behave like a conductor. Let me take some numbers and some specific examples to see what are the frequencies ranges over which certain mediums can behave like a dielectric or like conductor.

(Refer Slide Time: 00:19:56)



When $\sigma = \omega_T \epsilon = \omega_T \epsilon_0 \epsilon_r$

$$\omega_T = \frac{\sigma}{\epsilon_0 \epsilon_r} \quad \omega_T = 2\pi f_T$$

If $\omega > \omega_T \rightarrow$ Dielectric
 $\omega < \omega_T \rightarrow$ Conductor

Copper: $\sigma = 5.6 \times 10^7 \text{ S/m}$
 $f_T \approx 10^{18} \text{ Hz}$

Sea water: $\sigma = 10^{-3} \text{ S/m}$, $\epsilon_r \approx 80$
 $f_T = 225 \text{ KHz}$

So, let us say if I take a case of copper for which the conductivity is 5.6 into 10 to the power 7 mhos per meter and let us taking the dielectric constant of copper approximately 1, I can substitute this sigma in this, I will get a frequency corresponding to this omega so ω_T is 2 pi into frequency of this transition. So, for this I will get this frequency f_T which will be approximately 10 to the power 18 hertz whereas let us take another example like sea water for which the conductivity sigma is approximately 10 to the power minus 3 mhos per meter and dielectric constant for this is about 80 at low frequency we will get the quantity f_T for that that is equal to about 225 kilohertz.

So, what we see from here that if we take a medium like copper for which the conductivity is 5.6 10 to the power 7 mhos per meter then this frequency, transition frequency is 10 to the power 18 hertz that means the copper will behave like a conductor up to this frequency and beyond this frequency the copper will start behaving like a dielectric. If we go to the electromagnetic spectrum you will see this frequency will lie somewhere in x rays.

So, essentially up to that point that means the entire electromagnetic spectrum radio spectrum plus the optical spectrum plus the infrared spectrum all that frequency range if

you take the copper will behave like a conductor. Only if you go to the frequency which are x rays and gamma rays that's where the copper will start deviating towards the dielectric and the behavior of copper like conductor will be lost and that is the reason since at most of the frequencies of radio transmission we deal with the frequencies which are less than about 10^{12} hertz the copper is treated like a good conductor for that frequency range the copper behaves like a conductor.

On the other hand if I take a medium like sea water, we have this transition frequency which is 225 kilohertz. So, if you take the frequencies of few mega hertz or gigahertz, the sea water is like a dielectric. However if I go to a frequency which is less than 225 kilohertz, if I go to a frequency of 10 kilohertz then the sea water is more like a conductor it is not dielectric and you will see later on when we talk about the wave propagation for communication precisely that is the phenomena which is used for guiding the electromagnetic energy around the surface of the sea at extremely low frequency. So, the medium even if the conductivity is very small will behave like a conductor if the frequency is reduced to a significantly low value.

So, we see from here an important conclusion that any medium can act like a conductor or dielectric depending upon the frequency of operation. So, the absolute parameters don't really decide whether the medium will behave like a conductor or dielectric for a given medium parameters, the frequency of operation will decide whether medium would be more like a conductor or would be more like a dielectric. With this understanding then now we can go to propagation of electromagnetic waves in the medium with finite conductivity.

First, we will write down the general wave propagation and then we will take the two extreme cases, one with very low conductivity or when the medium is like a good dielectric other extreme when the conductivity is high enough so the medium can be treated like a good conductor. And in this two extreme cases the certain approximations can be made to the expression which will get for the wave propagation and one can really understand this phenomena better for these two extreme cases. So, a good dielectric for

which a conductivity is very low and for a good conductor for which a conductivity is very high.

So, as we said to investigate the wave propagation now we essentially have to solve the equation the Maxwell's equations and since we have already solved this problem for the dielectric medium, we have solved the wave equation for dielectric medium, we can take exactly the same analysis for dielectric medium and I replace the dielectric constant of the medium by the complex dielectric constant for the medium with finite conductivity.

(Refer Slide Time: 00:27:33)

Wave Equation:

$$\nabla^2 \begin{Bmatrix} \vec{E} \\ \vec{H} \end{Bmatrix} = -\underbrace{\omega^2 \mu \epsilon_0 \epsilon_{rc}}_{\gamma^2} \begin{Bmatrix} \vec{E} \\ \vec{H} \end{Bmatrix}$$

Propagation constant:

$$\gamma = \sqrt{-\omega^2 \mu \epsilon_0 \epsilon_{rc}}$$

$$= j\omega \sqrt{\mu \epsilon_0} \left\{ \epsilon_r - j \frac{\sigma}{\omega \epsilon_0} \right\}^{1/2}$$

Say as we know for the wave equation for a medium was written as del square, this quantity could be E or H that is equal to minus omega square mu epsilon₀ epsilon_r but now it is the complex dielectric constant times E or H. So, the wave equation as we have seen earlier for the dielectric medium, the wave same wave equation governs the electric and magnetic fields. So, essentially I am writing del square E is equal to minus omega square mu epsilon₀ epsilon_{rc} into E or for H it will be same thing for H and then as we have done earlier this quantity we defined as what is called gamma square, the square of the propagation constant in the medium. So, we can do the same thing here, we can

define this quantity as γ square. So, we have here propagation constant γ which is square root of $-\omega^2 \mu \epsilon_0$ to ϵ_r .

Let us assume that the media are non magnetic that means the permeability of the medium is almost like free space, so that is μ_0 , only the dielectric constant has now become complex because of the finite conductivity of the medium. So, what we can do, we can essentially now substitute into this the two complex dielectric constant which we have got earlier and we will get in general the expression for the complex propagation constant in the medium.

So, without solving the problem from the beginning as you have done for dielectric medium essentially now I can make use of that result which we have got for the wave propagation in dielectric medium and replace the dielectric constant by the complex dielectric constant in the medium. So, if I do that essentially we can write here so this minus square root will be j , this is ω square root of $\mu \epsilon_0$ to ϵ_r minus j sigma upon $\omega \epsilon_0$ to the power half. So, essentially what we have done, we have taken this complex dielectric constant which is ϵ_r minus j sigma upon $\omega \epsilon_0$ and substitute it in this expression.

So, essentially we got the quantity γ which is the propagation constant which is this line. So, this quantity now in general is a complex quantity. So, first thing we note is that as soon as the conductivity of the medium is introduced or if the conductive has now become finite then the propagation constant of the medium is no more purely imaginary quantity. What we saw if the medium was dielectric, if there was no conductivity the propagation constant was purely imaginary we had only phase constant and we saw ω square root $\mu \epsilon_0$ that is the phase constant.

However, when we are having this quantity now the conductivity of the medium, the propagation constant has become complex that means it has a real part and it has the imaginary part and the real part as we saw earlier like from the case of transmission line

that the real part of propagation constant essentially tells you the change in the amplitude of the wave as the wave travels or that quantity is called attenuation constant.

So, as soon as I have a finite conductivity in the medium, I have a attenuation of the wave because now the real part of the propagation constant is not zero. So, I have a finite value of the attenuation constant it is non zero and therefore when the wave propagates in a medium with finite conductivity, its amplitude reduces and this we can understand physically as follows that when there was no conductivity in the medium the wave was propagating so really we had electric and magnetic fields and we had a phenomena of wave propagation.

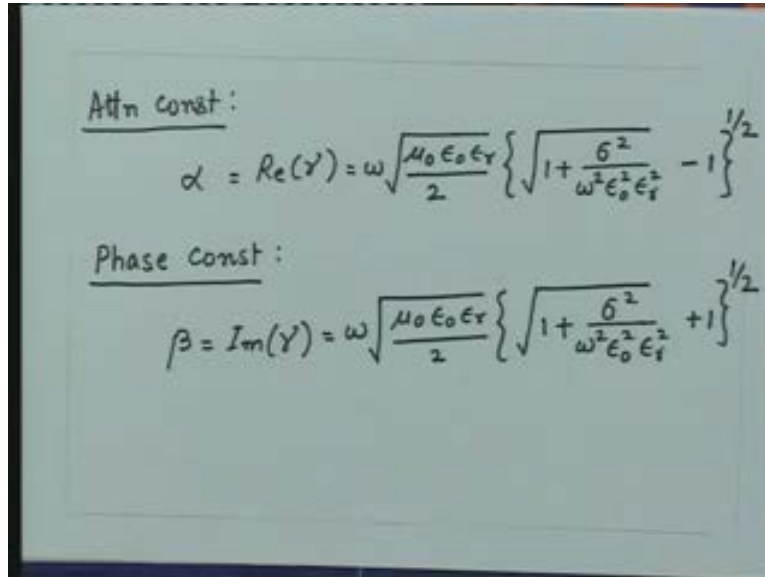
However, now when I have a finite conductivity into the medium this finite conductivity will give me the conduction current and when I have a finite conductivity we have finite resistivity also because whenever the conductivity is resistivity and once we have this conductivity, the conduction current is flowing and the medium has finite resistivity. So, we have essentially the ohmic loss into the medium we have $i^2 R$ loss in the medium because we have finite resistivity and we have finite current which is flowing in the medium.

As a result when the wave propagates, the part of the energy which the wave is carrying gets converted into the heat, it heats the medium because of ohmic losses and that is the reason when the wave propagates the amplitude of the wave reduces because the power which is carried by the wave reduces as it travels in the medium. So, physically it may extend that when we have a finite conductivity we must have attenuation of the wave because part of the wave energy will now go into the heating of the medium because of the ohmic losses.

If I separate out the real and imaginary part of this I can get the attenuation constant and we call this quantity as α that is equal to real part of γ and that is equal to $\omega \sqrt{\mu \epsilon_0 \epsilon_r} \div 2 \sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon_r^2}}$ divided by $\omega^2 \epsilon_0 \epsilon_r^2 \epsilon_r^2 - 1$ to the power half and

the phase constant beta that is equal to the imaginary part of gamma that is equal to $\omega \sqrt{\mu_0 \epsilon_0 \epsilon_r} \div 2 \{ 1 \pm \sigma \sqrt{\mu_0 \epsilon_0 \epsilon_r} \div \omega \}^{1/2}$ to the power half.

(Refer Slide Time: 00:32:54)



Att'n const:

$$\alpha = \text{Re}(\gamma) = \omega \sqrt{\frac{\mu_0 \epsilon_0 \epsilon_r}{2}} \left\{ \sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon_0^2 \epsilon_r^2}} - 1 \right\}^{1/2}$$

Phase Const:

$$\beta = \text{Im}(\gamma) = \omega \sqrt{\frac{\mu_0 \epsilon_0 \epsilon_r}{2}} \left\{ \sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon_0^2 \epsilon_r^2}} + 1 \right\}^{1/2}$$

So, the exercise is very simple you take this propagation constant gamma which is complex separate out the real and imaginary parts of this and we will get this quantity, the attenuation constant and the phase constant of the medium. Again we can note few things here and that is the attenuation constant is the function of frequency, it is of course the function of conductivity of the medium but also it depends upon the frequency that which we are operating. Same thing is true for the phase constant that has become a function of frequency. Earlier if you recall the phase constant was proportional to omega whereas when you are having a finite conductivity the phase constant beta is not proportional to omega anymore but rather it is a complex function of the frequency.

So, in the presence of finite conductivity the attenuation of the wave and the phase constant of the wave both become a complex functions of the frequency and also they depend upon the conductivity. Now with this general understanding, now we can take the two extreme cases that is if we take the conductivity which is very small compared to

$\omega\epsilon$ then we call that medium as the dielectric medium and let us call that medium as a low loss medium because as we saw that the loss is related to the conductivity, if the conductivity is more then as it decreases for dielectric medium then the conduction current will increase you have a finite resistivity, so the loss will increase. This is something interesting now that when you are analyzing the electrical circuits, we always found that if we have a higher conductivity there will be a less loss into the circuit. More conductivity means less resistivity and there will be less loss into the circuit.

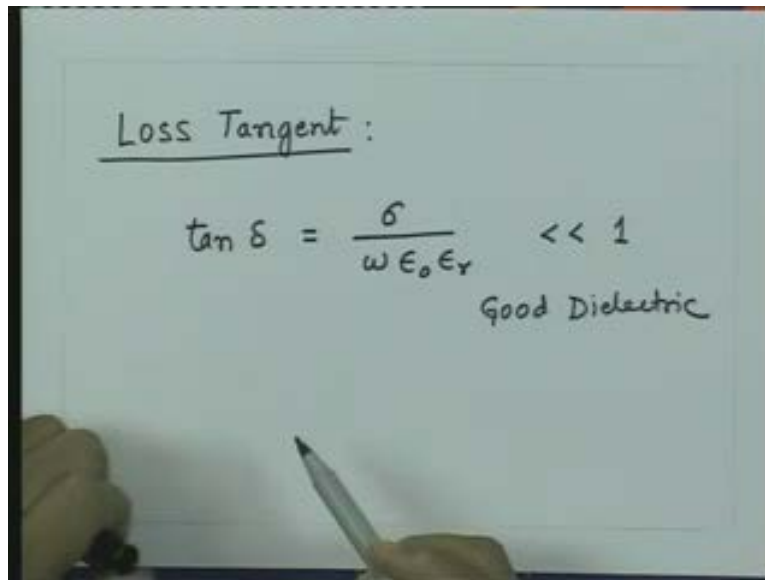
However, what we are seeing now is something opposite here, we find that when the conductivity is higher we have higher attenuation and then the more loss of the energy in the medium. How their behavior is changed from circuit to the waves or from the circuit to medium, how conceptually the conductivity which was supposed to do better in circuits now started doing worse when we went to the propagation of electromagnetic waves in this medium. The answer is very simple when we are dealing with the electrical circuits, we were dealing with the conduction current, we were dealing with the components which were more like a conducting and of components and in that situation when the conductivity is large it is infinite.

For a conductor if the conductivity is infinite there are of course there is no loss, the resistivity is 0 and even if the current flows there is no loss of power. However, when we come to dielectrics and if I keep the medium like a dielectric then an ideal dielectric without any conductivity again doesn't have any loss. So, if you have medium which is like a conductor then for no loss this conductivity must be infinite. If I take a medium like a dielectric then for no loss this conductivity must be 0, in both this situations there is no loss in the medium. However if you take a dielectric medium then higher the conductivity give higher loss. If I take an ideal conductor then lower the conductivity will give me more loss.

So, essentially what we see here since we are now dealing with the medium like a dielectric, the increase in the conductivity of the medium essentially gives me the ohmic

loss and because of you have the attenuation constant you have a power loss in the medium. There is a measure of this how much power is lost into the medium, essentially we define a parameter what is called the loss tangent which is nothing but the ratio of the conduction current and the displacement current \tan of that. So, we define a quantity what is called the loss tangent which is defined as \tan of δ which is equal to σ divided by $\omega \epsilon_0 \epsilon_r$.

(Refer Slide Time: 00:38:52)



A photograph of a whiteboard with handwritten text. At the top, 'Loss Tangent :' is written and underlined. Below it, the equation $\tan \delta = \frac{\sigma}{\omega \epsilon_0 \epsilon_r} \ll 1$ is written. To the right of the equation, the words 'Good Dielectric' are written.

$$\text{Loss Tangent :}$$
$$\tan \delta = \frac{\sigma}{\omega \epsilon_0 \epsilon_r} \ll 1$$

Good Dielectric

So, whenever we talk about a dielectric how good the dielectric is measured by this quantity what is called the loss tangent and the loss tangent is nothing but the ratio of the conduction current to the displacement current. Since we are talking about good dielectric we assume that this medium is predominantly dielectric that means this quantity conduction current is much much smaller compared to the displacement current. So, generally for good dielectric materials, this quantity is extremely small, so this is much much smaller for good dielectric.

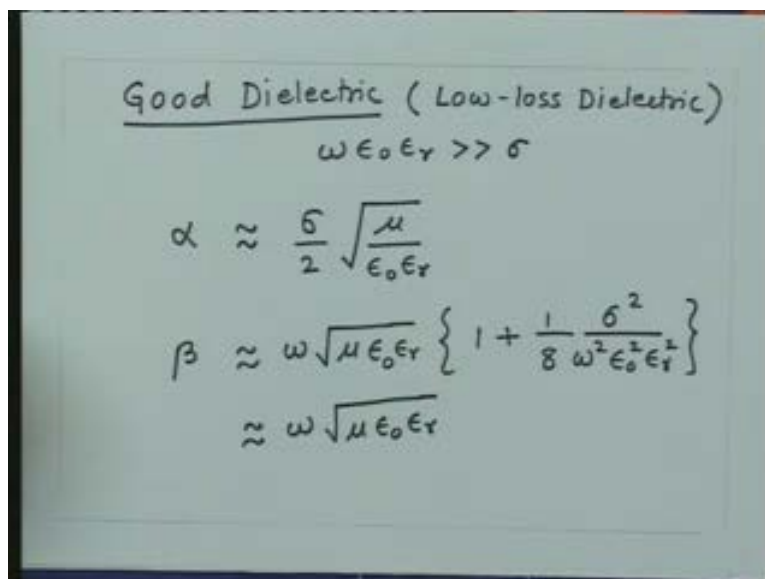
So, if you take a practical material which we want to use as a dielectric then this quantity generally is very very small and smaller the value of this quantity better is dielectric because less will be the loss into the medium. So, the loss tangent essentially is a measure

of how good the dielectric is and ideally we would like to get this quantity 0 but in practice this quantity may range from 10 to the power minus 4, 10 to the power minus 3 or some thing like that but smaller the value of this, better will be dielectric because in that case less will be the ohmic losses and less the wave will attenuate as it propagates in this medium.

So, whenever we have a material in the data sheet of the material the loss tangent is mentioned at certain frequency because as we see this quantity is a frequency dependent quantity, so we must know the loss tangent at the frequency of operation, if we know this value at certain frequency and if we assume that these quantities don't change at the function of frequency we can always scale this loss tangent at appropriate frequency. But this is the parameter which is the useful parameter for characterizing dielectric material whether it is the good dielectric or it is not a good dielectric.

Let us now take the two extreme cases of wave propagation and as I mentioned the two extreme cases would correspond to when the medium is very good dielectric that means sigma is much much smaller than omega epsilon, other extreme will be when sigma is much larger compared to omega epsilon and that time we will say this is a good conductor.

(Refer Slide Time: 00:43:37)



Good Dielectric (Low-loss Dielectric)
 $\omega \epsilon_0 \epsilon_r \gg \sigma$

$$\alpha \approx \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon_0 \epsilon_r}}$$

$$\beta \approx \omega \sqrt{\mu \epsilon_0 \epsilon_r} \left\{ 1 + \frac{1}{8} \frac{\sigma^2}{\omega^2 \epsilon_0^2 \epsilon_r^2} \right\}$$

$$\approx \omega \sqrt{\mu \epsilon_0 \epsilon_r}$$

So, let us say we consider first a medium which is good dielectric not ideal but a good dielectric, so let us say this is a good dielectric we call this medium also the low loss medium, low loss dielectric whereas we saw if the conductivity is small the losses will be small in this medium, so essentially for this we are saying that $\omega \epsilon_0 \epsilon_r$ is much much greater than the conductivity of the medium σ .

In this then I can make certain approximations to the attenuation constant and the phase constant that this quantity now is very very small compared to 1. So, what I can do is if I ignore this quantity with respect to 1 then of course I will not get σ dependent, so what I do is I just take at least this term here so I can expand this binomially and retain this term and the higher order terms, higher powers of this quantity will neglect and if we do that then we can get the attenuation constant and the phase constant as follows.

So, I will get α that will be approximately equal to σ upon $2 \sqrt{\mu \epsilon_0 \epsilon_r}$ and the phase constant β for this approximately would be $\omega \sqrt{\mu \epsilon_0 \epsilon_r} \left(1 + \frac{1}{8} \frac{\sigma^2}{\omega^2 \epsilon_0^2 \epsilon_r^2} \right)$ which if this quantity is very small we can even neglect this, so approximately we get $\omega \sqrt{\mu \epsilon_0 \epsilon_r}$.

So, if we take a medium for which σ is very very small compared to this quantity then α will be very close to 0 because this quantity is now very small but since we are now accounting for the losses in the medium, no matter what the small quantity is we would like to know that quantity, so we have this this quantity here. However, for the phase constant this quantity is negligible compared to 1 so we can say that the phase constant is $\omega \sqrt{\mu \epsilon_0 \epsilon_r}$.

This phase constant is same as what we are got for the loss less medium or ideal dielectric without finite conductivity. So, this case is very similar to low loss transmission line case if you compare with. In low loss transmission line case we have said that the phase constant is almost same as the phase constant of a loss less line and the attenuation constant is very small, so line can be treated like a loss less line only as and when the losses are to be calculated we have to take into account the attenuation constant α .

Precisely, same thing we can do for this medium also, so we can say that for a medium which is low loss or when this condition is satisfied, this medium can be treated like a loss less medium that means the medium without conductivity for all practical purposes.

As and when you have to really find out the amplitude of the wave that time essentially we have to make use of the attenuation constant α and find the reduction of the wave amplitude over certain distance. So, typical media which you see in practice which are good dielectric media, essentially we treat this media like loss less medium and find the propagation of the wave in this medium.

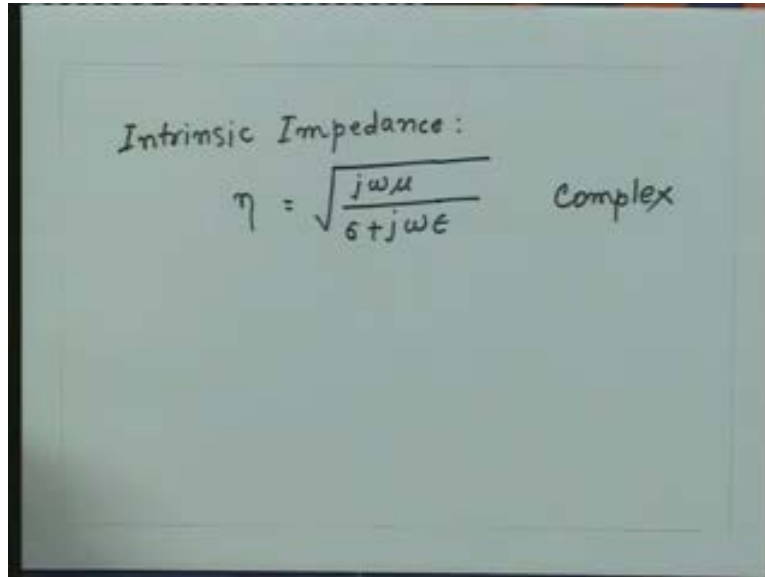
The important thing again to note here is that since the medium is treated like a dielectric, the loss is very small but whatever the small loss is given by the attenuation constant α , this α is proportional to the conductivity of the medium so that as I mentioned earlier if conductivity is 0, I have $\alpha = 0$ but α almost increases linearly as a function of this conductivity. So, a small values of the conductivity the relationship between the attenuation constant and the conductivity is linear but the wave amplitude goes exponentially with α where all those variations are e to the power minus αx .

So, essentially this quantity since is go linearly the wave decays very rapidly if the value of σ changes. So, any small change in the conductivity, increase in the conductivity may reduce the wave amplitude substantially over a given distance. Now as you have defined for a medium, the dielectric medium another parameter one was the parameter which was the propagation constant other parameter which you have defined is the intrinsic impedance of the medium.

We can define the same quantity for this media also so we can for this one, we can have the intrinsic impedance η and in this case we are talking about the general medium with finite conductivity. So this is or here if you remember the η was defined as square root of μ upon ϵ but that was essentially $j\omega\mu$ upon $j\omega\epsilon$ but now we are now having finite conductivity so we have σ plus $j\omega\epsilon$ and for a good dielectric in this case when this quantity is much much larger than σ , we can

make certain approximations to this, this quantity is much much larger than that but the fact is now when we have a finite conductivity the intrinsic the impedance of the medium is no more a purely real quantity.

(Refer Slide Time: 00:49:48)

A photograph of a whiteboard with handwritten text. The text reads "Intrinsic Impedance:" followed by the equation $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$ and the word "Complex" to the right of the equation.
$$\text{Intrinsic Impedance:}$$
$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \quad \text{Complex}$$

If you recall for the medium like free space, we had this number eta which was eta 0 which was 377 ohms that was a real quantity. So, the medium was a paring like a resistance when the wave propagated in the medium, it felt as if it is going inside a resistance of that intrinsic impedance and in case of free space the wave felt as if it is going into a resistance of 125 ohms or 377 ohms that is not true anymore when you are having a finite conductivity.

So, when now the wave propagates into the medium, it sees the impedance which is the complex quantity and that's what precisely we had seen for the transmission line case also that when the r and g were not zero for transmission line, the characteristic impedance of the transmission line become complex. Same thing is happening here that when we have a finite conductivity, we have the intrinsic impedance of the medium which is becoming complex.

So, let me summarize what you have done. We say when the medium is having a finite conductivity then substituting in the Maxwell's equation the conduction current density and simplifying we get a quantity what is called the complex dielectric constant of this medium. So, we say that the medium which is having a finite conductivity can be investigated by replacing the dielectric constant of the medium by the complex dielectric constant of the medium. We see that this complex dielectric constant depends upon the conductivity with the medium but also it depends upon the frequency of operation.

So, a medium can behave like a conductor or it can behave like a dielectric depending upon the frequency of operation. As we go towards the lower end of frequency, the medium has tendency to behave more like a conductor whereas if we go to the higher end of the frequency spectrum, the medium has a tendency to behave more like a dielectric. Then the wave equation can be solved for the complex dielectric constant medium and essentially by replacing the dielectric constant by complex dielectric constant we can get the attenuation constant and the phase constant of the medium. So, any medium which is finite conductivity can be investigated by using the concept of complex dielectric constant.