

Transmission Lines & E. M. Waves
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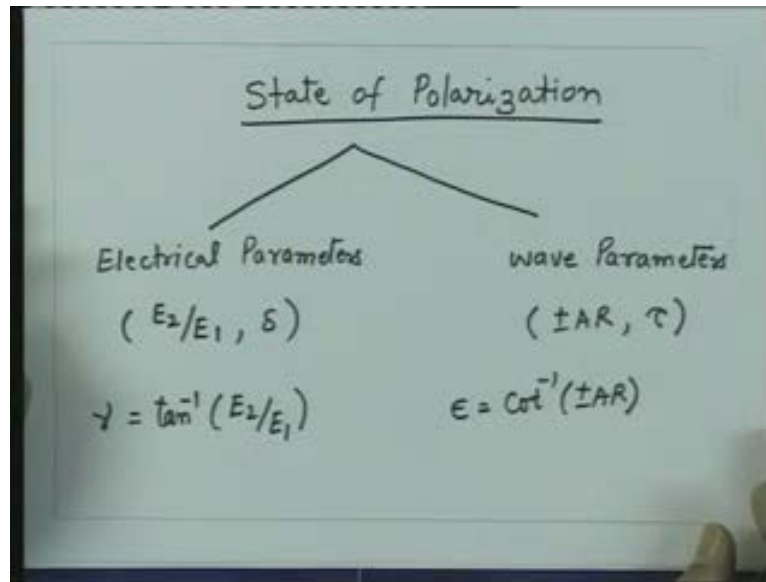
Lecture – 24

We are discussing the wave polarization of a transverse electromagnetic wave. We defined polarization as the behavior of electric field as a function of time at a given point in space. So, we saw that if you have a sinusoidally varying electric field then we have three types of polarizations the linear polarization, circular polarization and elliptical polarization. We also saw that linear and circular polarizations are the special cases of the elliptical polarization. We also saw that circular and elliptical polarization have what is called sense of rotation that means the way the ellipses and the circles are drawn, so elliptical and circular polarizations have a sense of rotation associated with that.

Today, we discuss the representation of states of polarization and then we will also discuss what is called the orthogonal states of polarization which one can use to generate any arbitrary polarization of an electromagnetic wave. So, we have a state of polarization and as we saw the state of polarization can be given by the shape of the ellipse which the wave draws and we saw that the shape of the ellipse is characterized by two parameters that is what is called the axial ratio and the tilt angle which the x axis makes with the major axis of the ellipse, the angle is tau.

Also we saw that there is a unique relationship between this wave parameters and the electrical parameters that is the ratio of the amplitudes of the electric fields of E_x and E_y components and the phase difference between them. So, a state of polarization can be defined either in terms of the electrical parameters which is the ratio of E_2 and E_1 and the phase difference delta or the two parameters of the ellipse that is the axial ratio and the tilt angle tau.

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We also saw that by assigning a sign to the axial ratio one can account for the sense of rotation for the ellipse in general. So, if the sign is positive then we say this sense of rotation is left handed, if the sign is negative then we say the sense of rotation is right handed. What we now do further that here we have one parameter which is a number other one is the angle, this is the ratio which is the number and an angle. We try to represent these quantities which are numbers also in terms of some angles and then we will see the utility of this that if we define both the parameters in terms of angles then maybe one can have a very compact representation of the states of polarization and that is what is called the Poincare sphere.

So, what we do is we define this quantity gamma which is an angle which is tan inverse of E_2 by E_1 and another angle epsilon which is called inverse of this axial ratio with the sign. So, now if I now look at the ellipse of polarization we have the axial ratio which ranges from 1 to infinity, the tilt angle will vary from 0 to pi, the delta which is the phase difference vary from minus 90 degrees to plus 90 degrees, if we take the sign result in to the amplitude and then this angle gamma will vary from 0 to 180 degrees or minus 90 to plus 90 degrees. So, we now have the range for all this parameters.

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Electrical Parameters	Ellipse Parameters
$0 \leq \gamma \leq \pi$	$-\pi/4 \leq \epsilon \leq \pi/4$
$-\pi/2 \leq \delta \leq \pi/2$	$0 \leq \tau \leq \pi$
(γ, δ)	(ϵ, τ)

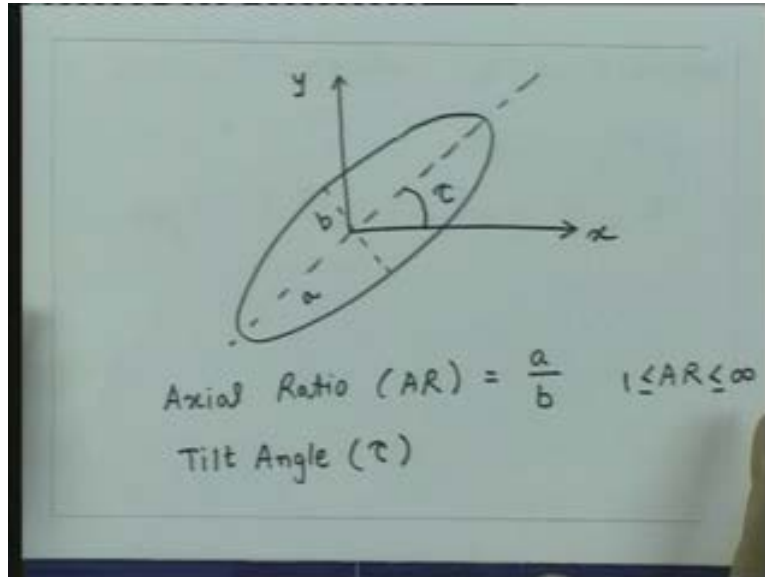
So, if I take electrical parameters, we have this quantity gamma which can range from 0 to pi. What we are doing here is that we are assuming that the phase angle between two components range from minus 90 to plus 90 degrees and whereas if you see the phase difference between the two electric quantities they can vary from 0 to 360 degrees. So, the account for that essentially we put sign along with this and make the phase difference ranging from minus 90 to plus 90 degrees. That is the reason this quantity gamma will vary from 0 to pi because this quantity can be even negative. The phase angle delta lies between minus pi by 2 to plus pi by 2.

So, we are having these two parameters which range, this ranges from 0 to pi, this ranges from minus pi by 2 to plus pi by 2. If I take on the other side the ellipse parameters, we have this axial ratio which can range from 0 to infinity so that means this angle epsilon can vary from 0 to 45 degrees with the sign which is plus minus. So, the range of epsilon is from minus 45 degrees to plus 45 degrees and the angle tau will go from 0 to 180 degrees as we saw for the ellipse.

So, we have here the range for this epsilon which will be from minus pi by 4 to plus pi by 4 and the tau will have a range from 0 to pi. So, as we saw in the case of ellipse when we

have ellipse like this then this axis, major axis the angle which it makes with the x axis can range from 0, so the ellipse can rotate from all the way up to this beyond that again the ellipse repeats itself.

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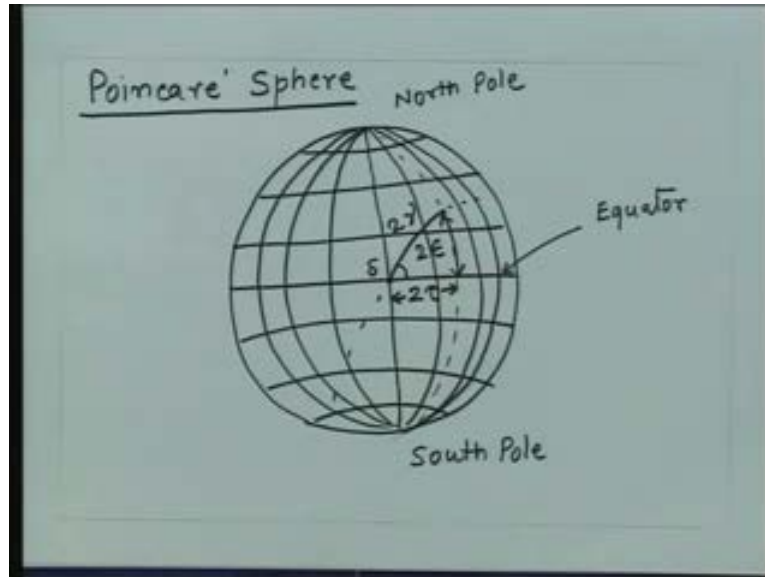


So, essentially the range for tau is from 0 to pi and as we defined the axial ratio which goes from 1 to infinity, so we have got the range for the epsilon which is from minus pi by 4 to plus pi by 4. So, now essentially we are defining the state of polarization of an electro magnetic wave by pair which is the angle pair which is gamma and delta and another pair of angle which is epsilon and tau. Once these things are defined then mathematicians have found a very nice way of compacting these angles.

So, what they realize is the what these angles are defined they can construct a sphere what is called the poincare sphere, every point on that sphere can be uniquely represented by either this pair or this pair and once you have the compact representation essentially we have got some kind of a visual representation of states of polarization. So, idea here is let us imagine a sphere and let us mark the angles on this sphere in terms of either this quantity epsilon tau or gamma and delta.

Let us say we have a sphere which is like the globe where you have the latitude and longitude lines drawn on it.

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So, let us say I have a sphere, this is a line which is the equator and then I have this is the longitude lines and so on. So, let me say I am having this sphere here on which we have drawn the latitude and longitude lines, this is the equator of the sphere and these are the poles of the sphere. So, we call this as like our globe this is my North Pole, this is the South Pole. And let us say I have this point here which is the difference point on the equator from which I have measured all my longitudes and the angle which is in the vertical direction that is measured in terms of latitude.

What is now realized is that if I take the angle which is 2ε 2τ where 2τ represents the longitude and 2ε represents the latitude of a point. So, if I say that a particular point if I take this here that is the longitude, I am passing through this. So, on the equator if I measure this angle, this angle is 2τ and this angle which is the latitude of this point is 2ε , so here this angle from here to here that is 2ε .

Now, since the range of these parameters ϵ is from $-\pi/4$ to $+\pi/4$, the 2ϵ will range from $-\pi/2$ to $+\pi/2$ that means all possible values of ϵ that means all possible values of axial ratio with the sense of rotation taken into account will lie on all this latitude points which will go from $-\pi/2$ to $+\pi/2$.

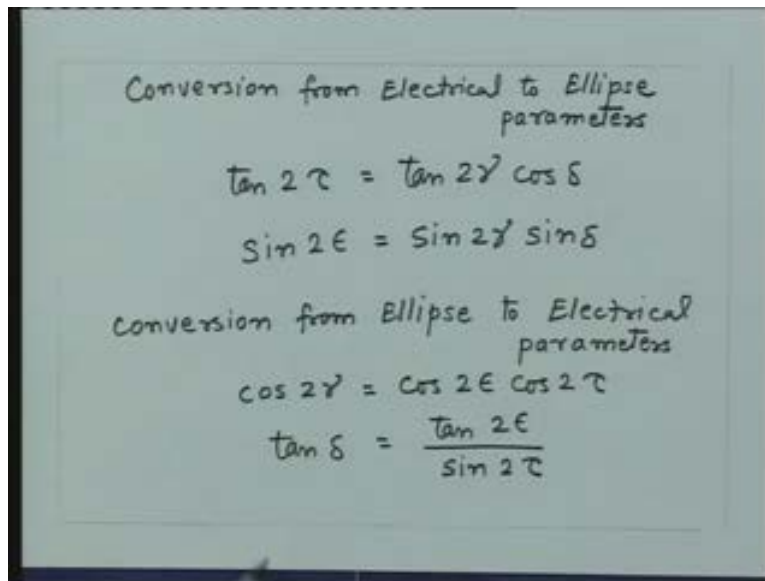
Similarly, this angle 2τ , so this is ranging from 0 to π as the τ varies from 0 to $\pi/2$ essentially the longitude of this point will vary from 0 to 360 degrees, so essentially we will have a rotation around this globe all the way up to this point. So, what that means is now that if I take this pair for representation, ϵ and τ where 2ϵ is the latitude and 2τ is longitude of the point then all possible combinations of ϵ and τ are covered by the surface of the sphere not only that every point which you are having on the sphere now is having a unique combination of ϵ and τ . What that means is now that every point on this sphere is representing a unique state of polarization, also all possible states of polarization are captured by the points on the sphere.

This is a very nice and very compact representation of states of polarization and we will see little later its utility but first let us understand what are the different special point which we are going to have on this sphere. One can say now what is the relationship between these quantities here which is γ and δ and ϵ and τ . So, if I take an angle if I consider now the horizontal plane which is passing through the equator and if I take a circle which is passing through the centre of the sphere and a plane like this which is passing through this point, the angle of the arc which is measured from this reference point up to this point that angle is 2γ .

So, this angle if I take this is that circle which will be passing through, this angle is equal to 2γ and the plane, this plane and now you are having two planes which is the equatorial plane which is this and a plane which is containing this arc passing through this point where I am defining the states of polarization, the angle between these two planes is given by δ . So, these angle here between the planes the two planes like that that angle is given as angle δ .

So, using now the spherical trigonometry essentially we can find out the relationship between epsilon tau, gamma and delta because we have marked all the angles in both the domain as an electrical domain which is represented by gamma and delta and ellipse domain which is epsilon and tau are marked on the surface of this sphere. So, we can have first the relation which is the conversion relation.

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Conversion from Electrical to Ellipse parameters

$$\tan 2\tau = \tan 2\gamma \cos \delta$$

$$\sin 2\epsilon = \sin 2\gamma \sin \delta$$

Conversion from Ellipse to Electrical parameters

$$\cos 2\gamma = \cos 2\epsilon \cos 2\tau$$

$$\tan \delta = \frac{\tan 2\epsilon}{\sin 2\tau}$$

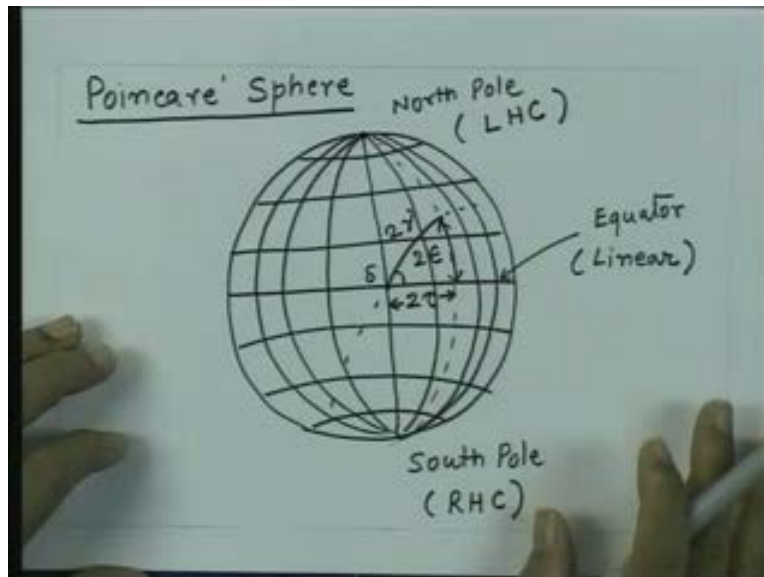
So, if you want to convert from the electrical parameters to ellipse parameters, we have a convergence relations, so we have conversion from electrical to ellipse parameters that is if we are given the quantities gamma and delta we want to find out what are the corresponding values of epsilon and tau. So, these relations are tan of 2 tau is equal to tan of 2 gamma cos of delta and sin of 2 epsilon that is equal to sin of 2 gamma sin of delta.

So, if we are given electrical parameters that we define now the amplitude ratio of the two fields which are going to excite this electromagnetic wave and if I know what is the phase different between these two electric fields, I can find out what will be the tilt angle and axial ratio of that ellipse.

As we mentioned last time in practice we may have a problem which is opposite that is we would like to generate a certain ellipse and we would like to know what should be the parameter which are the electrical parameters. So, we will also require a conversion from ellipse to the electrical parameters. So, when we require conversion from ellipse to electrical parameters and this we have $\cos 2\gamma$ is equal to $\cos 2\epsilon \cos 2\tau$ and $\tan \delta$ is equal to $\tan 2\epsilon$ divided by $\sin 2\tau$. So, using these relations either I can convert from the ellipse parameter to electrical parameters or from electrical parameters to the ellipse parameters.

While choosing the value of τ , ϵ or γ and δ however should we kept in mind that the range of these quantities is given by this, so depending upon the sign of these quantities, we will have multiple choice to choose for this values so far. So, we choose these values lie appropriately so that these values lie essentially in this range. So, by using these relations analytically one can convert from one state of polarization in one domain to another domain. Having done this now we will go back to our representation of the states of polarization on this sphere and the dimension the sphere is called the Poincare sphere. So, this sphere represents all possible states of polarization.

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Now, let us mark some of very specific states of polarization just to get later better field for where these states polarizations are distributed on this Poincare sphere. Let us say we are having a linear polarization, so come back here. If you have a linear polarization then the axial ratio for linear polarization is infinity because the minor axis for the linear polarization is 0. So, in this case if I take a linear polarization this ellipse is compressed so this b has become equal to 0. So, I have a tilt angle which is the inclination of the line but the axial ratio for this is infinity. So, for a linear polarization since the axial ratio is infinity we have this angle ϵ that is equal to 0 and tilt angle due to the inclination of the line which can orient from 0 to 180 degree.

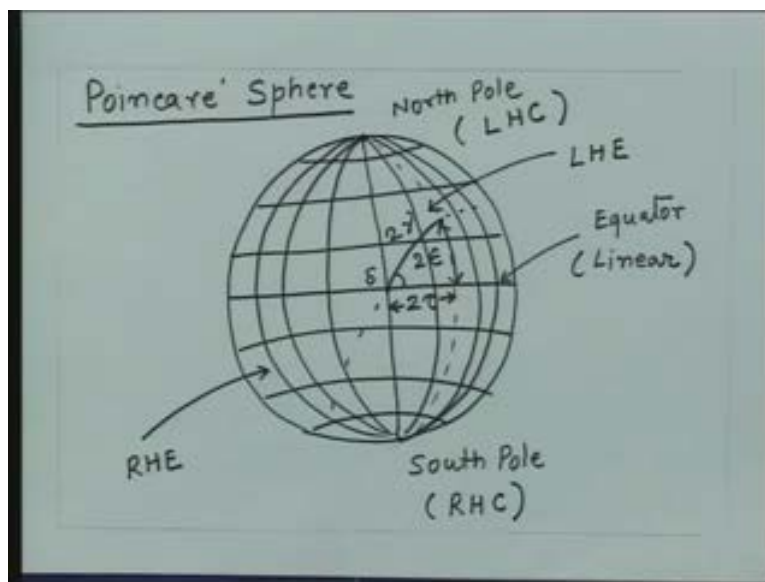
So, if I say my reference point which is for measuring the longitude is this point which is the reference point then this point corresponds to τ equal to 0 and ϵ equal to 0. So, this point here on the equator at a longitude which is the reference longitude, this point corresponds to τ equal to 0, ϵ equal to 0. So, τ equal to 0 means the tilt angle of the line is 0, ϵ 0 means the axial ratio is infinite so this is a linear polarization. So, this point corresponds to linear polarization with tilt angle 0 that means this point corresponds to the horizontally linearly polarized wave. So, this point here if I look at this point the reference point, this is horizontally linearly polarized wave.

Now, if I move on the equator if I go on changing the angle the ϵ still remains 0 and if since the ϵ remains 0 the axial ratio remains infinite that means the polarization remains linear. However, the tilt angle of this goes on changing and as I have moved when I go to here when 2τ become 90 degrees the τ will become 45 degrees that means the line will be tilted at 45 degrees with respect to the x axis. When I go all the way around to this on the back side of this which is this will become 180 degrees, so you will get 2τ equal to 180 degrees, so τ will be equal to 90 degrees that is the tilt angle will become 90 degrees or the line will become vertical.

So, if I start on this globe if I start from here with reference point the line was horizontal, as I move the lines slowly tilts like that and when it goes back of the sphere the line becomes vertical. So, on the equator the reference longitude point represents a

horizontally polarized wave whereas on the equator if I take the diagonally opposite point that point will represent a vertically polarized wave. Again, as I move further in the angle the angle will go on tilting and when it comes back the line is become again horizontally polarized. So, what we find that the points which lie on the equator of the sphere these points represent all linear states of polarization. We start with horizontal becomes vertical and again it comes back becomes a horizontal here. So, equator of this sphere represents linearly polarized waves.

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Let, us take some other special points let us say if I take the point which is the North Pole. Now, for North Pole, the 2ϵ has become 90 degrees plus 90 degrees so the ϵ has become plus 45 degrees. If ϵ is plus 45 degrees, the axial ratio for this is equal to 1 and if the axial ratio is 1 that represents the circle. Obviously, when it is a circle the tilt angular lost a meaning because the tilt angle is with respect to the major axis and once you are having a circle, the group major axis or minor axis are defined for it. So, at this point when 2ϵ becomes equal to 90 degrees, this becomes circle and obviously now the 2τ doesn't play any role because for this point this angle doesn't matter but we know for axial ratio equal to 1 with the positive sign that corresponds to the left handed circular polarization.

So, this point here the north pole of this globe represents left handed circular polarization. By the same token when I go to the south pole of this again 2ϵ now will become minus 90 degrees. So, ϵ is minus 45 degrees and for this the axial ratio is 1 for the angle is negative and we have seen that the negative angle is a sign for right handed sense of rotation. So, this point essentially represents the right handed circular polarization.

So, any point lying on the equator of this sphere represents linear polarization. The North Pole of this globe represents left handed circular polarization and the south pole of this globe represents a right handed circular polarization. Also we note that for any point which is lying on the northern hemisphere of this sphere has a value δ which is positive, this axial ratio is positive. What that means is that points which lie in the northern hemisphere essentially represent left handed rotation.

As we saw now our earlier discussion that if E_y leads E_x then the sense of rotation is left handed if E_y lags E_x then sense of rotation is right handed. So, in this case since δ is positive or ϵ is positive, we have a sense of rotation which is left handed but we have this quantity ϵ which is neither 45 degrees nor 0. So, if I take any point in the northern hemisphere in general point like this then it will represent an elliptical polarization but that elliptical polarization will be left handed elliptical polarization

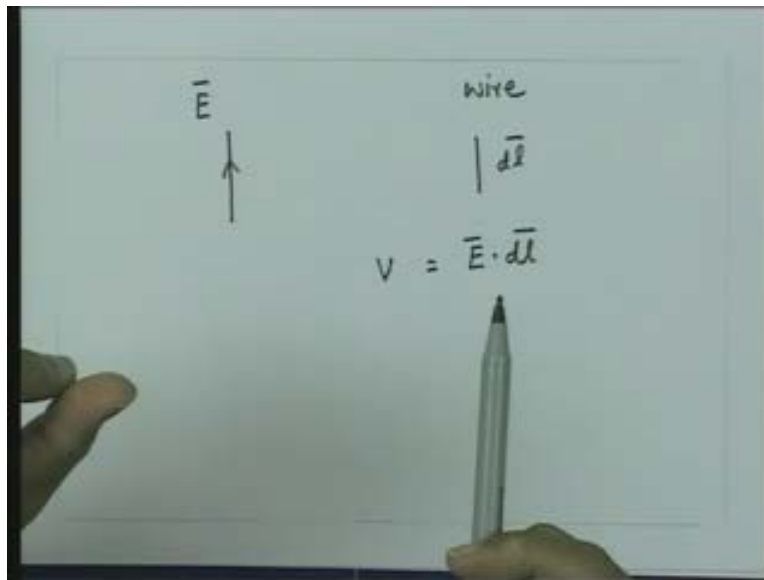
Similarly, if I consider any point which is lying on the southern hemisphere of this globe then the sense of rotation will be right handed. So, it will represent in general an elliptical polarization but the sense of rotation will be right handed. So, any point which lies in this hemisphere that represents left handed elliptical polarization whereas if I vary point which is southern hemisphere that will represent a right handed elliptical polarization.

So, what we now find is that this is the very nice compact representation of different states of polarization. So, we can really visualize how different states are distributed on this. Do we have the scientific use of this representation or it is just for only visual representation we have done this in fact we will see there are many nice things which can be done with the Poincare sphere. What are the things we notice here is that if we start

with this point for which the polarization is linear horizontal and as I move on this and go all the way back side of this sphere on this equator, the line becomes vertical. So, here I have a polarization which is horizontal, here I have a polarization which is vertical. Similarly, if I go to the North Pole which is the top point I have polarization which is left handed circular, if I go diagonally opposite point I have the polarization which is right handed circular.

Are these any special things or relation between these opposite points? The answer is yes. Imagine a situation like this let us say I have an electromagnetic wave which is coming with vertical electric field. So, let us say when the wave comes the electric field is vertical that means this wave is vertically polarized linearly polarized wave. Let us say I have a piece of wire which is placed vertical in this incoming wave, so we have $\vec{E} \cdot d\vec{l}$ which is the voltage induce in this wire.

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So, if the electric field was coming vertical like this and if I keep a piece of wire which is like that this wire, this is the E field, we will have $\vec{E} \cdot d\vec{l}$ produce that means the voltage produce across this wire. So, we will have from here $\vec{E} \cdot d\vec{l}$ where this length is dl , this vector is \vec{E} . So, this quantity will give me the voltage induced in this wire and if \vec{E} and $d\vec{l}$

are parallel so both are vertical then I will have simply E , magnitude of E multiplied by the length of this line that will be the voltage induced in this wire.

Let us imagine now that this wire is not vertical like that but suppose it is kept horizontal, so the field is coming vertical but the wire which is placed here is horizontal like this. Now since there is an angle between E and $d\mathbf{l}$ is 90 degrees the $E \cdot d\mathbf{l}$ is 0. So, although the field is existing, the wire is existing but there is no induced voltage because of this field in this wire. What that means is now that this wire does not respond to this electric field. So, when this kind of system is placed in this electromagnetic field, there is no response from the field to the wire, this thing now can be generalized to any systems. What we say if we have a system and if we are having electromagnetic wave.

When I keep this system in this incoming electromagnetic wave, some voltage or currents get induced in the system and there is a power transfer from the wave to the system. So, depending upon the properties of the system and the electric field of the incoming wave, there will be a certain amount of power exchange which will take place. As you saw in this case if the system was simply a piece of wire and if the wire was vertical, electric field is vertical there will be the voltage induced if the line was perpendicular to this, there is no exchange of power has taken place because no voltage is induced in the wire. If the wire was at an angle then I will have a certain voltage which will be $E \cdot d\mathbf{l}$ which will be smaller than this but which will be more greater than 0.

So, what one can now say is that we have a system which can maximally interact with an electromagnetic wave. So, in this case if the wire was vertical I have a maximum voltage induced that means I have a maximum interaction between this wave and this system. If the line was horizontal then the interaction is least that means there is no interaction, so wave simply passes through this without inducing any voltage, without any exchange of power. So, what we can say is that the system also has what is called a state of polarization.

Now, how do you define the state of polarization of a system? It is the state of polarization of that wave to which this system responds maximally. So, if I take a system and try with different all possible states of polarization, that state of polarization to which my system response maximally, I will say that is the state of polarization of the system.

So, now we are having two things, we are having the polarization of an electromagnetic wave. We also now are characterizing a system which is capable of receiving this wave and it also has so called state of polarization. If the two states of polarizations match then we are having maximum power exchange between the wave and the system, if there is no exchange of power then we will say these two polarizations are completely mismatched or these two states of polarization are orthogonal to each other. So, when the two states of polarization have maximum interaction, we say these states of polarizations are match, state of polarization whereas when there is no interaction between the two states we say that these two states are orthogonal states of polarization.

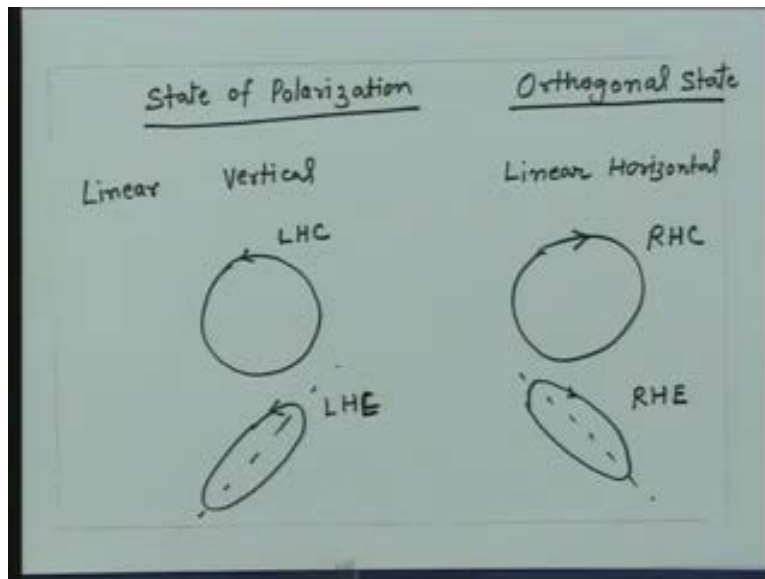
So, in this particular case if we take electric field vertical the state of polarization is vertical linear polarization. If I take a wire which is horizontal, the state of polarization of the system is horizontal linear polarization. These two states of polarization are orthogonal because there is no exchange of power between these two. If I take this thing also vertical then the two states of polarizations are matched and then I have a maximum exchange of power between the wave and the system.

Now, this concept can be generalized for any arbitrary state of polarization. What one can say is that this argument which we gave here is not necessarily true only for linear polarization. We can take any state of polarization and find another state of polarization to which this maximally responds or to which it doesn't respond at all. So, essentially for every state of polarization, we can have its corresponding orthogonal state of polarization. And what Poincare sphere does essentially it says that if I take the diagonally opposite point, diametrically opposite point on the surface of the sphere they represent orthogonal states of polarization. So, the usefulness of Poincare sphere one of the utility is identifying the orthogonal states of polarization and now we can go back and

verify whether this is really true, one thing we have already seen that if I start with horizontal polarization the diametrically opposite point is on the equator which is the back point and at which the polarization becomes vertical.

So, a horizontal linear and vertical linear polarization these are two orthogonal states of polarization by the same token we can have this state of polarization which is left handed circular and diagonally opposite of this point is the right handed circular polarization. So, a left handed circular polarization and right handed circular polarizations these are two orthogonal states of polarization. I can take any other point here and I can find out a corresponding point which is here that again will represent an arbitrary states of polarization. So, now if I specifically write how the orthogonal states of polarization will look like.

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So, we have state of polarization and we have orthogonal state. We saw if I take line vertical linear vertical I get an orthogonal set of polarization which is linear horizontal. So, two lines which are 90 degrees with respect to each other can represent orthogonal states of polarization though we are taking specifically vertical but there is nothing

special about vertical and horizontal. If I take two lines which are perpendicular to each other, at any angle these two states will represent the orthogonal states of polarization.

The second thing we saw is if I take a circular polarization and let us say my wave is going inside the plane of the paper, this is representing the left handed circular polarization. So, the orthogonal state will be a circle with a right handed polarization. So, for a linear vertical the orthogonal state is linear horizontal, for a circular left handed the orthogonal state is right handed circular and vice versa. What about the elliptical polarization? If I take some value here ϵ in northern hemisphere then the diagonally opposite point will definitely lie in the southern hemisphere. That means the axial ratio we have the same value but the sign of that will be opposite and this 2τ angle would have changed by 2τ plus 180 degrees.

What that means is that the tilt angle of the ellipse is changed by 90 degree because 2τ is change by 180 degrees and ϵ remains same that means axial ratio remains same but the sign changes because I have a diagonally opposite point. So, any two ellipses which are having same axial ratio but opposite sense of rotation and tilt angle differed by 90 degrees will represent orthogonal states of polarization. So, I have, if I take an ellipse which is like this and if I say this is the left handed elliptical this is the major axis now. If I take 90 degrees with respect to this the same ellipse, so the axial ratio is same for these two, the sense of rotation is opposite so this become right handed elliptical here.

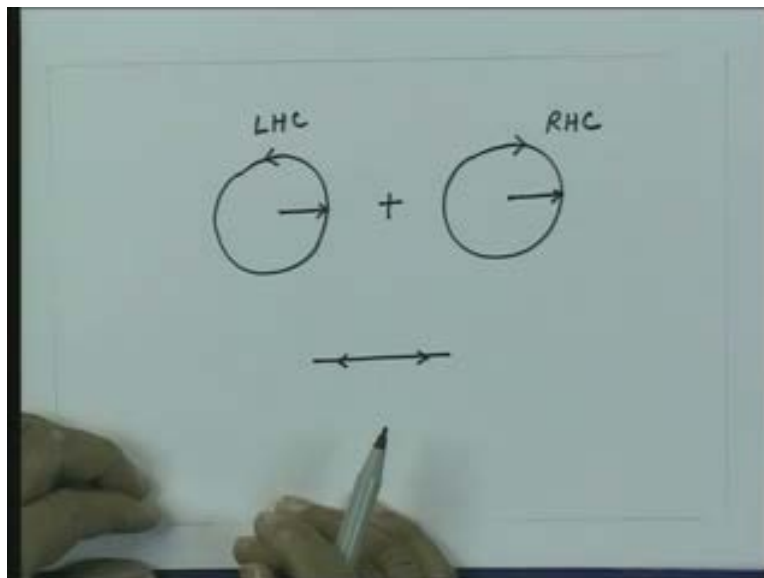
So, for these 2 ellipses the axial ratio is same, the sense of rotation is opposite and the tilt angle is changed by 90 degrees, these two represent the orthogonal states of polarization. So, in general we can say the 2 ellipses whose major axis are tilted at 90 degrees with respect to each other have the same shape but opposite sense of rotation will constitute a pair of orthogonal states of polarization and then the special cases of that would be the circular polarization and the linear polarization.

So, essentially we can now define the orthogonal states of polarization by using the Poincare sphere and once we have an orthogonal pair we can treat this more like the

orthogonal coordinate system for representing a polarization and then any arbitrary state of polarization can be now generated by combination of two orthogonal states of polarization. Now we will go back and see why the two horizontal and vertical polarized waves E_x and E_y could generate any arbitrary ellipse because in fact we were talking about the two states of polarization which are orthogonal, we had the x polarized wave or E_x wave for which the electric field was horizontally polarized, we had a E_y field for which the electric field was vertically polarized. So, in fact we were dealing with the orthogonal states of polarization and that is the reason by combination of these two, we could generate any arbitrary elliptical state of polarization.

Then it is immediately apparent that if that is so there is nothing special about horizontal and vertical fields which could generate the elliptical or the general any polarization. Any pair of orthogonal state can be used or generating any arbitrary state of polarization. So, instead of these two we could have as well use two circular polarization one left handed, one right handed and by combination of these two we could generate any arbitrary state of polarization.

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So, let us say suppose I take two circular waves which are rotating like that. So, this is let us say left handed, this is right handed and let us say both of them have equal amplitude but they are rotating in opposite directions. So, this is left handed circular, this is right handed circular and let us say at some instant of time these two vectors were in that direction. After sometime this vector would move here, this vector will go like that so the amplitude resultant vector these two are, these angles has become here, these angles has become here. So, the super position of these two vectors again will lie in this direction this amplitude will reduce.

When the vector rotates to this angle 90 degree this vector would have rotated to 90 degree they will completely cancel each other. So, I will say if I take this two amplitude equal and let them super impose in space, the resultant electric field will simply move in the horizontal plane so it will draw a linear polarization. So, if the two amplitudes are equal and if I combined this essentially I have created a linear polarization which is horizontally linearly polarized wave. So, we see here that now I am taking my primary polarization or circular polarization and by combining two circular polarization, I can generate a linear polarization.

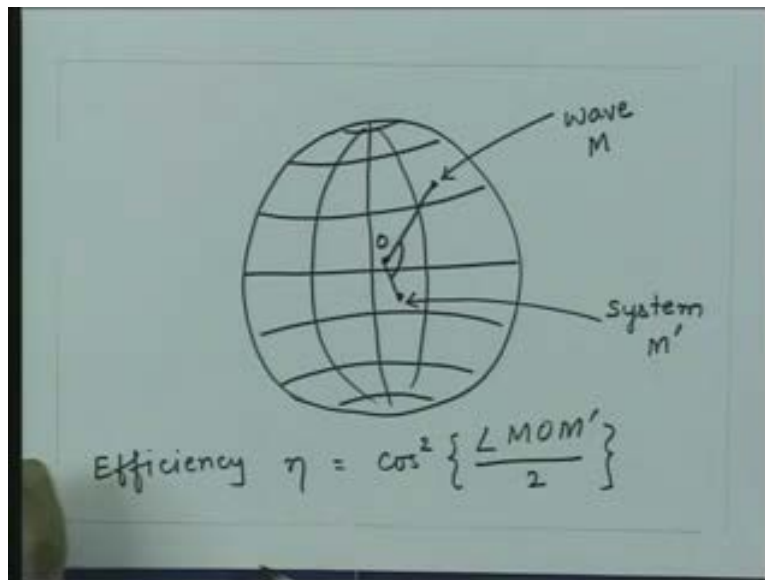
So, one can now go and try some different values of this combinations, the ratio between these two amplitudes also at some instant of time the two need not be in same direction. So, if I take this is here and this is here and try how difference states of polarization can be generated. So, what we emphasis now here is that by using these two states of orthogonal polarization we can generate any arbitrary state of polarization and first they are orthogonal states now in practice we can find its utility because if you are having two orthogonal system they don't interact between each other.

So, in any communication system there is now a possibility to send the two signals at the same time at same frequency but by using two orthogonal states of polarization because even if we are mixed in space at the receiver I can always put two systems which are having orthogonal states of polarization and I can separate out the signals. So, essentially by using now the polarization we can increase the capacity of sending information by

factor of two because we can put independent information in two states of polarization and since we have a mechanism of separating out the states of polarization by choosing a system with particular polarization, we can always separate out the information.

So, essentially by using now the two orthogonal states of polarization the transmission capacity of a system can be increased in principle by factor of 2. Well, the Poincare sphere also does something and that is not on the extreme case like the orthogonal state of polarization but one can ask if I have two states of polarization how much is the efficiency of power exchange between the wave and the system.

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So, what one can do from here this is the Poincare sphere and let us say this is my equator lines, these are latitude lines here and these are the longitude lines. Let us say I have some points here which represent the polarization of the wave that we denote this point as some point m . Let us say I have a system whose state of polarization is defined and we already have said we defined the state of polarization of a system is that polarization to which the system response maximum.

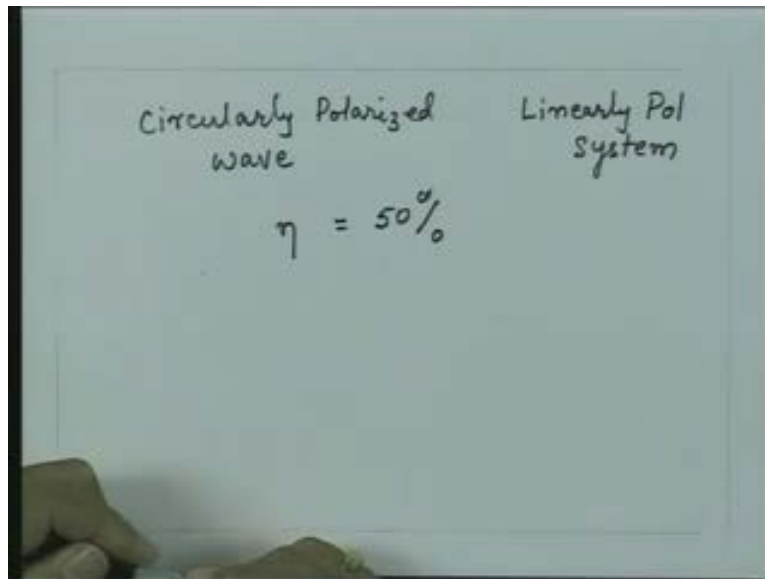
So, let us say I have a point here which is the state of polarization of my system and let me call this as some m prime and that is the center of the sphere. So, essentially these two points, this point represents state of polarization of the wave which is impinging on a system whose state of polarization is given by this and we asked a question how much exchange of power it will take place, what are the efficiency of the power exchange between these two systems? This efficiency we can get essentially for finding this angle which these two points obtain at the center of the sphere. This angle let us say the center was of this sphere was o , so this angle is $m o m$ prime this angle. The efficiency of power exchange is \cos^2 of this angle $m o m$ prime divided by 2.

So, the Poincare sphere can also be used to find the interaction efficiency between the wave and a system. So, if I just mark these two states of polarization on the Poincare sphere, find the angle which subtends at the center of the sphere, half of that angle cosine square that quantity gives me a quantity which is called the efficiency of exchange of power between the wave and a system.

Let us try to see some specific cases how the exchange of power will take place between the wave and a system. Let us say if I have a system which is linearly polarized and let us say a wave impinging is circularly polarized. So, a linearly polarized will lie somewhere on the equator, it could be horizontal or any angle but it will lie on the equator. A circularly polarized wave will lie either on this pole or this pole depending upon the sense of rotation. So the angle between this and this is always 90 degree, so any point on the equator and the pole the angle is 90 degrees. So, this angle $m o m$ prime for a linear polarization and circular polarization is always 90 degree. So, if I have a wave circularly polarized and system linear polarized or vice versa, the angle $m o m$ prime is 90 degrees. So, this angle this quantity is 45 degrees, so \cos^2 of 45 degrees that will be half so we have 50% efficiency.

So, what we see from here that if I take circularly polarized wave and linearly polarized system or vice versa then the efficiency of power exchange η is 50%.

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So, in this case even if the orientation of this line changes that means the state of polarization, the linear polarization changes but as long as this wave is circularly polarized and this is linearly polarized need not be horizontal or vertical, the efficiency will always remain 50%. So, if I have a situation where the field vector is varying at the function of time but it remains linear and if I use the receiving system which is circularly polarized I will always have a constant output because the efficiency is always 50% instead if I use for a linearly polarized system if I use the linearly polarized wave and if the angle between them keeps changing, the output will keep changing because the dot product between E and $d\mathbf{l}$ will go on changing

This is the situation we have precisely in satellite communication that because of the ionosphere in between we have a rotation of the electric field linearly polarized field what is called the faraday rotation and if we put an receiving antenna which is linearly polarized, the output goes on changing as a function of time because these two fields keep changing angle with respect to each other instead if we have put a circularly polarized antenna for reception and if this wave is coming linearly polarized even if it

change the direction, the efficiency is always 50% and we get a guaranteed output so we do not see any fluctuation in the output.

So, we saw now by using the concept of polarization many nice innovative things can be done in communication systems and many factors essentially decide what should be the polarization of the receiving system, what should be the polarization of the transmitting or the receiving electromagnetic wave. So, we find that the polarization of an electromagnetic wave place a very very important role in designing the communication systems. So, before we get into the design of the communication system we must understand these aspects of electromagnetic wave very thoroughly.