Transmission Lines & E. M. Waves
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Lecture – 23

We are investigating one of the very important aspects of a transverse electromagnetic wave and that is the wave polarization. We define the wave polarization as the behavior of the electric field vector as a function of time at any point in space. So, we saw that if you take a transverse electromagnetic wave, the direction of electric and magnetic field vectors are perpendicular to each other and also they are perpendicular to direction of wave propagation. However, the direction of the electric and magnetic field vectors can change the function of time and that's the phenomena which is captured by what is called the wave polarization.

So, any electric field vector can be thought of as a combination of two perpendicular electric field vectors and here we consider the x oriented and the y oriented field vector. Again following the right handed coordinate system if this is the x axis, this is the y axis then the z axis is coming out of the paper. So, in the analysis of the wave polarization we assume that the wave is propagating in positive z direction. That means the wave is now coming out of the paper, it is traveling upwards and then the two electric fields which are sinusoidally varying at the function of time may have different amplitudes and they may have a phase difference between them.

So, this electric field has a maximum value which is E_1, so this vector essentially oscillates in the horizontal direction whereas this vector oscillates in the vertical direction with amplitude E_2, this oscillates with amplitude E_1 that at any instant of time the total electric field is the vector sum of these two electric fields, so we can find out any instant of time E_y and E_x and by vector radiation of these two we can find the total electric field at that instant of time.

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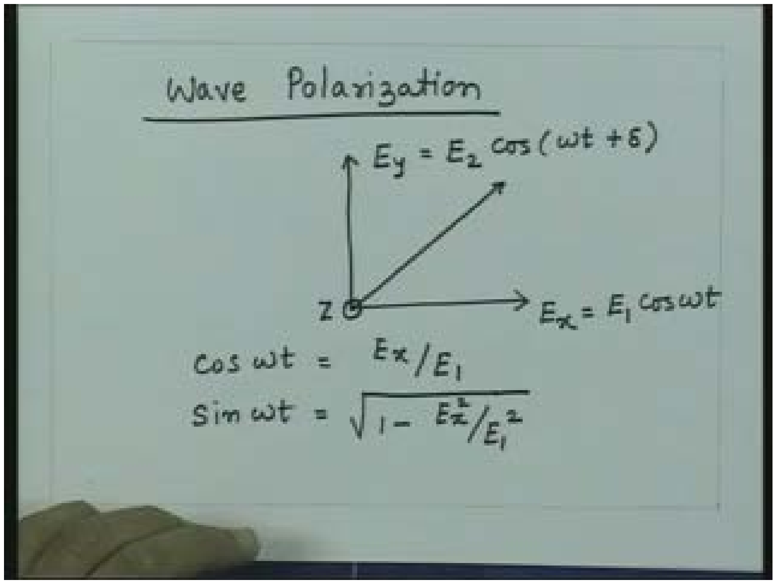

Now, if we are interested in the behavior of this electric field as a function of time and then if I treat this electric field like an arrow then the tip of the arrow, the way it varies the function of time in this electric field amplitude space that will be the state of polarization for this electromagnetic wave. So, essentially we are looking for a locus of the tip of the electric field as a function of time. So, we eliminate the time parameter from these two the components.

So, from here we have cosine of omega t which is E_x upon E_1, from here we can find out the sine omega t equal to the square root of 1 minus E_x square upon E_1 square and then for this E_y if we expand this cosine function we can get E_y that is E_2 cos of omega t cos of delta minus sin of omega t sin of delta. I kept in this E_2 down here so I can write this as E_y upon E_2 that is equal to cosine of omega t which is E_x upon E_1 and I can substitute for sine of omega t from here. So, this is E_x upon E_1 cos of delta minus square root of 1 minus E_x square upon E_1 square sine of delta. To remove this square root sine we can bring this term on this side and take a square on both sides, so we can get here E_x upon E_1 cos of delta minus E_y upon E_2 whole square that is equal to 1 minus E_x square upon E_1 square sine square of delta.

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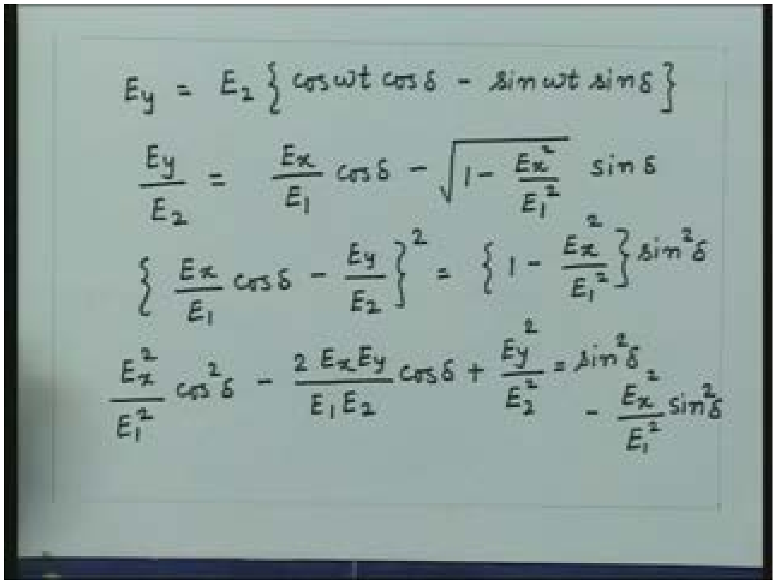

I can expand this again, so I get here E_x square upon E_1 square cos square delta minus 2 E_x E_y upon E_1 E_2 cos of delta plus E_y square upon E_2 square that is equal to this quantity which is sine square delta minus E_x square upon E_1 square sine square delta. Now, I can bring this term on this side and that will give me E_x square upon E_1 square into cosine square delta plus sine square delta that will be equal to 1, so I have to bring this term on this side and add with this, I will get only E_x square upon E_1 square.

So, simplifying this finally I get an equation which is E_x square upon E_1 square minus 2 E_x E_y upon E_1 E_2 cosine of delta plus E_y square upon E_2 square that is equal to sine square delta. So, the equation of the locus which the tip of the electric field vector draws in the amplitude electric field amplitude space is given by this equation and immediately we note that this equation is essentially an equation of an ellipse. So, in general the tip of the electric field vector then draws an ellipse in the electric field amplitude space as a function of time.

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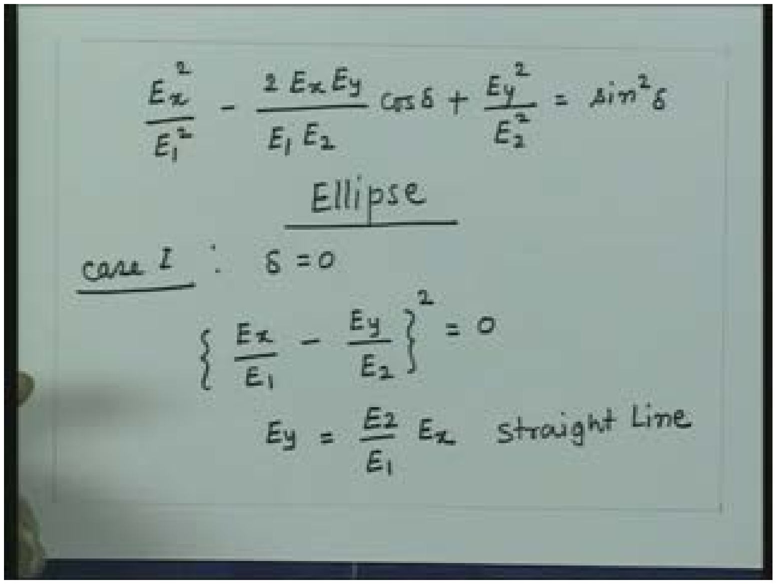

So, if I treat the electric field vector as we saw as an arrow, this tip of the electric field vector will rotate on this circumference of this ellipse as a function of time. How many times it will rotate? It will make number of frequencies times rotation per second. So, if I have a wave which is at 1 gigahertz, it will be making 10 to the power 9 rotations per second. So, in general if I am having electromagnetic wave we can say is a state of polarization is elliptical because the tip of the electric field vector draws a figure which is an elliptical figure.

Then there may be some specialized cases we can investigate that we are having two parameters now that is the amplitudes of the two electric fields and the phase difference between them. Keep in mind here that we are interested only in the shape with the tip of the electric field vector drawn at the function of time, the size of this ellipse doesn't matter. So, as long as the shape of the ellipse is same that means this orientation is same this actual ratio is same, it represents the same state of polarization. So, the absolute value of E_1 and E_2 do not matter as far as the state of polarization is concerned, what matters is the ratio of these two quantities, E_1 and E_2 and the phase difference between the two electric fields E_x and E_y.

So, essentially for defining the state of polarization we have two parameters, one is the ratio of E_2 and E_1 and the phase difference delta. So, we can ask if I change this two parameters what are the different shapes this ellipse can draw. So, we can take the special cases, so this is ellipse this equation. So, I can take a case one and that is if the phase difference between these two electric fields is 0, so let us say delta is 0 and the ratio E_2 by E_1 could be anything it can range from 0 to infinity but when delta is equal to 0, this quantity will be 0, this quantity will be 1 and then this becomes a complete square.

So, essentially in this case the equation will reduce to E_x upon E_1 minus E_y upon E_2 whole square that is equal to 0 or if I write the equation from here I will get this will be E_y is equal to E_2 upon E_1 into E_x. This is now is the equation of a straight line. So, if the phase difference between the two electric field is 0 then the tip of the electric field vector draw this straight line as a function of time and then we can see from here that let us consider a cosine function where t equal to 0 both this will be maximum.

So, this amplitude will be E_1, this amplitude will be E_2 and then as time progresses both this vectors will start reducing, after the quarter cycle this amplitude will become 0, this amplitude will become 0 so both the field will go to 0 and then this will reverse the sign, this will reverse the sign so the point will, resultant vector will go here and so on and so on. So, as the time progresses the tip of the electric field vectors starts from maximum value which will be square root of E_1 square plus E_2 square and then it will move along a line, go to 0 after quarter cycle when both the components go to 0, go to opposite direction maximum after half cycle again next quarter cycle it will go to 0 and so on.

So, the point this tip of the field vector essentially will move along a line which is this straight line whose equation is given by this. So, the slope of this line now E_2 by E_1 so the two amplitudes of this ratio of these two decides the slope of this line. So, we can have within this the special cases when E_2 is equal to 0 that time the slope of the line is 0, so the line is horizontal.

If E_1 is 0 the slope is infinite, so the line becomes vertical or in general the line will be oriented with respect to x and y axis or some arbitrary angle which is tan inverse of E_2 by E_1. Since, the tip of the electric field vector is now drawing a straight line, we call this polarization as the linear polarization.

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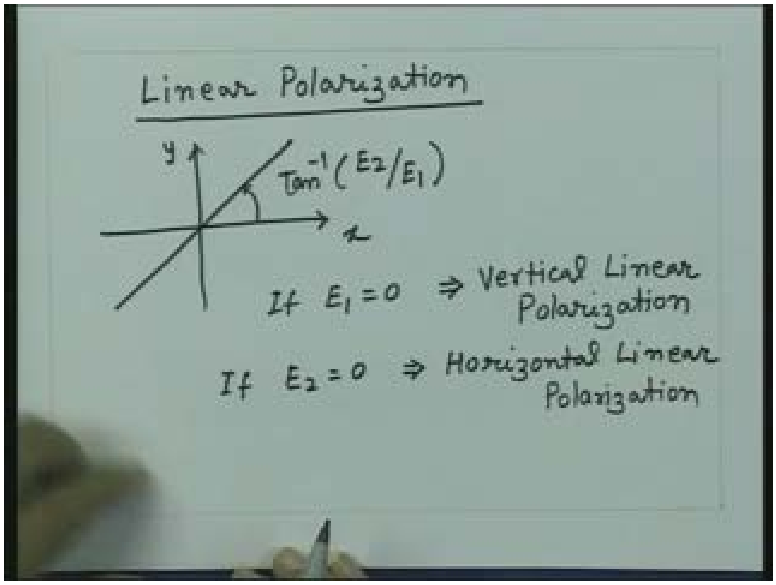

So, one of the cases of the wave is what is called a linear polarization and the condition to opt a linear polarization is that the phase difference between the two electric vectors should be 0. So, the slope of this line which this makes with the x axis, this is x, this is y that is given this angle is tan inverse of E_2 by E_1.

So, as we saw if E_1 is 0 we get the slope 90 degrees, the now line is vertical in that case we get a polarization which is linear and we call that polarization as the vertical linear polarization because line is oriented in the vertical direction. So, we have in this case a vertical linear polarization. Similarly, if E_2 is equal to 0 that gives me the line which is horizontal so we call that polarization is horizontal linear polarization. As a special case when E_1 is equal to E_2 the line will be at the 45 degrees with respect to x and y axis.

So, in general we see that if the phase difference between the two electric field vectors is 0 then you always get a polarization which is linear polarization. This is one of the special cases of the general ellipse.

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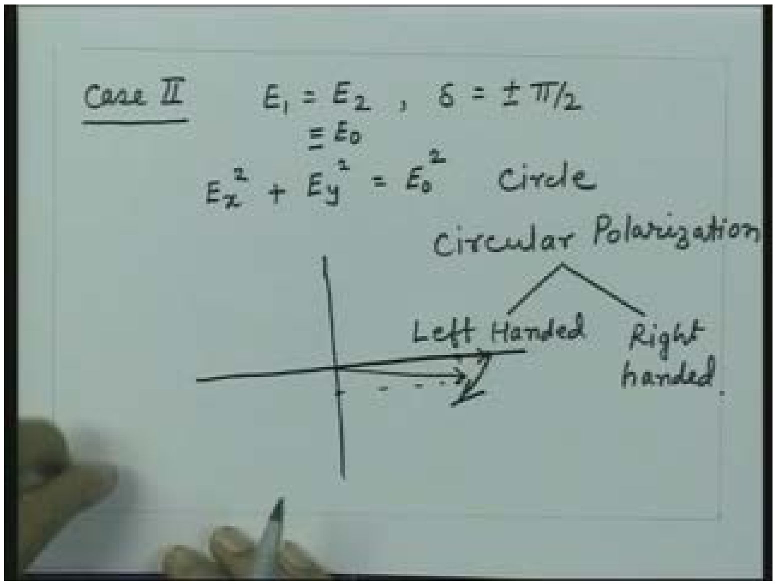

Second special case we can have is if E_1 is equal to E_2 and if the phase difference between the two is plus minus pi by 2. So, delta is equal to plus minus pi by 2. If I go by this equation again and substitute this value for E_1 equal to E_2 and delta is equal to plus minus pi by 2, this quantity will be now 0 because delta is plus minus pi by 2, this quantity will become equal to 1 then E_1 is equal to E_2 let us say that is equal to E_0. So, this equation now becomes from here E_x square plus E_y square that is equal to E_0 square.

So, in this special case when the two amplitudes are equal and when the phase difference is plus minus pi by 2, though in general equation of ellipse reduces to equation of a circle. So, this is a circle and that is the reason this state of polarization is called the circular polarization. So, in this condition we get a polarization what is called a circular polarization. That means the electric field vector draw the circle that means the amplitude of the electric field remains constant that the function of time only its orientation keeps on changing as a function of time. We can go back to our original figure and try to see this how it draw the circle as a function of time. So, let us say the phase difference between these two now is pi by 2. So, that means when this point is maximum, this point is 0 then as the time progresses this will come down let us say the delta is plus pi by 2.

So, when this was maximum this point would be zero and this point is reducing, this just comes down like that. If I find out now the vector sum of these two electric fields I will get the resultant vector whose magnitude will remain constant because if delta is pi by 2 this quantity has become sine of omega t. So, resultant magnitude will be cos square omega t plus sine square omega t which will be equal to 1. So, the magnitude of the electric field vector will remain constant and then as time changes the direction of this vector will keep changing.

So, when the two amplitudes become equal and if delta is plus minus pi by 2, we get this special state of polarization which is called circular polarization. Now whether delta is plus pi by 2 or minus pi by 2, the shape which the tip draws is a circle. Does that mean that the sign of this doesn't come into picture? If I take plus pi by 2 or minus pi by 2 would that not matter to a state of polarization or would that mean that they represent the same state of polarization, the answer is no. Although, the shear which is drawn by the tip of the electric field vector is circle, the way this circle is drawn that will depend upon the sign of this phase difference delta. Let us see how.

So, let us say I consider delta is let us say plus pi by 2. So, when this thing is maximum, this thing will be passing through 0 and then this will become negative, this will start reducing. Since, t equal to 0 if I take this function, this will be now amplitudes are equal so this amplitude is 0. So, at t equal to 0 since delta is pi by 2, this is sine of omega t minus sine of omega t. I will get the amplitude to be 0, so at t equal to 0, the E_x is here but the E_y is here.

If I take a time little later from 0, this vector would have changed from here to here but this vector would have gone somewhere here. So, t equal to 0 the resultant vector was lying somewhere here, if I take a time little later this x component is coming here, y component is here so my resultant vector direction would be somewhere here. So, that means as the function of time if delta is positive as you have taken here plus pi by 2 the vector has rotated from here to here which is like that.

On the other hand if this delta was negative then the electric field this will be lagging behind this, so this will be omega t minus pi by 2 and in that case when this is maximum, this would be 0 but little later this point when it comes here this point would have gone up, it will be increasing. So, if delta was negative I would get a resultant vector which would be shifted from this to this whereas if I take delta positive then the resultant vector would shift in the downward direction.

So, now we see the implication of the sign of delta that is if the y component of the electric field leads the x component then we get a rotation of this field vector in the clockwise direction whereas if delta is negative then we get a rotation which is in the anticlockwise direction. Of course again the clockwise and anticlockwise in three dimensional space is a relative thing, we have to define still some difference directions if you want to talk about clockwise and anticlockwise directions. So, what we do is we define the reference direction that is the direction of wave propagation which is in this case is the one which is coming out of the paper that is positive z direction.

So, whenever we define the rotation of the field vector as a function of time, we take the reference x y z coordinate system. We assume that the wave is traveling in the positive z direction and then we say that if delta is positive, we will have a rotation of the electric field vector which is clockwise if delta is plus, it will be anticlockwise if delta is negative. Generally instead of talking in terms of clockwise and anticlockwise, we prefer to talk in terms of the rotation towards your right hand or the left hand. So, in this case if the wave is coming like that, when delta is positive the electric field vector will rotate like this which will be on my right. If delta is negative then the vector will rotate like that which will be towards my left. Again the left and right will depend upon what reference direction I am taking.

So, we now we have to take definition due to define some references and then we can talk about this right hand, left hand or the clockwise and anticlockwise direction. So, the convention which is followed for defining the rotation of the electric field vector is that if you look in the direction of the wave propagation, so if observer orients himself in the direction of the wave propagation that means he sees the waves receding from him and then if the vector rotates towards its right hand we say that wave is right handed polarization and in this case if the shape drawn is circular then we say that wave is a right handed circularly polarized wave.

Similarly, if the wave if you orient yourself in the direction of the wave that means wave is residing from u and if the electric field vector rotates towards your left hand then you say that this wave is left handed polarized and left handed circularly polarized if the shape drawn by the tip is a circle. So, the circular polarization can be classified into two categories and that is a left handed circular polarization and the right handed circular polarization. So, we have two possibilities here this left handed and right handed.

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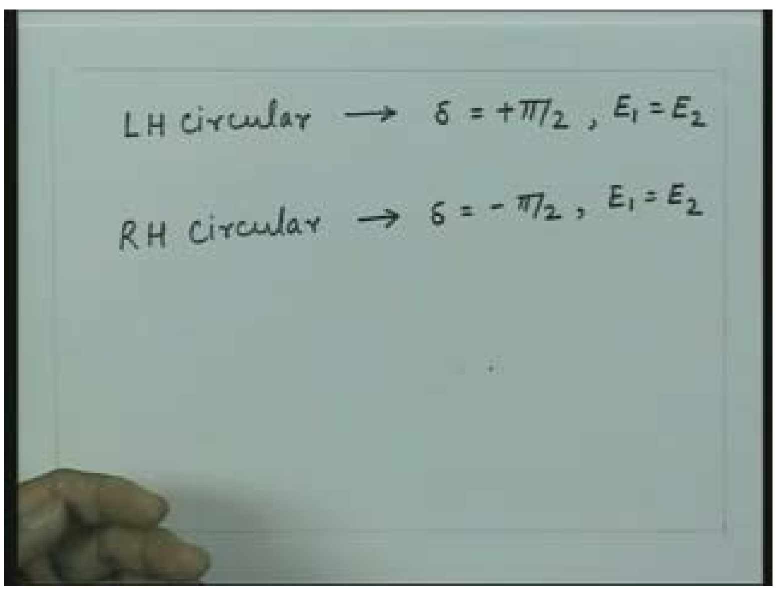

So, in general circular polarization we have LH circular, we have RH circular and for this the condition is that now if the wave is receding so that means wave is coming like that, so first I orient myself so that I look in this direction. If I do that then this rotation will be towards my left hand and this rotation as we saw corresponds to delta positive, so when delta is plus pi by 2 that was the rotation, if delta was delta is negative this was the rotation. So, that means any value of delta which is negative that means E_y lagging E_x gives me a rotation which is right handed rotation.

Similarly, if delta is positive that means E_y leads E_x then the rotation will be the left handed rotation. So, we see from here the left handed circular polarization corresponds to delta is equal to plus pi by 2 and E_1 equal to E_2 and right circular polarization corresponds to delta is equal to minus pi by 2 and E_1 is equal to E_2. So, when we defined the state of polarization, only the shape with the tip of the electric field vector draws is not adequate because it does not define uniquely the state of polarization. The shape with the tip of the electric field vector draws combined with the way it is drawn or the sense of rotation that defines completely the state of polarization.

As the general case then when none of these conditions are satisfied that we saw for circular polarization in the case two, so in general when E_1 is not equal to E_2 delta is not equal to plus minus pi by 2 or 0 then we will have in general the ellipse drawn.

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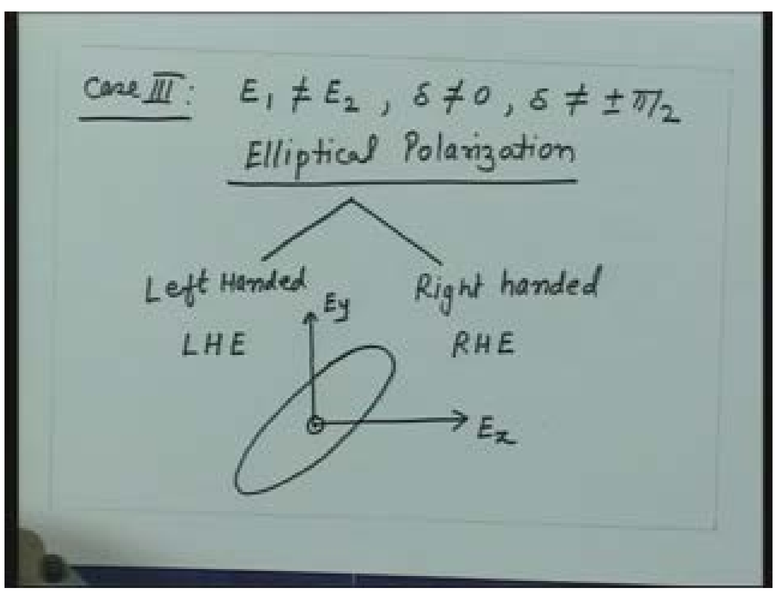

So, the third case which means the general case where E_1 is not equal to E_2, delta is not zero or delta is not equal to plus minus pi by 2. In this case we will have in general the elliptical polarization. Again as for the sense of rotation is concerned the argument which we had in case of the circular polarizations are valid, that is if delta is positive then it will have a sense of polarization which is left handed, if delta is negative then we will have a sense of polarization which is right handed. So, again as we did in case of circular polarization, we can classify even the elliptical polarization into two cases and that is left handed elliptical and we can have right handed elliptical.

So, I can have left handed elliptical polarization and we can have right handed elliptical polarization. So, in general in this case the shape will be drawn which will be an ellipse is your E_x is your E_y and this is the direction of the wave which is coming out of the paper. So, if the ellipse is drawn like that in the anticlockwise direction then that will be right handed polarization, when if the ellipse is drawn in the clockwise direction that will be the left hand polarization.

Keep in mind the wave is coming out of the plane of the paper, so first we have to orient yourself so that you see in the direction of the wave then you see whether the rotation is towards your right hand or towards to your left hand and that gives you the right handed or the left handed sense of polarization. The special case of this elliptical polarization we say well this quantity E_1 is not equal to E_2 but let us say delta was plus minus pi by 2 in that case your general equation which you have here is delta is plus minus pi by 2. So this quantity is 1, this quantity is 0 but E_1 is not equal to E_2.

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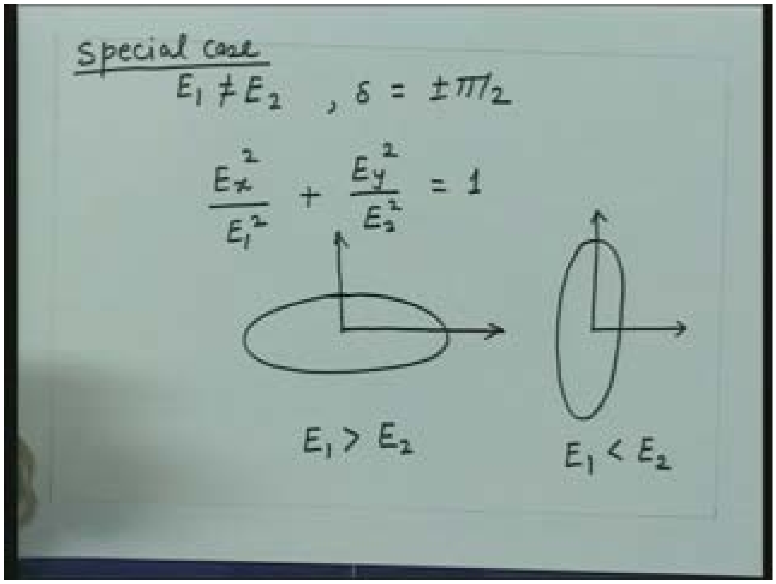

So, the special case of this ellipse if you take the equation of this ellipse will be if E_1 is not equal to E_2 but delta is equal to plus minus pi by 2. This is a one of the special cases. In that case your equation of ellipse will simply become E_x square upon E_1 square plus E_y square upon E_2 square that will be equal to 1 and we know the property of this ellipse that for this ellipse, the axis of the ellipse that major and minor axis are aligned along the coordinate axis. So, this ellipse looks something like this or it will look something like that depending upon whether E_1 is greater or E_2 is greater.

If E_1 is greater than E_2 I will get an ellipse which will look like that, if E_1 is less than E_2 we get the ellipse which will look like that but in both the cases the axis x and y or E_x and E_y are oriented along the major and minor axis of this ellipse.

So, a special case for the elliptical polarization where the major and minor axis of the ellipse are oriented along the coordinate axis will correspond to delta x plus minus pi by 2 and again plus minus will be referred whether the rotation is left handed or the rotation is right handed.

So, now in general we see that the polarization of a wave is elliptical and the other two cases which you have seen earlier the linear and the circular, they are in fact the special cases of general elliptical polarization. If we take an ellipse and if we compress this ellipse, this ellipse becomes a line so you get a linear polarization. If I stretch the ellipse so that the major axis becomes equal to minor axis for the ellipse that becomes a circle. So, even the first two cases which we discussed are the special cases of this general polarization of ellipse which is the elliptical polarization.

So, in practice you can have variety of polarization and we will discuss little later depending upon the application some polarization might have advantage over the other. So, you may see in practice the linear polarizations, you may see in practice the circular polarizations and in general we may see in practice the elliptical polarization. So, let me just summarize what we saw here now. So, we got in general three types of polarization linear, circular, elliptical. The linear polarization, the electric field drawn is along a straight line so it oscillates on this line. A circular we may have like that. Let us assume now the wave is going away from you which is going inside the paper. So, let us say this is the direction of the wave.

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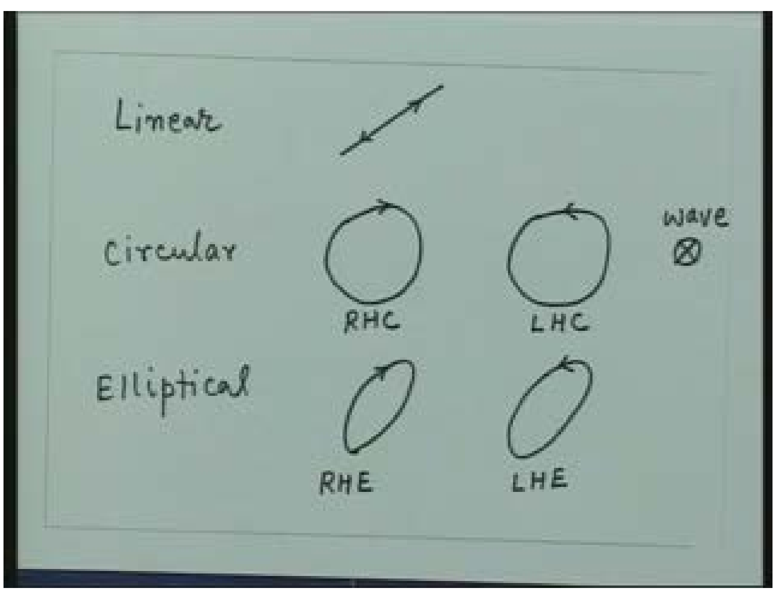

So, wave is now going inside the paper that means wave is receding from you. So, if I look at this circle, this rotation now is towards my right because I am seeing in the direction of the wave which is receding from me so this gives me the right handed circular polarization. Similarly, this gives me left handed circular polarization and then you may have an ellipse which in general would be like that. They are again this will give me the right handed elliptical polarization and left handed elliptical polarization.

So, now if we define analytically certain parameters of the ellipse we can actually capture the parameter what is called wave polarization by some analytical quantity for the parameters of the ellipse. As we already mentioned the size of the ellipse doesn’t come into the picture, so if I want to characterize an ellipse which will correspond to a state of polarization then essentially there are two parameters which are required to define the characteristics of an ellipse and that is what is the orientation of this ellipse with respect to let us say x axis and what is the bulge of this ellipse which is captured by the ratio of the major and minor axis.

So, if I define the axial ratio for this which is the major and minor axis and if I define the orientation of the major axis with respect to the x axis by these two parameters the ellipse can be uniquely defined. So, essentially now what we do is having understood that it is the general behavior of the tip of the electric field vector is a function of time, we try to define the state of polarization by an analytical parameters and that is by the parameters of an ellipse.

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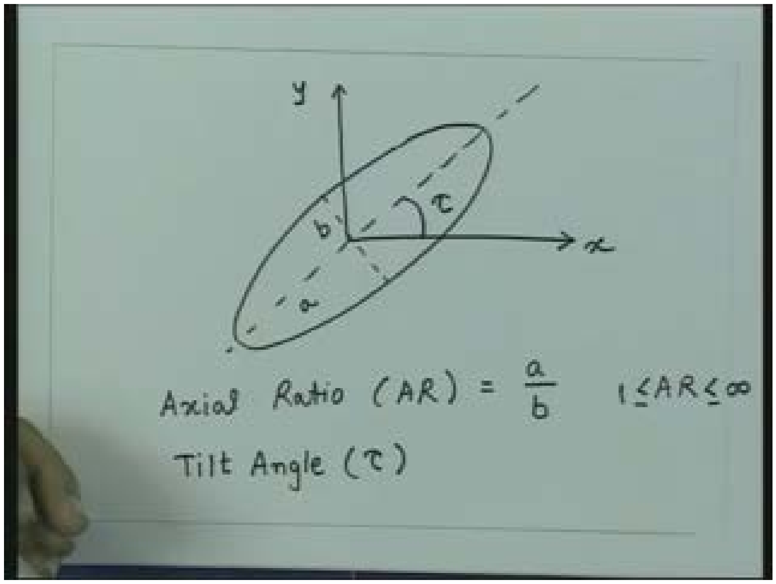

So, let us say we have an ellipse is like that, this is the x direction, this is the y direction, this is the major axis of the ellipse and let us say the semi major axis is given by a and the semi minor axis for this ellipse is given by b. So, we define now a quantity which is the ratio of the major and minor axis which is called the axial ratio of the ellipse. So, we define axial ratio let us call it AR that is a by b and the angle which the major axis makes with the x axis that angle let us say that angle is given by tau and that angle is called the tilt angle of the axis, so we have a parameter called tilt angle tau.

Now, the extreme cases we are having on a and b as we saw as that if the ellipse is compressed into a line, the b will become 0 if we stretch so that this ellipse become a circle then a will become equal to b, so this quantity a upon b essentially can range from 1 to infinity. So, you are having the range here for AR which will be 1 to infinity. The 1 would correspond to the circular polarization, the infinity will correspond to linear polarization.

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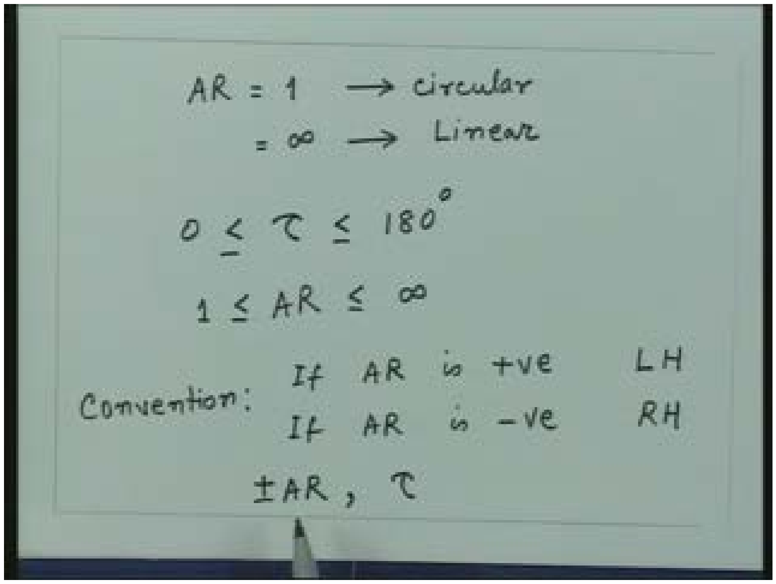

So, this thing so from here we get AR is equal to 1 that gives me circular equal to infinity that gives linear and the tilt angle tau if you rotate this axis, this start from 0 here you rotate this axis when tau becomes equal to 180 degrees that is a maximum ellipse you can have that because beyond that there is symmetry there. So, the range of tau is from 0 to 180 degrees and the range of the axial ratio AR is from 1 to infinity. So, parameter tau can go from 0 to 180 degree and as you mentioned earlier the AR will be 1 to infinity.

So, if I defined these two parameters the tilt angle and the axial ratio then we can define the shape of this ellipse uniquely. However, you recall that only shape of this ellipse is not enough we have to capture the rotation of the ellipse also that means the way the ellipse is drawn whether it is this way or that way that also you needed to define the state of polarization uniquely. These two quantities which we are defined here the axial ratio and tilt angle they define only the shape of the ellipse, there is no mechanism here to define the way the ellipse was drawn.

So, what we do to put that information of way the ellipse is drawn into these parameter, we assign a sign to the axial ratio and follow a convention that if the sign is positive then there is one sense of rotation, if the sign is negative there is another sense of rotation. So, what we now follow a convention is if the axial ratio here is positive, time artificially putting a sign on the axial ratio. If the axial ratio is positive then we say that sense of rotation is left handed, if the axial ratio is negative then we get the sense of rotation which is right handed.

So, I now define a convention here that if AR is positive that gives you the left handed. If AR is negative then it gives me the right handed. So now the value of AR, the bulge of the ellipse we get from this number which can range from 1 to infinity then depending upon the sense of rotation we assign a sign to this, so this value would be ranging from plus 1 to infinity or minus infinity to minus 1 depending upon which sense of rotation I have. So, if the sense of rotation is left handed then the AR would lie between 1 and infinity, if sense of rotation is right handed the value would lie between minus infinity and minus 1.

So, essentially now I have got these quantities, so complete description of state of polarization now can be given by plus minus AR and the quantity is the angle tau. So, now I can treat the state of polarization by a pair of this two quantities AR which is sign plus the tilt angle. So, I can now analytically treat a pair of these two numbers to define uniquely the state of polarization of a wave. We could have done the similar thing in terms of the electrical parameters also. So, instead of looking at the shape with the tip of the electric field vector is drawn if I had gone to my original figure here also I have two parameters which is a ratio of these two E_2 by E_1 and delta. So, I could have taken these two parameters also E_2 by E_1 and this pair delta with this ratio and delta that will also define uniquely the way the ellipse is drawn.

So, now I have two pairs one is the ratio of this two amplitudes and the phase difference and the sign value of axial ratio and the tilt angle of the ellipse. We can call these parameters as the wave parameters and these parameters as the electrical parameters because they are defining the amplitudes and the phase difference between the two electric fields. So, a state of polarization now can be defined in two ways and they are equivalent either I can define the state of polarization in terms of this wave parameters which is axial ratio and tilt angle or we can define the parameters in terms of the ratio of these two and the angle delta.

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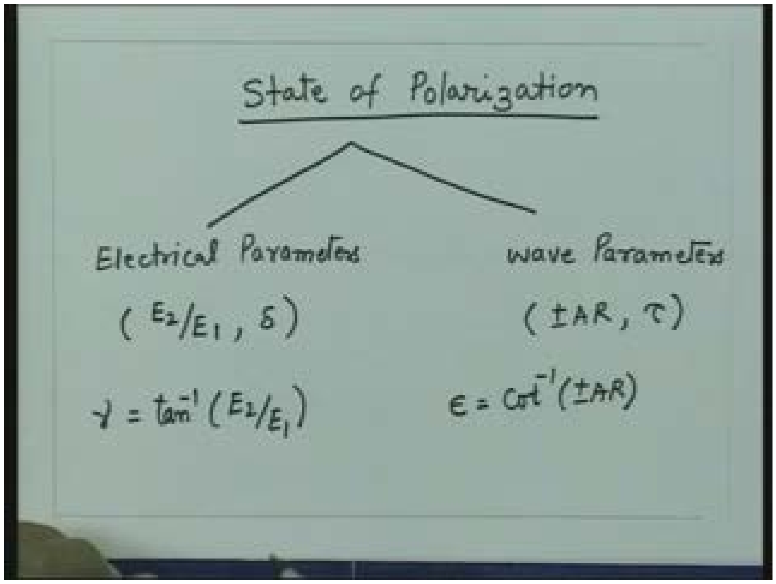

So, what we say that a state of polarization can be defined by two pairs. We say this is electrical parameters or wave parameters. The electrical parameter the pair is E_2 by E_1 and delta and for wave parameters the pair is plus minus axial ratio and the tilt angle tau. Why we require this equivalence between these two are description of the state of polarization in the two domains the electrical parameter domain and the wave domain is for simple reason that many times we would like to generate a state of polarization and we would like to know what way this polarization should be generated by exciting two electric field which are horizontal and vertically polarized. So, if I had two systems which are generating linear polarization, one which is horizontal and one which is vertical.

One can ask a question what should be the amplitude of these two and the phase difference between them so that I can generate a specific ellipse which I wanted to generate when the wave was launched. So for generating a particular shape of ellipse or a particular wave of the elliptical polarization, one would ask what parameter should be set electrically while generating this wave. So, we need a relationship between this and this.

Conversely one can ask a question more like a analysis question that if I had two linear polarization vertical and horizontal and if I excite in certain proposition with certain phase difference what kind of ellipse will be generated. So, this problem from here to here will be more like the analysis problem but from here to here will be more like a synthesis problem because this will define the need which we want for polarization and then we would ask what should be electrical systems parameter so that that particular state of polarization will be generated when the wave is launched.

So, we will see the relationship between these two parameters. However, what we do here these two quantities the axial ratio is a number whereas this is an angle, this quantity is a ratio and an angle. For defining the state of polarization more compactly what mathematicians have done is they have defined all this quantities in terms of angles. So, what is done is you define a parameter called gamma which is tan inverse of E_2 by E_1 and you define cot inverse of plus minus AR as the angle corresponding to this.

So, what we got here the ratio is now also defined in terms of this angle gamma, the axial ratio also is defined equinity in terms of this angle we call this angle let us say epsilon. So, the state of polarization electrical domain now is defined by pair of angles gamma and delta or in the wave parameters it is defined in terms of two angles epsilon and tau. We will follow this convention and when we meet in the next lecture essentially we will see how the state of polarization is compactly represented in terms of this pair of angles and then we will also discuss the conversion from one parameter to another parameter and then we will also go into understanding how orthogonal states of polarization generated once we have a understanding of the state of polarization in this two domain.