Transmission Lines & E. M. Waves
Prof. R. K. Shevgaonkar
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture – 22

In the previous lecture we started discussing the solution of the Maxwell's equations in an unbound medium. We saw that the simplest solution which can exists in an unbound medium is an electric field which is constant in a plane containing the field vector and have a variation perpendicular to the vector.

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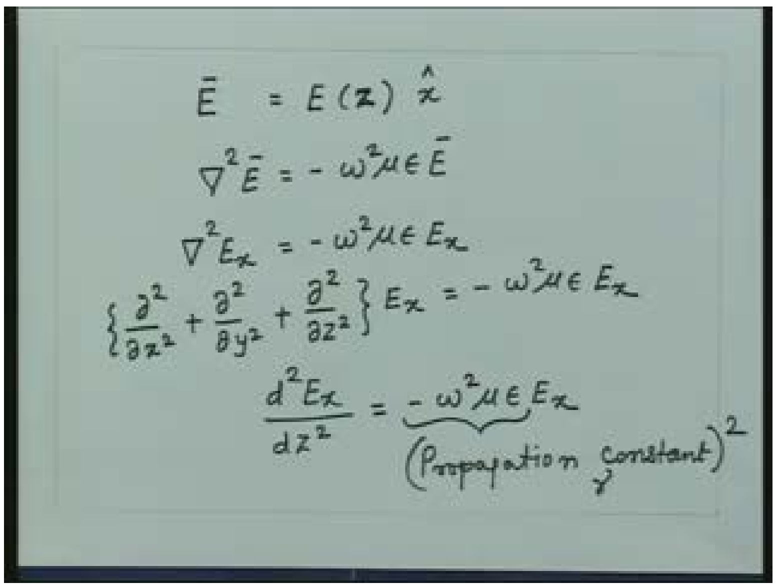

So, without losing generality we consider the electric fields which were x oriented and they had a variation only as a function of z that maybe assumed that the electric field was constant in the xy plane. Then substituting this electric field in the wave equation essentially we got a one dimensional equation which was identical to the transmission line equation and well then we defined this propagation constant in this case which was purely imaginary, see we got only the phase constant in this case which was beta that is same as omega square root mu epsilon.

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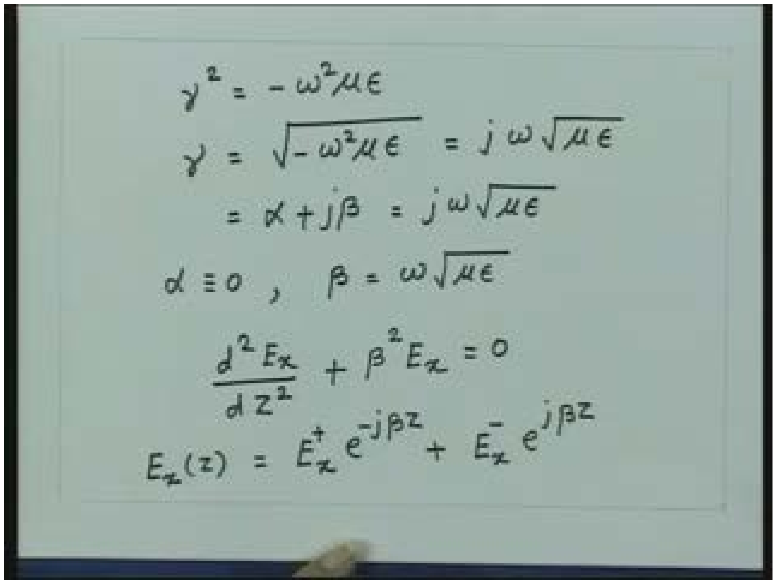

And then this differential equation which was second order differential equation had in general has a solution which was representing essentially two waves traveling in opposite directions, the positive z and the negative z directions with amplitudes E_x plus and E_x minus. Now, we can substitute this solution of the electric field again in any of the Maxwell's equations to find out the relationship between the electric field and the magnetic fields. Say as we did earlier we can take the curl equation and substitute the electric field, so we can get here x y z.

Now, since the electric field is the function of z only d by dx is 0, d by dy is 0 and d by dz and however x component of the electric field there is no y component in z component and that should be equal minus j omega mu into h. And E_x now E is a solution which is this so it has two components which are representing two traveling waves.

So, I can substitute now this E here so this will be d by dz of E_x which is E_x plus E to the power minus j beta z plus E_x minus E to the power j beta z that is equal to minus j omega mu and the components the magnetic fields, so that is hx x plus hy y plus hz z.

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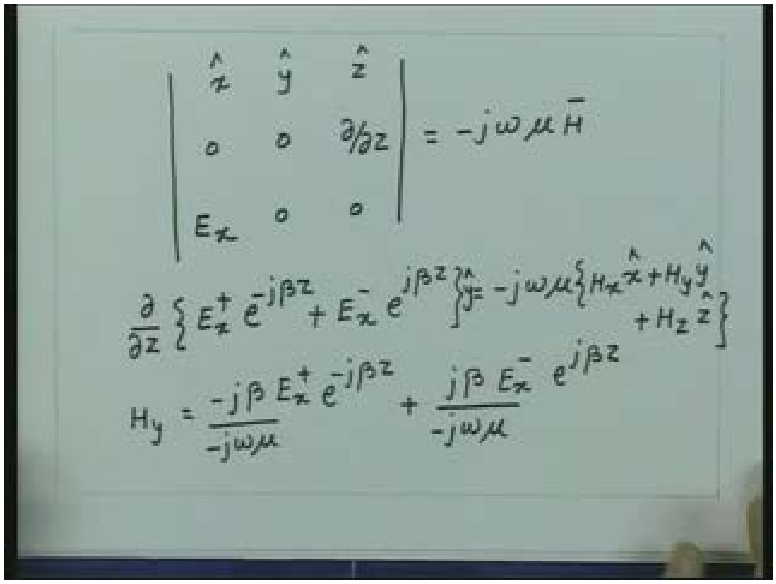

And this component is we have seen earlier this is d by dz of x, this is the y component, so essentially this is the y component. So, as we have seen earlier the H_x in this case is 0, the H_z is 0 and essentially the y component is given by d by dz of this point. So, I can now expand this and I can get from here H_y that is equal to, differentiating this with respect to z that will be equal to minus j beta E_x plus divided by this quantity which is minus j omega mu which is divided by minus j omega mu e to the power minus j beta z plus j beta E_x minus e to the power j beta z divided by minus j omega mu. So, the magnetic field which is having now the y component also has variation which is wave variation because this term e to the power minus j beta z represents the traveling wave in positive z direction, this will represent the traveling wave in negative z direction, so H_y also has two components corresponding to a two traveling waves.

So, from here the magnetic field now if I cancel out this term minus j, I will get the y component of the magnetic field that will be beta upon omega mu E $_x$ plus E to the power minus j beta z minus beta upon omega mu E minus E to the power j beta z, so Hy which has now two components corresponding to a two traveling waves. So, the magnetic field for this wave if I take the traveling wave which I can take like a forward traveling wave in positive z direction.

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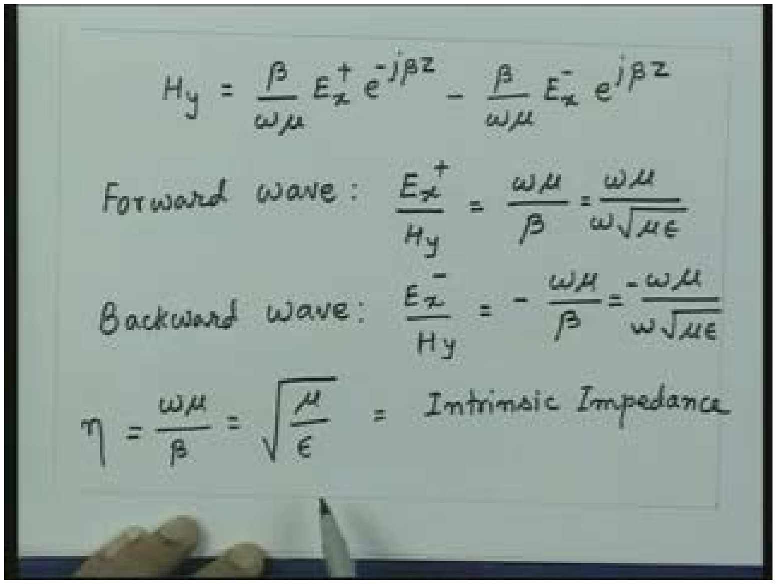

The H_y upon E_x plus or E_x plus upon H_y that quantity will be so far let us say forward wave which is given by this term. I have E_x plus divided by H_y that will be equal omega mu upon beta and for a backward wave which is given by this term, the ratio of E_x minus by H_y that will be equal to minus omega mu upon beta. I can substitute for this beta which is the phase constant and we saw earlier this is omega square root mu epsilon. So, I can take this value of beta and substitute into this. So, this quantity will be omega mu upon omega square root mu epsilon and that will be equal to omega mu upon minus omega mu omega square root mu epsilon.

Omega will cancel in this case, I will get a quantity square root of mu upon epsilon, here I will get a quantity minus square root of mu upon epsilon. Now, if I look at this quantity omega mu upon beta which is from here square root of mu upon epsilon. So, we have here omega mu upon beta that is equal to square root mu upon epsilon and we know the units of this mu which is Henry per meter, this is farad per meter or we can go by the units of this quantity that the electric field has the unit volts per meter and the magnetic field as unit of amperes per meter. So, this quantity what we have here has units of impedance is the volt upon current. So, this quantity has unit of impedance. Whatever this quantity is, it is related to the medium parameters of mu and epsilon and also has the unit of impedance. So, this quantity is similar to what we had got for transmission line which was like characteristic impedance.

There also we had a ratio of the voltage and current for traveling waves which were characterized by the medium parameters, inductor transmission line parameters here we have medium parameters. So, this quantity is identical to the characteristic impedance as we have seen on transmission line. However, for the wave propagation in the three dimensional space, this quantity is referred to as the intrinsic impedance of the medium and normally it is denoted by eta. So, we have this quantity eta which is square root of mu upon epsilon and is called the intrinsic impedance of the medium and this quantity play the same role in this three dimensional propagation as the characteristic impedance used to play in one dimensional transmission line case.

So, we have a characteristic impedance which is now decided by the medium parameters. If I take a specific case that to the medium was the free space if we take unbound medium like free space, we know the value of mu which is mu 0, we know the value of epsilon which is epsilon 0, so I can substitute for this. So, if my unbound medium was free space then mu is equal to mu 0 is 4 pi into 10 to the power minus 7 Henry per meter and epsilon is equal to epsilon 0 that is 1 upon 36 pi into 10 to the power minus 9 farad per meter.

If I substitute this value for mu and epsilon in the expression for the intrinsic impedance then I get eta and for free space I can denote this quantity as eta not that is equal to square root of 4 pi into 10 to the power minus 7 divided by 1 upon 36 pi into 10 to the power minus 9, so that is equal to 120 pi ohms.

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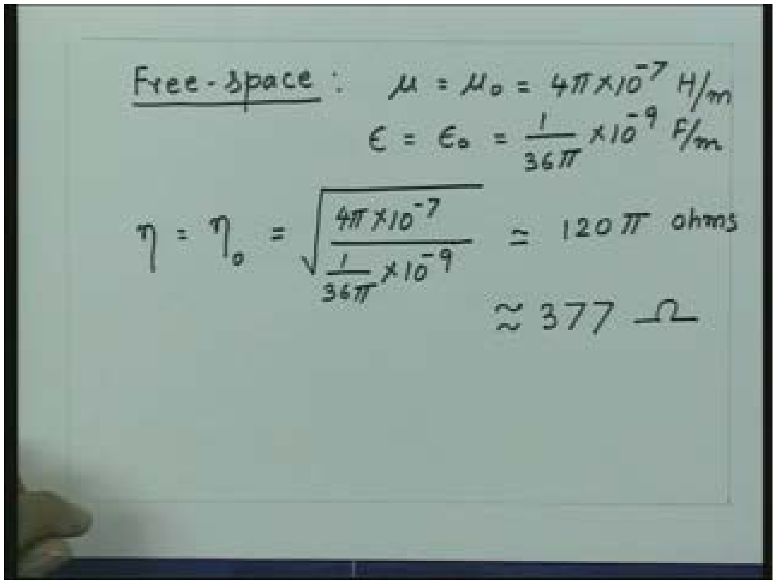

If I take the value of pi 3.14, this quantity is approximately 377 ohms. When you are discussing transmission lines, we are seeing that the ratio of the voltage and current for a traveling wave is always equal to the characteristic impedance. The same statement now we can make for the traveling wave in the three dimensional space that for this wave the ratio of the electric and magnetic field for a traveling wave is equal to the intrinsic impedance of the medium and for free space, this intrinsic impedance is 120 pi or 377 ohms. So, as we use the terminology that a forward traveling wave always sees the characteristic impedance.

We can say here also that a forward wave in this medium, unbound medium always is an impedance which is the intrinsic impedance of the medium and that is for free space 377 ohms. That means if I look at the vacuum, the free space then the waves is as if it is going into an impedance of 120 pi ohms or 377 ohms. So, a medium like vacuum or free space appears like a impedance when the wave travels in this medium. So, a free space appears like a resistance, so if the wave travels in this essentially the wave is delivering power to this medium.

So, if I see from the system which is generating this wave it is as if the system has delivered power to the space. So, the space appears like a resistance which consumes power, so if I see from the system point of view which was generating a wave, the wave was given from system to the space so the space has consumed the power. So, the power was given to a resistance which is of a value 120 pi or 377 ohms. Now, this is very interesting that when in the medium there is nothing there, if the free space that medium appears like a resistance of a value of 377 ohms.

So, later on when we go to the discussion on antennas or other propagation that time this concept will be very handy that the space will be treated like an impedance of 377 ohms to which the power is to be delivered. So, from here then we see that once we get this parameter which is the intrinsic impedance of the medium, the ratio of the electric and the magnetic field is equal to the intrinsic impedance of the medium which is eta and that is true for both the cases however there is a negative sign here. And if you remember similar thing we had for the transmission line case also that a forward traveling wave used to see an impedance which is characteristic impedance but the backward traveling wave used to see negative of the characteristic impedance and that time we had understood that if the power is traveling backwards it is as if the power is supplied to the generator which is equivalent to a negative resistance.

So, the negative sign was equivalent to the reversal of the power flow. However, in this case when we are talking about this quantity E_x and H_y, we have a negative sign here and that should also referred in some sense to the same phenomena which we have seen on transmission line. So, this we will see little later whether it would represent the flow of power backwards and so on.

However at this point we can say that the ratio of electric and magnetic field is eta if I take E_x as the electric field and H_y as the magnetic field. However, if I take x oriented electric field and y oriented magnetic field but if the wave was traveling in the backward direction then the impedance seen by the wave is negative.

So, for a electric and magnetic field which are now perpendicular to each other, one has x component one has y component, if the wave is forward traveling wave then the ratio of these two is equal to the characteristic impedance or the intrinsic impedance, if the wave is traveling backwards then the ratio of these two is negative of the intrinsic impedance. Although, we have taken the electric field here which is x oriented, we could have taken actually the electric field in any direction in the xy plane because what we have seen is the electric field should be constant in a plane containing the vector and once we say that plane is the xy plane, the electric field could orient anywhere in the xy plane. That means if I consider this plane of the paper like xy plane, any arbitrary direction of the electric field satisfy this condition that it is constant in the plane containing the vector.

So, here we have done the analysis for the x oriented field but I could have taken even the y oriented field and all arguments which are there will be value. So, essentially we can say that in general if I had electric field which was oriented in any arbitrary direction, we can always decompose this electric field into two components is x component and y component and then substitute into this equation and from there again I can find out the relationship between the electric and magnetic field components.

So, as we did the analysis for the x oriented field if I have done the analysis for the y oriented field I would again get the equation very similar to this. So, if I consider y oriented field that is my E is E_y function of z and this is oriented in y direction and if I substituted this into the wave equation I would again get the identical equation as this with E_x replaced by E_y. So, for this also I will get a solution that will be your E_y as a function of z will be E_y plus e to the power minus j beta z plus E_y minus e to the power j beta z. After this point everything is identical to what we got for the x oriented field.

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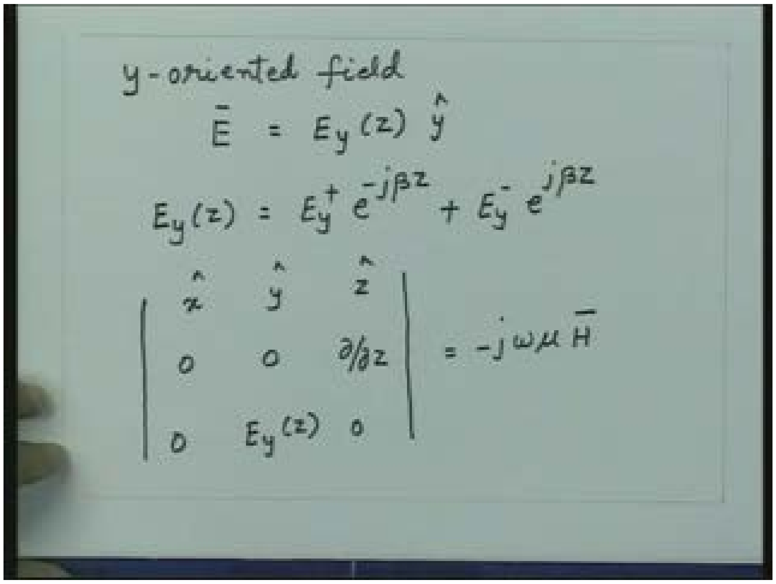

However, when we find out the relationship now for the E_y with the h, now in that curl the x component is 0 and electric field is now y component, so essentially we have to, here we had x component. We saw x component, now we have to substitute for y component and do the same thing as we did for the x oriented electric field. So, if I substitute now for the y component the curl equation I will get here x y z. Again field is not varying in the xy plane, it is varying only in the function of z but the field is y oriented, so I have here E_y as a function of z, 0 that should be equal to minus j omega mu into h.

So, now the magnetic field will not be y oriented because the y component will be 0, the magnetic field will be x oriented and that will be d by dz of E_y. So, from here I can get this will be this minus this, so minus d by dz of E_y. So, it will be minus d by dz of E_y to the function of z that will be equal to minus j omega mu into H_x x plus H_y y plus H_z z. Again what we will see here is that this component is now the x component, so this is the x. So, I have H_y is equal to 0, H_z is equal to 0 and I have only the x component. So, from here I get H_y is equal to 0, H_z is equal to 0 and the x component which is minus d Ey z dz that is equal to minus j omega mu into H_x.

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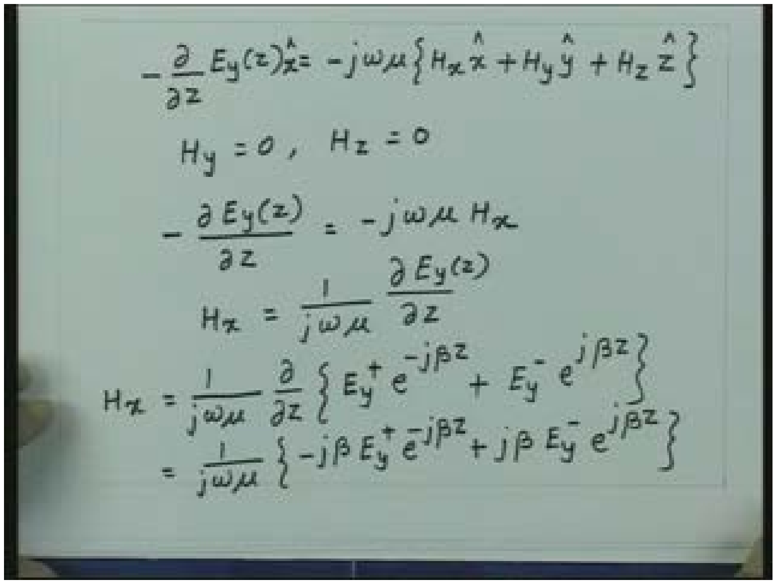

So, essentially H_x is equal to 1 upon j omega mu d E_y by dz function of z. And if I substitute now for E_y from here I get H of x that is 1 upon j omega mu d by dz of E_y plus e to the power minus j beta z plus E_y minus e to the power j beta z. So, that is equal to 1 upon j omega mu minus j beta E_y plus e to power minus j beta z plus j beta E_y minus e to the power j beta z which simplifying gives me H of x is equal to minus beta upon omega mu E_y plus plus beta upon omega mu E_y minus e to the power j beta z. This has the term e to the power minus j beta z.

So, again I got the two traveling waves for the magnetic field x component also and if I take a ratio of this is we have taken in the previous case for the two traveling waves, for the forward wave I have E_y plus upon H_x that is minus beta upon omega mu which is nothing but the intrinsic impendence of the medium with negative sign and for backward wave I have E_y minus upon H_x that will be beta upon omega mu is equal to eta. So, for the forward traveling wave now if I had taken earlier where the electric field were x component and the magnetic field were y component, this quantity was plus eta whereas if I take the electric component as y and the magnetic field component as x then the ratio of this is negative eta.

So, earlier when the quantity is less scalar for transmission line though direction of these quantities were not coming into picture.

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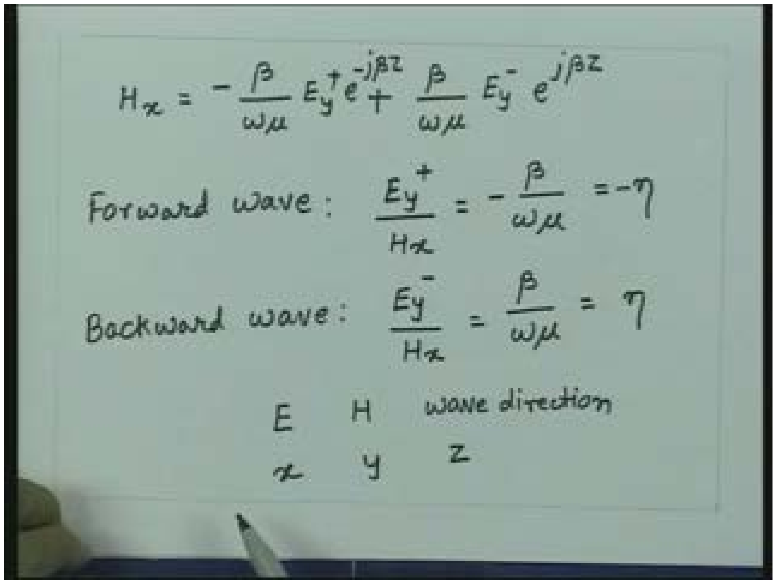

However, what we see now is that when we interchange x and y for E and H, the sign of this the ratio of these two changes from positive to negative. What that essentially means is that the sign or the ratio of these two or the sign of this quantity not only depends upon the direction in which the wave is traveling but also the orientation of the electric and magnetic fields. So, essentially what we have now is that you have E H and the direction of propagation and if you consider this as sequence and if you follow the right handed system, if I have the E to H in the right sequence, so I have E_x and H_y the ratio of these electric and magnetic field is equal to the positive intrinsic impedance but if I take a ratio of E_y and h_x where I am going opposite to the sequence of xy then the ratio become negative because now I am going opposite direction from the right hand rule.

So, if I take a sequence your E to H to the direction of propagation and if I take this quantity as x y and z, if I have from E to H in the right sequence as x and y then the ratio of E and H will be positive. If I am going opposite to this then the ratio of this will be negative. Now, remember here that when we had the electric field which was x oriented, the magnetic field was y oriented and when we have the electric field y oriented, the magnetic field is x oriented that means that electric and magnetic fields are perpendicular to each other and also these fields are having a wave motion which is in z direction, so this is the wave direction. The electric field, the magnetic field and the direction of the wave propagation they are perpendicular to each other.

This wave then is called the transverse electromagnetic wave because you are having electric field which is perpendicular to the direction of propagation, the magnetic field is also is perpendicular to the direction of propagation and also we note from here that electric and magnetic field also per perpendicular to each other. That means E H and the direction of propagation essentially form this 3 coordinate orthogonal axis and if I go from E to H in the right handed sense then I get the direction of the wave propagation. So, if my fingers of my right hand point from E to H then my thumb gives me the direction of the wave propagation.

Similarly, if I go from wave direction to e, the sequence follows x y z, x y z. So, if my fingers point from wave direction to the electric field, my thumb must point in the direction of the magnetic field. So, now the right handed sequence for E H and wave propagation is E H and direction of wave propagation in the sequence x y and z. So, the ratio which we are seeing for the electric and magnetic fields that now if it is follow this sequence then the ratio of electric and magnetic field is positive intrinsic impedance. If you go against the sequence then the ratio is negative intrinsic impedance for the forward traveling wave and the opposite will be true for the backward traveling wave. So, if I take E_y minus upon H_x which is the backward traveling wave, so there is a negative sign because of the backward movement of the wave and also this is going against the sequence they are going from y to x, it should give me one more negative sign, so the ratio of these two is the positive intrinsic impedance.

So, while dealing with this vector fields in three dimensional space, you have to consider two things. One is the orientation of the fields and their sequence and the wave propagation direction. The two together we will decide the ratio of the electric and magnetic fields whether it should be positive eta or it should be negative nevertheless, the total electric field which we are going to see is the magnitude of the electric field which is the E_x and E_y.

Similarly, this is true for the magnitude which is going to get from H_x and H_y for the magnetic field. So, the ratio of the total electric field and total magnetic field is always positive value of eta because the magnitude of the electric field will be square root of E_x square plus E_y square.

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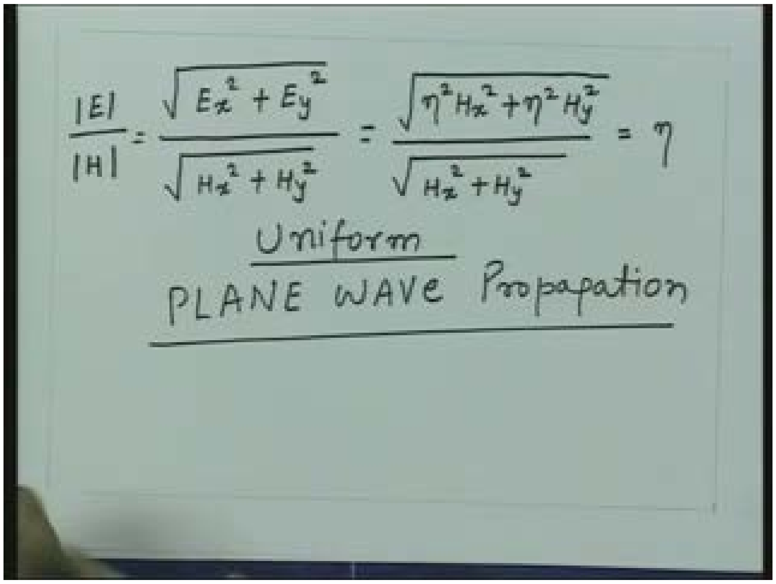

Magnitude of the magnetic field will be H_x square plus H_y square. So, if I say I take a ratio of the magnitude of the electric and magnetic field at any point then I get this. I can substitute for E_x and H_y, I can get here this is square root of eta square H_x square plus eta square H_y square divided by square root of H_x square plus H_y square that is equal to eta.

So, what that means now is that ratio of the magnitude of the electric and magnetic field is always equal to the intrinsic impedance of the medium which is decided by the medium parameters. So, that means in this medium if I know the electric field magnitude then we can calculate uniquely the magnitude of the magnetic field because we know the medium properties, we know the medium parameters like permittivity and permeability from there we can find out what is the intrinsic impedance of the medium. So, the magnitude of magnetic field is not an independent quantity.

If I know the electric field magnitude and if I know the medium parameters, the magnetic field is uniquely defined, this magnitude is uniquely defined. Also if I know the direction of the wave propagation, the electric field, the magnetic field and the direction of the wave propagation are perpendicular to each other and as we said the sequence is from E to H to the wave propagation that means if I know the direction of wave propagation and the direction of the electric field then the direction of magnetic field is uniquely defined because I know if I go from E to H, my finger points from E to H I must get the direction of the wave propagation which is given by the thumb. So, once the medium properties are known, medium parameters are known and if the direction of wave propagation is known, the magnetic field is completely dependent quantity on the electric field or vice versa.

So, we don't have to define the electric and magnetic field separately, we can just define the vector electric field and the magnetic field is completely characterized because the ratio of the electric and magnetic field should be equal to the intrinsic impedance and the magnetic field should be perpendicular to the electric field so that when the fingers point from E to H, the thumb should point in the direction of the wave propagation. That is the reason whenever we do the wave analysis, invariably we talk only about the electric fields because we know that if the electric field is described, the magnetic field is automatically described because magnetic field is completely dependent on the electric field.

So, as and when you require a knowledge of the magnetic field, we can always derive explicitly the knowledge for the magnetic fields but we don't have to carry separate information for the magnetic fields. So, now onwards when we talk about the wave propagation in this medium, we describe the characteristic only for the electric field and as and when we need the knowledge of the magnetic field we can extract it from the knowledge of the electric field. So, this phenomena which we discussed till now is giving you the electric field and the magnetic field which are perpendicular to each other and they are constant in a plane perpendicular to the wave propagation. This phenomena then represents what is called a uniform plane wave. So, this wave, transverse electromagnetic uniform plane wave is the simplest solution which you get for the fields in an unbound medium.

Precisely, that's how it was investigated but originally the light propagation was understood and that's what Maxwell's showed by analyzing that he got the velocity of light correctly by solving this problem by showing that light is a transverse electromagnetic wave. So, since for most of the bulk media, the size of the medium is much larger compared to the wavelength of light unless you go to the structure where the size becomes comparable to the wavelength, we can treat the propagation of light like the propagation in an unbound or very large medium. And that is the reason the light which is an electromagnetic wave is treated as a transverse electromagnetic wave.

So, those applications where the structure becomes small like optical fibres where the size of the structure is compatible to the wavelength, the light is treated like a transverse electromagnetic wave but we will see later if the size of the structure becomes compatible to the wavelength then this transverse nature of electromagnetic wave is lost and there we will have more complex phenomena which we will investigate little later. So, what we have discussed up till essentially is what is called a plane wave propagation or uniform plane wave propagation.

Now, if I look at this electric and magnetic fields for the plane wave propagation the analysis tells me that the electric and magnetic field are perpendicular to each other and they are perpendicular to direction of propagation like this. However, nowhere it is said that this electric and magnetic fields cannot change the direction at the function of time. What that means is some instant of time with the electric and magnetic fields are like that and for another instant the electric field changes direction like this and if the magnetic field also changes direction by same amount then whatever relationship which we got between electric and magnetic fields is still valid. So, they satisfy the solution of the wave equation and the Maxwell's equations. So, the solution which we have got says that they must be, at every instant of time they must be perpendicular to each other and they must be perpendicular to the direction of the wave propagation but the solution doesn't say what should be the behavior of electric and magnetic fields as a function of time. In fact this electric field might change the magnitude as a function of time and if the magnetic field changes the magnitude exactly in same proportion and they remain perpendicular to each other they will satisfy the solution. So, that it is possible that the electric and magnetic field both might vary as a function of time and still they will be satisfying the solution if you have investigated. So, in general term we can say a uniform plane wave may have an electric and magnetic field variation. So, they are uniform in a plane but the value of the electric and magnetic field at different times maybe different.

Since, we are taking about the time harmonic fields we can assume that this electric field will be varying sinusoidally. So, we are having some kind of a periodic behavior of the electric and magnetic fields. So, we have possibility that now this electric field in general which is the component of the two orthogonal components x and y, it might vary the function of time and corresponding to the magnetic field also will vary as the function of time.

So, we assume here that the electric and magnetic fields both vary sinusoidally, there maybe possibility that they may behave completely randomly also but whatever is the situation at every instant of time, at every point in space they are perpendicular to each other electric and magnetic fields and their ratio of the magnitude of electric and magnetic field is equal to the intrinsic impedance of the medium.

Now, since we are saying that the electric field can vary at the function of time at particular location in space and if I treat this electric field is like a vector like an arrow, as the function of time the tip of the arrow might be tracing a curve in a plane perpendicular to the wave propagation. Then we can say this behavior if we capture by a parameter what is called the polarization of the wave.

So, we defined the polarization of the wave is the direction of the electric field or if we treat the electric field is like an arrow, the shape which the tip of the electric field vector draws in a plane perpendicular to the direction of the wave propagation as a function of time, we call that as the polarization of the electromagnetic wave. So, now we investigate what is called the polarization of a uniform plane wave which is the variation of the direction of the electric field and its magnitude as a function of time.

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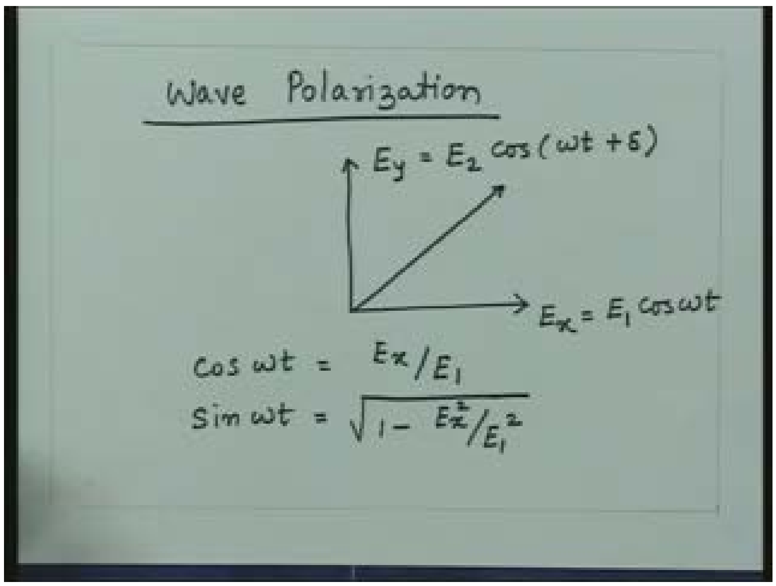

So, we take a discussion on a subject what is called the wave polarization and this is a very important phenomena for any electromagnetic wave because any wave you take, it will have a direction and magnitude of the electric field and in general it will vary at the function of time. So, every way we will have a definite what is called a state of polarization. So, two things might happen when the wave is traveling that at a given point in space the electric field vector might orient and change in amplitude as function of direction. This phenomena we called as wave polarization. Other possibility is that the wave propagates at a particular location the electric field is same but as it moves the electric field might change the directions is possible. This phenomena is not polarization, this phenomena we will see later is what is called the Faraday rotation. So, the polarization is the orientation of the electric field as the function of time at a given point in space and at from point to point things might change but at given location in space, we may say that is the polarization of the wave or that is the state of polarization of the wave.

So, any wave which is generated by a signal will always have a definite polarization. So, let us look at this phenomena which is very important phenomena of what is called the wave polarization. Let us see since the electric field as you are taken in the plane xy plane, the wave is propagating in the z direction can be oriented in any arbitrary direction at any instant of time. So, I can say I can resolve this electric field into two components or alternatively I can say that I can generate any arbitrary electric field by combination of two orthogonal fields, one in oriented in x direction other oriented in y direction.

So, I can say that let us say I have two fields, one is oriented in x direction and I have another field which is oriented in y direction, under combining these two fields I can always get any arbitrary field in this xy plane where the wave will be propagating in the z direction perpendicular to the plane of the paper. So, conversely I can say that any arbitrary wave I can generate by having two sinusoidally varying electric fields, one oriented in x direction, one oriented in y direction.

So, let us say I have this one which is given as some E_1 cosine of omega t and omega is the angular frequency. So, this wave oscillates like this, this amplitude goes maximum of E_1 comes here then change the direction, goes up to minus E_1 and so on. So, by treat it like an arrow, the tip of the arrow will be oscillating like this as a function of time.

Similarly, I can take this electric field which is y oriented and let us say it has a magnitude E_2 frequency is same omega t but it is possible that it might have a phase difference in time with respect to this field, so we can have this has given a sum quantity delta. So, now we are having two fields which are oriented in two direction x and y directions, they have different amplitudes E_1 and E_2 and they may have a time phase difference which is delta. If delta is positive then this field is leading with respect to this and if delta is negative this field will be lagging with respect to this then at any instant of time I can have a resultant field which is a combination of these two.

So, at every instant I can find out this value, I can find out this value vector layer these two components and I will get the resultant electric field which will be this. Then the shape with the tip of the electric field will draw at the function of time maybe if I just take at different times and find I will get some curve here that curve will represent the state of polarization of that wave which is generated by combination of these two perpendicularly polarized wave.

So, I can say that whatever general thing I am talking about is equivalent to as if I have two waves, one for which the electric field vector remains in x direction like that, I have another wave which is having same propagation characteristic but for which the electric field vector was like this it's a vertical. And if I launch these two waves simultaneously into the medium then at any location in space I will always get the vector sum of these two electric fields and I will get the resultant electric field.

So, if I have to find out now the locus of the tip of the electric field essentially we eliminate the time parameter in this and from here we get the locus of the tip of the electric field vector that gives me the equation of the curve which the tip of the electric field vector would draw. So, at any location this is the time variation which is I am going to get and by eliminating the time parameter essentially I can get the locus of this curve.

So, from this equation I have cos omega t that is equal to E_x upon E_1 and from here I will get sin omega t that is equal to square root of 1 minus E_x square upon E_1 square. What we will do in the further analysis that expanding this quantity you will get the term which will be cosine omega t and sine omega t, so we will take now from here cos and sin omega t substitute for that in this equation and that will give me elimination of the time parameter and whatever equation I will get then in the amplitude space for the x and y oriented fields I will get the equation that will represent the equation of the curve drawn by the tip of the electric field vector.

So, when we meet next time essentially by eliminating the time you will get the equation and then we will see what are the different steps of polarization which a transverse electromagnetic wave can generate and what are the implications for transmitting the signals efficiently from one point to another point.

So, in communication in general when we transmit an electromagnetic wave this state of polarization or the nature of the electric field play the very important role and that will decide how much power will be transferred from the wave to the system and vice versa. So, in the next lecture we will investigate in detail this important parameter what is called the wave polarization of a transverse electromagnetic wave.