**Transmission Lines and E. M. Waves**
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**Lecture – 20**
**Boundary Conditions at Media Interface**

So now we are having overall, the quantities like: the charge density which means volume charge density, we have a quantity like current density which means conduction current density, we have a quantity like displacement current density then we have got surface charge density and then we have got the surface current density. So one may say these are the sources which are related to the fields which are the electric and magnetic fields.

So in general then one can establish relationship between these quantities which we can call as sources to the fields which are electric and magnetic fields and these relationships are called the boundary conditions.

So now what we do is we go back to the integral form of the Maxwell's equations, take the sharp boundaries media interfaces that means across the boundary the medium property suddenly change from one side to other and then we establish relationship between the electric and magnetic fields in the two regions. These boundary conditions are essential when we solve the electromagnetic problems in various media like let us say coaxial cable or wave guides or optical fibers. So whenever we solve the phenomena of electromagnetics generally these media they constitute these discrete boundaries and we require the relationship of electric and magnetic fields across a boundary what are called boundary conditions so these boundary conditions are essential in solving the electromagnetic problems in physical structures.

So let us say I have an interface now media interface and the two sides have differential material properties so in general let us say I have a medium 1 here, I have medium 2, so in general I can have permeability of the medium mu 1 here, permittivity epsilon 1 and the conductivity could be sigma 1. For medium 2 I can have permeability mu 2 permittivity epsilon 2 and the conductivity sigma 2. Now I can write down the different integral equations or integral form of Maxwell's equations across this boundary.

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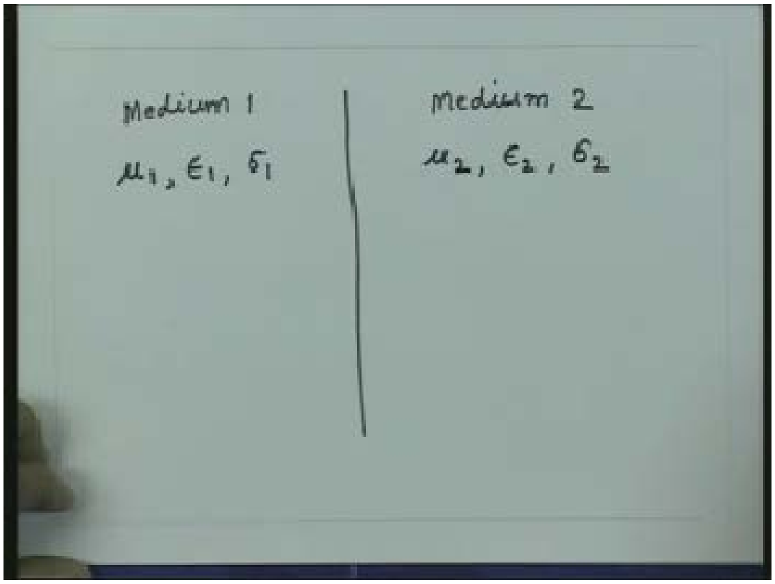


So if I apply the Gauss' law across the boundary, if I consider an area if I consider a box around this boundary so let us say I take a box which is which is like that (Refer Slide Time: 3:59) and from the Gauss' law it says that if the total displacement which is coming from this box will be equal to the charges enclosed and if I make the size of this box swing to 0 I will get the relationship between the displacement vectors on the two sides.

So the displacement vector might be coming in this direction, it might be coming from this direction, it might come from this direction, it might come this direction. So if I say the displacement vector here is having two components say D n1 let us say this is tangential component which is D t1 similarly here if I take the component which is D n2 and this is D t2 so any general displacement vector in medium 1 close to the interface I can resolve into two components so I have what is called a tangential component and a normal component; normal to the interface, tangential to the interface. Similarly, I can have a tangential and normal component of the displacement vector in medium 2.

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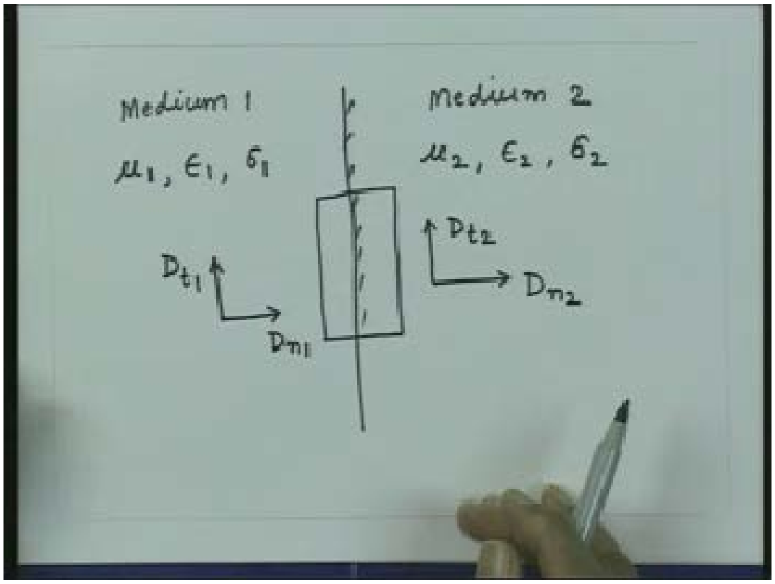


Now the normal displacement coming from this box the box is perpendicular to this so this I am seeing essentially the end view of the box. When the box size goes to zero the displacement coming out of this side will go to zero because this length is going to go to zero so the net displacement coming from this side of the box will be equal to zero, similarly the net displacement coming from this side of the box will be equal to zero. So the displacement which will be coming out of this box when this goes to zero will be difference of these two which are the normal components because this is these components are tangential so this is not representing the outward electric displacement.

So, from the Gauss' law if I take the net normal displacement from this box that is equal to the total charges enclosed within this box; so if I consider a medium when the box goes to zero there are two possibilities: one is there is no charge here in this region and in that case this quantity whatever this quantity is going in must be coming out because the Gauss' law says the divergence of D should be equal to 0 there are no charges now. So, if there are no charges in this region in the limit when the box size is zero you have a continuity of a normal component of the displacement vector.

So however the other case could be that when this box goes to zero I have a surface charge here and if there is a surface charge here then the difference of these two should be equal to this charge because this is the charge which is enclosed by this box. So what we will have is from the Gauss' law we get that D n2 minus D n1 is equal to the surface charge density. But if there is no surface charge then this quantity should be equal to zero and in that case D n2 should be equal to D n1. So this is if surface charge is present and this quantity will be equal to 0 or D n2 equal to D n1 that is if no surface charge.

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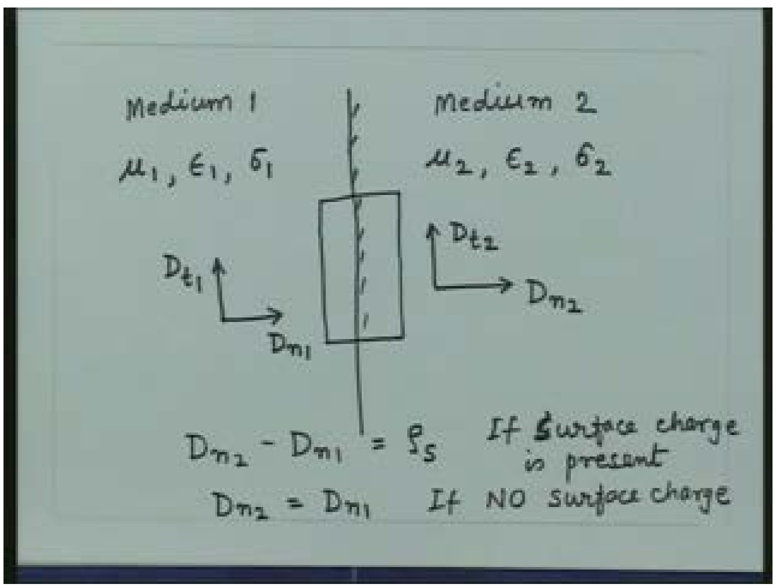


So this is one boundary condition which we have from the Gauss' law that the normal component of the displacement vector is continuous across a boundary if there is no surface charge and in the presence of surface charge the difference of the normal component of the displacement vector is equal to this surface charge density. This is this is boundary condition 1. So we have a boundary condition on the normal component of the charge density.

Same thing we can do for the magnetic flux density also. We can take again the same box like this and instead of displacement vector we can have the magnetic flux density which is B and since we do not have the free charges that there is no nothing like surface charge for the magnetic fields so this quantity will be identically zero so the normal component of the magnetic flux density will be always continuous.

So we have from this Gauss' law two things that is what is called the boundary conditions. It says, first condition says normal component component of D is continuous if no surface charge. In the presence of surface charge the difference of the normal component of the D is equal to surface charge. So in the presence of surface charge we have D n2 minus D n1 that is equal to the rho x.

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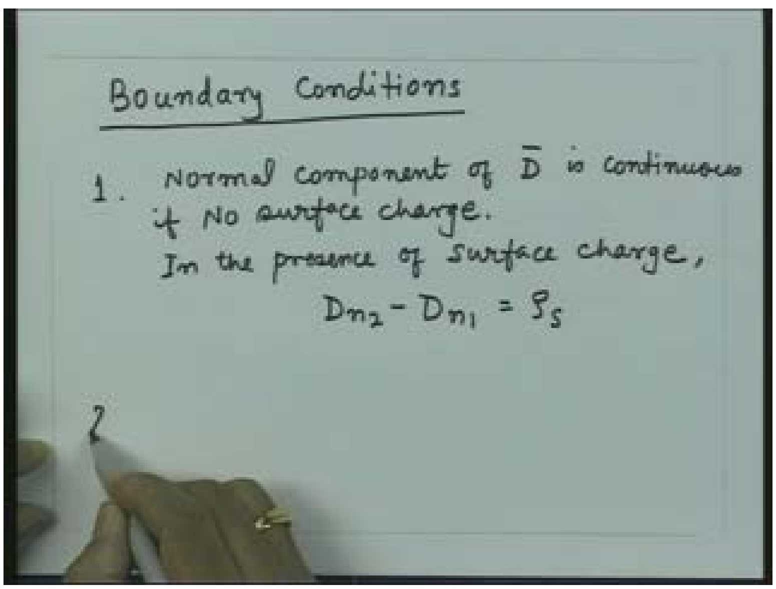


The second boundary condition which we have for the magnetic flux density as I mentioned this quantity which is the magnetic charge density is always zero because we do not have free magnetic monopoles so this quantity is always zero for the magnetic flux density. So we say that a general condition for the magnetic flux density is normal component of B is continuous.

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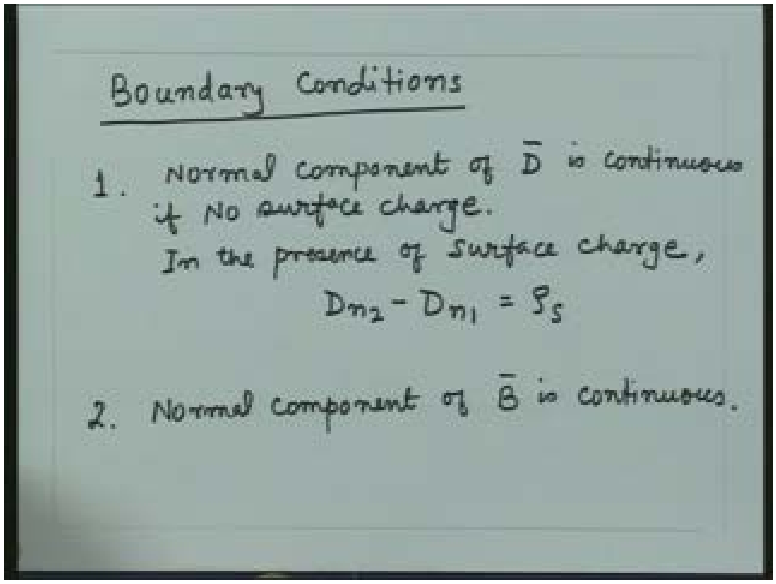


So we have now these two basic boundary conditions on the displacement vector D and the magnetic flux density B. Magnetic flux density B satisfies the condition that its normal component is always continuous whatever the media these continuities are whereas the displacement vector if there are surface charges there is a discontinuity in the normal component and that is equal to surface charge density. If there are no surface charges then the normal component of displacement vector also is continuous across the boundary.

So now, while using the Ampere's circuit law and the Faraday's law across the media interface we will get to more boundary conditions on electric and magnetic fields. So let us say I have a dielectric medium and in medium 1 we are having a electric field which I can resolve it into two components the normal component which can be given by E n1 and the tangential component which can be represented by E t1 and medium two again I can represent the normal component is E n2 the E t2. If I now take a loop across this boundary so this is the loop (Refer Slide Time: 13:03) and I want to apply the Faraday's law across this loop so essentially if I find out the line integral that is the electromotive force around this loop that must be equal to the rate of change of the magnetic flux enclosed by this loop.

Now since we are talking about the finite magnetic flux densities and also its rate of change the rate of change enclosed by this loop auto magnetic flux that is a finite quantity. So if I take now the line integral of the electric field across this loop essentially this is the normal component so on this wall the line integral will be zero for the normal component so I will get the line integral contribution for this side that will be E t1 multiply by length of this loop.

Similarly, on this side I will get the line integral contribution which is E t2 multiplied by the length. However, if I take a loop which is in the clockwise direction then the direction of the loop or the line integral which I am taking that is opposite to the electric field direction and that is that I have a minus sign here so in the limit when I make this loop shrink to a thin sheet across this line the line integral contribution which is coming from these two sides will go to zero so we will have a contribution to the line integral which will be from this side and from this side. So this total line integral (Refer Slide Time: 14:45) when the size of this loop goes to zero will be E t1 into L minus E t2 into L and that is equal to the rate of change of the magnetic flux enclosed by the loop.

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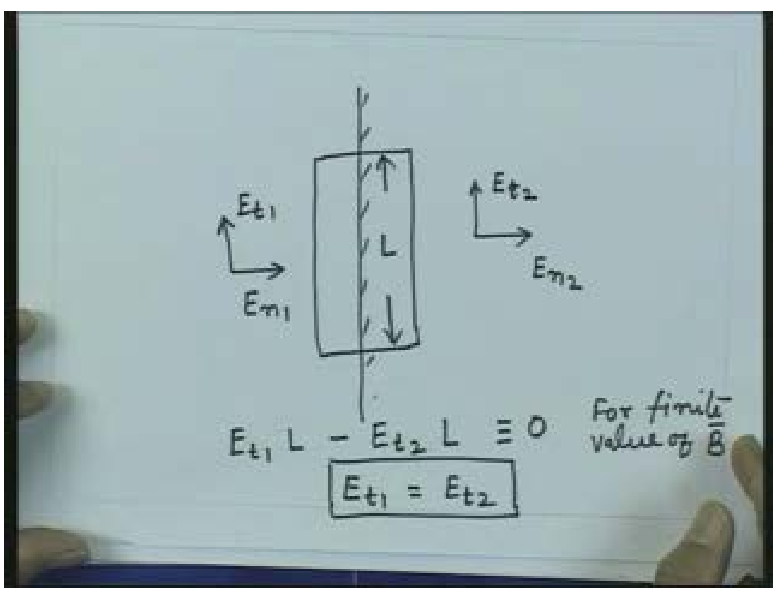

Since the flux density is finite, as the area of the loop goes to zero the flux enclosed by the loop will go to zero so this quantity will be identically equal to zero. So what we get from here that the tangential component of electric field is continuous across the boundary. So, irrespective of what the boundary is the tangential component of electric field will always be continuous across the boundary. That is because the magnetic flux density is always finite in the region enclosed by the loop.

The same thing we can do for the Ampere's circuit law and for the magnetic fields. So let us say now I have an interface and I just consider a loop which is covering two sides of this interface. I take a magnetic field which has two components: normal component H n1 and tangential component H t1, I have a magnetic field in the medium 2, here again normal component is given by H n2 and tangential component is given by H t2; I apply now the Ampere's law around this loop. So I find out the line integral of H around this loop which is the magneto motive force around this loop and that should be equal to the total current enclosed by that loop. So this is medium 1, this is medium 2 and if I find out line integral as we did in the previous case the line integral contribution is going to come from this side and from this side, the contribution when the size of the loop tends to zero the contribution coming from these two sides will go to zero.

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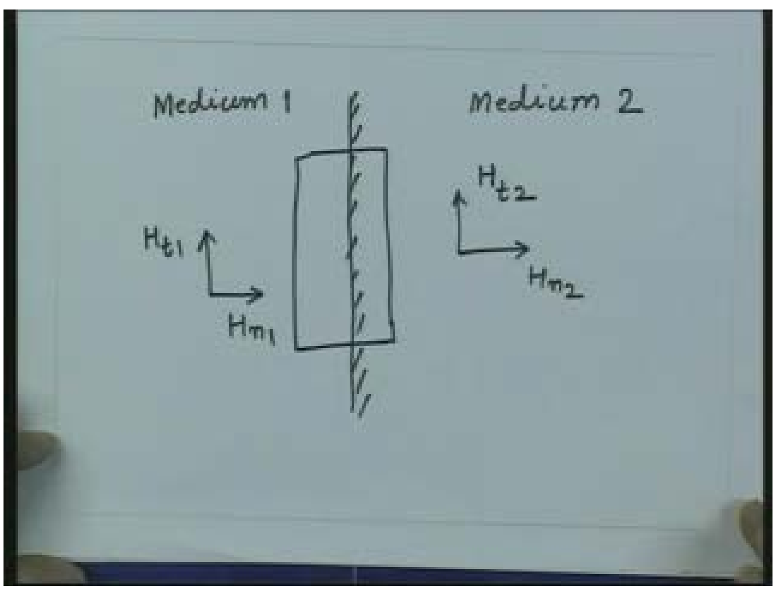

So essentially we are having now the line integral which is H t1 minus H t2 into L which is the length of this loop that should be equal to the total current enclosed by this loop in the limit when the size of the loop goes to zero.

Now as we have seen there are three possibilities for the current to be enclosed by this loop: one is we have a conduction current density in this (Refer Slide Time: 17:31) so the total current enclosed by the loop will be conduction current density multiplied by the area of cross section of this and for finite conduction current density if the area goes to zero then the current enclosed by this loop will go to zero. So the current enclosed by this loop in the limit when the loop shrinks to a line for finite conduction current density that current enclosed will go to zero.

The other possibility is that I have displacement vector in this which is perpendicular to the plane of the paper. If I am having time varying fields then I will have the displacement current density which if I multiply again by the area of the loop I will get the displacement current enclosed by the loop, I have possibility that the conduction current density J multiplied by the area of the loop that is one contribution plus I have a displacement current density dD by dt multiplied by the area of this loop, third possibility is I may have surface current here and when the loop shrinks to a line the surface current is still enclosed by this loop so I may have a surface current which is on the surface of this interface.

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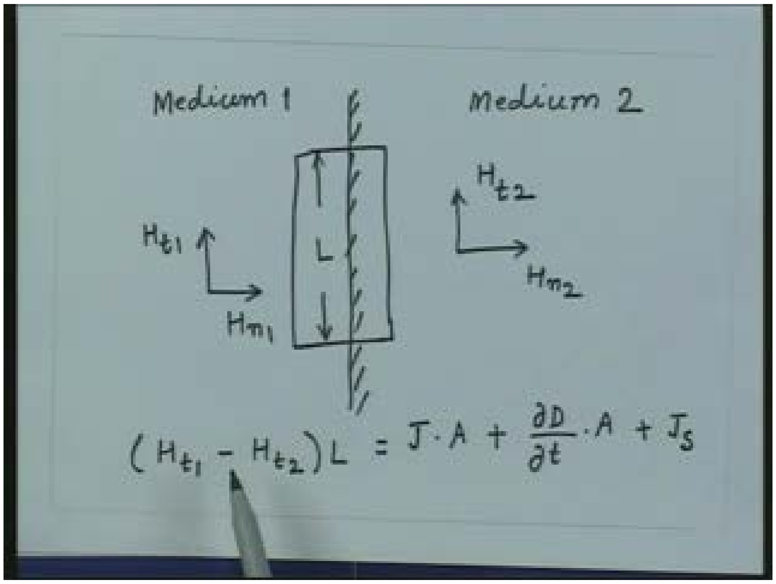


When area tends to zero when the loop shrinks to the line this quantity will tend to zero, this quantity will tend to zero for finite value of displacement vector that it is for finite value of electric field, so in the limit when the size of the loop goes to zero both this quantity will go to zero. However, this quantity will not go to zero (Refer Slide Time: 19:26) because this quantity is a surface quantity. So in the limit when A tends to 0 this will be equal to only the surface current density J s.

So from here essentially what we find is that the difference of the tangential component of magnetic field that is equal to the surface current density; note here this area A is this L into the width W, so when we have area going to 0 this W is going to be tending to 0 so there will be L here which will cancel with this so I will get a relation which means H t1 minus H t2 that is equal to the surface current density.

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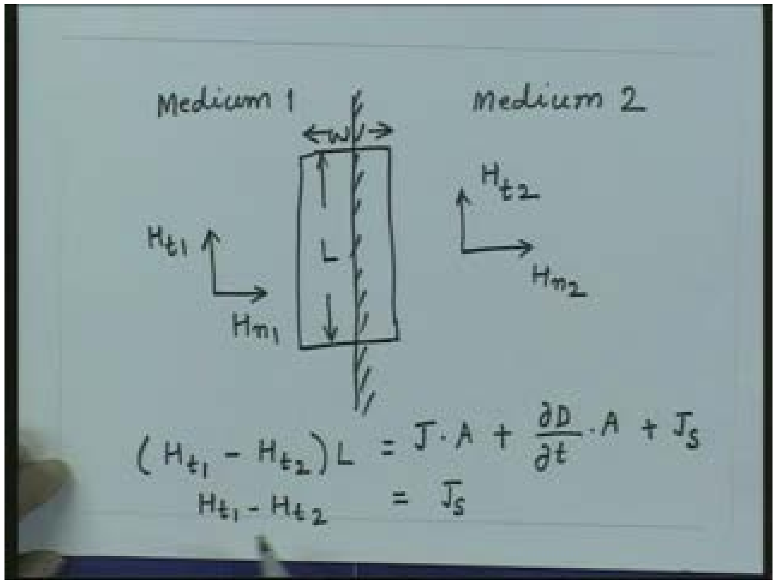


If there is no surface current density then the different between the tangential components of magnetic field will be equal to zero. Or in other words, the tangential component of magnetic field also will be continuous across the boundary. So, using these four Maxwell's equations in the integral form applied to media interface we get the so-called boundary conditions.

So what we have done; we started with the four basic laws; applying these basic laws and using the vector identities and using the integral vector theorems we could write these laws in the integral form and by using the integral theorem we could convert these laws or the expressions in the integral form to the differential form.

So we had Maxwell's equation in the integral form, we had the Maxwell's equation in the differential form. We also mentioned that the differential forms of the Maxwell's equations are the point relations. However, these forms cannot be used in those situations wherever you are having media discontinuities because they require space derivatives and when the medium properties are discontinuous the space derivatives are not defined. So in those cases we can apply the integral form and we apply the integral form to the media interfaces and we got what is called the boundary conditions.

Now this boundary condition it can be written more compatibly that if I have total magnetic field in the region 1 and region 2 then the tangential component can be obtained as a cross product of the normal to the interface and then the total magnetic fields. So many times this boundary condition is written as unit vector cross H 1 minus H 2 that is equal to the surface current density.

So here (Refer Slide Time: 22:26) n is the normal to the interface, H 1 and H 2 are the total magnetic fields so this is H 1, this is H 2 and the cross product of the unit normal to the interface and the magnetic field use a component which is the tangential component. So the same thing which we have got here can be written in the form of the vector magnetic field as that the n cross H 1 minus H 2 that is equal to the surface current density.

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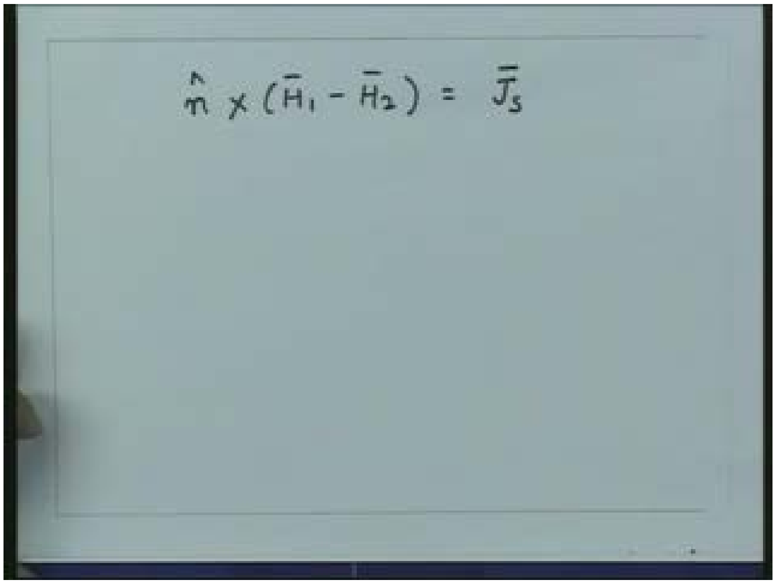


So with this now we can summarize the four boundary conditions for the dielectric media as we have been discussing. So we can write down the four boundary conditions for the dielectric media. So if I say I have a medium which is dielectric dielectric interface that means I have a medium on both sides of which I have a dielectric. The conductivity may be finite, may be zero but the conductivity is not infinite which means there is no conductor on either side of the boundary; in that case then I have the boundary conditions the first boundary condition as we got is D n1 minus D n2 that was equal to this surface charge density. Condition 2 was B n1 is equal to B n2. The third boundary condition was E t1e is equal to E t2 and the fourth condition was that n cross H2 minus H1 that is equal to the linear surface current density.

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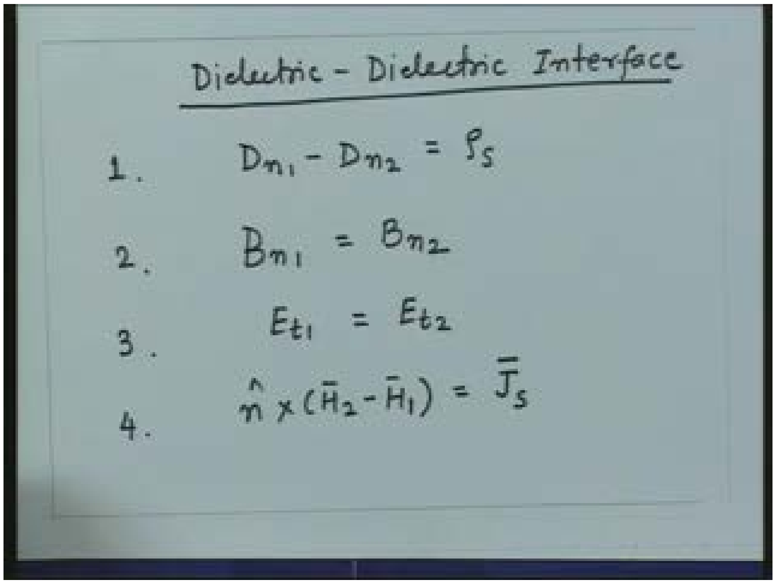


So these two boundary conditions: 2 and 3 they can be applied in any situations whether you have surface charges or surface currents. So invariably you will see when we do the analysis we apply these boundary conditions that is normal component of magnetic flux density is always continuous, the tangential component of electric field is always continuous across the boundary.

The normal component of the displacement vector maybe continuous if this quantity is zero may not be continuous if this quantity is not zero. Similarly, the tangential component of magnetic field will be continuous if surface current is not there so unless we have a knowledge whether we have a surface current and surface charges these boundary conditions cannot be applied. But these boundary conditions can always be applied because this does not require the knowledge of the surface condition, this does not require knowledge of surface current, it does not require knowledge of surface charges. So these two boundary conditions are always very reliably applied without knowing the complications of the surface conditions.

Now if I have the media which is dielectric to conducting media and therefore we will see whenever we have the transmission structures like coaxial cable or wave guides or transmission lines we always have the conducting media separated by dielectric. So we want like to have the boundary conditions on the dielectric to conducting media.

Firstly, we will note that if I have one of the medium which is conductor; so let us say I have now a boundary and on this side of this is conductor and this is dielectric so I have a medium which is dielectric conductor interface, from this side I have certain dielectric properties, the conductivity on this side may be zero may not be zero, but this side the conductivity is infinite so I have since this conductor the conductivity sigma 2 is infinity.

Now if the conductivity of this medium is infinite and if you want to have the finite current densities the conduction current density is sigma into the electric field so the conduction current density will be sigma of this medium which is infinite into the electric field. See if I take any finite electric field the conduction current density will be infinite in the medium.

So if we say that we have the finite current density finite conduction current densities for ideal conductor the electric field must be identically zero otherwise for even arbitrarily small value of the electric field the conduction current densities will be infinite, there will be infinite current flowing in the medium so that is the reason when the conductivity tends to infinity the electric field in this region must tend to 0. So we have in this region the electric field E equal 0 identically zero when you are having the conductivity infinite.

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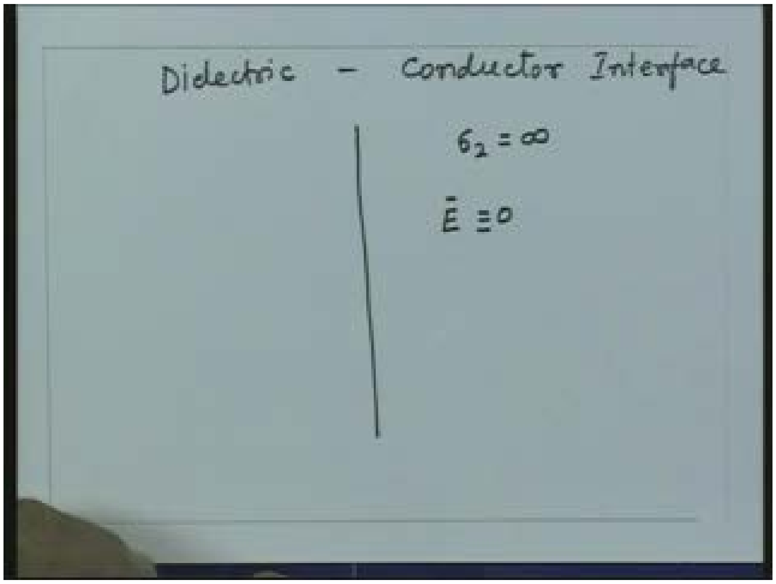


Also we have seen from the Maxwell's equations that the electric field is related to the magnetic field and vice versa. So if I have a time varying electric field and if it is identically zero in this region then the magnetic field also will be zero in this region. So I have a magnetic field zero inside this if the fields are time varying, the magnetic field is zero so electric and magnetic fields both will be zero if you have time varying magnetic fields and conductivities infinite. So our ideal conductor the fields do not exist inside the medium. However, imagine a situation that I apply a very arbitrarily small value of electric field which is in the direction perpendicular to the interface it will dry with the charges inside the conductor to the surface so you can have accumulation of charges on the surface of the conductor.

Similarly, it is possible when I making time varying fields there might be currents which might be flowing along the surface of the conductor. So, when I have an ideal conductor inside that the fields are zero but I can very well have the surface charges, I can have the surface currents. So now I have a situation; I have here the electric field which is E 1, I have a magnetic field which is H 1 and the electric field E 2 is 0 the magnetic field H 2 is also 0 for time varying fields.

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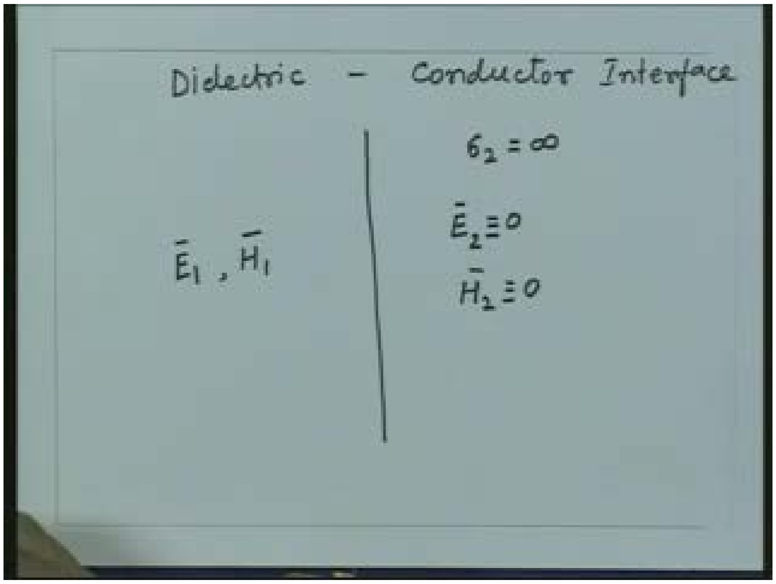


Then I can go back and apply the boundary conditions which I have got here which are in general. So since this quantity is electric field is 0 in this medium and epsilon times or the permittivity times the electric field is the displacement vector so D n2 is identically 0 inside the conductor.

So now I have the boundary conditions for this. So in this case the boundary conditions would be that D n1 is equal to the surface charge density because D n2 is zero; B n1 that is equal to 0, I have E t1 that is equal to 0 because E t2 is 0 and I have n cross H1 that is equal to the surface linear surface current density. And in this situation when there are no surface charges and surface currents then the normal component of the displacement vector is 0, the normal component of the magnetic flux density is 0, tangential component of electric field is 0 and the tangential component of the magnetic field is also 0 because there are no surface currents.

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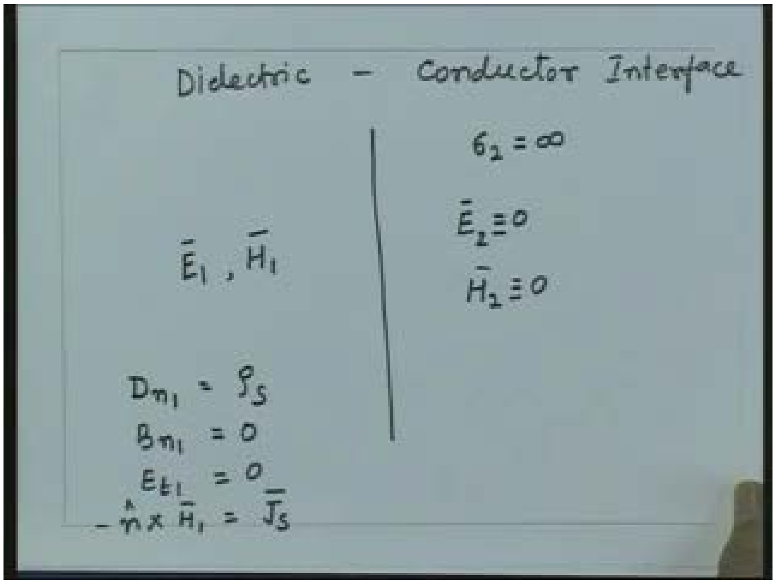


So depending upon the situation whether we have the dielectric dielectric interface or we have dielectric conductor interface we can appropriately apply these boundary conditions. This now gives us the framework for analyzing any electromagnetic problems in three dimensional space.

So what we have done, staring... let me just summarize what we have done starting from the basic laws. We ((…32:07)) what the Maxwell’s equations and essentially we are now going to make use of the Maxwell’s equations in differential form to analyze the electromagnetic problem in three dimensional space and then we got the boundary conditions which will be useful when we try to solve the electric and magnetic fields across the dielectric to dielectric or dielectric conductive interface.

Let us now solve some problems related to the time varying charges and time varying fields.

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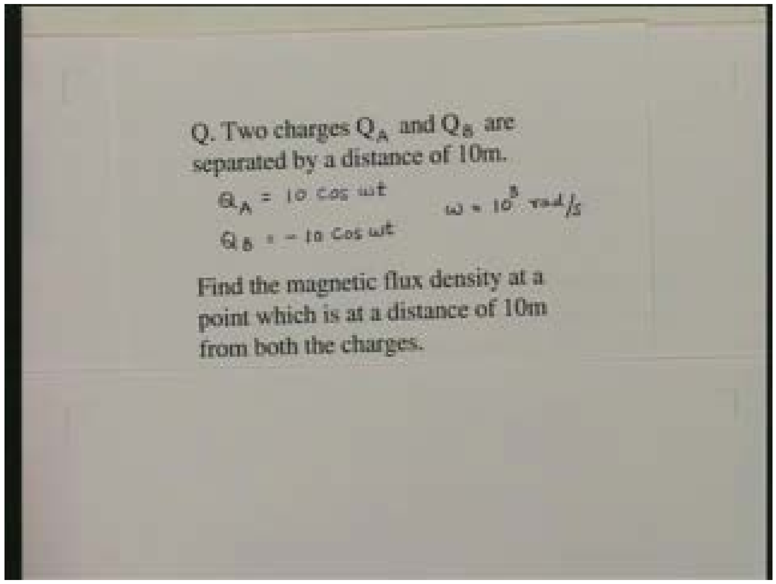


So let us consider a problem here. There are two charges Q A and Q B which are separated by a distance of 10 meters and QA has an amplitude of 10 Coulombs and it is sinusoidally varying with the frequency omega. Similarly, the charge Q B has an amplitude which is minus 10 and it also varies sinusoidally with the frequency omega. Then omega is equal to 10 to the power 4 radiance per second. You have to find the magnetic flux density at a point which is at a distance of 10 meters from both the charges.

So note here the charges are separated by a distance of 10 meters and we are also interested in finding out the magnetic field at a distance of 10 meters from both the charges. So the situation is something like this.

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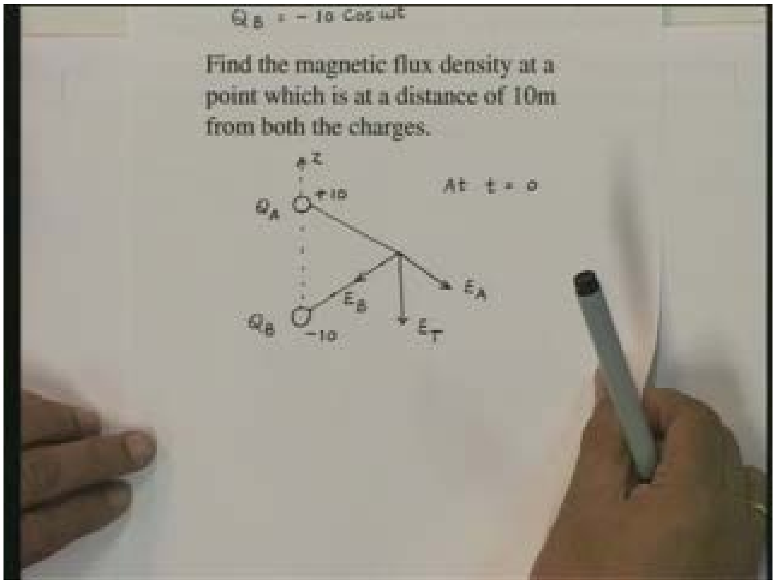


Let us say if I consider at a instant t equal to 0 then the charge Q A will be plus 10 Coulomb and the charge Q B will be minus 10 Coulomb. Without losing generality let us say these charges have an axis in the line joining them which is oriented in the Z direction. So at T equal to 0 then we have a situation that Q A is plus 10 Coulomb, Q B is minus 10 Coulomb this distance between them is 10 meters and then you have to find out now the magnetic field at this point P which is at a distance of 10 meters from both the points that means it is along this line of symmetry so this is 10 meters and this distance also a 10 meters.

Now firstly since we are having time varying charges here we can find out the electric field due to these two charges at this location P so you will get an electric field which will be time varying then we can go to the Maxwell's equations and from there we can find out the corresponding magnetic field. So at this instant if you find out the electric field at this point P since this charge is positive there will be electric field which will be oriented in this direction and let us say that electric field is E A; due to this negative charge the electric field will be oriented in this direction let us say that is given by E B. So then we have the electric field E A that is equal to 10 cos omega t divided by 4pi epsilon 0 to r square where r in this case is 10 meters. Similarly, we can find out the electric field due to charge B and that will be equal to minus 10 cos omega t divided by 4pi epsilon 0 r square.

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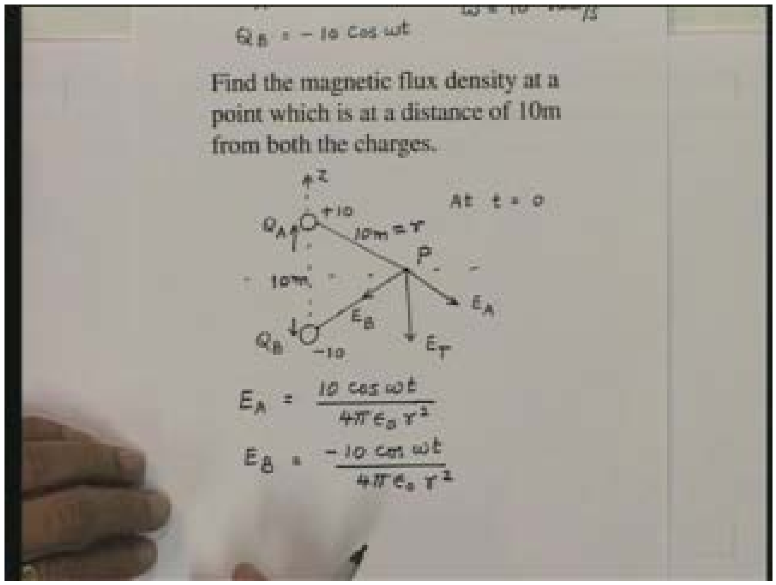


Now, magnitude-wise the E A and E B are equal. So I can resolve these two fields into two components: one component is this component (Refer Slide Time: 36:17) and one component is this component. Now since the two magnitudes are equal this angle let us say theta so the field E A cos theta will be in this direction and E B cos theta will be in the opposite direction and since these two are equal these two fields essentially will cancel each other. So we will have essentially a resultant field which will be oriented in this direction and which will be two times the electric field E A or E B multiplied by sign of theta. So then we have the total electric field E T at this location that will be equal to E A sin theta minus E B sin theta which is equal to two times this quantity multiplied by sin theta. So that is 20 cosine of omega t divided by 4pi epsilon 0r square sin of theta.

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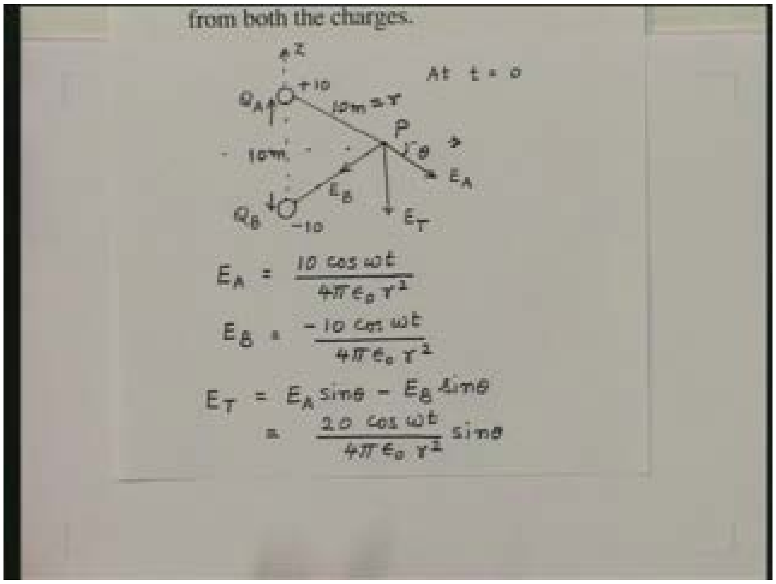


Now note here this quantity is theta here so this angle is also theta. This distance is 10 meters and this distance is 5 meters so we have sin of theta which is 5 upon 10 so it will be theta is equal to 30 degrees or sin theta is equal to 1 by 2. So from here we essentially get sin of theta that is 5 upon 10 which is equal to 1 upon 2 or giving this quantity theta which is equal to pi by 6.

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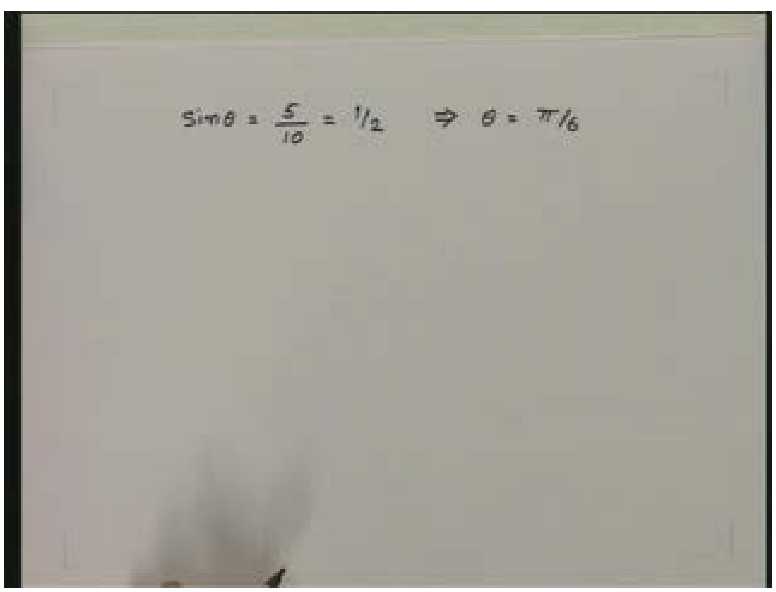

So the total electric field which you are getting now at this location p (Refer Slide Time: 38:28) which will be oriented in the negative z direction; so if you want to write down the total vector electric field at this location we can write the electric field E which is equal to 20 cos of omega t divided by 4pi epsilon 0r square. And since this field is oriented in the negative z direction I can put a negative sign here with z cap multiplied by sign of theta which is 1 by 2. So we get the electric field which is minus 10 cos of omega t divided by 4pi epsilon 0r square z cap.

Once I know the vectorial magnetic field then I can apply the Maxwell's curl equation and find out the magnetic field due to charges. So essentially we get now the H which is 1 upon j omega mu 0; here mu 0 is a permeability of the medium to del cross E. See if I write down the curls for this that will be equal to 1 upon j omega mu 0 to d E z by dy x cap minus E z by dx y cap where E z is this quantity here (Refer Slide Time: 40:32).

Now when I differentiate this quantity with respect to x or y I have this term here 1 over r square so I require d by dx of 1 upon r square which will be nothing but minus 2 r to the power minus 3 dr by dx and since r is square root of x square plus y square this quantity will be x upon r. So you get d by dx of 1 upon r square that is minus 2 x upon r to the power 4. Similarly, we can get d by dy of 1 upon r square and that will be equal to minus 2 y upon r to the power 4.

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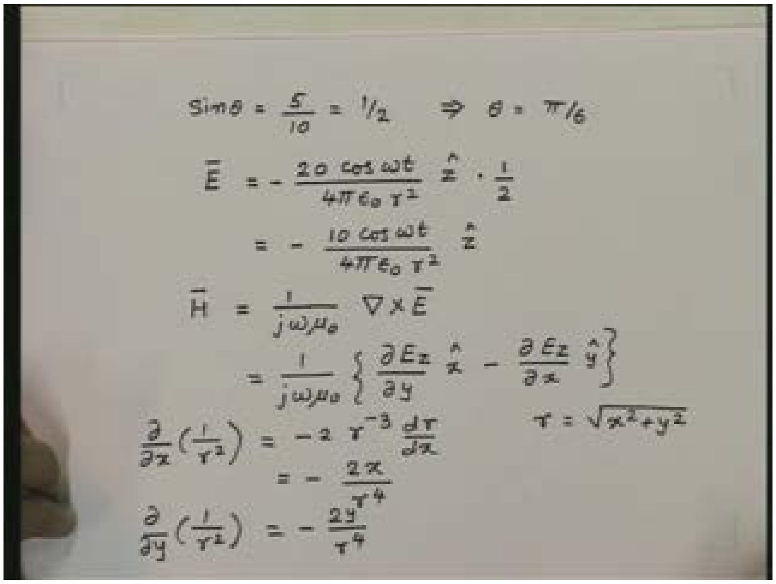


So I can take this quantity and substitute into this and then I can get the vector magnetic field H that will be equal to 1 upon j omega mu 0 minus 10 cos omega t divided by 4pi epsilon 0 minus 2y upon r to the power 4 x cap plus 2x upon r to the power 4 y cap.

Now at point P where we want to find out the field (Refer Slide Time: 42:48) here this quantity is r for the total 1 so I can find out from here this quantity which is r which is equal to 10 and x is equal to this square minus this square, so x will be equal to square root of 75. So at point P we get x which is equal to square root of 100 minus 25 which is equal to square root of 75 which is equal to 5 root 3.

So, for these locations first of all if I consider the coordinate system at this point here then I can just substitute that y is equal to 0 for this point and then from there I can find out the total electric field. So I can substitute now for y equal to 0 and x equal to a quantity which is equal to r for this coordinate system, I can get the total electric field E or total magnetic field H that will be equal to j cosine of omega t divided by 75 root 3pi omega mu 0 epsilon 0.

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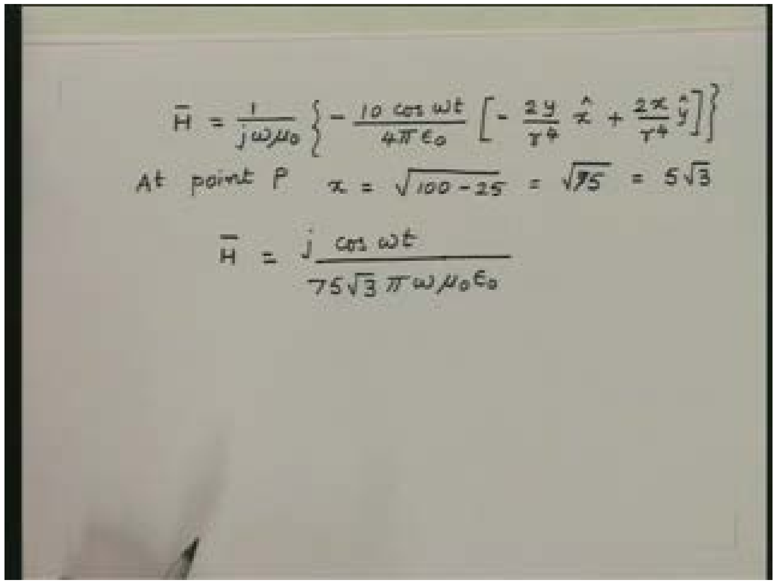


So leaving aside the numerical part what essentially we find from this problem is that when we are having the time varying charges they produce both electric and magnetic field and that is what we have seen from the Maxwell's equation that whenever we are having time varying fields the electric and magnetic fields coexists. So here once we know the time varying charges first we can find the electric field at a point, once you know the electric field that location then we can find out from the curl equation the corresponding magnetic field.

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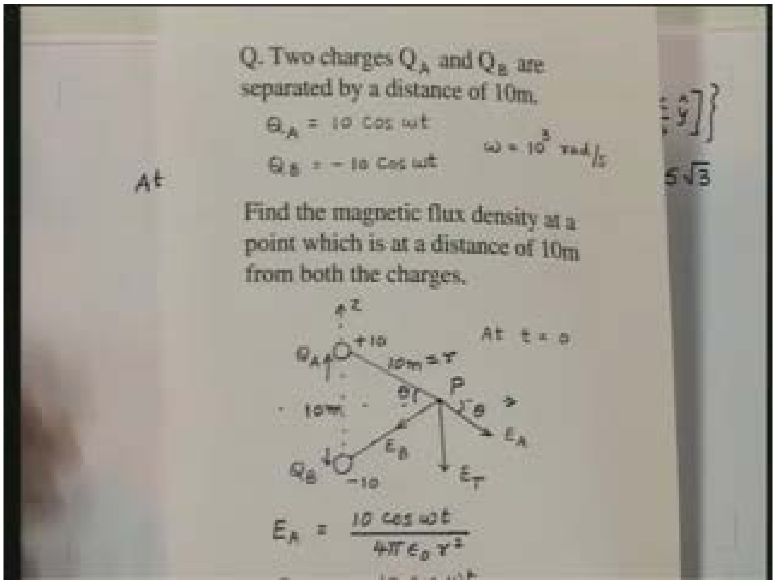


Let us consider now one problem based on the boundary conditions.

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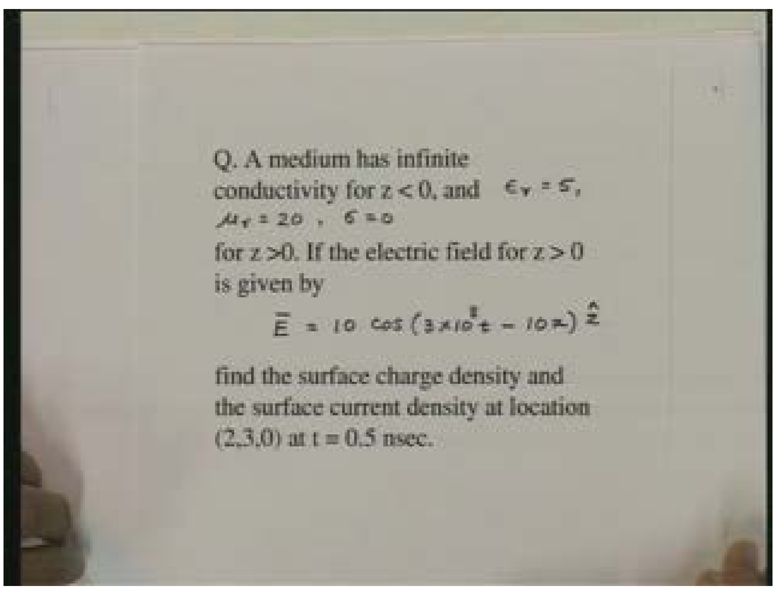


We have seen that the boundary conditions are nothing but the application of the laws in the integral form across a boundary. See the laws are express in the differential form; we get what are called the Maxwell's equations, if the same laws are applied across a discrete boundary then we get what are called the boundary conditions.

So let us consider now a problem that there is a medium which has infinite conductivity for z less than 0 and for z greater than 0 we have the dielectric constant which is 5, relative permeability which is 20 and the conductivity is 0. If the electric field for z greater than 0 is given by a time varying field find the surface charge density and the surface current density at location 2, 3, 0 at t equal to 0.5 nanoseconds.

So here the situation is as follows. You are having medium so this is z equal to 0, below this thing we are having the infinite conductivity and since we are now considering the time varying fields the electric or magnetic field cannot exist inside a conducting medium so the electric fields are identically zero in this medium. Then the z direction is given by that so the electric field is normal to this surface so this is the way the electric field will be orient in the z direction.

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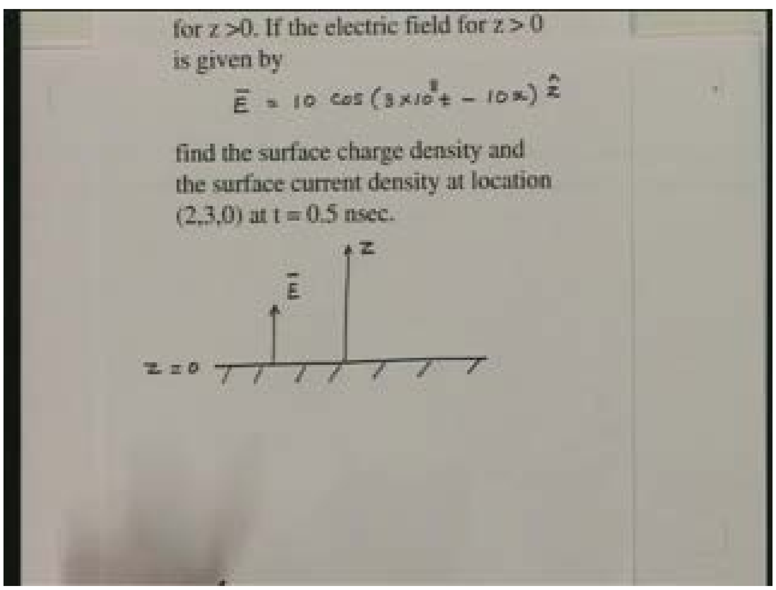


Now the surface charge density essentially is related to the normal component of the electric field. Now in this case the electric field which is given here is already normal to this conducting surface so the surface charge density in this case rho s that will be equal to rho 0 epsilon 0 epsilon r into the normal component of the electric field E n.

If I substitute now E n which is same as this quantity E then I get epsilon 0, epsilon r is given as 5 so it is 5 into the electric field which is 10 cos of 3 into 10 to the power 8 t minus 10x.

So if I substitute now the location which is 2 comma 3 comma 0 since this is not depending upon the y coordinate or the z coordinate, essentially the s coordinate which is 2 will play role here so I can substitute s equal to 2 in this and I can substitute t equal to 0.5 nanoseconds so x is equal to 2 meters and t is equal to 0.5 into 10 to the power minus 9 seconds. If I substitute that into the expression I get the surface charge density rho s that will be 2.387 into 10 to the power minus 10 Coulomb per meter square.

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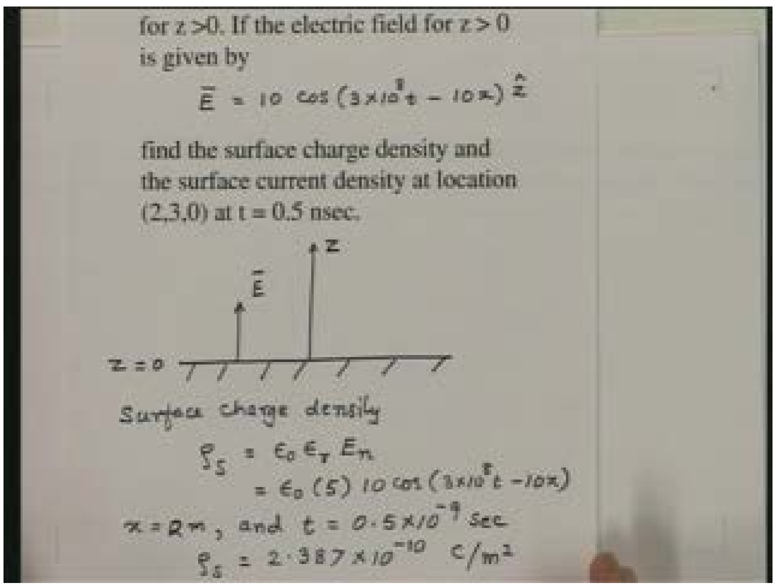


The second thing you have to find here now is the surface current density at this location. And as we know for a conducting boundary the surface current density is related to the magnetic field. So if I knew the tangential component of the magnetic field on the conducting boundary then I can find out what will be the surface current on this conducting boundary.

So firstly we can use now the curl equation to find out what will be the magnetic field on the surface of this boundary. So we have E cross del cross E that is equal to x cap y cap z cap d by dx d by dy d by dz and E has a component which is only z component so this quantity is 0, x component of E is 0, y component of E is 0 and I have z component E z.

Now we know from the Faraday's law that del cross E that is equal to minus mu 0 mu r dH by dt. So the magnetic field H will be equal to minus 1 upon mu 0 mu r integral del cross E dt.

Now since E is only a function of y dz by dy; now since E is not a function of y dz by dy is identically 0 so essentially we have this del cross E will be equal to minus d Ez upon dx y direction and if I substitute for this E then I can get this quantity that is equal to minus 100 sin of 3 into 10 to the power 8 t minus 10 x y and therefore the magnetic field for this could be 100 upon mu 0, mu r this minus sign cancels with this so you have a plus sign integral of sign 3 into 10 to the power 8 t minus 10 x y cap dt. This integration is very straightforward with time so you get the total magnetic field which is minus 100 upon mu 0 mu r cosine of 3 into 10 to the power 8 t minus 10 x divided by 3 into 10 to the power 8 y cap Ampere's per meter.

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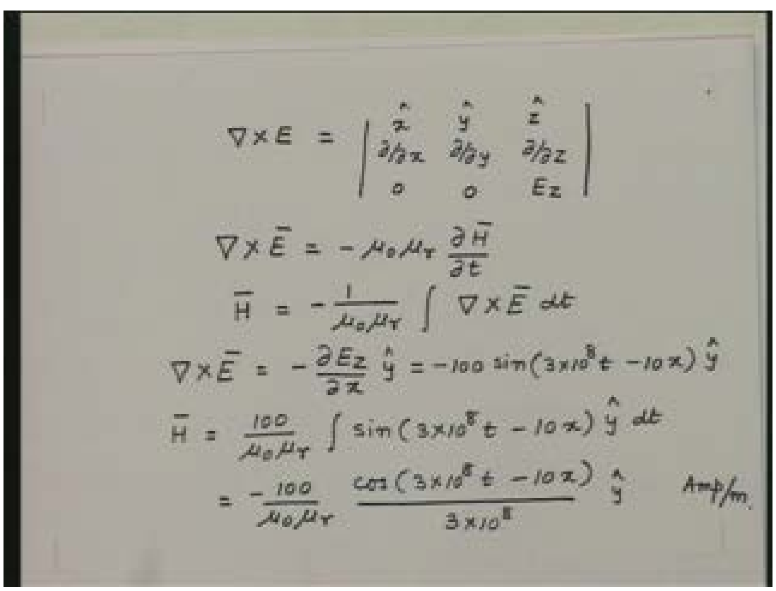

So now I know the magnetic field which is y oriented so if this direction is z then the y oriented magnetic field will be tangential to this conducting surface and then you can find out n cross h which will give you the surface current.

So from here we can get the surface current J s that is n cross H that is equal to z cross... the y component of the magnetic field which is minus 100 upon mu 0 mu r cosine of 3 into 10 to the power 8 t minus 10 divided by 3 into 2 to the power 8 y.

Now the z cross y will give the current which will be in minus x direction so the minus sign will cancel so you will get the surface current J s the vector quantity that will be equal to 100 upon mu 0 mu r cosine of 3 into 0 to the power 8 t minus 10 x divided by 3 into 10 to the power 8 oriented in x direction Ampere's per meter. Then the surface current density at location x is equal to 2 meters because we have to find out in the quantities here at this location here (Refer Slide Time: 55:26) 2 comma 3 comma 0; again we note here this is having a variation only as a function of x so y coordinate and z coordinate do not come into picture and if I substitute x equal to 2 and t equal to 0.5 nanoseconds t is equal to 0.5 into 10 to the power minus 9 seconds then we can get the surface current J s that will be equal to 7.2 into 10 to the power minus 3 x cap Ampere's per meter.

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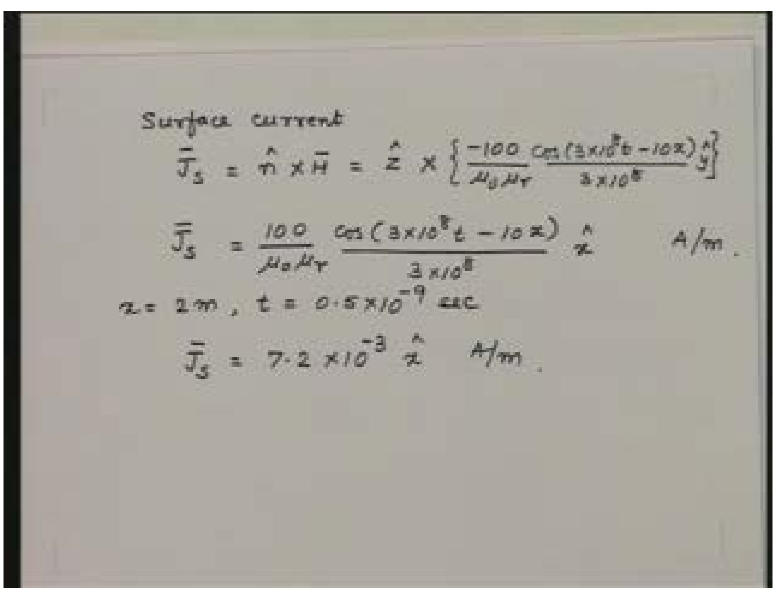

So in this case the electric field was given close to the conducting boundary. From the knowledge of the electric field you find out what is the magnetic field, then we find out what is the tangential component of the magnetic field, then using the boundary condition that n cross H gives the surface current you find out the surface current on the conducting boundary and then its location x equal to 2 meters and time t equal to 0.5 nanoseconds you find the surface current on the conducting boundary.

So for the time varying cases either one can specify the electric field or the magnetic field near the conducting boundaries or dielectric boundaries and then by applying the same physical laws in the integral form and applying the boundary conditions then we can get the quantities on the surface.