Transmission Lines and E.M. Waves
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Lecture-2

Welcome, the first topic which we discuss in this course is Transmission Lines. The Transmission Lines is a special case of electromagnetic waves and essentially it deals with time varying voltages and currents.

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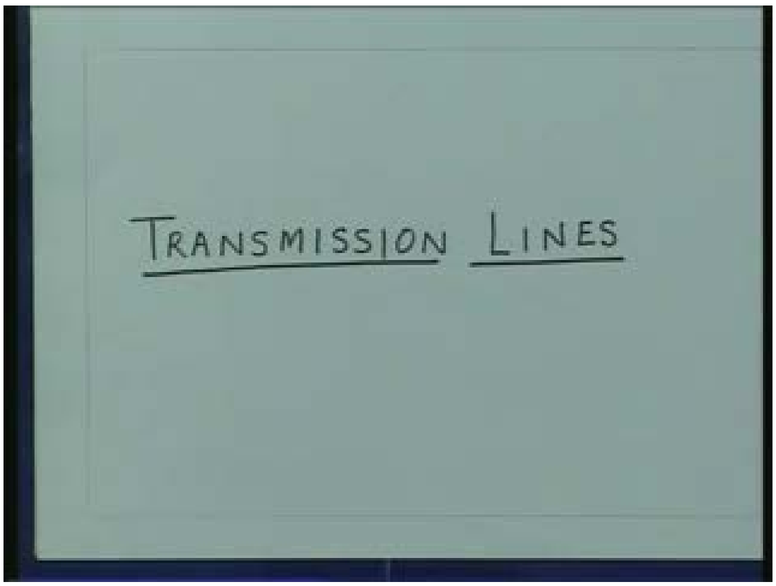

We saw in the previous lecture that the Electromagnetic Waves are a subject which deals with time varying electric and magnetic fields. However there is special case of that where still the concept of voltage and current is valid we can discuss that special case called Transmission Lines.

The idea in this lecture is to still deal with the quantities like voltage and current with which we are quite familiar with but slowly introduce the concept of space in the circuit analysis.

As the name suggest Transmission Line is a medium which can transfer power from one point to another. So we can have a variety of structures which can efficiently transfer power from one point to another and some of them are like a Co-axial structure where you are having the concentric cylinders a inner rod here, there is outer shell and you can apply a voltage between the inner and outer conductor and the energy will propagate along the length of this structure called the Co-axial cable.

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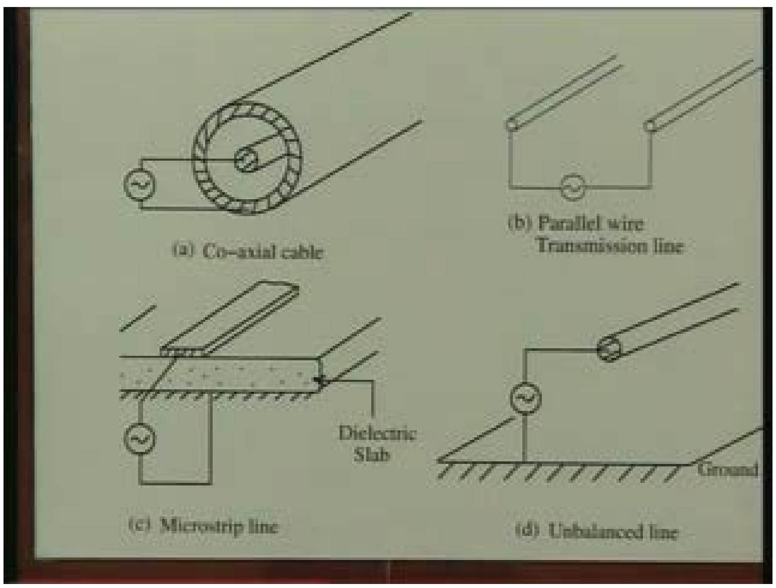

We can have a structure where the two conducting wires or rods can run parallel to each other. We can apply a voltage between these two rods and the energy will flow along the rods.

We can have a structure where we have a dielectric slab one side of which we have a conducting surface and then we are having a conducting strip on the other side you can apply a voltage between these two the lower surface and the upper step and the energy will flow along the strip inside the dielectric.

We can have a structure where we have a conducting rod which is placed above the ground surface and the voltage can be applied between the rod and the ground surface.

All these structures can efficiently carry power along the structure. The common feature of all this is essentially we are having a two conductor system across which we apply the voltage and specifically we deal here with time varying voltage as the result of that the current flows and there is a power flow along this structure. So generically all this structures can be characterized by two conductor system and precisely that is what we do when we do the Transmission Line analysis that to start with we do not go into details of the structure whether it is a Co-axial cable or whether it is a Parallel wire Transmission line or whether it is a Microstrip Transmission Line or a Unbalanced Transmission Line.

We treat Transmission Line as a two conductor system in which we apply an energy source at one end and we apply a load at the other end. Then we do carry out the analysis of power transfer from the source to the load. However before we go in to the analysis of this Transmission Line we can ask some very basic questions. One of the questions is we are quite familiar with the circuit laws at low frequencies if the circuit is given to us we can apply the Kirchhoff's law we can solve the loop and node equations we can get the voltages and currents in at different branches and at different nodes of a circuit and the circuit analysis is complete.

At low frequencies when we do the circuit analysis we take into consideration only the electrical value of the components. For example, if somebody says the value of resistance is 1Ω (ohm) or if the value of inductance is point one Henry or if the value of the capacitance is 0.1 microfarad no other information is needed for circuit analysis because at low frequencies no other information is needed. If the electrical value of the component is correct the circuit analysis will be correct and complete. However as we increase the frequency the scenario changes significantly.

The question one can ask is if I increase the frequency would the simple application of Kirchhoff's laws across the circuit element be invalid and the reason for asking this question is very straight forward that consider a two conductor system which is just a set of two conducting wires I apply a voltage source at one end of this structure let us say this voltage is V

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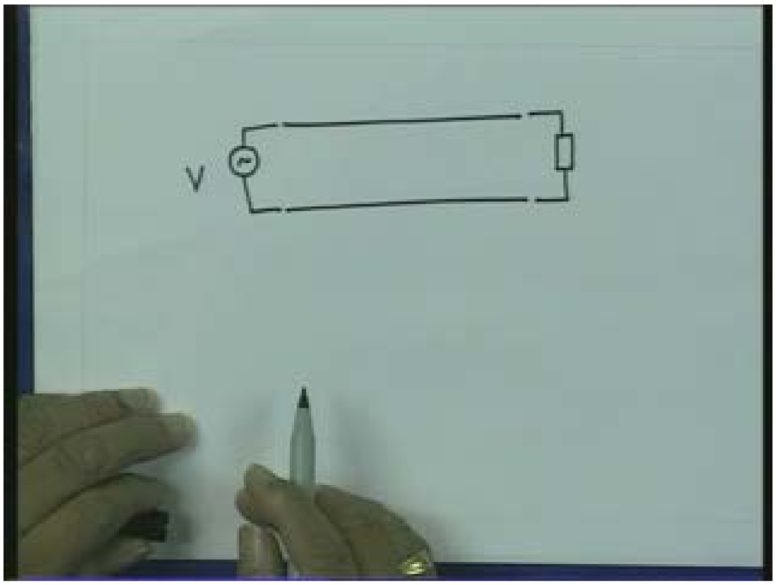

and I have connected some impedance at the other end of this structure and specifically we apply let us say I have a sinusoidal voltage at some frequency f at the input of this structure. So let us say I have a sinusoidal signal applied at the input of this structure so this voltage which I am applying at this end of the line is sinusoidal which is having a frequency f. Then the time period of this signal will be one over frequency so we have time period $T = 1/f$ so this time from here to here is one time period.

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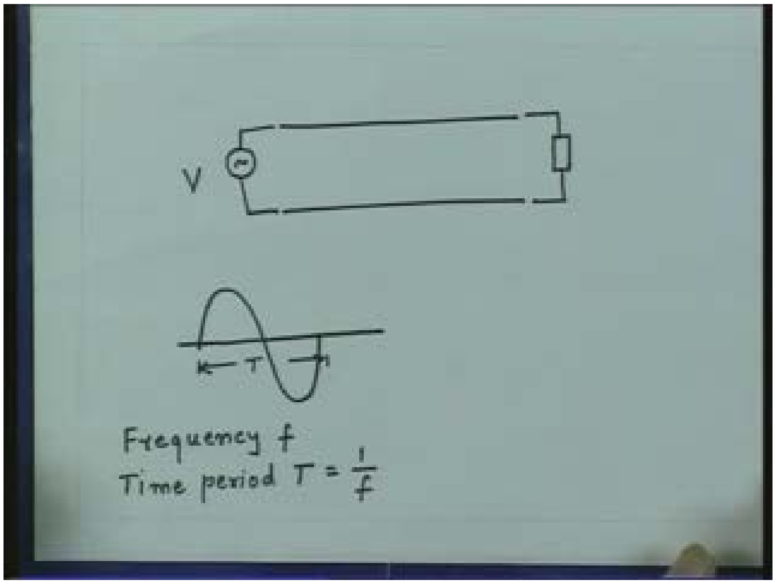

Now let us say at some instant of time some voltage volts connected across this end of the line let us say this voltage is denoted by this point here V_P. The fact is that no signal can travel with infinite speed in any physical system. This voltage V_P does not occur instantaneously at this end of the line. So let us say if this end is denoted for the line by A and A' and this end is denoted by B and B'. The voltage V_P if applied across A A' at some instant of time then the voltage V_P will not appear at B B' at a same instant it will require a finite delay to appear at this location B B'. This finite delay is called the Transit Time of the signal from this end of the line to the other end of the line.

So let us say if the length of the line is given by L and this signal or this voltage travels with a speed from this point to this point with velocity V then we have Transit Time t_r which is equal to $\frac{l}{v}$ where v is the velocity of this signal over this structure. So this quantity is the velocity.

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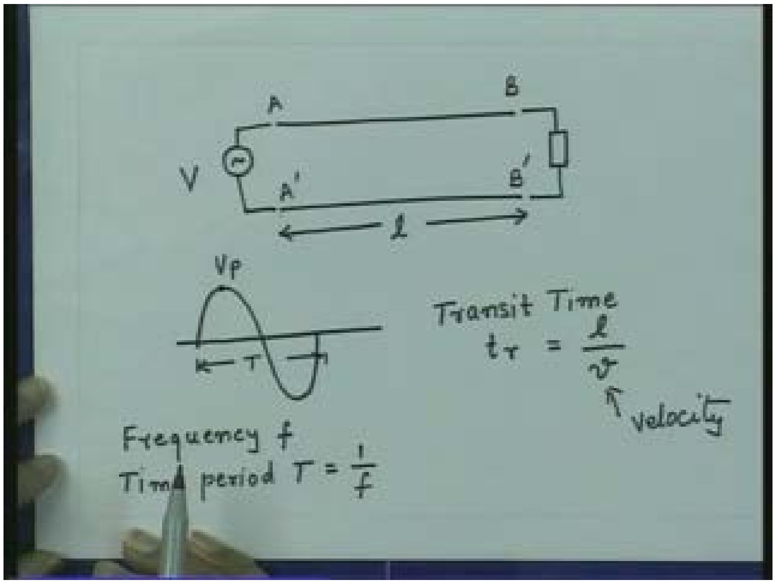

So now if I apply a time varying signal then the voltage require a finite time to travel from one end of the structure to the other end of the structure. However during this Transit time the voltage at this location is not going to remain at V_P this signal is continuously varying so during this time t_r the voltage at this location would change to some other value let us say that changes to this point Q so this point is P and this point is Q. So the voltage at this location V_Q will appear at this location A A' but at that instant if I see the voltage at B B' that will be V_P.

So what we see is when the voltage at this point is V_Q the voltage at this point is V_P and in other words there is a potential difference between the two points A and B. And the difference is related to the length the larger the length more is going to be the delay or the Transit time and the voltages will be differing significantly between these two points. If I take the length which is very small then this point is very close to this point so the point P and Q will be very close to each other and the voltage difference between these two will not be substantial.

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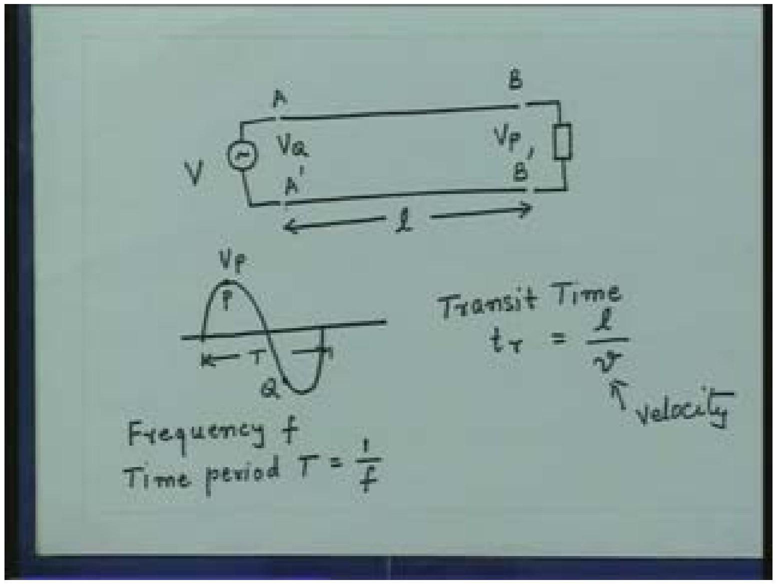

However, no matter how small a length I take there will always be a difference in the potential between any two points in the line. This effect is called Transit time effect. And the role of this effect becomes more and more important as you increase the frequency. Then the question one can ask is, when can I neglect this transit time effect and when you have to incorporate this transit time effect. The question essentially lies in the discussion which we had if the potential difference between these two points is negligibly small then we say that transit time effects can be neglected, If the voltage difference between these two points is significant then we say the transit time effects cannot be neglected.

So essentially what we say is that if the Transit time is much smaller compared to the time period of the signal then we say that the transit time effects can be neglected. However when the Transit time becomes comparable to the period of the signal then we have to incorporate the effect of the Transit time. In other words what we are now saying is for carrying out the circuit analysis the size of the structure or the space has started playing a role and we can find a condition when the role of the space becomes important. But it is very clear as we increase the frequency the time period becomes small so for the same Transit time the relationship between the Transit time and period will become much larger or comparable to T. So the role of the Transit time becomes more and more important as the frequency increases.

When you go to a frequency which is extremely small the time the time period is very large and the Transit time will be negligible compared to the time period. So essentially when we carry out the circuit analysis at high frequencies the Transit time has to be taken into account, without taking the Transit time the circuit analysis is inaccurate and incomplete.

The condition for including the transit time effect is that when the time T is much greater compared to Transit time you can ignore transit time effect.

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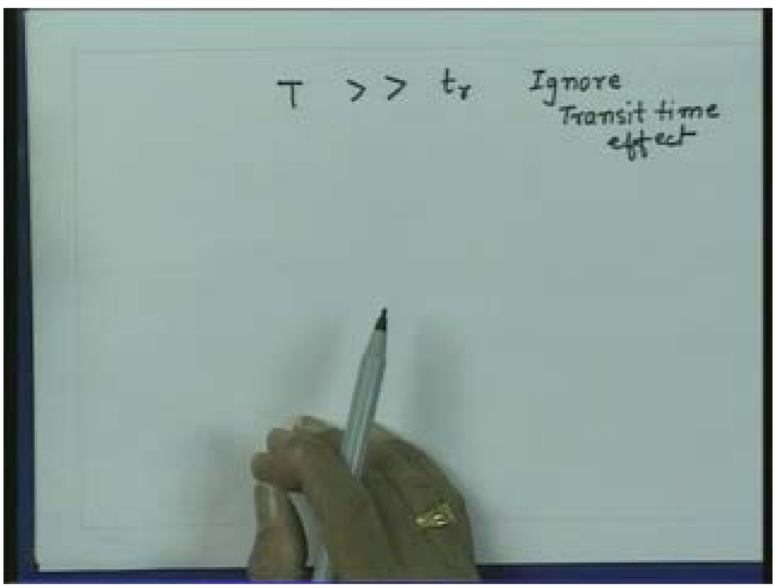

However when T becomes comparable to the Transit time that time we have to incorporate the Transit time effect into the circuit analysis. I can substitute for the T and the Transit time from here so T = $1/f$, the Transit time is length of the line divide by the velocity so the condition is,

$$T >> \frac{l}{v}$$

Or, $$\frac{1}{f} >> \frac{l}{v}$$

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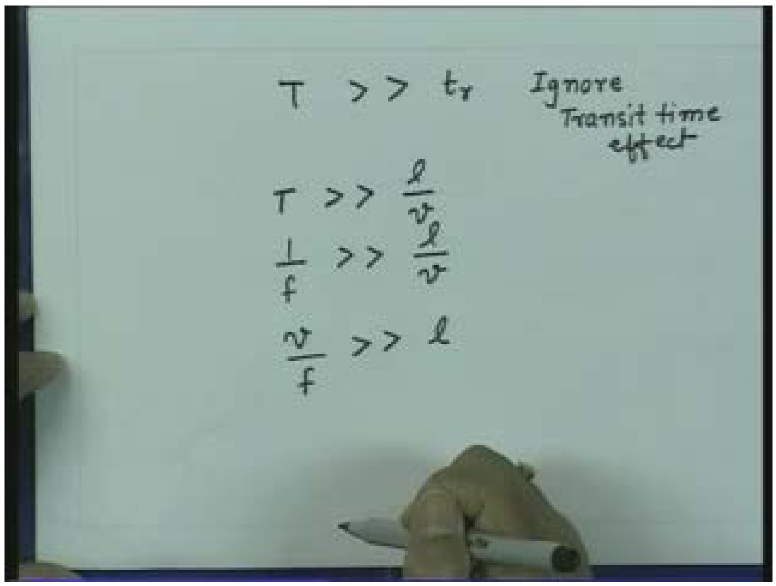

I can bring v this side so this becomes velocity $\frac{f}{v}$ should be much greater than the length of the line.

But we know from our very basic physics that the velocity divide by frequency is something called wavelength so this quantity is wavelength. So we get wavelength $\lambda = \frac{f}{v}$ should be much greater compared to the length of the structure.

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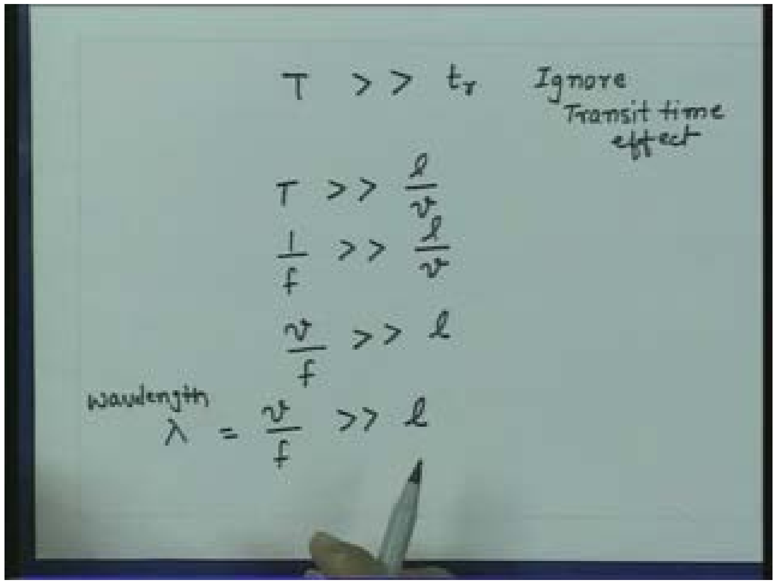

So if the wavelength of a signal or the voltage which is I am applying across the line is much larger compared to the physical size of the line then the Transit time effects can be ignored. However whenever the wavelength becomes comparable to the physical size of the element then the Transit time effects cannot be ignored I have to take the Transit time effect into consideration.

So as I mentioned as we increase the frequency the wavelength becomes smaller and smaller and the same physical length of a wire becomes comparable to the wavelength and then we have to take the Transit time effect into consideration.

Now we have to do the analysis of the Transmission Line taking into account the effect of Transit time. Just to elaborate on this point what we are now saying is while carrying out a circuit analysis the size of the electrical component plays a role. As we saw earlier at low frequency in the circuit analysis if the electrical value of the component was right we get an accurate circuit analysis.

However as we increase the frequency the size of the electrical component plays a role which means let us say in a circuit I had a resistance of 1Ω if I make that resistance in a size of one centimeter then I will get some performance, if I make the size of the same resistance over one meter I will get another performance. So though electrically the resistance had a value of one ohm since the physical sizes are different for these two resistances depending upon the frequency of operation I will get different circuit performances.

This is the very important aspect of the circuit analysis rather now we have drastically changing our approach to the circuit analysis where up till now the circuit analysis was not taking the space into consideration.

As soon as we come to the regime where the Transit time effects becomes important then the space has to be brought into the analysis of the circuit. However while carrying out this analysis we still do not want to depart too much from our analytical approach for low frequencies so still we would like to use the voltage and current laws. But we would like to accommodate the changes which we need because of the Transit time. One thing will be immediately clear that if I take this piece of wire if I divide this wire into very small sections then I can always look at a very small section and that small section will be much smaller in length compared to the wavelength. So the Transit time effects can be negligibly small over the small section of the wire so as the whole Transit time effect cannot be ignored.

But if I go to the small section of this wire the transit time effects can be negligibly small so I can still apply more or less the lump circuit analysis or low frequency circuit analysis. However if I want that whatever analysis we would carry out now is valid at any arbitrarily large frequency which means at any arbitrarily small value of λ then I must take a small section of this wire which is approaching to zero.

So if I take a length of this wire or length of the structure and make it oppose to zero then for any arbitrarily high value of frequency or any arbitrarily small value of λ the Transit time effects will be negligibly small and I can still apply the low frequency circuit analysis to this small element. Precisely this is the approach essentially we take.

We takes this structure divide the structure into small elements, apply the voltage and current laws over the small element and let that small element size goes to zero so that whatever relationship we get between voltage and currents is valid at any arbitrarily high frequency. And that gives you essentially the relationship of voltage and current for a transmission line in the presence of Transit time effects.

However before we do that we have a small problem and the problem is normally when we define the electrical parameter for any component we define that parameter for entire component.

If I take an inductor I know the inductor value as a whole, if I take a resistance I know the resistance value as a whole. However since I am now going to divide this structure into small segments knowing the value as a whole does not serve any purpose. I should know the value of that small element and that is the reason when we carry out this analysis of Transmission Line at high frequency, it does not make any sense to define the electrical parameter for the entire length.

It will be rather more useful if we define this parameter per unit length of this wire. So now whatever are the parameters of this we define these parameters per unit length of this wire. Before we go into this analysis, since we found out a voltage difference between these two points and these are the two conductors which are ideal conductors we may wonder at this point if this is an ideal conductor there is no resistance here. Then, what is making the potential difference to lie between these two points? We know if this is an ideal conductor there is no potential difference across the conductor. However now what we should remember is we are talking about the time varying signals and we are talking about the high frequencies.

So as the frequency increases the same two conductor system as the ac the electrical element, you have the magnetic fields around this conductor as the ac current flows in this.

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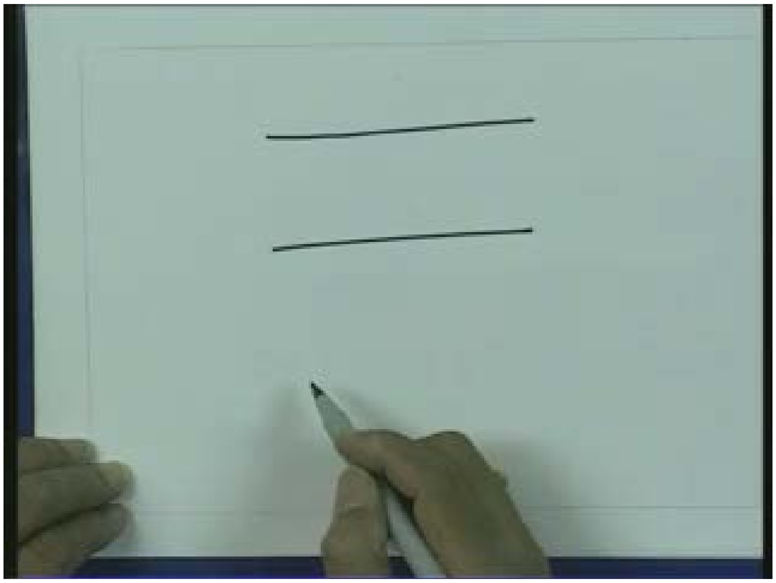

So you have the magnetic fields which are around these conductors and you have the electric field between the two conductors.

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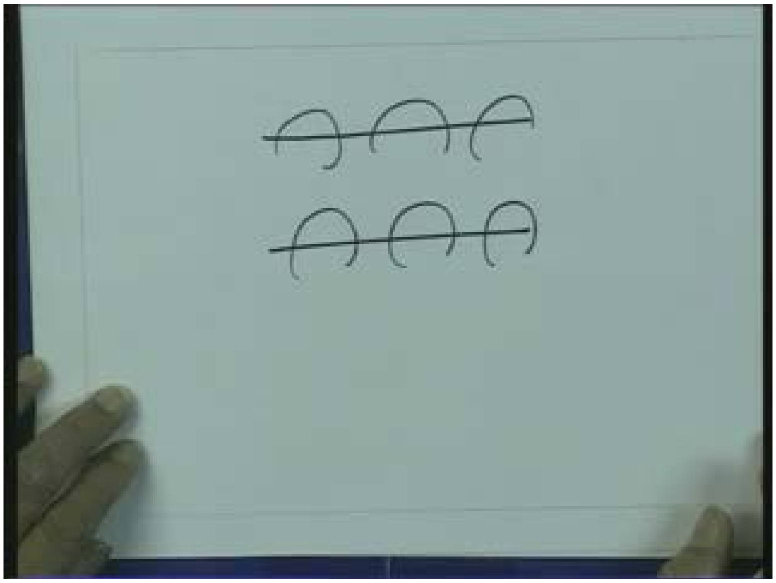

These magnetic fields linkage with the current that gives you the inductance of this wire and the separation between these two conductors by dielectric essentially gives you a capacitance between the two conductors. So we have an inductance along the conductance and we have the capacitances between the conductors. And this inductance or capacitance is not located at any point but they are distributed all along the length of this structure.

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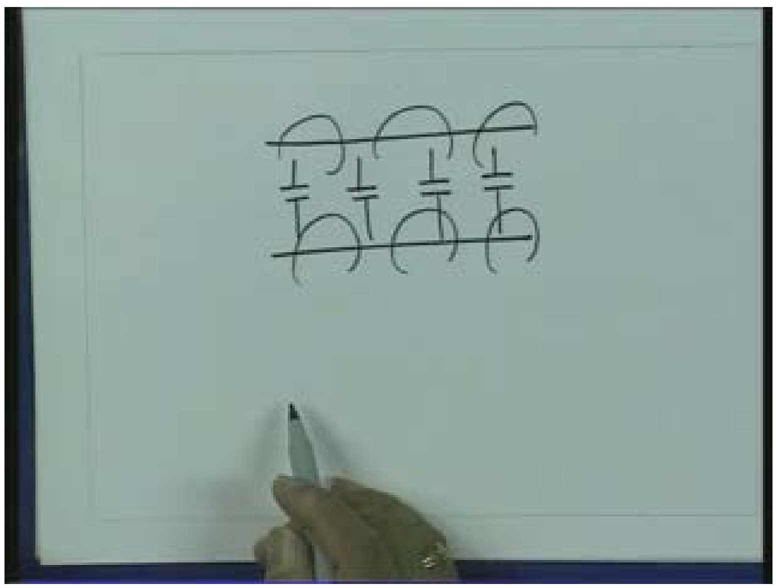

These parameters are called the distributed parameters of the line and the analysis which we are going to carry out for the Transmission Line is called a distributed circuit analysis. So these elements are called the Distributed Elements.

So as I mentioned these distributed elements are not located at any point but they are distributed all along this structure. And the better way to define this parameters are the value of this parameters per unit length of this line, that is what we do when we do the analysis of Transmission Line that we define this parameters per unit length.

Then coming to the point that why there was a potential difference between these two points when their resistance of this wire is zero is that as the frequency increases the reactance increases due to these inductance and capacitance. So the reactive elements start playing important role and that is the reason we have a potential difference. So although we do not have a resistance or potential difference there is reactance now and its value is going to increase as we increase the frequency.

So from the finite time point of view there is a potential difference because of there is a finite delay between these two points from the electrical circuit point of view I have the potential difference because the reactive elements now start playing more important role. So the dominance of the reactance is at high frequencies and the Transit time effect they are thus same phenomena or rather they are the two faces of the same coin.

So you can look at the phenomena from the transit time point of view or you can look at the phenomena from the dominance of the reactance point of view. Now at high frequencies when we carry out the analysis inclusion of the reactive component is important rather we will see later that the resistance of this lines is negligibly small and they are the reactance's of the inductance and capacitance, they only play a very important role in transferring power from this end of the line to the other end of the line.

Now what we are saying is we define this parameters the inductance capacitance and also we can say that this conductors can in general have some resistance, also the medium which is separating these two conductors is not ideal there is some small leakage current which can flow in that dielectric material which can be accounted for by introducing a conductance between these two conductors.

In general we can characterize this structure by resistance, inductance, capacitance and conductance and we can define these quantities per unit length of this line. So what we do now is the distributed elements which we define they are defined over per unit length of the line. So, essentially this structure is equivalent too.

You have the inductance, the resistance, the inductance, the resistance, the inductance, the resistance in a series and then you have a capacitance, a conductance, a capacitance, a conductance and so on. So generically any two conductor system can be represented something like this where the inductance which I represented only in one half is the total inductance which is for the both the conductors of the Transmission Line.

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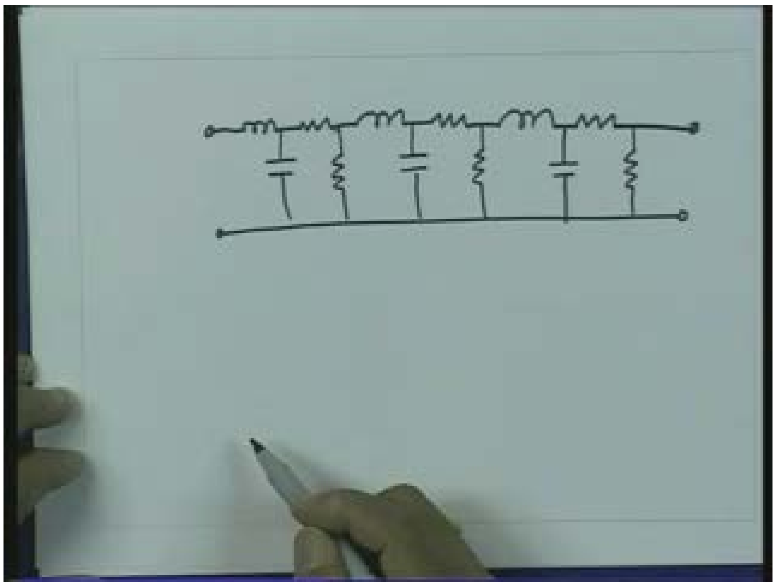

Now if I measure this quantity the inductance per unit length of the line which is the value of the distributed element per unit length same is true for Resistance, Capacitance, and Conductance and so on.

So now I define the quantity R which is Resistance per unit length of the line, L is the Inductance per unit length of the line, C is the Capacitance per unit length of the line and G is the Conductance per unit length of the line. The units for R are $\frac{\Omega}{m}$, the L will be Henry per meter, C will be Farad per meter and G will be moles or Simon per meter.

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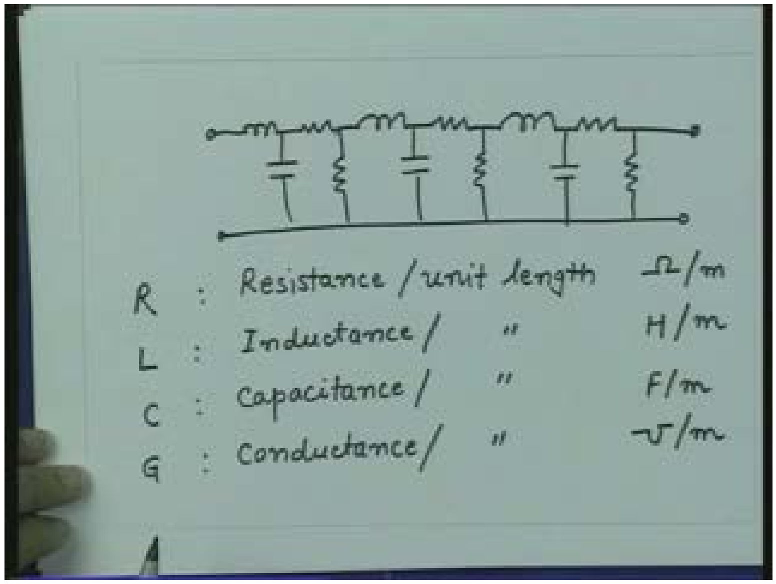

Now I have defined these quantities which we call as the Primary constants of the line which are the values of the Resistance, Inductance, Capacitance and Conductance per unit length of line. So these parameters are called the Primary constants of the line. So for any transmission line we can define these primary constants.

Once we know the primary constants then the analysis of the Transmission Line can be carried out. As we discussed divide this Transmission Line into a small segments, write down the Kirchhoff's law the voltage and current and then carry out the analysis in the limit when the size of this segment goes to zero. So the analysis is valid for any arbitrarily high frequencies or any arbitrarily low wavelength.

So we divide the Transmission Line structure into small elements. Let us take any one of the elements let us say its length is given by Δx where x is the distance along the Transmission Line.

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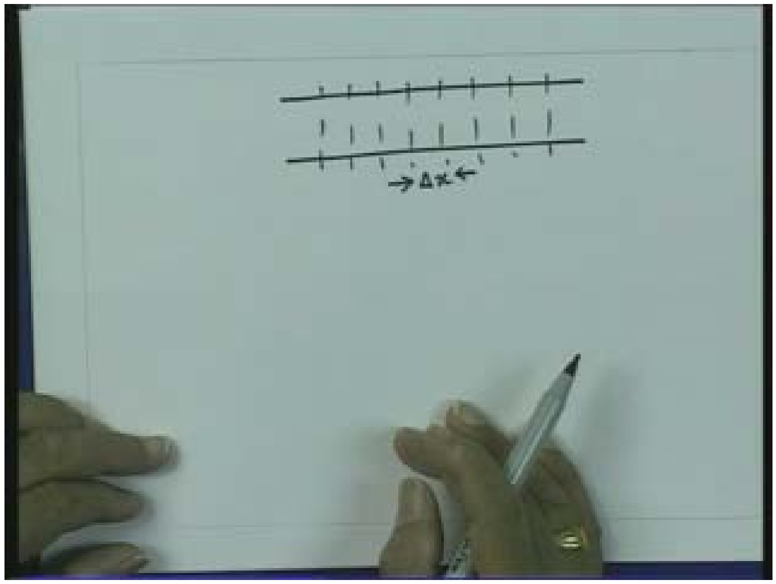

Then this small element can be represented by a combination of Inductance, Resistance, Capacitance and Conductance with appropriate values and since I know these values per unit length I can multiply that by Δx. So I know this value for this small element so this small section of the Transmission Line is equivalent to a Inductance, a Resistance, a Capacitance and a Conductance and the value of this will be the value of the line per unit length multiplied by the length.

So this is $L\Delta x$, this is $R\Delta x$, this is $C\Delta x$, and this is $G\Delta x$. So this infinitesimal section of transmission line can be represented by this lumped circuit and we can carry out the analysis of this small section by applying some voltage V at this end. Now some current is going to flow in this circuit as you apply a voltage V between these two terminals. Let us say that current is I. when the current flows into this element which is the series R of this network there is going to be a small potential drop across this so the voltage at this location will not be same as V. So let us say that voltage will be $V + \Delta V$.

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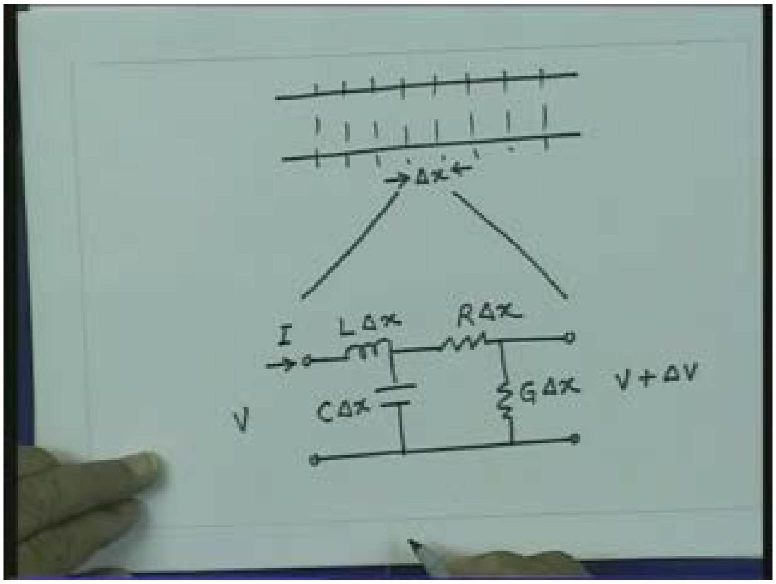

Similarly the current which I am going to get here is not same as I because part of current is going to be passed through this parallel R. So the current which you get at this location will be different than the input current let us say that is $I + \Delta I$. So the change in the current from this location to this location is the current which is passed through these arms, change in the voltage is the potential drop across this element since the section is very small this change in the current and voltages are very small.

If I take the first order approximation to this circuit the change in the voltage and current can be written on this circuit as ΔV will be equal to the impedance of this circuit which is $R\Delta x + j\omega L\Delta x$. Since the frequency is f, the angular frequency is ω which is 2πf so the reactance of this element will be $\omega L\Delta x$. So I can write the change in voltage which is $-(R\Delta x + j\omega L\Delta x)I$ and the change in current ΔI will be equal to $-(G\Delta x + j\omega C\Delta x)V$ where the negative sign indicates the change in the voltage and current are negative and that we can see from here since the direction of the current flow you have taken is this, this current will be less than the current at this point and similarly there is a potential drop across this element, the voltage at this location will be small compared to this. So this change in voltage the way we have defined that this voltage is V + ΔV and current is I + ΔI I, these quantities are going to be negative.

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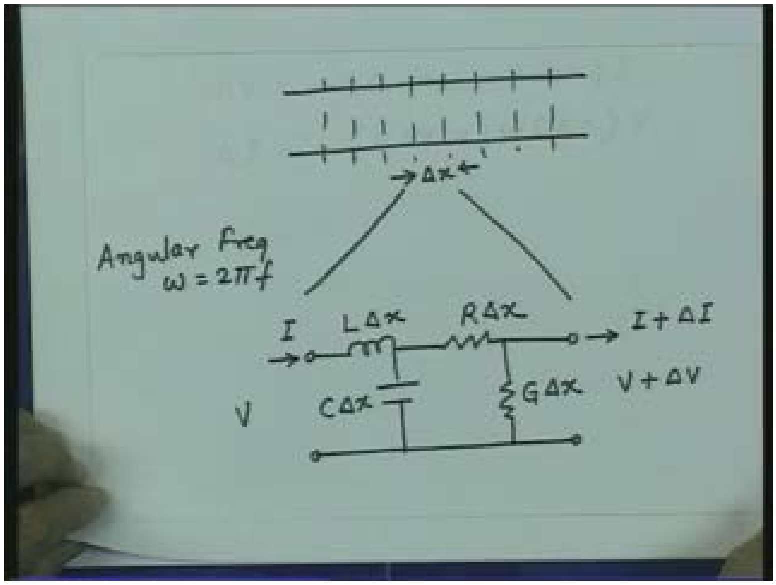

So that is the reason why we have a negative sign here. Now I can take Δx common and on the other side and as we said earlier if this model whatever we have taken is valid at all frequencies at any arbitrary high frequency then the size of that element must tend to zero. That means we have to now carry out the analysis of this in the limit when Δx tends to zero.

So if I take that then I can take $\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x}$ and we know from our basic calculus that this quantity is nothing but $\frac{dV}{dx}$. So this is equal to $\frac{dV}{dx}$ which is again equal $-(R + j\omega L)I$.

Similarly from this equation if I take a limit $\lim_{\Delta x \to 0} \frac{\Delta I}{\Delta x}$ that will be $\frac{dI}{dx}$ which will again equal $-(G + j\omega C)V$.

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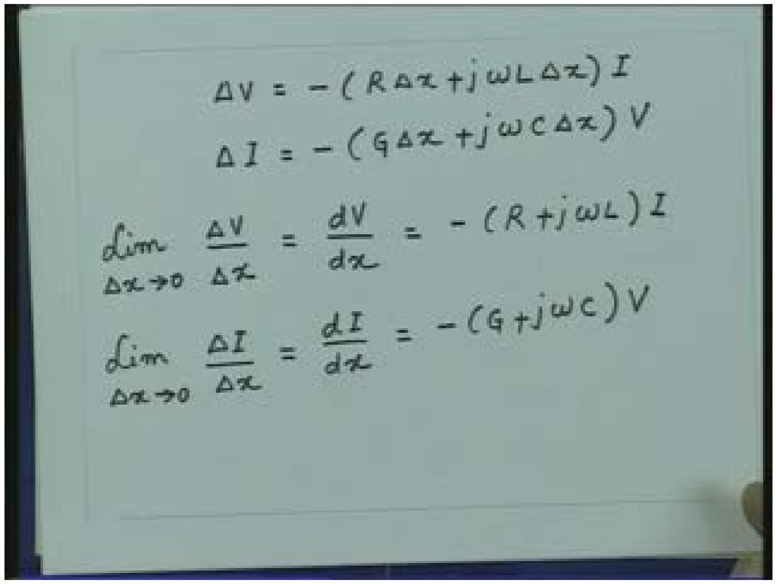

So now what we see is the voltage and current when we take this limit that the size of the element tends to zero are not related through algebraic equation but they are related through a differential equation.

Here the derivative of the voltage as the function of space is related to the current and the derivative of current versus space is related to the voltage. So essentially these two are the coupled equation where the voltage is related to current and the current is related to voltage. The basic relations which govern the voltage and current on this Transmission Line are in the form of this coupled first order differential equations.

So the major departure from our circuit analysis on the low frequency to high frequency is the low frequency we were dealing with algebraic equations. However, when we go to high frequencies you have to deal with differential equations.

How to solve these equations? First thing we can do is we can decouple these equations so I can take differential of any of these equations once more with respect to space and substitute from the other equations. So by doing this I take a derivative of first equation so I will get $\frac{d^2V}{dx^2}$ that will be minus $-(R + j\omega L)\frac{dI}{dx}$ and I can substitute for $\frac{dI}{dx}$ from this second equation.

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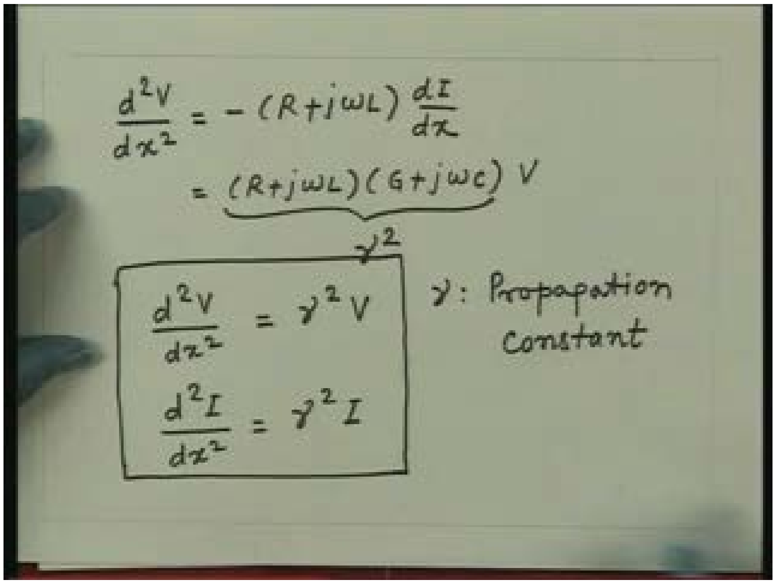

so I get $(R + j\omega L)(G + j\omega C)V$ where the two negative sign cancels each other and say you have positive sign here. Now this quantity whatever it is, is some kind of a characteristic quantity for a Transmission Line because it depends upon Primary constants of Transmission Line R, L, G and C and it is also depends upon the frequency of operation.

Let us denote this quantity by γ^2. So let us say I define this quantity as γ^2, we will see what the physical meaning of this is but at this point this quantity is some characteristics quantity of the Transmission Line. So in short we can write this equation as $\frac{d^2V}{dx^2} = \gamma^2 V$.

Exactly similar step if I had taken for the current equation that if I had differentiated this with x and substituted from V then I get identical equation for the current also. So I would get $\frac{d^2I}{dx^2} = \gamma^2 I$. Now these are the equations which govern the voltage and current on the Transmission Line.

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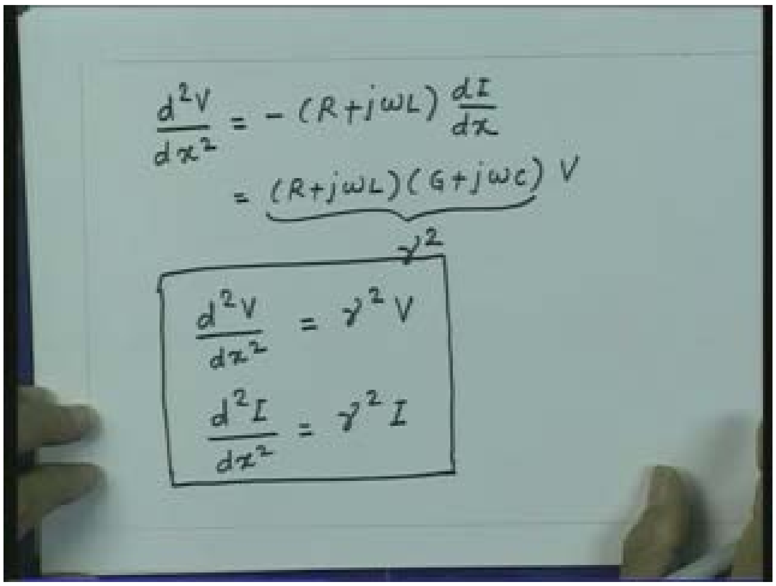

The quantity gamma is called the Propagation constant and we will see why it is called so.

Now, the quantities which we have taken here V and I when we write this voltages and currents are the peak quantities or the rms quantities. If I wanted to have the instantaneous value of the voltages and currents all of these quantities have a harmonic time variation that means though we have not said explicitly all this voltages and currents have a variation of $e^{j\omega t}$ where ω is the angular frequency and t is the time.

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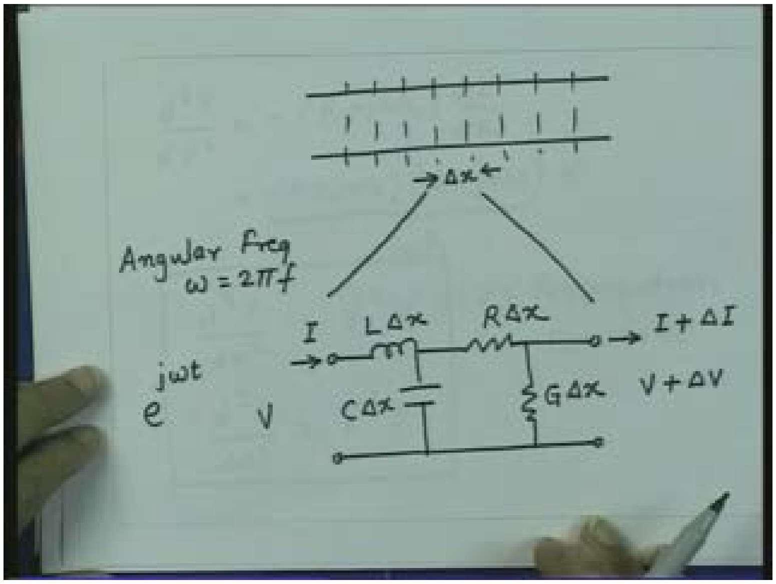

So, at any location if I find out the voltage and current varying sinusoidally as a function of time with an angular frequency ω. so if you have to get the instantaneous value of the voltage and current at any location and at any time essentially this value of voltage and current has to be multiplied by $e^{j\omega t}$ so we get the instantaneous value of the voltage at that location.

Now you can see the variation of the voltage and current. Essentially we have to take the solution of this equation and this equation has a very simple second order differential equation with constant coefficient where γ is constant for given line and given frequency then the solution of this can be written very easily. And the solution for this equation let us say you take the first equation the voltage equation so the solution for this can be written as V(x) is some arbitrary constant which we have to elaborate into $e^{-\gamma x} + V^{-} e^{\gamma x}$.

If you have to find the instantaneous value of this then all this quantities have to be multiplied by $e^{j\omega t}$.

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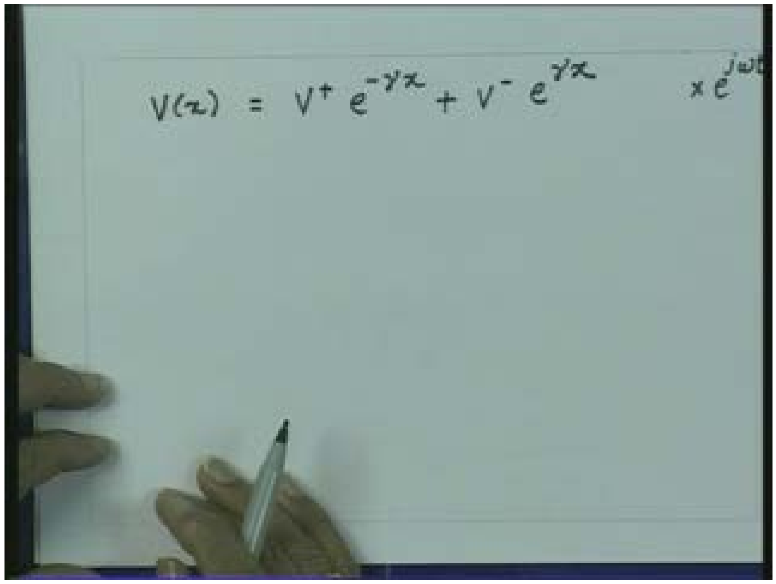

So if I am not interested in this instantaneous value if I want to find out let us say the rms value or the peak value then I can just use this equation if I have to find out the instantaneous value then I have to multiply each of this term by $e^{j\omega t}$.

Now the voltage or current on the line is a composite phenomena of space and time where it depends upon x it also depends upon the value t. So if I write down the voltage as composite phenomena of space and time then that will be $V^+ e^{-\gamma x} e^{j\omega t} + V^- e^{\gamma x} e^{j\omega t}$.

Here the quantity V^+ is a complex quantity and it represent the complex value of the voltage at x equal to zero location and the phase of this quantity is essentially the phase in time. So at location x = 0 if I calculate this value that will be the complex value of the voltage.

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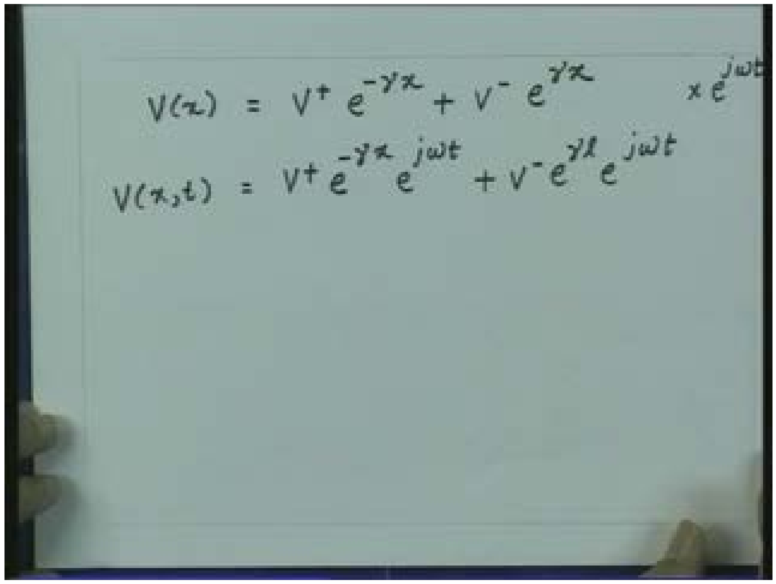

Now if I look at the expression there are two terms here and physically these two terms represent different phenomena and this is what essentially we try to see.

Let us consider the first term which is this term. So now we concentrate on a term which is $V^+ e^{-\gamma x}$ and also let me explicitly write down this γ as its real and imaginary part. So let us say in general the γ is a complex quantity so this has a real part α and an imaginary part β. So now here I can write explicitly $V^+ e^{-\alpha x} e^{j(\omega t - \beta x)}$

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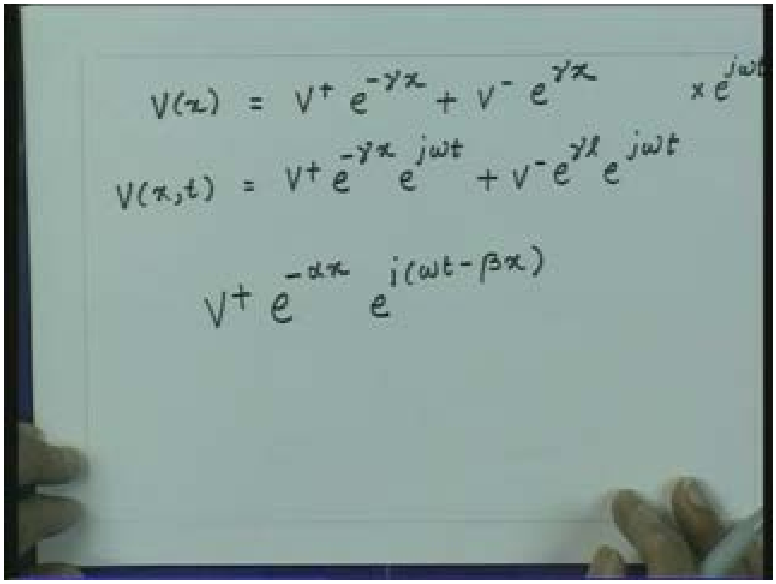

So first term here has a voltage which amplitude varies exponentially along the line and the phase of this voltage is a combination of space and time. This is a component which is coming because of phase, this is the component which is coming because of time.

Now the first term of this voltage is representing some thing whose amplitude exponentially varies along the length and the phase of width is a combination of space and time.

Let us take a simple case and let us say I have a structure for which α is zero and also without loosing generality let us say this V^+ is a positive real quantity. So if I assume that V^+ is real and positive and alpha is zero the first term is for $\alpha = 0$ and V^+ is real, this term is $V^+ e^{j(\omega t - \beta x)}$.

Now I can ask that what this is. This term representing physically which is a combination of phase and time, what this is the meaning of this phase? If I take the voltage and plot as a function of space and time, if I take some value of time and I ask what the voltage variation is. So the voltage will be real part of this quantity so if I want to see the voltage essentially I have to see the voltage which is the real part of $V^+ e^{j(\omega t - \beta x)}$ which is V^+

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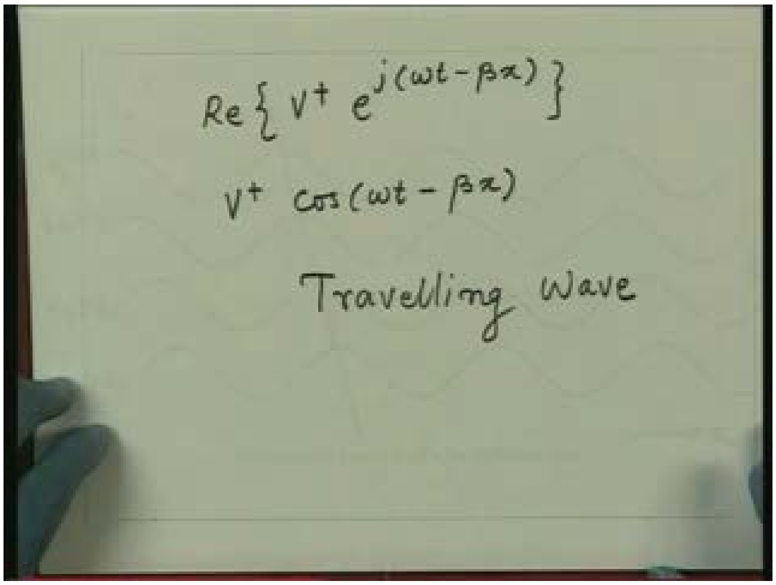

because V^+ is the real quantity that is what we assume, $\cos(\omega t - \beta x)$. So if I take some value of t and vary x I will get a function which will be sinusoidal function, if I change the value of t to some positive value I will get another function which will be shifted in x. so if I look at this function, this function essentially in space will look like

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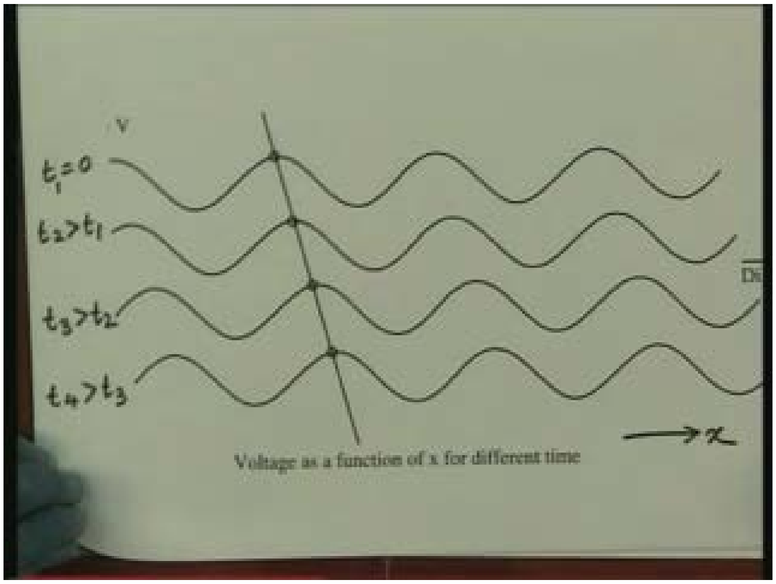

So this is the direction x and here let us say this is $t_1 = 0$, I have $t_2 > t_1$, $t_3 > t_2$and $t_4 > t_3$ and so on.

So if I take this term and plot for different values of t first take $t_1 = 0$ then t_2, t_3 and t_4 the function essentially will look something like this. So if I concentrate on any one point of this function let us say peak of this function and as the time increases the point will shift right wards it is shifting from here to here to here to here. If I look at these phenomena as total phenomena of phase and time it will appear as if every point on this function is moving rightwards.

So this phenomena is called the wave phenomena or Traveling wave phenomena. so the term which we have got here which have the phase which is the combination of space and time essentially represents a Traveling wave phenomena. And since here we are having a variation of voltage this term essentially represent a traveling wave in the positive x direction the wave is traveling from left to right.

If I do the same thing for the second term of the solution which is V^{-} $e^{\gamma l}$ $e^{j\omega t}$ will have a phase term for which the sign of β will be changed so it will be ωt + βx. Then that will represent a phenomena which will be a phenomena traveling in the opposite direction this is x again this is $t_1 = 0$, $t_2 > t_1$, $t_3 > t_2$ and so on

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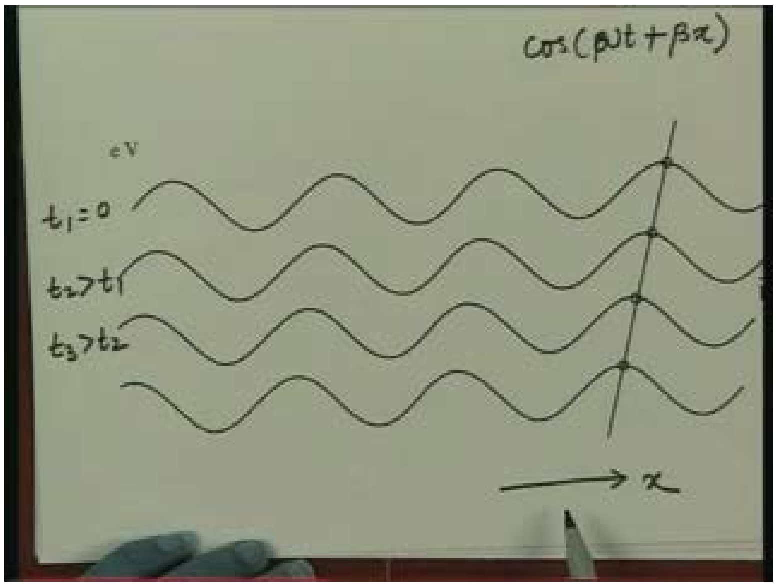

Now in this case when your function is the second term of the solution where the function is cos(ωt + βx) the point will appear moving leftward or in the negative x direction but in both the cases the phenomena appears to be moving on the Transmission Line. So what we see in general is that as soon as we introduce a Transit time effects on the Transmission Line the voltages and so will happen to the current will exist in the form of traveling waves on the Transmission Line.

So in general whenever we are having high frequency analysis the voltages and currents have to be visualized in the form of the waves. So now we see from this lecture that a departure from the lumped circuit analysis to the distributed circuit analysis radically changes the approach to the circuit analysis and now all the voltages and currents exist in the form of waves on the electrical circuit. Thank you.