Transmission Lines and E. M. Waves
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Lecture – 19
Maxwell's Equations

In the last lecture we got the mathematical formulation of the physical laws of electromagnetics that were the Gauss' law, Ampere's circuit law and Faraday's law of electromagnetic induction. The compilation of these physical laws in mathematical form is called the Maxwell's equations. So Maxwell essentially compiled these laws, gave the mathematical representation; however, while doing this we found a difficulty. He found there was inconsistency in the Ampere's circuit law. Let us understand what was the difficulty which was encountered by Maxwell while compiling these equations.

So let us consider a closed surface of certain volume let us say V and let us say these are charge distributed inside this surface then there is a possibility that the charges might leave this surface and when the charges will leave there will be a rate of change of charge so there will be current flow from this surface. So we will have a current density distributed on the surface and whatever charges will be leaving in the form of current that should be reflected in the reduction of the net charge inside this surface.

So let us say we have the conduction current density on the surface which is given by J, then as we saw from the concept of divergence that if you take this vector field J the divergence if J essentially is a net flux coming out of this volume. So if I just take a divergence of J that essentially should represent like the outflow of this vector field which is the conduction current density. Before getting into that let us say that I have a small infinitesimal area on this surface which if given by da. So, if I take the dot product of conduction current density and the area that gives me the outward current from this infinitesimally small area and if I integrate that over the entire surface I get the net outward current from this surface.

So I have here from this surface the net outward current which is nothing but integrated over the closed surface let us say the surface is S, the conduction current density over that infinitesimal area. So this integral as we saw earlier also that it represents the total current coming out of this surface. As we know the current is nothing but the rate of change of charge and since this current is coming outwards from the surface it should be equal to the rate of charge from this volume and since the current is coming outwards the net charge must be decreasing inside this volume.

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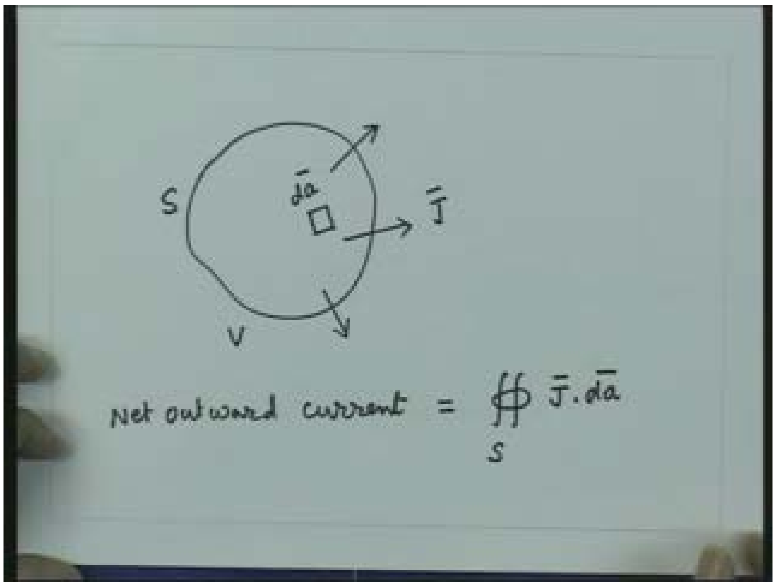

So if I say that this volume is having a charge density which is given as rho and this is the conduction current density then the net decrease or rate of decrease of charge inside this volume must be same as the total current which is coming from this surface. So if I have the charge density rho then the total charge enclosed by this surface will be the integral of rho over the volume. So if I take now then rate of decrease of charge rate of change of charge in the volume that quantity will be equal to minus d by dt for rate of change of the total charge enclosed in this volume so that is the integral rho into d.

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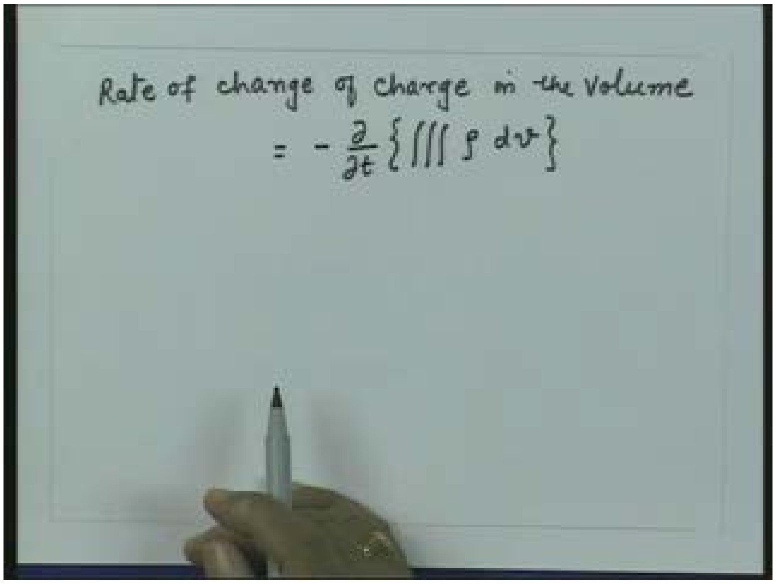

What we are now saying these that, simple by definition of the current, that is the current leads the rate of change of charge, this net flow of the current from this surface that should be equal to the rate of change of charge from this is volume. So essentially from this we get that the closed surface integral over S, this is over volume J dot da that is equal to minus d by dt of integral over the volume rho into dv.

If I say the volume is not changing with time as we assumed earlier we can change the charges are changing as a function of time, the volume is fixed, we can interchange the integral dt so that quantity that we can get as minus integral over the volume d rho by dt into dv.

Now as we have done earlier we can convert this surface integral (Refer Slide Time: 7:43) into the volume integral by applying the divergence theorem. So this thing by applying divergence theorem we can write as... from the divergence theorem or using divergence theorem we can get this integral as the volume integral V del dot J dv that is equal to minus the triple integral over the volume d rho by dt over the volume.

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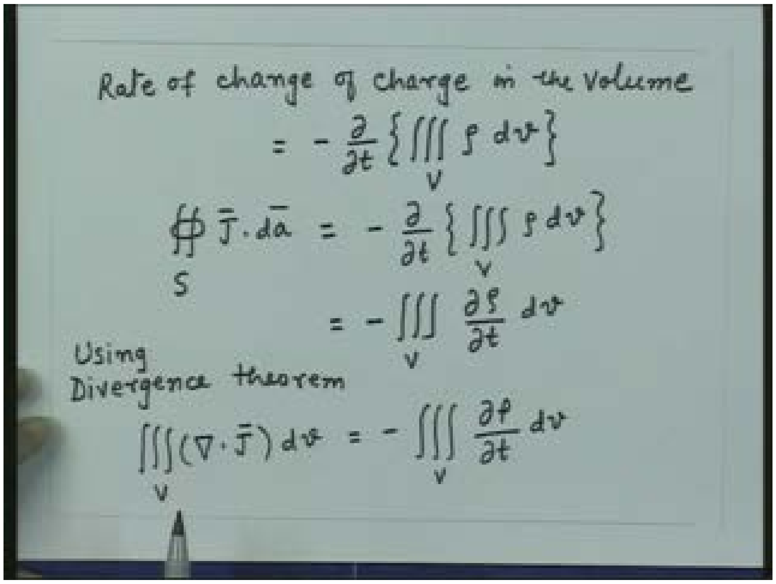

Now I can bring this term on this side and write the integral as the triple integral over the volume del dot J plus d rho by dt over the volume that should be equal to 0. And as we did in other equations, since this relation should be true for any arbitrary volume this quantity must be identically zero so the integrand must be identically zero so from here essentially we get that del dot J that is equal to minus d rho by dt; this equation is called the continuity equation.

So if we have the time varying charges then this quantity is the finite quantity and then the divergence of the conduction current density is not zero; if we take a static case where the charges are not changing then the divergence of the conduction current density is identically zero.

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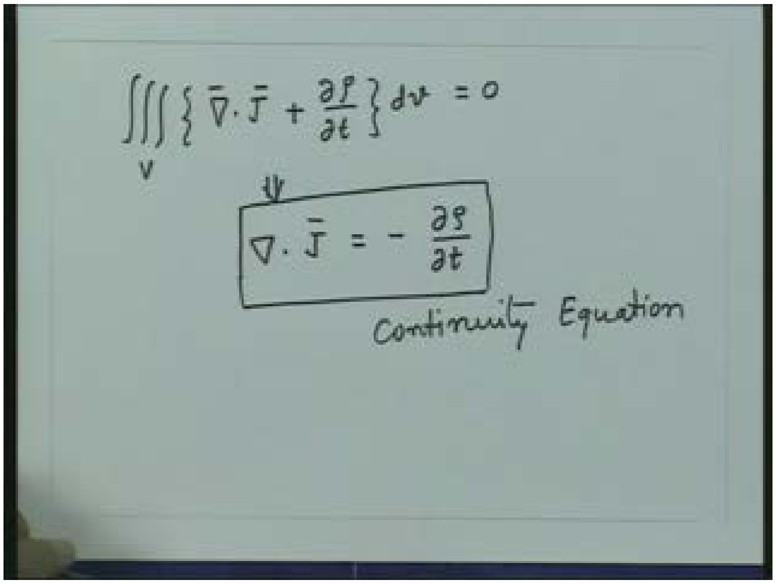

And that makes physically sense because if you are saying the divergence of J as we saw earlier from the concept of divergence the divergence tells you the net quantity coming out of this volume and if the net current is coming out of this volume that means there must be... the net charges must be leaving this volume so there must be a change in the total charge or the charge density inside that volume. If the charges are not changing inside the volume then whatever charges are entering in the volume those charges must be leaving so the net charge inside the volume remains same and in that case the divergence of the conduction current density is equal to zero because there is no net flow from this volume for the current.

So whenever we are having in general the time varying quantities the quantitative equation must be satisfied by the conduction current density and the charges. Precisely this was the equation which led to difficulties for compiling the other equation by Maxwell. Let us see what the problem is.

We have seen earlier that from the Ampere circuit law we get an equation which is del cross h that is equal to J (Refer Slide Time: 11:16) so the curl of the magnetic field is equal to the conduction current density. If I simply apply the vector operation on this without getting into physical aspects we can simply say I can take the divergence of this equation on both sides so I can say divergence of curl of H is equal to divergence of J. So if I just take divergence I will get from this relation del dot del cross H that is equal to dell dot J. We know in the vector product we can interchange the sign dot and cross so this thing can be also written as: del cross del dot H equal to del dot J.

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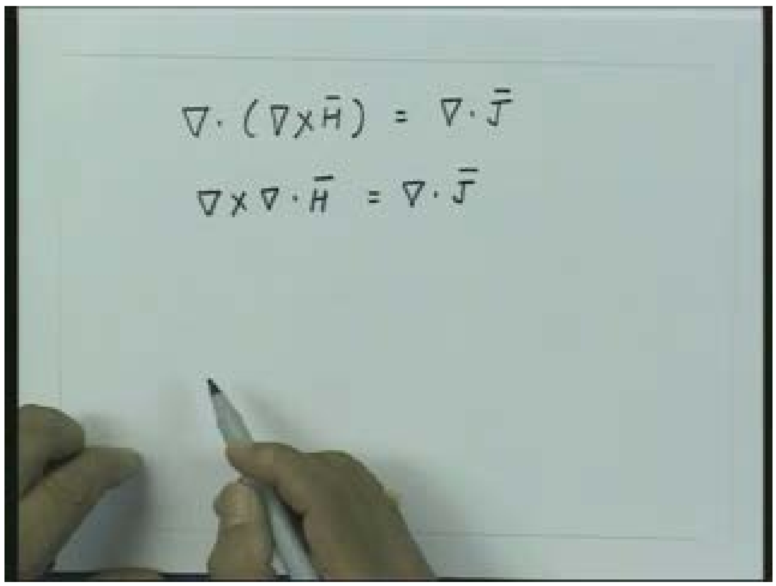

Now this quantity is identically zero so this quantity is identically zero. What that means is from the Ampere circuit law that the divergence of J is identically zero; that is a precise problem because it is inconsistent with the continuity equation.

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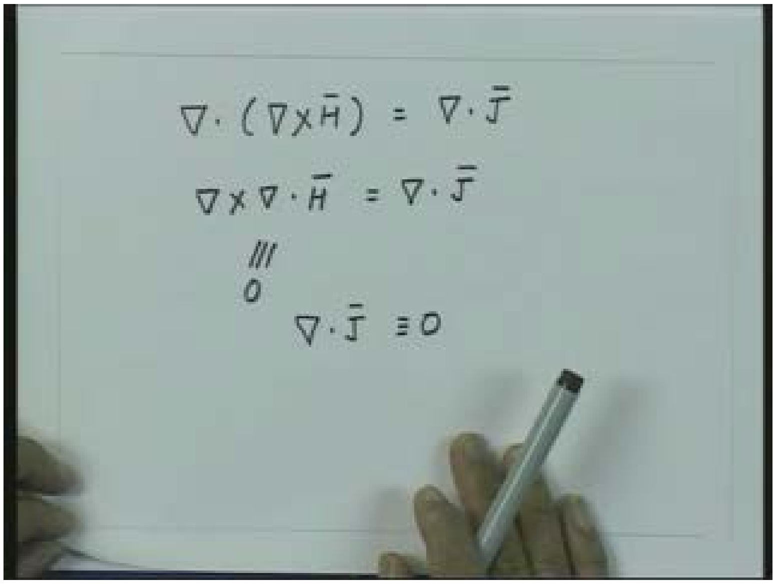

The continuity equation says that the divergence of J has to be negative of rate of change of the volume charge density whereas the Ampere circuit law says that the divergence of J must be identically equal to 0. So basically the Ampere's circuit law does not satisfy the continuity condition; precisely that was the difficulty which was encountered by Maxwell and He had difficulty by introducing the concept of what is called the displacement current density.

So what He said is if I have this rho... and I go to one of the Maxwell's equations and if I replace this rho from the Gauss' law which says that the rho is the divergence of the displacement vector D so I can write this quantity as del dot D from the Gauss' law. See if I substitute for the charge density rho from the Gauss' law we have del dot D is equal to rho so I can substitute for rho in this equation so I get del dot J is equal to minus d by dt of del dot D interchanging the operators space operator and time operator this will be minus del of dot of dD by dt.

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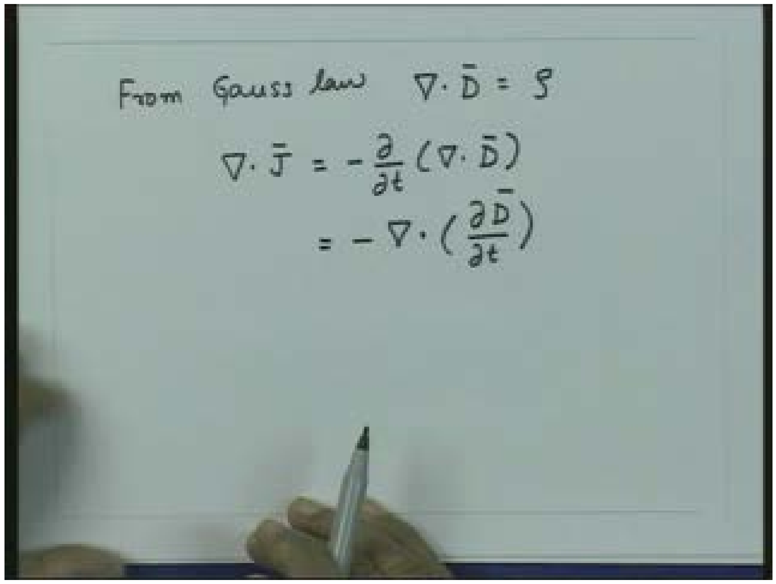

I can bring this thing on the left side and integrate over volume. This is del dot J plus del dot dD by dt dv that should be equal to 0. From here again I am using the divergence theorem; this thing can be written as over the surface A J plus dD by dt into da that should be equal to 0. What we are now saying that is that the current which is coming from this surface which we saw the closed surface is not only this current which is the conduction current but there is some quantity which is also having dimensions which are the current density dimensions. However, this is the rate of change of the electric displacement.

So dimensionally if I look at this quantity this quantity is same as the current density. However, this quantity does not depend upon the conductivity of the medium. We have seen earlier that the conduction current density is related to the conductivity of the medium; from the Ohm's law it is sigma into E; however, even if the conductivity of the medium is zero then we have a quantity which is rate of change of the displacement D and that quantity is equivalent to some current and since this is related to the displacement vector D we call this as the displacement current density. So this quantity (Refer Slide Time: 16:38) is the new quantity which was introduced to satisfy the continuity equations and that is called the displacement current density.

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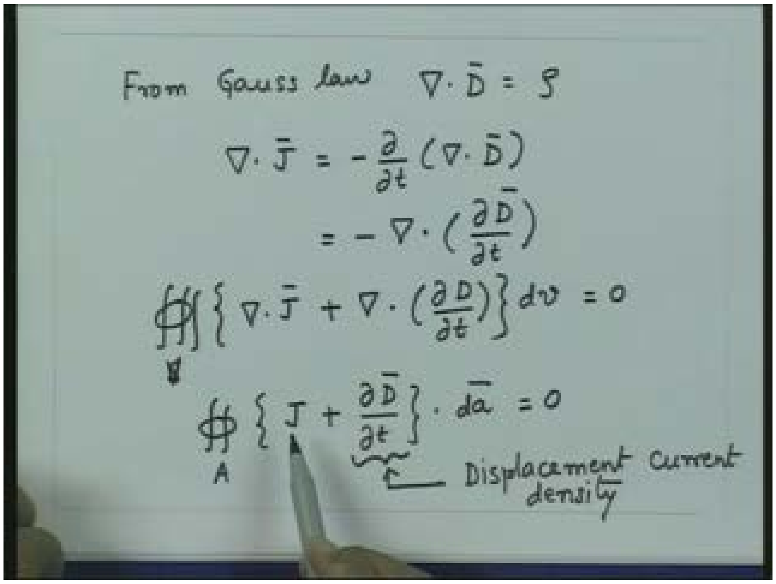

Once we say that then what we are saying is now the net current which we are talking about is not only the conduction current but it is the conduction current plus the displacement current. And then Maxwell said: if you incorporate this term appropriately for defining the total current then the Ampere's law can be modified to say that the magneto motive force around a closed loop is equal to the total current enclosed by that loop which includes the conduction current as well as the displacement current.

So earlier since this concept was not there the current was always meant by the flow of the charges. However, in this case there is no flow of charges that even if you... I do not have charges still I will get this quantity because this quantity D is related to the electric field. So, if I have an electric field which is time varying this term will be equivalent to a current flow. So this quantity which is simply representing the rate of change of electric field represents the quantity which is equivalent to current.

So, if I take the total current which is a combination of the conduction current which is because of the moment of the charges and because of the time varying electric field, this total current is equal to the magneto motive force around the closed loop. And once you do that then this equation of the Ampere's circuit law is modified because now this current is being changed to the conduction current density plus the displacement current density.

So, once you get that then the Ampere's law now has been modified is modified to del cross H that is equal to the conduction current density as we had earlier plus we are having now an additional term which is dD by dt. That essentially resolves the difficulties which were faced by Maxwell and now this gives you the complete description of the phenomena of electromagnetics.

So essentially now the four equations which you have got: the Gauss' law equation, the Gauss' law equivalent applied to the magnetic fields for magnetic charges, the Faraday's law of electromagnetic induction and the Ampere's law appropriately modified to accommodate what is called the displacement current density makes the complete set of equations which represent the static and time varying electric and magnetic fields. So these equations are called the Maxwell's equations.

So as we mentioned earlier the Maxwell's equations can be written in the differential form or they can be written in the integral form and depending upon the suitability the equations can be used either in differential form or they can be used in the integral form. So we finally write the four Maxwell's equations. We can write this in two forms so we can get these equations in differential and integral form.

So I write here the differential form or I can write this equation in the integral form. So if I go to the the first equation which is the Gauss' law that gives you del dot D that is equal to rho. The same equation when written in the integral form it says that D dot da over the surface that is equal to the rho into dv. The second equation when applied to the magnetic charges that gives me del dot B is equal to 0 and that can be written in the integral form S B dot da that is equal to 0.

The third equation which is Faraday's law of electromagnetic induction that gives me del cross E that is equal to minus dB by dt and the integral form of it can be written as integral over the contour E dot dℓ that is equal to minus dB by dt dot da.

And finally the Ampere's law which we have got after introducing displacement current density that is del cross H will be equal to J plus dD by dt and the integral form the same thing is over this contour H dot dℓ that is equal to the total current enclosed so that is J plus dD by dt into da.

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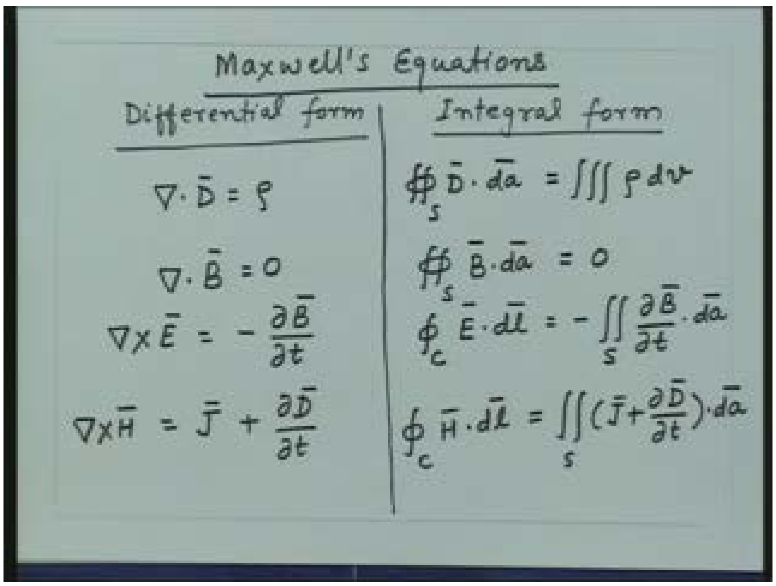

So now these are the set of equations which governs the total phenomena of electromagnetics for static as well as time varying fields. So once you have these generalized equations which are for time varying fields we can reduce the equation for the static fields that is by putting all time derivatives to zero; I will get the equations for the static, electric and magnetic fields. So depending upon whether the permittivity or the permeability of the medium is varying as a function of space that means if the medium is homogeneous I will use this equation, if the permittivity is constant in the function of space then I can write the same equation as we saw earlier; I can take epsilon now and I can say del dot E is equal to rho upon epsilon and so on.

So you may have various forms of these equations depending upon the condition applied to the medium whether we are dealing with the electric fields, magnetic fields or we are dealing with the displacement vector and the magnetic flux densities and so on. And also as we said if we are talking about the static fields then the time derivative should be put identically to zero so this quantity will be zero for the static field, this quantity will be zero for the static fields.

So if we consider the static fields then this quantity will be equal to zero so the curl of the electric field will be always equal to zero. So we will see depending upon application later on that we may put this quantity equal to zero if you are dealing with the static fields. However, in this particular course which is a course on electromagnetic waves we are dealing with the quantities which are time varying.

The static fields you would have studied in the earlier course which is now electromagnetic fields. So here we assume that all the fields and all the quantities which we are dealing here they are time varying in general. So the quantity rho, the quantity B, the quantity D, J, H all these quantities are essentially time varying quantities. So we will look for the solution of these equations later for the time varying fields.

Now as we mentioned earlier that this equation which is in differential form cannot be applied in a situation where medium has discontinuity. That means if we talk about the media interfaces where the medium property suddenly change the permittivity may change or permeability may change and there the derivatives the space derivatives cannot be defined because the medium is discontinuous so the differential form of this Maxwell's equations is not useful in those situations.

However, as we had mentioned, the integral form is always useful and this can be applied in a situation. However, when we apply the integral form to the discrete media interfaces we get a relationship between the quantities D B E H in the two media just across the interface and that relationship we call as the boundary condition. So the same set of equations when written in differential form they give you what is called point relation so these relations are valid at every point in space. The equation in integral form when applied to the discrete media interfaces they give you what is called the boundary conditions.

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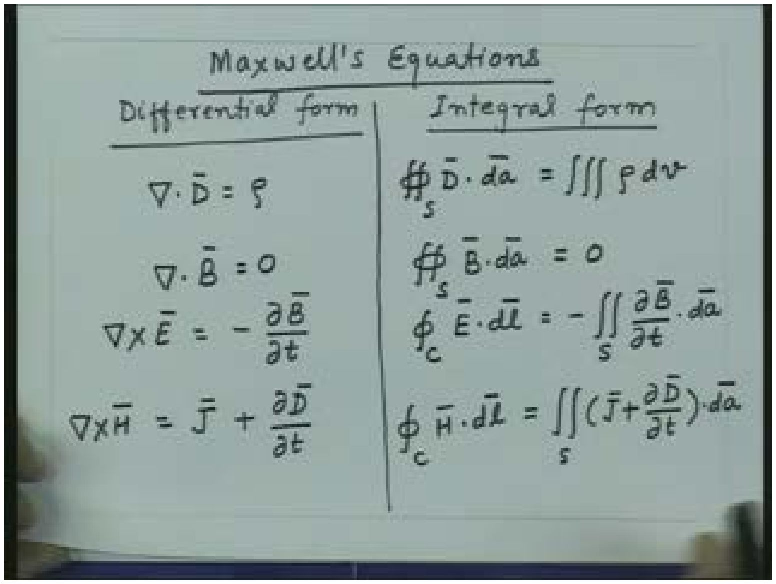

However, before we go into the boundary conditions let us introduce a concept of what is called the surface current and surface charges and the idea is as follows.

Let us say I have a surface which is having let us say thickness some D so the thickness for this is D and let us say I have a charge density inside this slab which is given by rho; I have a charge density which is rho which as we saw is Coulomb per meter cube that is the density which you have, so all over these charges inside is distributed to this density.

If I have considered an area which is unit area on the surface so let us say this length is 1, this is also 1; so if I consider a volume of unit area on the surface of the sheet the total charge which will be in this volume which is having a height of D and area 1 will be the charge density multiplied by the volume. So I have total charge in this area in unit area slab that will be equal to rho into the volume of this which is 1 into 1 into D so that is equal to d. So if the thickness is given then I can say if I see from the top that is the charge I am going to see which is: rho into D in this area which is 1 into 1.

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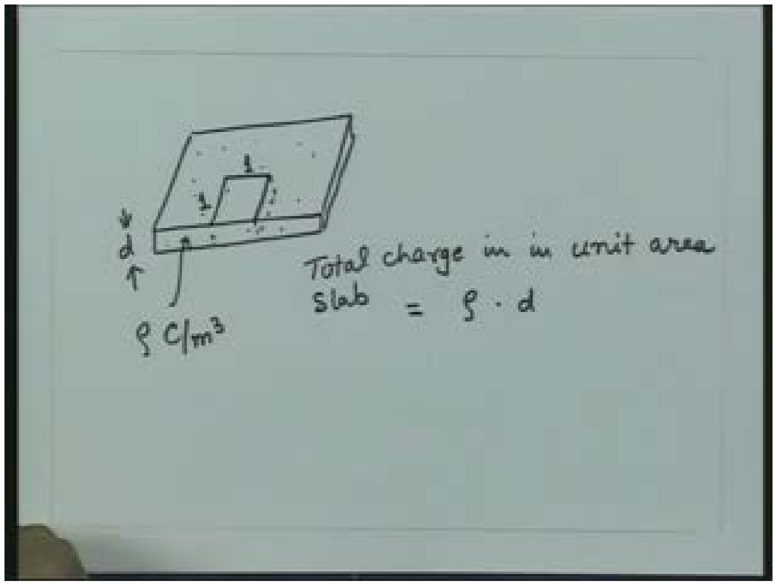

Now let us see if I reduce the thickness of this lab and go to a limit when d tends to 0 and suppose the rho increases to infinitive when this thing happens when d tends to 0 so you will get a quantity when this is tending to infinity, this is tending to zero so the product of these two still might tend to a finite quantity.

So if I have a situation where the volume charge density rho is infinity but the thickness to which this density is confined is zero I will see a net charge which is just on the surface because the charge is now confined to a thickness of zero width. So, since the thickness is zero the charge will be just lying on the surface and that charge now is in this area of unit area. So if I take this quantity rho into d and take a limit when d tends to zero, I get a quantity what is called a charge distributed on the surface and that density will be surface charge density. So the units for rho will be Coulomb per meter cube I am multiplying by this so I will get now a charge which is per unit area and this charge truly is confined to the surface of the sheet. So we say rho s is in the limit when d tends to 0 if I calculate this quantity rho into d that is what is called the surface charge density.

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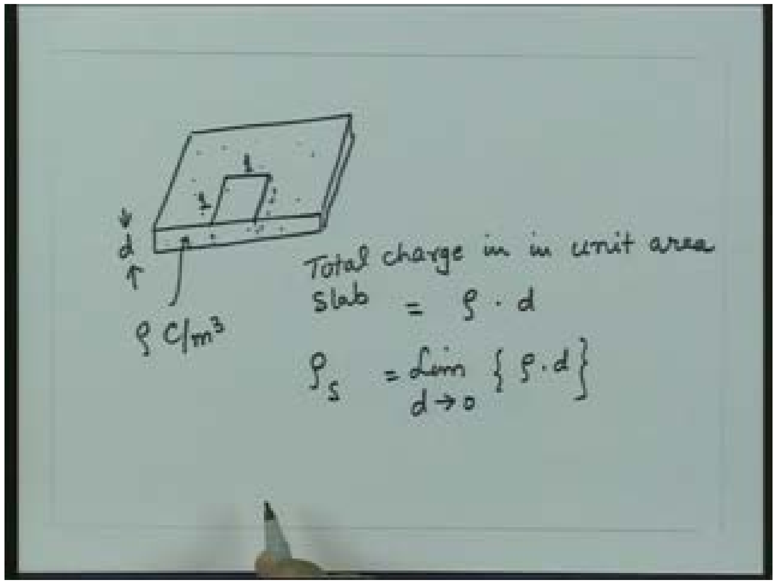

So the two things you should note here that if I go from this volume charge density rho to the surface charge density when d tends to 0 if two surface charge is there that is equivalent to the infinite volume charge density. If that infinite density is confined to zero thickness that gives me the distribution of charge truly on the surface and that is what is called surface charge density. So we have here what is called surface charge density and the unit for this will be Coulomb per meter square. So we have this quantity which will be Coulomb per meter square.

So, later on we will see that in situation like conducting boundaries where the conductivity becomes infinite you might get the charge density which you... the volume charge density which will be infinite and these charges truly will be confined to the surface and that time this concept of surface charge density will be useful. So at the moment without getting into which media will give me surface charge density we can say principally we may have charges distributed truly on the surface within a zero thickness and that charge density we will call as a surface charge density.

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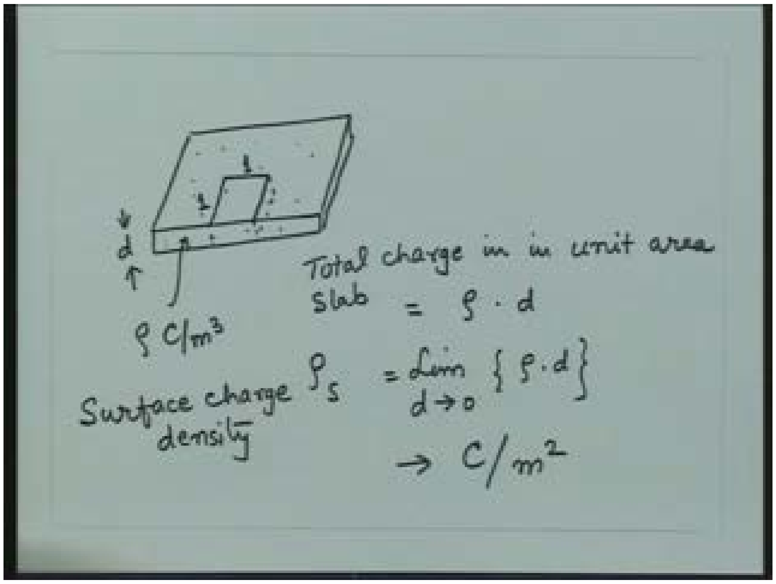

A similar thing we can do for the current also. So let us say I have again this slab of thickness d and let us say this is carrying a current which is J. Now let us consider now like this (Refer Slide Time: 33:17) so the current which is slowing now in this will be this is let us say this is unity, this is unity so the total current which will be flowing through this will be J multiplied by the cross section of this which if I see from the f side the cross section will be d into 1 so the total current which I am going to see under this area that will be J into 1 into d that is a current which is confined to this.

So, for the conduction current density J the current which will be flowing in the direction of J within the unit area on the surface will be J into d. So if I say this is the conduction current density I will say now the current under unit area on the surface that quantity will be equal to J into d into 1 which is the area of cross section is 1 into d so that will be the current which will be under this.

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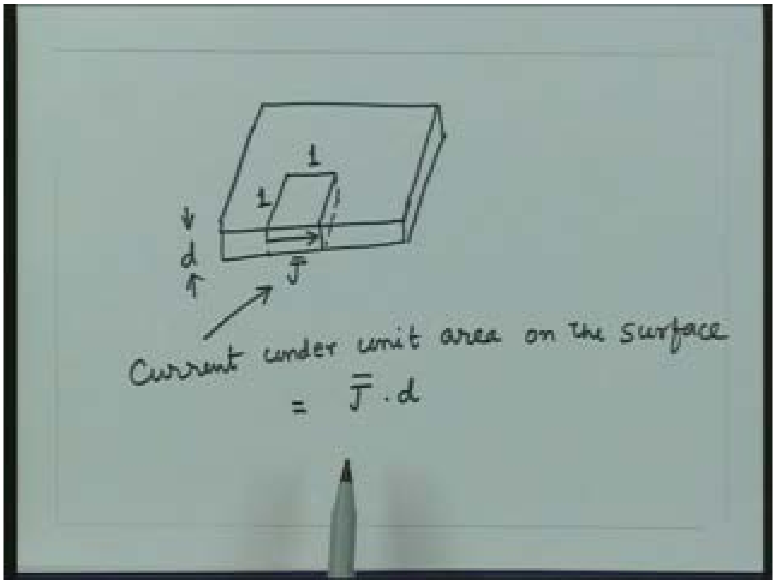

Now again if we make this d tending to 0 and if J goes to infinitive then again this product J into D will be a finite quantity so you will have a current which is truly flowing on the surface and that current we call as the surface current. So if I take a limit, so if I define what is called surface current density and that is given by J s that is equal to the limit when d tends to 0 J into d times d is not a dot product it is just the J multiplied by the area and direction of J s is same as the direction of J so this quantity will give me a current which is flowing on the surface.

And again as I mentioned, this will be true provided this quantity of the conduction current density J is infinite then and then only when d goes to 0 this product will be finite and we will have the surface current density. This again will be applicable to the boundary which are conducting boundaries so when the conductivity of the medium becomes infinite that time the current which will be sigma times E for finite electric field will become infinite and then you will have what is called the surface current.

Since the unit of the conduction current density is Ampere's per meter square the unit for this will be the Ampere's per meter square multiplied by d which is the length so the unit for J s will be Ampere's per meter; that is the reason this quantity is also called linear surface current density because this is now as the per unit length you are defining this current.

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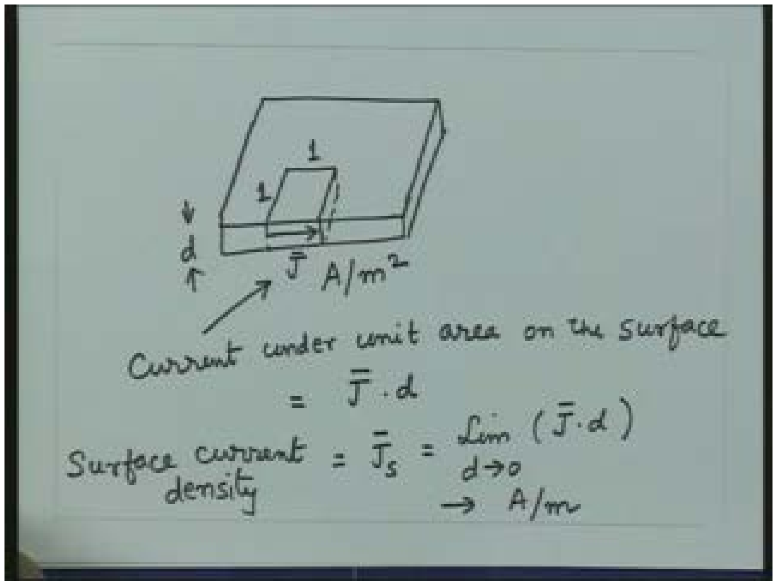

So now we are having overall, the quantities like: the charge density which means volume charge density, we have a quantity like current density which means conduction current density, we have a quantity like displacement current density then we got surface charge density and then we have got the surface current density. So one may say these are the sources which are related to the fields which are the electric and magnetic fields.

So in general then one can establish relationship between these quantities which we can call sources to the fields which are electric and magnetic fields and these relationships are called the boundary conditions.

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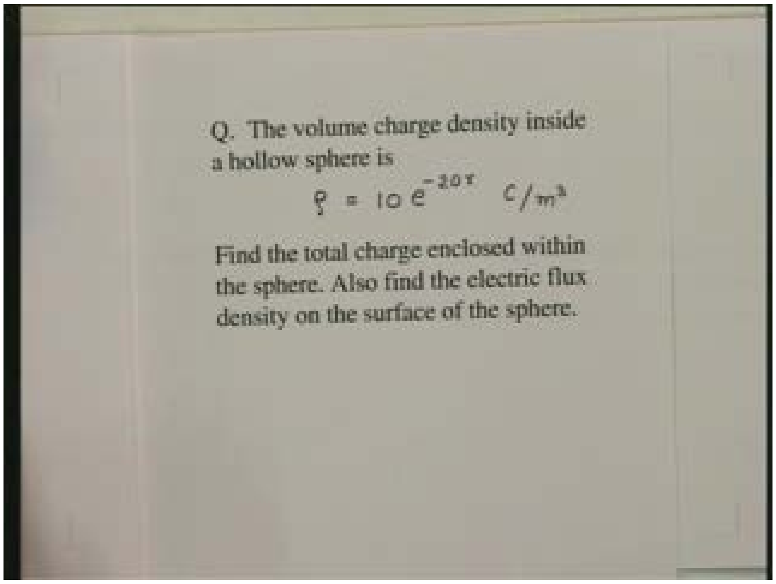

Let us now solve some problems based on the Maxwell's equations and also on the boundary conditions. So let us consider a very simple problem based on the Gauss' law. So we see here the volume charge density inside the hollow sphere is given as the rho equal to 10 into e to the power minus 20r Coulombs per meter cube. Find the total charge enclosed within the sphere. Also find the electric flux density on the surface of the sphere.

So, finding the total charge enclosed inside this sphere is very straightforward. So the total charge enclosed inside this sphere Q that will be equal to the integral over the volume of this sphere to rho into dv. If we write down this volume into the spherical coordinates this is equal to the phi going from 0 to 2pi, theta going from 0 to pi and then r which is the radius of the sphere which is 2 meters so it is going from r is equal to 0 to 2 rho r square sin theta dr d theta d phi.

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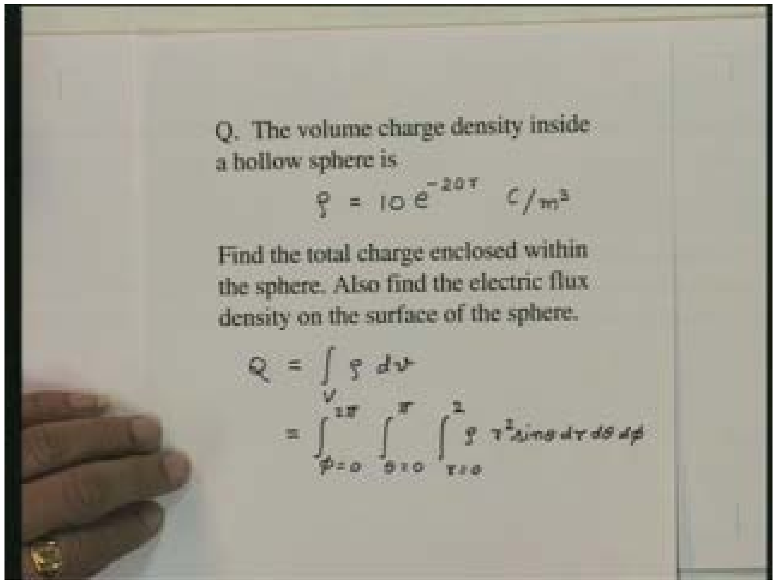

So if I substitute now for the charge density inside this and integrate over this volume essentially we can write down is the integral 0 to 2pi, there is a circular symmetry over phi, then we are having integral for theta which is going from 0 to phi sin theta d theta and then we have integral here which is from 0 to 2 so integral 0 to 2pi then we can substitute for rho from here that is 10 e to the power minus 20r into r square dr. This integral is very straightforward that will give me equal to 2pi. Similarly, the sin theta integral is also very straightforward, will be minus cos theta and if I substitute for the limits of theta and pi then I can get the total integral for this that will be equal to pi divided by 100 Coulomb.

So the total charge enclosed inside the sphere of radius 2r this circular symmetry with the charge density which is given by this is pi upon 100 Coulombs.

Now, to find out the electric flux density on the surface of the sphere essentially we can make use of the Gauss' law which states that the total electric displacement from a closed surface is equal to the total charge enclosed by that surface.

And in this case this surface is a spherical surface and if the charge is distributed in a spherically symmetric manner the electric displacement will be uniformly distributed on the surface of this sphere. If I consider this sphere like that the electric displacement will be uniform all across the sphere.

So now we can find out the electric displacement on the surface of this sphere which is nothing but the total charge enclosed which is this charge Q which is pi upon 100. So if you want to find out what is the electric flux density on the surface of this sphere essentially we have 4pi r square where r is the radius of this sphere multiplied by the density is equal to the total charge enclosed which is Q which is equal to pi upon 100.

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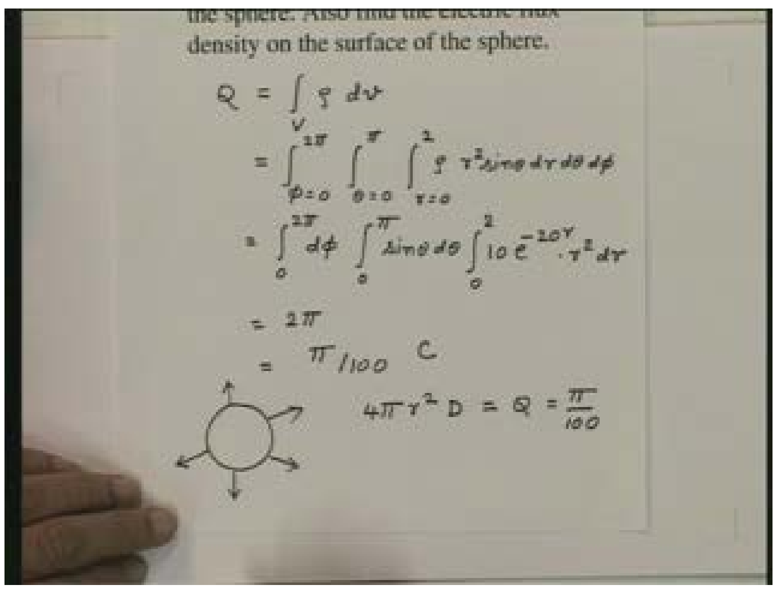

So from here we can calculate the electric for density on the surface of this sphere which is equal to Q upon 4pi r square and substituting for the value of Q which is pi upon 100 and r equal to 2 meters we get electric flux density on the surface of this sphere equal to 6.25 into 10 to the power minus 4 Coulomb per meter square.

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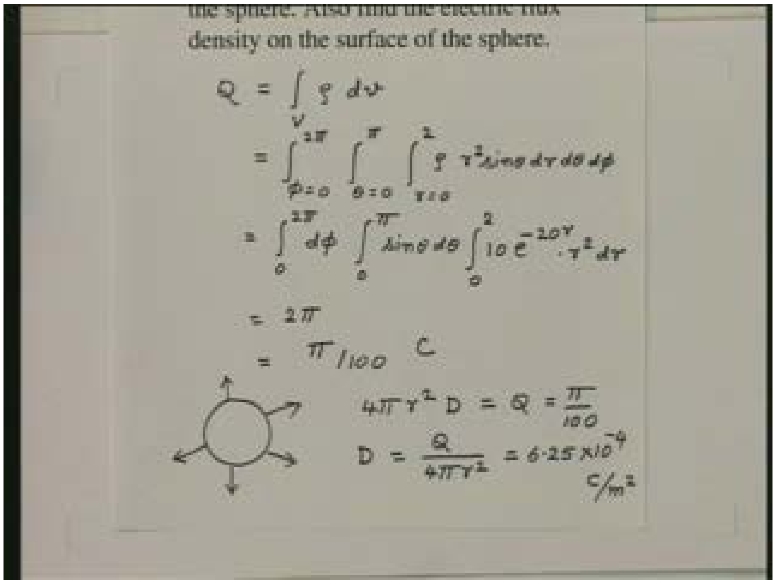

So in this simple problem the electric charge density was given inside a spherical volume and we were to find out the total charge enclosed in that volume and also the electric flux density on the surface of this sphere.

Let us consider another problem. Let us say now the problem is that the electric flux density is given as: D is equal to x cube x cap plus x square y z cap.

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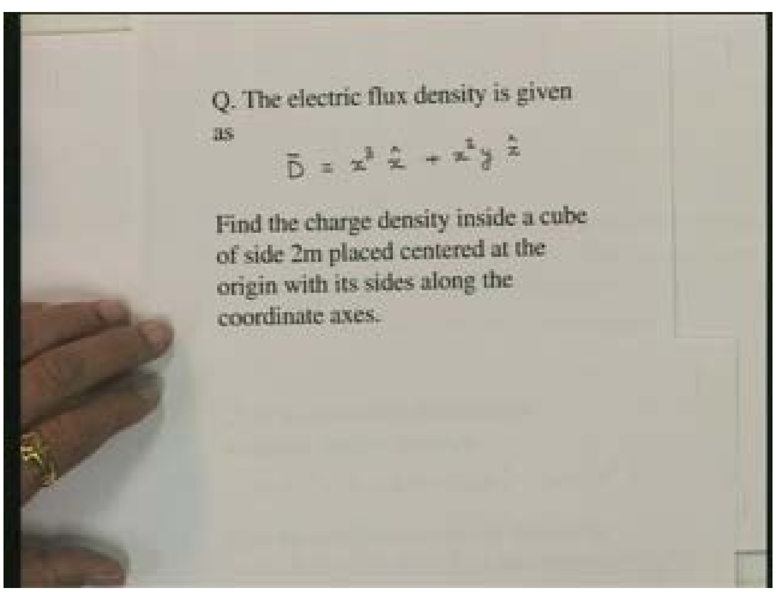

 So now the electric field or the electric displacement density is given in the vectorial form and you have to find the charge density inside a cube of side 2 meters placed centered at the origin with its side along the coordinate axes.

So now we can apply the Gauss' law in the differential form to find out first the charge density and once you get the charge density then we can find out the charge enclosed inside the cube.

So firstly we can go... we know from the Gauss' law that the del dot D is equal to the volume charge density rho. Expanding this in the Cartesian coordinate system we get d Dx by dx plus d Dy by dy plus d Dz by dz that is equal to the volume charge density in that region. Substituting now for the D which is given here where the x component of D is x cube, y component of D is 0 and z component of D is x square y. If I substitute this into the expression essentially we find that rho is equal to d by dx of x cube, the y component is 0 so this quantity is 0 plus d by dz of x square y. Even this quantity is 0 (Refer Slide Time: 45:12) because d by dz of this quantity x square y will be 0 so essentially rho will be d by dx of x cube which is equal to 3 x square.

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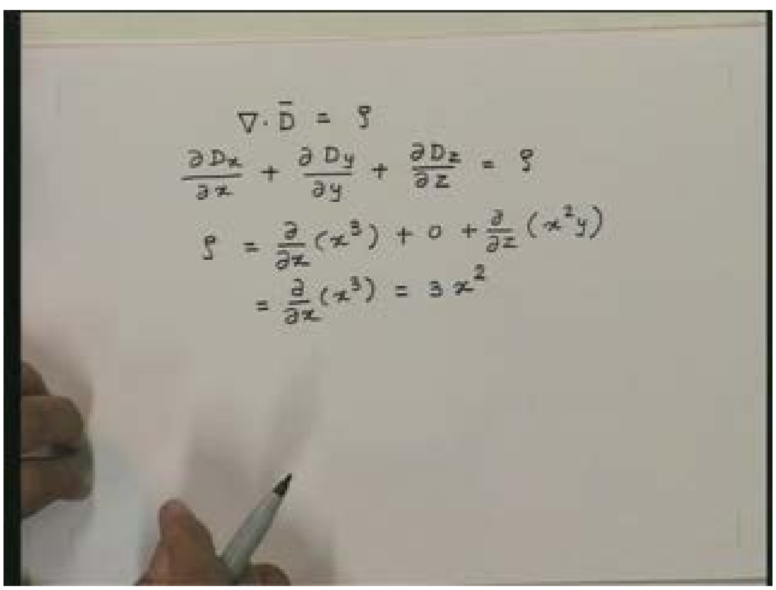

So now inside the volume of the cube the charge density essentially varies only as a function of x and the charge density is constant as a function of y and z. So now if I have a coordinate system a cube essentially is placed something like this (Refer Slide Time: 45:50) around the center and then you want to find out what is the total charge enclosed inside this cubical volume.

So you can get the total charge enclosed Q that will be integrating this charge density over the cubical volume that will be equal to minus 1 to 1 minus 1 to 1 minus 1 to 1 rho into dx to dy dz. Substituting for rho in this that will be minus 1 1 minus 1 to 1 minus 1 to 1 3 x square dx dy dz which we can write this quantity integration versus vector y will give me 2 versus to z will give 2 so essentially we have here 12 minus 1 to 1 x square dx which is nothing but equal to 12 into 2 upon 3 which is equal to 8 Coulomb.

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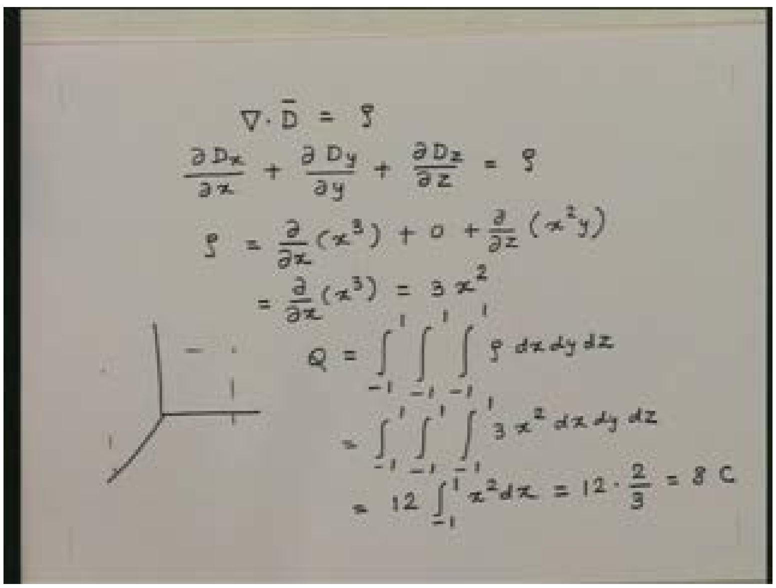

So, in this problem the electric displacement density was given in the vectorial form and by applying the same Gauss' law in the differential form essentially we could find the total charge enclosed inside a volume.

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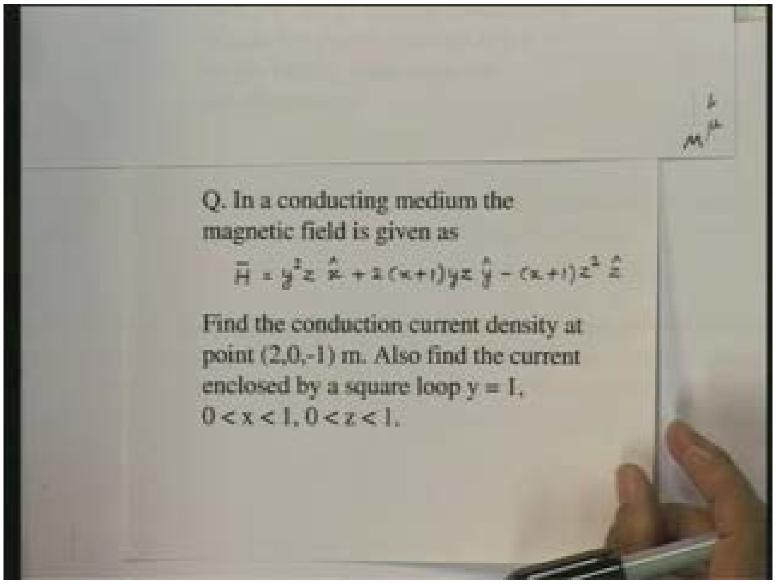

Let us consider now a problem which is related to the magnetic fields. So let us say now the problem is in a conducting medium; the magnetic fields is given as H is equal to y square z x cap plus 2 into x plus 1 yz y cap minus x plus 1 z square z cap. Find the conduction current density at 0.20 minus 1 meters. Also find the current enclosed by a square loop which is defined by y equal to 1 x between 0 and 1 and z between 0 and 1. So, essentially this loop is lying in that x, z plane at a height of y equal to 1. So we have to find now the conduction current density at a point because of this magnetic field which is given in the space.

So essentially we want to use the Ampere's law in the differential form to find out the conduction current density. See if we go to the Ampere's law we know that the Ampere's law is given J is equal to del cross H. in the Cartesian if we expand this this curl operation that will be the determinant x y z d by dx d by dy d by dz and if I write down now the component of the magnetic field which are given here which is y square z 2 into x plus 1 yz minus x plus 1 z square then you can get here y square z 2 into x plus 1 yz minus x plus 1 z square. And if I expand this essentially we get the conduction density in the vectorial form which will be minus 2 x plus 1 y x cap plus y square plus z square y cap.

So at location 2, 0, minus 1 the conduction current density J will be equal to... if I substitute now x equal to 2 y equal to 0 and y equal to minus 1 the conduction current density will be 1.0 y cap.

Once I know the conduction current density then I can go and find out what is the total current enclosed by the loop which is defined by y equal to 1 and x going from 0 to 1 and z going from 0 to 1. See if I look at this in the y z plane this is the... or xz plane this is z is going like that, x is going like that so x is going from 0 to 1, z is going from 0 to 1 and now the conduction current density is oriented in the y direction. So that means if I go by the right hand rule the z or the fingers if I point from z to x the thumb should point to the y direction so essentially the conduction current density within this is essentially given by that, the current is coming out of the plane of the paper.

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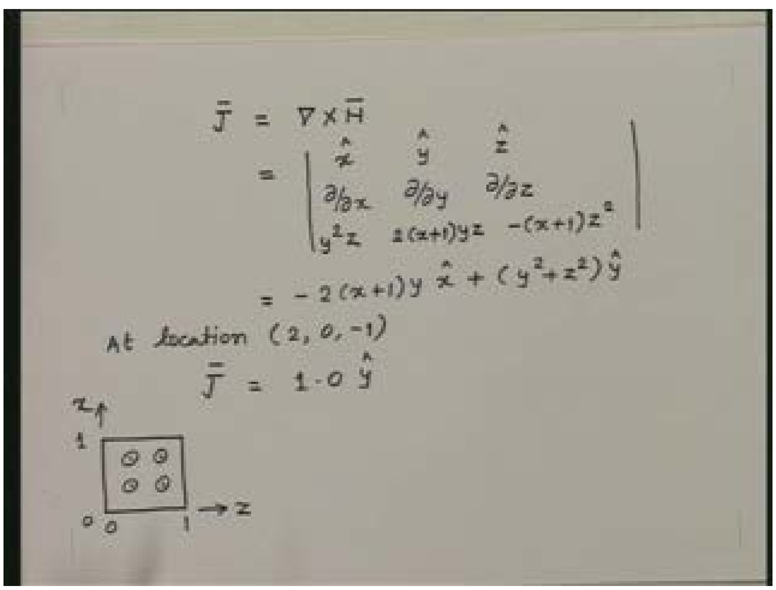

So now the total current enclosed by this I that will be integral J dot ds which is nothing but the integral J dot y cap to dx dz. If I substitute now for the limits the x from 0 to 1, z goes from 0 to 1 and the current now will be only J y because dot product of J and y cap will be J y dx dz at y equal to 1. So substituting now for the conduction current density in this essentially we get this integral with the same limits 0 to 1 0 to 1 1 plus z square because we are substituting y equal to 1 in this dx dz. If I solve this integral I get the total current enclosed inside this loop which will be 4 upon 3 Amperes.

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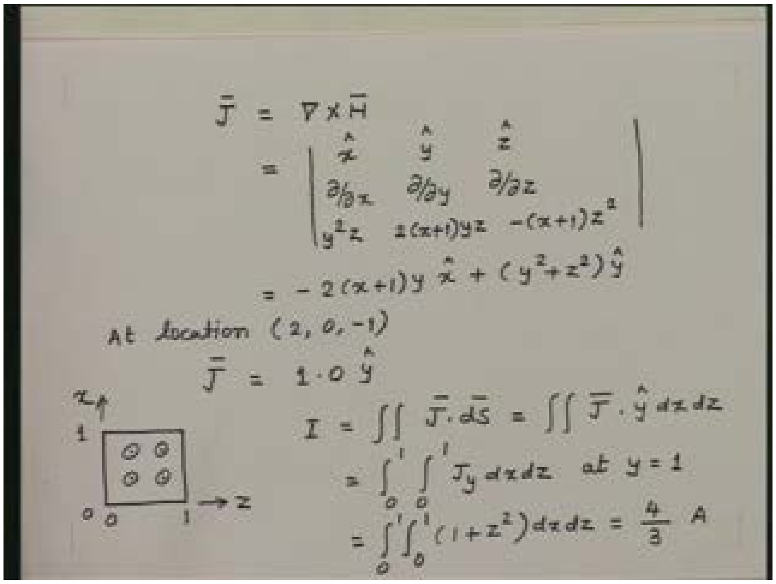

So in this case we are using the Ampere's law the differential form. The magnetic field was given in the differential form. then by using the Ampere's law in differential form we find out from the curl the conduction current density and once you know the conduction current density then we can find out by integrating over the area of the loop the total current enclosed by the loop.

So these are some of the very simple problems which essentially give me some feel for how to apply in practical life the laws the physical laws like Gauss' law or the Ampere law either in differential form or in integral form.