Transmission Lines and E. M. Waves
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Lecture – 18
Basic Laws of Electromagnetics

We saw in the earlier lecture that the origin of the electric and magnetic fields is a charge. The effect of a charge is felt by a quantity what is called an electric field. However, when the same charge is kept in motion is constitutes a current and then the presents of current is felt by a quantity what is called the magnetic field. So we can take current as the origin of the magnetic field. And today we try to establish the relationship between the current and the magnetic field and then also define the parameter what is called the magnetic flux density and its relation to the magnetic field and then we will go to some other laws like Ohm's law and other laws of electromagnetic and formulate these laws in the mathematical form what are called the Maxwell's's equations.

So if I have a current element which is the small piece of let us say wire which is carrying some current I, the length of this current element let us say given by dℓthen at some point in the space which is as a distance r from this current element let us say this is denoted by vector r, the magnetic field due to this small element let us denote it by dH where d gives you the incremental value of the magnetic field so dH gives me the magnetic field due to the small current element that is equal to I dℓ cross r unit vector in this direction divided by 4pi into r square.

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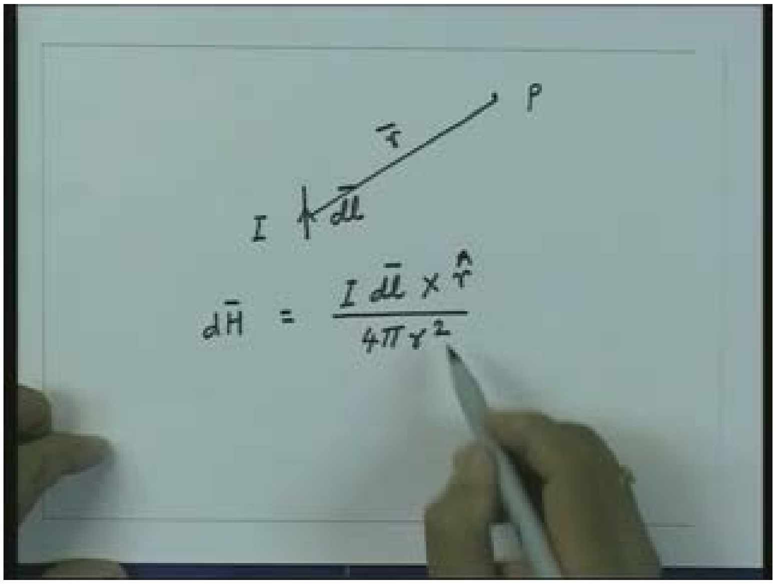

So the magnetic field at this location point p let us say is given by this expression so the magnetic field is proportional to the current; it is also proportional to the length of the wire over which the current flows and it is inversely proportional to the square of the distance from the element. dℓ cross r unit vector that is the cross product of dℓ and the unit vector in this direction r direction so if I apply the right hand rule the dℓ cross r gives me a direction which goes inverts the paper. So at this location (Refer Slide Time: 4:08) magnetic field will be going inside the paper so which we denote by the cross so the direction of the magnetic field is inside the paper at this location and the magnitude is given by magnitude of this expression.

Since the dimension of dℓ is length and you are having a length square the denominator, the unit of the magnetic field is the current per unit length so that is amperes per meter. This is the magnetic field strength at this location p due to this current element. Many times the magnetic field strength is simply called magnetic field and what you find is that this quantity is not related to on any of the medium parameters (Refer Slide Time: 4:58); as long as this current is r this length is dℓ, no matter what medium is surrounding at this current element the magnetic field produced at this point will be less.

The quantity then which relates to the medium parameter is the magnetic flux density. So the quantity which we define now is what is called in the magnetic flux density and that is denoted by B that is equal to a medium parameter what is called the permeability of the medium times the magnetic field strength. So that is H. So this quantity mu is called permeability of the medium.

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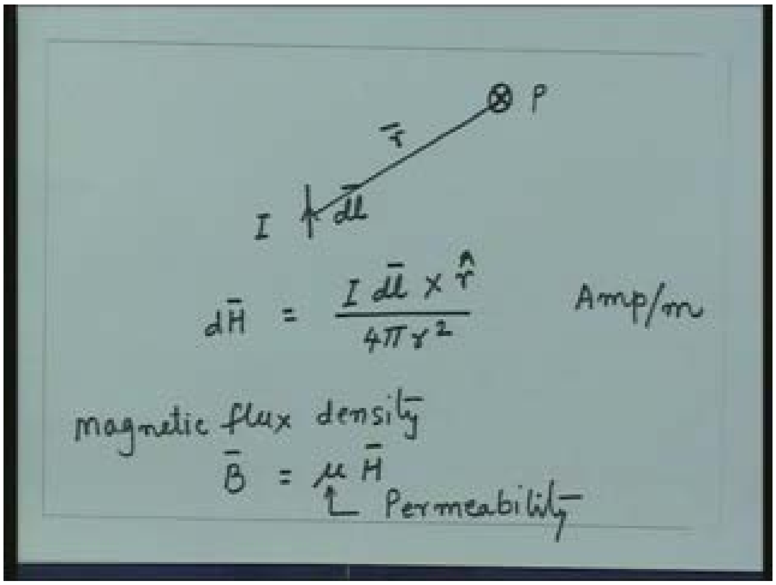

Depending upon the magnetic properties of the material the value of the permeability changes so that for free space the value of permeability is denoted by free space, the permeability mu is denoted by mu 0 and its value is 4pi into 10 to the power minus 7 Henrys per meter.

There has been defined in the case of the electric field or the media for the electrostatics; if I have a material whose permeability is mu the ratio of mu and mu 0 that is the ratio of the permeability of that medium whose permeability of the free space is called the relative permeability of the medium. So we have a parameter, relative permeability of a medium which is the medium parameter that is mu r is equal to mu divided by mu 0.

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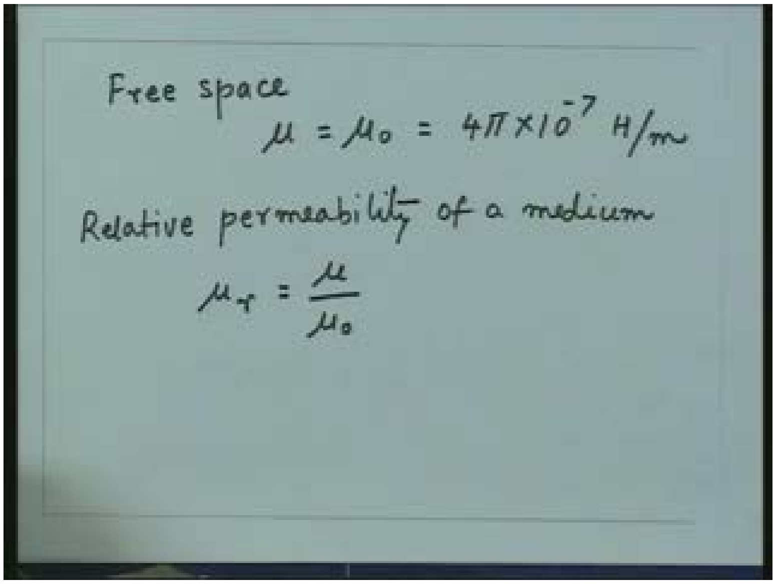

If you take magnetic material typical value of mu may range of 10 or even sometimes 100. So the magnetic flux density at any location B is related to the permeability of the medium and the magnetic field strength which is related to the current which is producing that; unit for B is tesla or more commonly used is weber per meter square. So essentially this quantity B tells you that density of the magnetic lines of forces at a particular location. So this quantity is essentially telling you some kind of vectors which are oriented and that is the reason this quantity is a vector quantity; the same is true for the magnitude field that we are finding the effect of this current at this location so we can define the magnetic field even alternatively as we define in terms of the electric field that is the force experienced by per unit charge due to the presence of a charge. The same thing we can define for the magnetic field also that is the force which will be experienced by a unit magnetic pole if it is placed in the vicinity of a current (Refer Slide Time: 8:47) so essentially it is the magnetic field or magnetic field strength and the magnetic flux density or the vector quantity.

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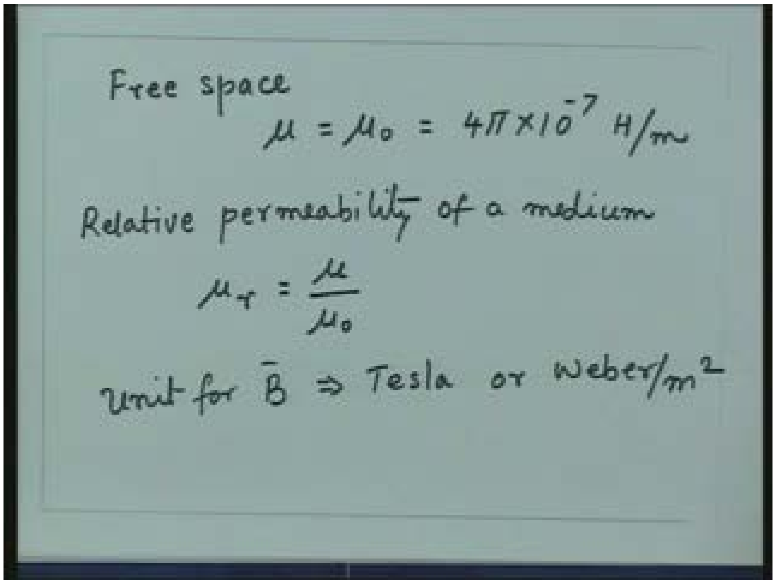

The next thing which we have in our discussion is what is called the Ohm’s law and that is the relation between as we commonly know between the voltage and current flowing in a medium or in a circuit. However, in the general form of Ohm’s law is the relationship between a quantity what is called the conductance current density and the electric field.

See if I take a very general medium where the conductivity of the medium is not constant, then it is not very meaningful to define the total current for that medium. So normally what we do we define a quantity what is called the conduction current density. So if I have let us say some conductor like that (Refer Slide Time: 9:48) and let us say the medium property which is the conductive area of the medium vary inside this then I can define the quantity what is called conduction current density. So it is the current flowing per unit area of this conductor.

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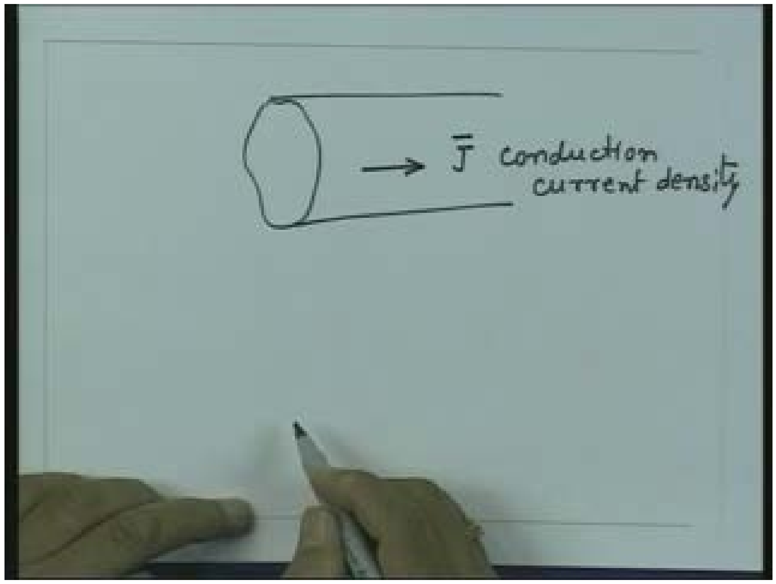

So again, since it is having a direction in which the current is flowing this vector quantity so it is the amperes per meter square that is the unit of this Ampere's per meter square and this current is now flowing because you are having some electric field which is related to the potential. So we have seen that the electric field is related to the potential gradient of potential as a potential difference; so if you go from the circuit Ohm's law we have a relation between the voltage and current which is the proportionality relationship and we have a parameter proportionality constant which is what is called the resistance. Here we have a parameter which is the conductivity of the medium which is inverse of the resistivity of the medium. So essentially we have a relationship between the electric field which is driving this current and the conductance current density.

So the Ohm's law in general form is the conductance current density J that is equal to the conductivity of the medium times the electric field. Again in both the cases; as we saw in the previous case the relative permeability of the medium mu or mu r and the conductivity of the medium sigma in general could be 3 by 3 matrices. So if you take a medium which is isotropic medium then the quantity mu r and sigma are the scalar quantities whereas if the media or anisotropic medium that means if the properties of the medium depend on the direction then in general mu r will be a 3 by 3 matrix, the conductivity sigma will be a 3 by 3 matrix. So this quantity conductivity in general could be a 3 by 3 matrix for anisotropic medium. And meaning of this is exactly same as we saw in the case of electric field that there if the medium is isotropic then the direction of the displacement vector and the electric field are same; same thing will happen in the case of the magnetic field also that if the medium is isotropic then the direction of the magnetic field and the magnetic flux density that will be same. But if the medium is anisotropic then these two directions will not be same. The same is true for the conduction current density and the electric field but if the medium is anisotropic then the direction of the electric field is not same as the direction of the conduction current density.

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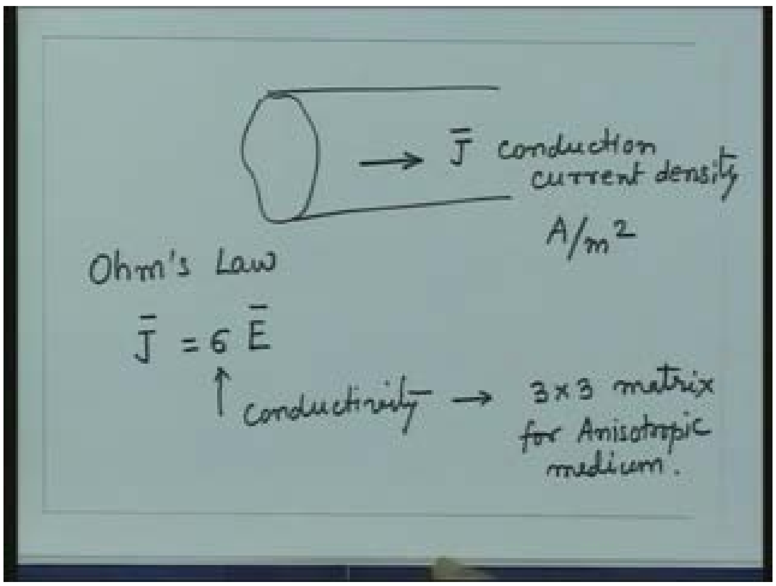

So in general if I have anisotropic medium then the direction of the electric field and the conductance current density will be different. With this basic introduction of the parameters now we can go to the physical laws which give you finally the Maxwell's equations.

Now there are two ways to look at Maxwell's equations: one is that you treat Maxwell's equations or mathematical postulates and the physical laws or the experimental verification of those postulates. The other way of looking at it is we have physical laws and then try to represent these physical laws in appropriate mathematical form that gives you the Maxwell's equations.

Both have its merits. That is if I say that the Maxwell's equations are the mathematical postulates then they are very exact. So what we say is if we take this as the postulates then all the experimental errors and all that do not come into picture so these are very exact forms of these equations and then these are simply validated or verified by the experimental laws.

However, one can ask a question how do you reach to this postulate?
One cannot simply come up with these mathematical forms without having any background. So certainly the background was in the physical laws. So, further the physical laws came; just the curiosity of the relationship between the magnetic and electric fields relate to these physical laws and then the mathematical formulation guided us to get this mathematical form what is called the Maxwell's equations.

See if I go by this picture that the Maxwell's equations were guided by the experimental laws then the advantage is that we will always try to see the phenomena in physical terms. So if you have any electromagnetic phenomena and if you say that the origin of this lies in the physical laws it will be always useful to look at every phenomena of electromagnetics physically. However, if you treat these equations more like mathematical postulates then one may get lost into the other mathematical manipulations of these equations.

So from the exactness point of view the mathematical postulate is it has merit. But seeing what physically is happening, first understanding the physical laws and then getting the Maxwell's equations as a merit. What we will however do here we will state for the physical laws and then using the vector identities and the theorems which you have mentioned earlier: the divergence and stokes' theorem, we will try to get the mathematical form and then we will get finally the Maxwell's equations.

So the very first law which governs the relationship between the displacement vector or the electric field and the charge distribution is what is called the Gauss' law. The Gauss' law states that the total displacement coming out of a closed surface is equal to the net charge enclosed by that closed surface. So what that means is if I have a closed surface like that and let us say we have the charger distributed inside this closed surface I find out the net displacement which is coming out of this surface and that net displacement will be equal to the total charge or the net charge enclosed by this surface that is the Gauss' law.

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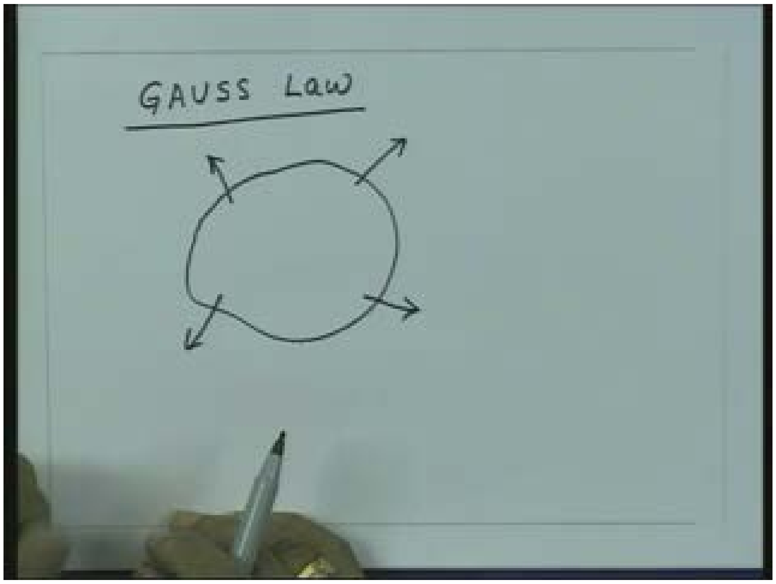

In general, the displacement vector will be varying as a function of location on the surface; also it will be oriented in different directions. But the net displacement vector which is coming out of the surface will be the component of this normal to the surface at that location. So if I consider let us say a small incremental area on this surface (Refer Slide Time: 18:18) let us say it is given by da so da has as you saw the area has a direction which is given by its normal so this is the normal to the surface which is outward normal and then any location here... let us say this is the direction of the displacement vector. So the outward displacement which will be coming from this small area will be the dot product of these two so it is D dot da that is the component of the displacement vector in the normal direction.

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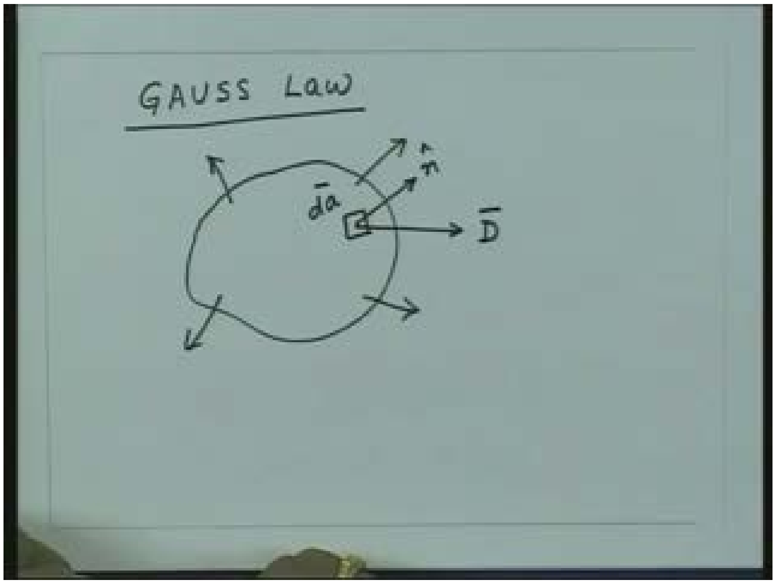

Once I get that I can add all those contributions from all the area on the surface and I will get the total displacement coming from this closed surface. Then let us say in general we are having charges which are distributed inside this surface and that can be characterized by the charge density inside this surface.

So let us say I have a charges which are distributed in this closed surface (Refer Slide Time: 19:38) which are denoted by the volume charge density denoted by rho. So this is representing the density of the charges inside this volume so its units will be Coulomb per meter cube. So if I add up all the charges inside this volume I get the total charge enclosed by this surface.

So, integrating the displacement vector over surface I get the total outward displacement from this closed surface. If I add up all the charges which are lying inside this volume I get the total charge enclosed by this surface. Then from the statement of the Gauss' law these two are equal.

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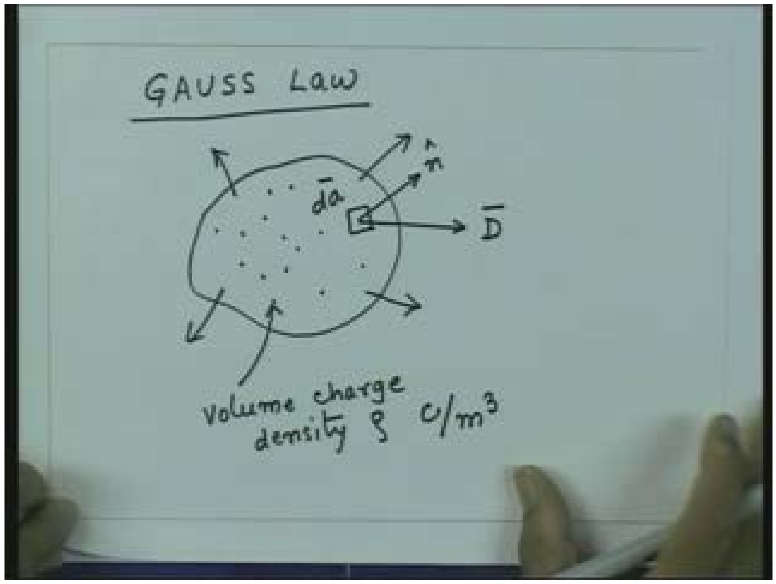

So if I write down mathematically this Gauss' law can be written as follows: integration of the displacement vector over the total surface will be D dot da integrated over the entire surface closed surface so which I can write as closed surface D dot da where this surface is denoted by S (Refer Slide Time: 21:21) and this volume is denoted by let us say V. So this quantity represents now the total outward displacement from this closed surface. From the Gauss' law this is equal to the total charge enclosed by this surface. So, if I find out the total charge in this volume: rho into dv over this volume V that gives me the total charge enclosed by this surface.

So mathematically the Gauss' law can be written as simply as finding out the total displacement integrated over the surface and the total charge integrated over the volume.

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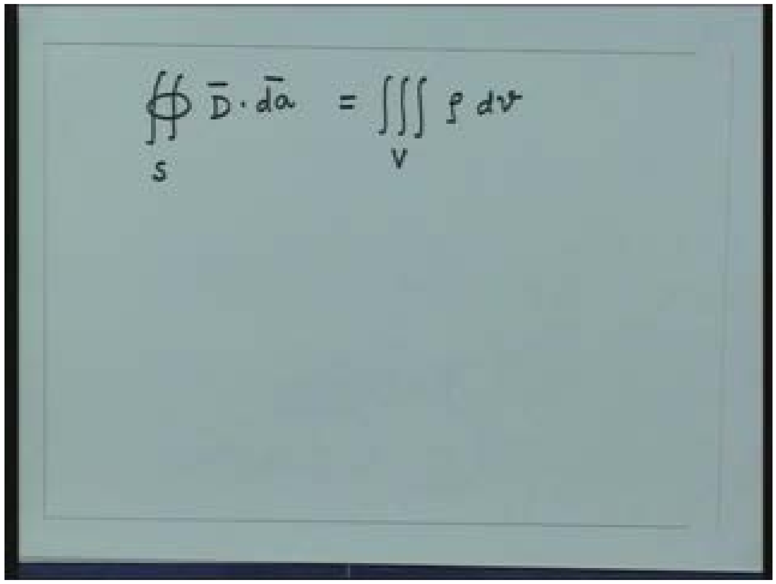

Once I get that then I can apply a divergence theorem to this Gauss' law statement and I can convert this surface integral to the volume integral. So, as you have seen from this divergence theorem that this thing can also be written as the triple integral over the volume del dot D integrated over the volume so that is equal to the volume integral rho into dv so this is from the divergence theorem.

See if I apply divergence theorem to this integral this integral now is converted from the surface integral to the volume integral. I can bring this term on this side and I can write this is over this volume V del dot D minus rho dv that is equal to z. Now this relationship or this integral must be true for any arbitrary volume that can happen provided this integrand (Refer Slide Time: 23:47) is identically zero. So this essentially gives you del dot D is equal to rho.

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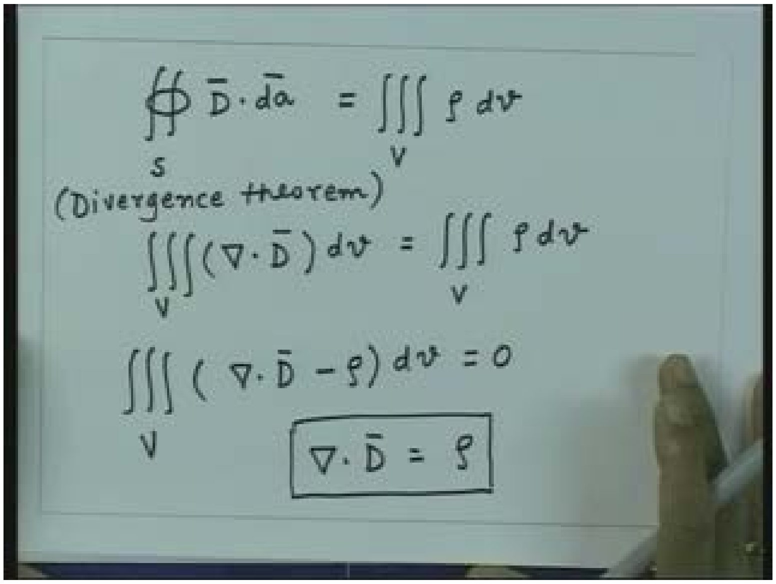

So the Gauss' law if I write in the integral form it is this but the same Gauss' law can be written in a differential form which is this (Refer Slide Time: 24:12). So, if I take divergence of the displacement vector at any location then this is related to the volume charge density at that point. This relation is a point relation. That means we can apply, go to any point find out what is the volume charge density at that location, if I know the displacement vector at that location then find the divergence of the displacement vector at that location I can get the volume charge density.

So the Gauss' law can be applied depending upon the situation in the integral form and it can be applied in the differential form. Later on we will see that differential form has limitations because differential from assumes that we have continuous media so the properties of the medium do not suddenly change, there is no discontinuity in the medium properties so whenever we are having boundaries where the properties of this media suddenly change, that time the differential form is not applicable because the derivatives do not exist and in that situation we go back to the original form of this law (Refer Slide Time: 25:29) which is the integral form of the law and that will be true in other laws also which we will discuss in the following. So in all situations the integral form of the law is applicable. However, if the medium is more continuous then you can apply the differential form of the law also.

Now this law in general is true so the divergence of the displacement vector is the volume charge density. However, we have seen that the displacement vector is related to the electric field so we have the relationship D is equal to epsilon into the electric field.

If we assume that the medium is homogeneous that means epsilon is not a function of space then I can substitute for D into this and epsilon is not a function of space, epsilon can be taken out from here so the Gauss' law can be written as del dot of epsilon into E that is equal to rho. For homogeneous medium epsilon is not a function of space so this epsilon can be taken out (Refer Slide Time: 27:06) so I get del dot E that is equal to rho upon epsilon.

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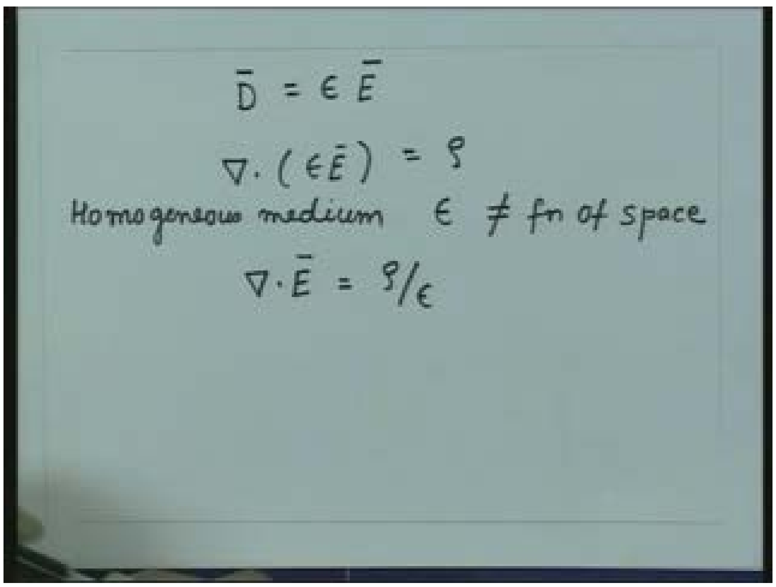

So we can apply this Gauss' law for a homogeneous medium for the electric field directly instead of going to a displacement vector. And later on we will see when we solve the problem of electromagnetics for time varying fields. Essentially we try to establish relationship between the electric and magnetic fields rather than displacement vector and the magnetic flux density so that time this form in the homogeneous medium this form will be more useful.

Now the same Gauss' law, now if you apply to the magnetic field and the magnetic charges... see if you say that the net magnetic flux coming out of a closed surface is equal to the net magnetic charge or magnetic poles enclosed by this surface that will be the Gauss' law for the magnetic fields. However, we know there are no isolated magnetic poles, the magnetic poles are always found in pairs. That means no matter what volume I take I will always have equal number of north and south poles. So, in a closed surface the net magnetic charges will always be zero.

See if I apply the Gauss' law for the magnetic charges or magnetic poles then there is no net charge enclosed by any closed surface. So this quantity will be always equal to zero (Refer Slide Time: 28:58). So I can make a statement with the Gauss' law or the magnetic field or the magnetic flux density and that is the net flux coming out of a closed surface is always equal to zero because there are no net charges enclosed by the surface, the net charge is always zero.

So I can write down the Gauss' law for the magnetic field and that is the net flux coming out of the closed surface S is identically 0. Again if I apply divergence theorem to this I will get del dot B dv is that equal to 0 and again if it has to be true for any arbitrary volume then we get del dot B is equal to 0.

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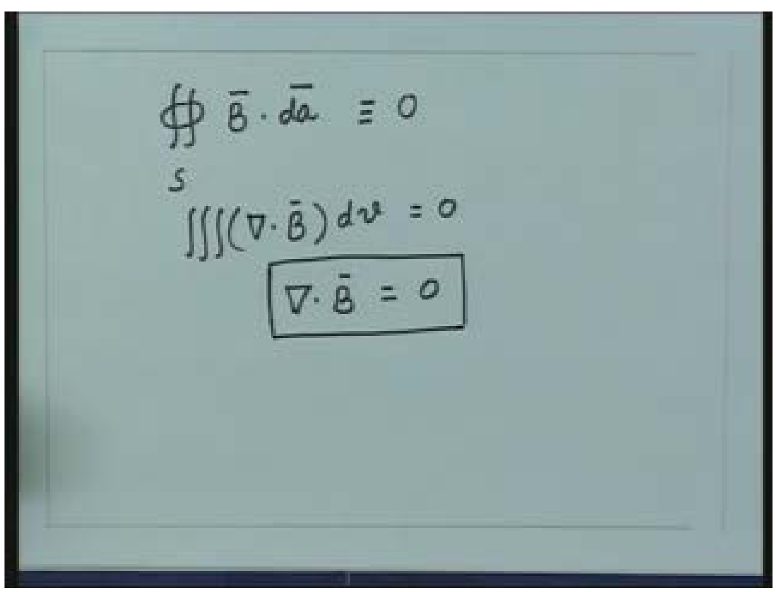

So the Gauss' law when is applied to the electric charges we get del dot D equal to rho whereas if you apply that to the magnetic charges we get del dot B that is identically go to zero because there are no free magnetic monopoles possible in time. The next law which we have which relates now the electric magnetic fields are what are called the ampere law and the Faraday's law of electromagnetic induction. The Ampere's law is what is also called the Ampere's circuit law which states that the magnetic motive force around a closed loop is equal to the total current and closed by that loop.

So the third law which we have is called the Ampere's law or Ampere's circuit law which says that if you have a loop or let us say have some current which is flowing in this current this current is going to produce a magnetic field; if I calculate the total magneto motive force around this loop which is the line integral of the magnetic field around this path then that line integral is equal to the total current enclosed by this loop that is the Ampere's circuit law.

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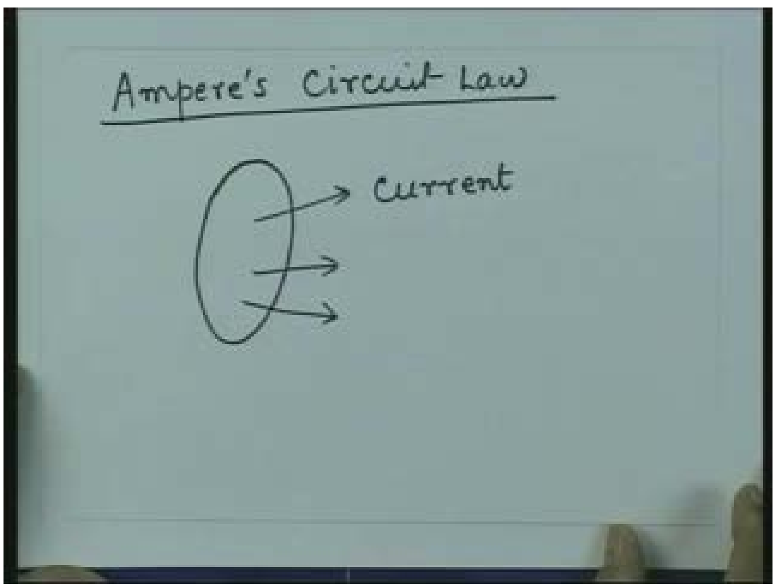

See if I say that the element on this is given by some dℓ around this path and I have some magnetic field in this region which is H some magnetic field which is H the magneto motive force is equal to the line integral of this over this closed path H dot dℓ is over the length here, this is over this contour C (Refer Slide Time: 33:04) so this is the contour C, this quantity is equal to the total current enclosed by this loop. So if this current let us say is I this quantity is equal to I.

However, if I take a loop of any arbitrary size it is possible that the current may not be uniform inside this loop it may be varying from location to location, what it requires is the total current; however, if the current is varying we can write this current in the form of the current density integrated over the area of cross section.

So if I know the conduction current density inside this loop let us say it is J and if I find out what is the net current which is coming out of there which is nothing but j integrated over this area that gives me a total current i. So I can write this also as the integral over this area over this area A this is J dot da. So J dot da gives me the current flowing through a small area here which is da, this is the direction for the area and that is the direction for the J so J dot da gives me the current flowing perpendicular to that area at this location, if I add upon those currents that will give me the total current enclosed by this loop.

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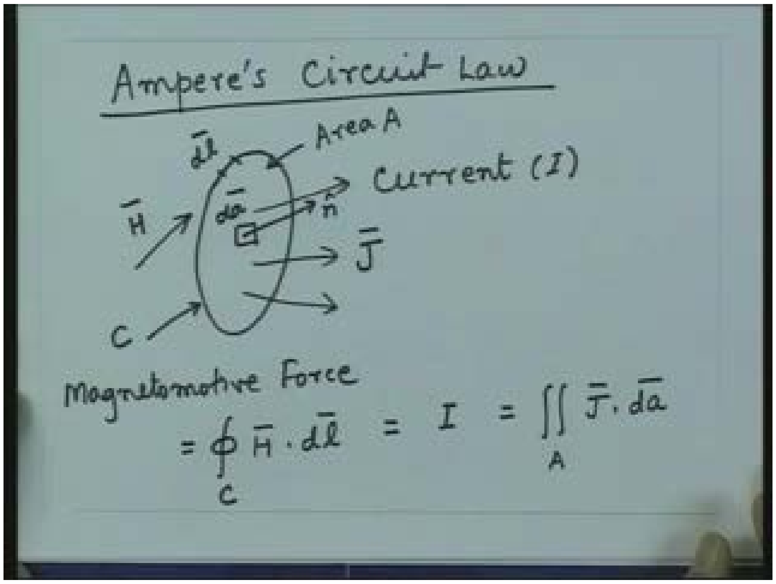

So, from the Ampere's circuit law the total magneto motive force is equal to the total current enclosed by the loop which is nothing but the integral of the conduction current density over the area of that loop. This is the integral form of the Ampere's circuit law. As we did in the previous case we can I can apply the integral theorems and can convert this relation in the differential form.

So we can convert this line integral or the contour integral into the surface integral and that we can do by applying the Stokes' theorem. See if you apply Stokes' theorem the H dot dℓ that is equal to double integral del cross H dot da over this area A these by the Stokes' theorem that is equal to the total current enclosed which is integral J dot da. So this quantity is equal to double integral A J dot da. Again we can bring this term on this side so this thing will be written as double integral A del cross H minus J dot da that is equal to 0.

Since this relation should be valid for any arbitrary area this quantity should be identically zero (Refer Slide Time 37:36) so this gives essentially del cross H that is equal to J. So the Ampere's law in the integral form is this whereas the same law by applying the Stokes' theorem we get in the differential form and that is that. So essentially the curl of H is equal to the total or the conduction current density at that location.

So, again as we saw in the previous case (Refer Slide Time: 38:15) this is applicable over a finite area whereas this relationship is a point relationship. See if you go to a particular point and if you know the magnetic field at that location the curl of that magnetic field gives me the conduction current density at that location.

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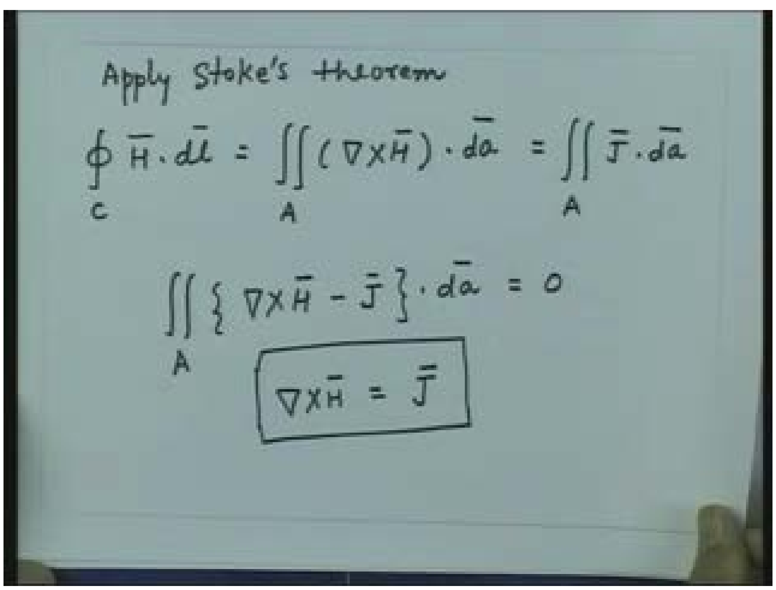

Lastly we go to the fourth law which relates the electric and magnetic fields and that is the Faraday's law of electromagnetic induction. So we have Faraday's law of electromagnetic induction. The Faraday's law electromagnetic induction states that the total EMF around a closed loop is equal to rate of change of magnetic flux enclosed by the loop as a direction of this EMF is such that the magnetic field produced by this current because of the EMF opposes the original magnetic field.

So if I have let us say a loop like that (Refer Slide Time: 39:42) and let us say we have some magnetic flux density which is given by B we can find the total flux by integrating this magnetic flux density over this area. The rate of change of this flux density is equal to the total EMF produces around this loop.

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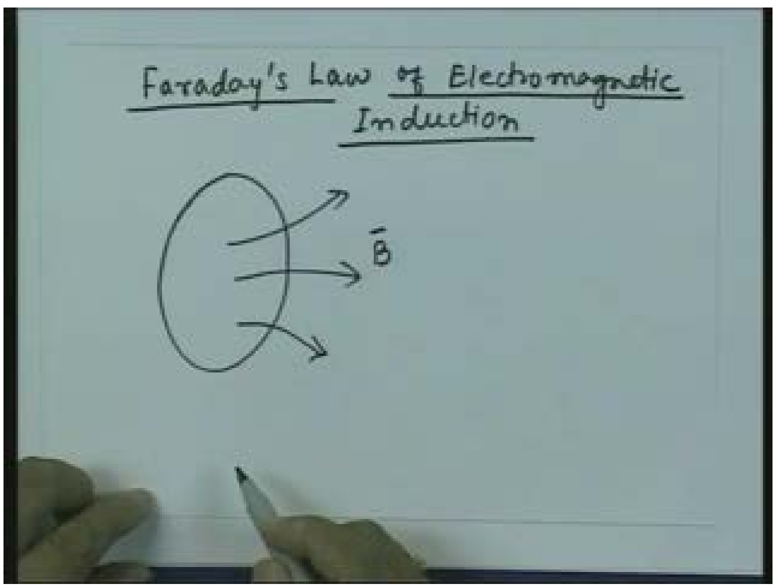

So if I have the electric field in this region which is given as E and if you have an element here which is dℓ the E dot dℓ is the EMF produced in this small segment, because of this EMF the current will flow into this and the current will flow is such a way that the magnetic field produce by this current will oppose this magnetic field. So mathematically if you write the EMF produced because of this electric field is line integral over this contour C this contour C E dot dℓ that is equal to minus d by dt of the total flux let us say that quantity is phi. So phi is total flux enclosed by the loop.

So if I know the total flux enclosed by this loop I take its time derivative. The negative sign essentially is to account for the fact that the EMF produced is such that the magnetic field produced by that current because of that EMF will oppose the original magnetic field. So this negative sign is more like a resistance produced to the original magnetic field and its role is very important. Suppose the negative sign was not there what that means is the magnetic fields will produce EMF which will produce a current which will produce the magnetic field which will add up with the original magnetic field so this magnetic field will increase, if the magnetic field increases the EMF will increase so essentially there is no stabilizing element here, the thing will go indefinitely growing. So essentially this negative signs stabilizes. What that means is whenever the magnetic field tries to induce the EMF in the loop and the current flows the magnetic field produced by that current tries to oppose this so this phenomena cannot go on increasing indefinitely. So this magnetic field produced by the current produced by the EMF essentially tries to provide a resistance with the original magnetic field that is what essentially this negative sign represents.

Now since the total magnetic flux enclosed by this loop can be found out by integrating the magnetic flux density over this area, I can write this as minus d by dt of the total flux and which you can find out by integrating this so it is integral over this area B dot da. If I say here (refer Slide Time: 43:30) incremental area is da then B dot da gives me the flux perpendicular to this, I integrate over the total area that gives me the total flux.

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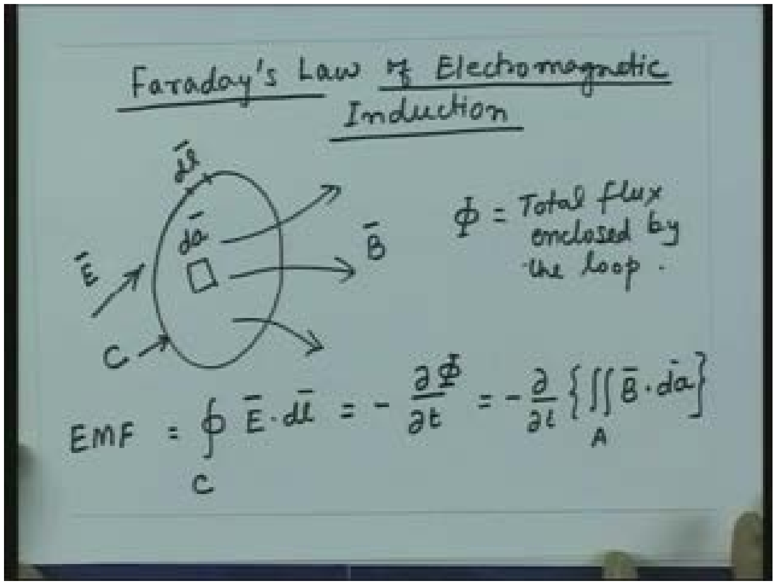

So the Faraday's law of electromagnetic induction in the integral form now is written as integral C E dot dℓ that is equal to minus d by dt of B dot da. Now this is the general relationship that is we have a rate of change of the total flux enclosed by this loop. Now the flux enclosed by the loop might change as a function of time in two ways: one way is that the magnetic flux density changes as a function of time so B is a function of time, loop is fixed it does not change and as a result... so the B changing as a function of time so the flux enclosed by this loop changes as a function of time. So I have a rate of change of the total flux enclosed.

Other possibility is the B remain same but the area of the loop changes as a function of time and because of that the flux enclosed by this loop changes as a function of time and again I have the EMF produced. or I can have combination of both that the magnetic flux density might change as a function of time and at the same time the loop size also might change as a function of time.

So if we keep the magnetic flux density constant B constant and where is the area we get a change in the EMF that is what is called the generator action that is what we do in a generator; the magnetic flux density is constant and by rotating in the loop in that magnetic field essentially the projected area of the loop changes and because of that we get the induced EMF.

When the loop area remains constant and the magnetic flux density changes that is what is called a transformation; that is what happens in transformer, you have coils, the size of the coils is not varying as a function of time but the magnetic field which is induced in those pores that is varying as a function of time and because of that we have the induced EMF.

In our further analysis we are interested in the time varying fields that means time varying magnetic fields, time varying electric fields so we assume here that the space is not changing that means the size of the loop is not changing, the magnetic flux density is changing and as the result we say that the area of the loop is not a function of time. So having said that we are interested in time varying fields and not the time varying space we essentially can bring out this derivative inside this because now the area is no more a function of time. So I can write down in this case this is minus integral dB by dt dot da.

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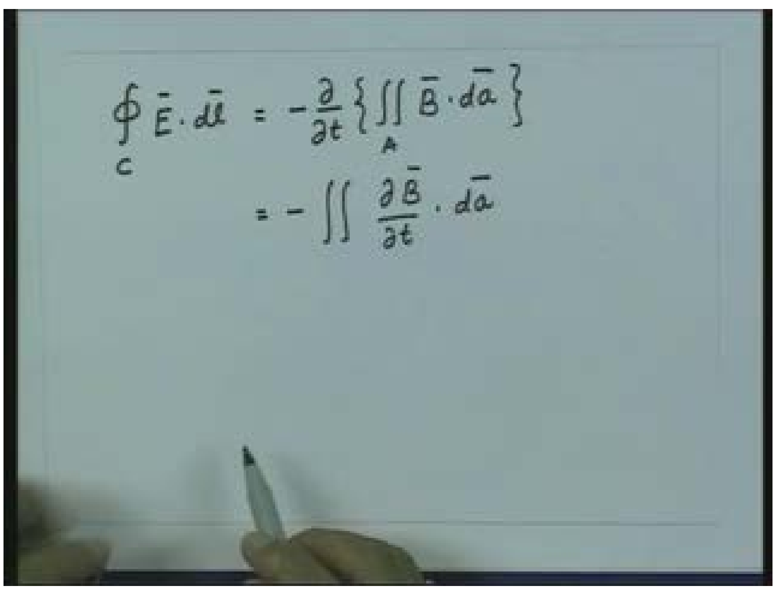

Once we do this then as we did in the previous case again I can apply the Stokes' theorem, convert this integral into the surface integral. So if I apply the Stokes' theorem then I get double integral del cross E dot da and this whole area (Refer Slide Time 47:59) A that is equal to minus dB by dt dot da.

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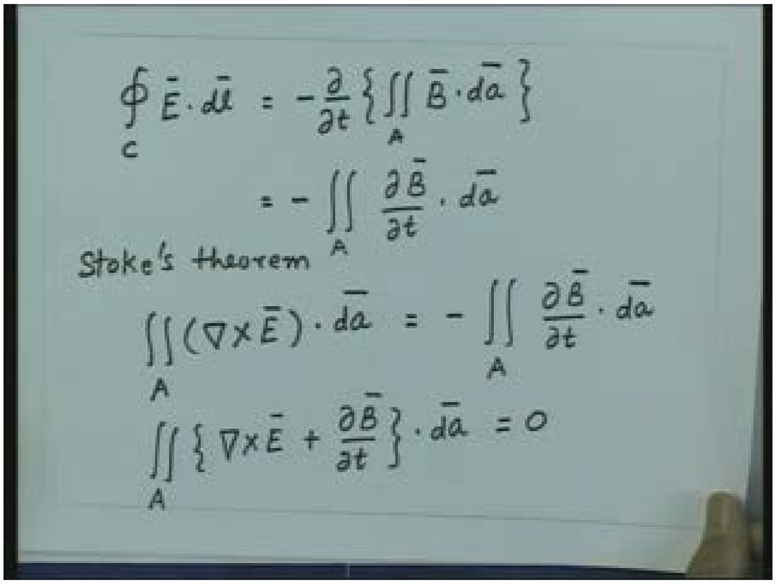

Again as we did earlier you bring this term on this side and or any arbitrary area A this is del cross E plus dB by dt dot da that is equal to 0. And since again this relationship should be valid for any arbitrary area this integrand should be identically zero so from here we get del cross E that is equal to minus dB by dt.

So the integral form of the Faraday's law of electromagnetic induction is this (Refer Slide Time: 49:17) whereas the differential form of the Faraday's law of electromagnetic induction is this (Refer Slide Time 49:21). And again as we said this is the point relationship, so if I know now the electric field at a particular location then the curl of that electric field gives me the rate of change of the magnetic flux density at that location.

Again if you consider a medium which is not varying as a function of time that means the medium properties which are captured by the permeability of the medium is not a function of time I can write for B as mu into H and if mu is not varying as a function of time I can write here del cross E that is equal to minus d by dt of mu into H which is minus mu dH by dt.

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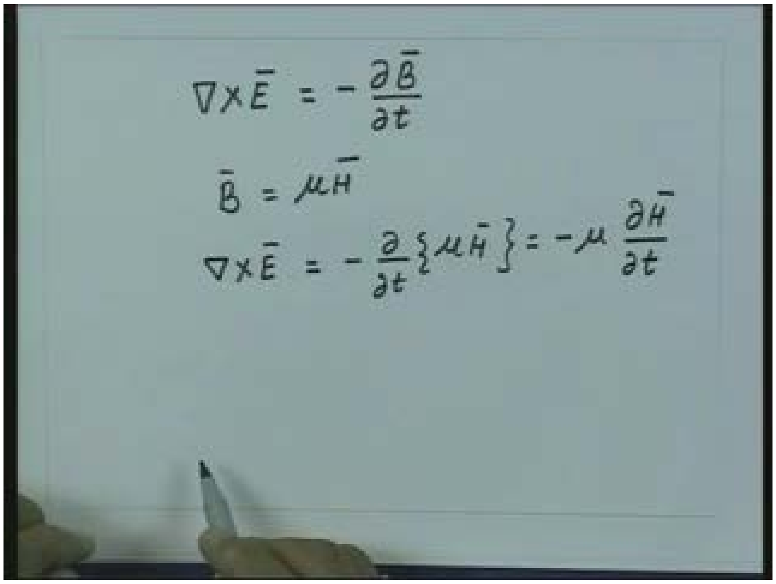

So, for a non-time varying medium I have the curl of electric field that is equal to minus mu times the rate of change of the magnetic field strength at that location. So essentially what we saw that if you get these four basic physical laws: the Gauss' law applied to the electrical charges, the Gauss' law applied to the magnetic charges, the Ampere's circuit law and the Faraday's law electro magnetic induction and write these four laws in the mathematical form using the vector algebra and the theorems which we have discussed, these four laws can be written in the integral form but by applying these integral theorems the same laws can be written in the differential form.

Next time when we meet we will see the inconsistency in some of these laws which essentially was rectified by Maxwell. So, after rectification of the discrepancies between these laws we get the final set of equations which govern the relation between the electric and magnetic fields and that will be the Maxwell's equations.