Transmission Lines and E. M. Waves
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Lecture – 16
Basics of Vectors

Till now we discuss one of the various special cases of magnetic waves that is transmitted lines. We introduced the concept of space in the circuit analysis and we saw naturally the electrical quantities like voltage and current exist in the form of waves on the electrical circuit. However, the concept of voltage and current is applicable to the bound structure like electrical circuits where you have conductors separated by dialectics like coaxial cable, parallel wire transmission lines and so on. If I go the media which are infinite in extent or semi infinite in extent or if I consider a medium which is only dialectic then the concept of voltage and current is not very attractive.

In fact in many applications it is very difficult to define these quantities like voltage and current. in this situation essentially we will have to go to the more fundamental quantities and that is electrical magnetic fields. So having now got some field for the wave phenomena I will bit for a special case like voltage and current wave now we will make a departure to the more generalized phenomena of electromagnetic waves and that is the waves in the form of electrical magnetic fields.

So, here onwards, now we discuss the phenomena of the electromagnetic waves in the form of electric and magnetic fields. You will appreciate that whatever we have done so far the analysis for voltage and current that was essentially dealing with the quantities which was scalar quantities, voltage is a scalar quantity, current is a scalar quantity. If we however go to now jumble fields like electrical and magnetic fields these quantities are vector quantities. So essentially now we have to deal with the analysis of this quantities electric and magnetic field not in a one dimensional structure like transmission line but in three dimensional space. So to get the formulation for the vector fields like electric and magnetic fields let us first revise our concepts of the vector calculations and vector algebra.

Before we go into the vector algebra and vector calculus, let us first start on how we can represent the three dimensional space. Well, ultimately we have to represent this quantity electric and magnetic fields in the three dimensional space. So there are three major coordinate systems which are used for representing this electric and magnetic fields and these coordinate systems one of them is what is called the Cartesian coordinate system which is having three orthogonal axis x y z.

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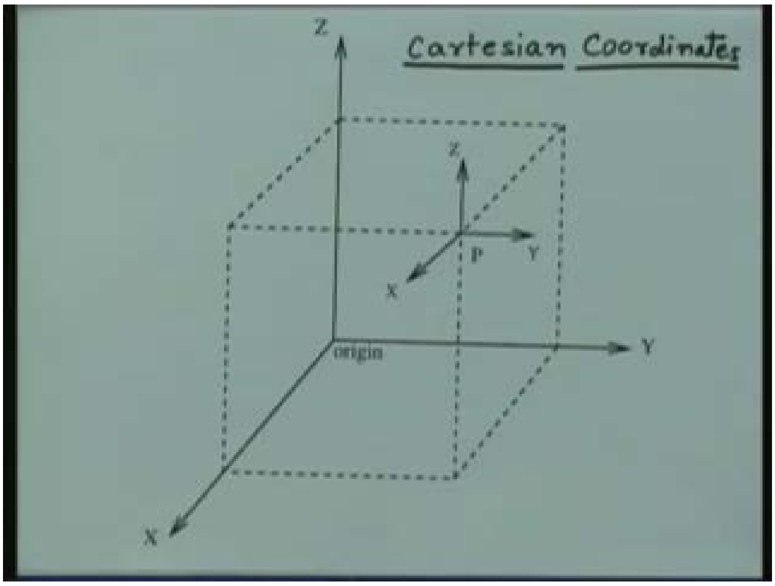

So if i imagine an unbound space three dimensional space like a box then the three axes will be the three edges of the box. So we have here x axis y axis and z axis and the sequence is x to y to z. So when we write a coordinate of a point in the three dimensional space we write this as x comma y comma z. While defining this axis we follow certain conventions and one of the conventions we will follow throughout our discussion and that is this coordinate system is a right handed coordinate system. What that means is if we point our fingers of right hand from going from x to y the thumb should point in the direction of z. If I point my fingers from y to z then my thumb should point in the direction of x and if I point my fingers from z to x my thumb should point in the direction of y. So this convention later on when we want to point the vector operators will reserve some of the ambiguities for defining the direction of the vectors.

So in this case we visualize the three dimensional space as a box we follow this right hand convention. as you can see here (Refer Slide Time: 6:04) if I take the horizontal plane which is x y and if I point my finger from x to y then my thumb will point out upwards so the z direction is upwards. Then at each location I can define a vector which is represented by an arrow and we will come to that what convention we will follow in this course for defining or writing a vector in three dimensional space. So these three vectors which we call as the component of a vector at that location that will be pointing in the three coordinate axis.

So you will have for any vector a component along the direction of x which will be x component, a component of the vector along the direction of y which will be y component and a component along z will be called z component. So any vector now can be resolved into three components. So a vector essentially can be represented by a set of three elements. First element of that set will denote the component of the vector in the x direction, the second element will represent the component in y direction and the third component will represent component in z direction. So whenever we write a vector now in three dimensional space just for visualizing a vector or electric or magnetic field which are vector quantities in fact we require a whole lot of imagination.

You can write down the mathematical expressions but ultimately it will be good idea to visualize this vector in three dimensional space. And if you do that and if you develop a practice of visualizing these fields or vectors in three dimensional space then the subject of electromagnetic waves will be more fun than a burden of mathematics. So the idea here is to visualize these vectors or whatever phenomena we analyze immediately in three dimensional space and when we do that then there will be much more physical insights in the problem of electromagnetics than simply getting lost into the mathematics.

So idea here is to get a physical field for the vectors and then whenever we solve a problem we make sure that we do not lose touch with the physical aspects though we will be doing rigorous mathematics which will be the vector calculus and vector algebra.

So the simplest coordinate system which we see here is a Cartesian coordinate system and the import feature of this coordinate system is no matter when you go in the space the direction of these vectors along x, y and z direction they remain the same. So if I take a point from here to here the x vector will still point this way, y will point this way and z will point this way. That may not be true for when we go to the other coordinate system. So we will write down essentially the vectoral relations for the Cartesian coordinate systems because that is the coordinate system which is rather easier to visualize and then as and when we require other coordinate system we will use the vector identities in that coordinate system. The second way of defining the coordinate system is what is called as cylindrical coordinate system.

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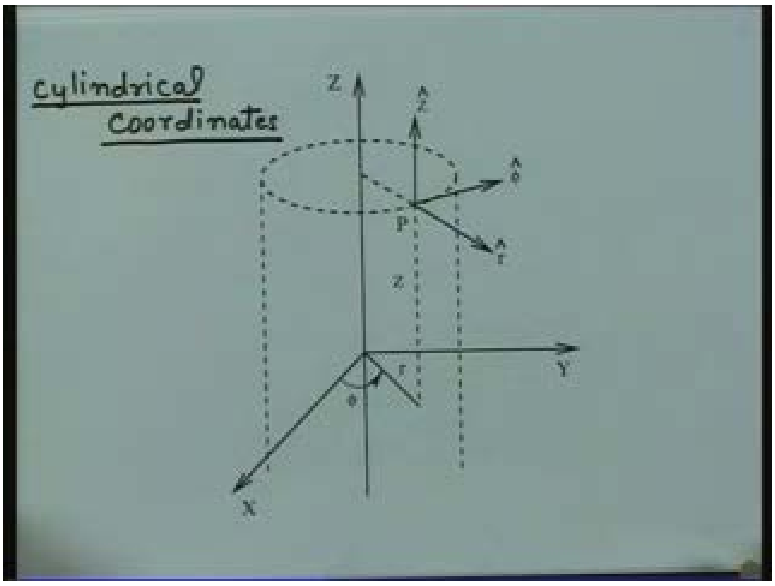

That is if you imagine the three dimensional space like a big cylinder then this is the axis of the cylinder (Refer Slide Time: 9:50) and the space is imagined like a cylinder. See if I write the same Cartesian coordinate system xyz the plane passing through xy will be perpendicular to the axis of the cylinder. So this plane xy will be perpendicular to the axis of the cylinder. Then a point p the coordinate of a point we can find out as the radius vector if I draw by perpendicular from this p on this xy plane and measure the radial distance of this projection of the point on the xy plane that distance we call as r. The angle which this radius vector makes with the x axis as we call angle phi and from this point which is the reference point as the origin the distance which we travel along the axis of the cylinder we call as the z point.

So we have got a coordinate system in this case with a sequence which is r, phi and z. Again we follow the right handed coordinate system. So, if I write a vector then from r to phi if I point my fingers from r to phi I must get z direction, if I point my finger from phi to z I must get r direction and so on. However, how do I define this direction the phi. The direction r and z it looks quite straightforward from here that if I take an arrow which is pointing in the direction of z that is the z vector. Similarly, if I take a vector which is pointing in the direction of the radius vector r, that is the r vector. Phi vector is the vector which is tangential to the surface of the cylinder which is passing through this point p. So if I take this cylinder and if I draw tangent at this point p to this cylinder this direction of this vector (Refer Slide Time: 12:05) is the vector phi.

So in this location I have one vector r which is coming radially outward from here, the tangential vector through this cylindrical surface will be phi and the vector which is along the axis of the cylinder will be z and from here we can see the relationship between the Cartesian coordinate system and the cylindrical coordinate system. So whenever we have rectangle geometry we use the coordinate system which is the Cartesian coordinate system. However, if we have a geometry which is cylindrical in nature like a coaxial cable, optical fiber, circular wave guides or many other structure where the geometry looks more like a cylinder that time the coordinate system will be the cylindrical coordinate system.

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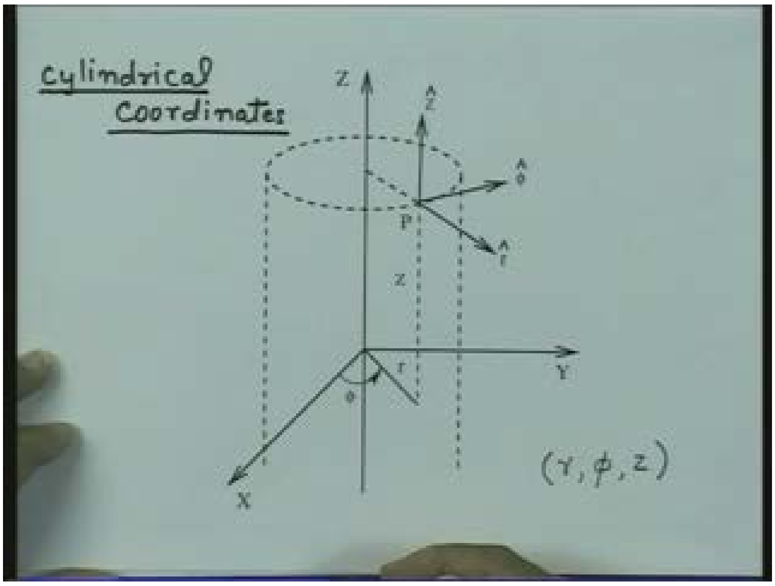

Of course we can always analyze the problem in any coordinate system which you like. But there will be ease in analyzing the problems in identical coordinate system if the structure looks like a cylinder. So generally when we do the analysis we first choose appropriate coordinate system and then we solve the problem of electromagnetics in that coordinate system. The one thing you should note compared to the Cartesian coordinate system in this coordinate system is: in Cartesian system, as we saw, the direction of xyz component of the vector they remain same everywhere in space, no matter where the point moves the x always orients in the same direction physically. However, if I go in cylindrical coordinate system the z vector is always in the same direction no matter where I go but the r vector and phi vector they will keep changing directions as I go to different locations. see if I go to let us say a point on this cylinder somewhere here (Refer Slide Time: 14:13) right in front then the radius r vector will be coming towards you, the phi vector will be perpendicular to you, if I go to this point on this cylinder the right most point then the r vector will be perpendicular to you and the phi vector will be going inside the plane of the paper.

So as we see that in this coordinate system though vector components r, phi they change their orientations physically at different locations' space but the vector direction z remains constant at every point in space. The third coordinate system which is what is called as spherical coordinate system.

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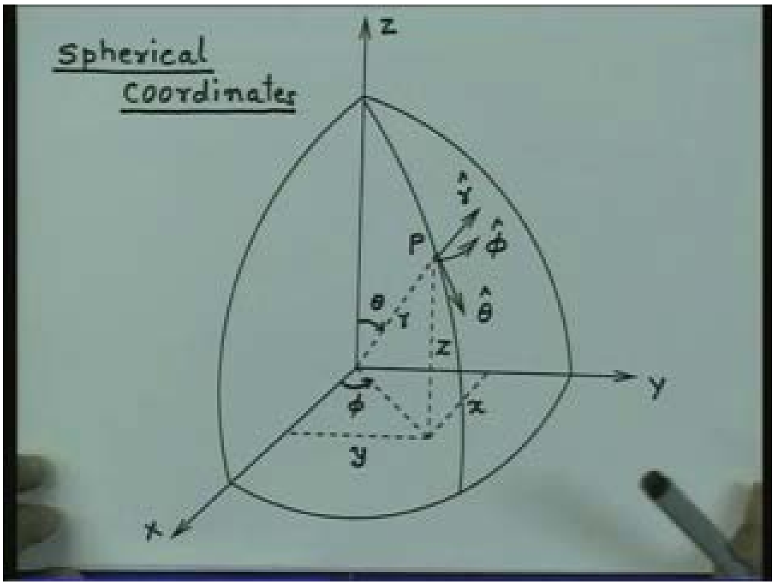

In this situation if you imagine the three dimensional space like a big sphere, so you have some center of the sphere what we call as origin and if you would consider a sphere... here I have shown only one octant of a sphere; so if I take a sphere and mark a point p on the surface of a sphere then this location of this point can be now written in three quantities, the radial distance from the center of the sphere that is origin and the vector which are tangential to the surface in two perpendicular planes. So let us say if this point was p I draw by perpendicular from this p on the xy plane, the radius vector which we have from the origin to this point where the perpendicular is dropped if I measure the angle from x axis of that vector that angle we denote as phi.

So we have this vector here which is a distance r from the origin so this is the first coordinate, the second coordinate is if I draw a tangent to this surface of the sphere in a plane which is passing through the top most point of the sphere here where the z axis meets the origin and point p you will see a cut in that in the sphere and that cut will be this cut (Refer Slide Time: 16:53). If I draw a vector in that plane tangential to the surface at point p that defines the direction of vector theta and angle theta is the angle which this radius vector makes with the z axis.

So in the coordinate system we have r, theta, phi that is the sequence so r is the radius vector radial distance from the origin, theta is the angle which is measured from the z axis from the radius vector and phi is the angle which is measured from the x axis of the radius vector which is formed by the projection of this point p on the xy plane. So in this case now the point is defined by the radial distance and the two angles.

Compare this with the Cartesian and the cylindrical coordinate system. In Cartesian coordinate system the location was defined by three distances. When we go to cylindrical system it was defined by two distances and one angle. If I go to the spherical coordinate system then the location of the point is defined as one distance and two angles. Again in this case the direction we defined that sequence that r... if I make my fingers point from r to theta my thumb must point at the direction of phi, if my fingers point from theta to phi my thumb must point in the direction r and so on.

So see here, if I consider this point p the r will be radially outwards vector on this case, if I draw normal to the surface of the sphere that will denote the direction r, if I draw a tangent as I saw here in this plane so if I take this radius vector at an angle theta and if I draw a vector perpendicular to this vector going away from this theta that will be the positive direction of vector theta and if I take a tangent to this surface in a plane parallel to the xy plane in the direction of phi that vector will be called the phi vector. So you can see here this angle is theta, this angle is phi. So if I put my fingers like this (Refer Slide Time: 19:35) my thumb will be pointing in the direction of r. If I go from r to theta which is like this then my finger will be going inwards that is in the direction of phi.

So we have marked here three vectors r, theta and phi for a given coordinate p, they are marked in such a way that they follow the right hand rule. So whenever we draw a coordinate system whether it is Cartesian or cylindrical or spherical first we must develop a habit of writing the right handed coordinate system. Because when we do the vector analysis we will follow certain conventions and those conventions are all with the understanding that we are following the right handed coordinate system. If we change our coordinate system all those conventions will go wrong and the directions of the vector will go wrong. So that is the reason, whenever we draw let us say coordinate axis if we say in this direction x in this direction y then the direction of z should be let us say the xy lies in the plane of the paper, if I point my fingers from x to y I must get the direction of z which is the direction of my thumb.

So if I put my finger x to y like this then the thumb points downwards that mean the z axis must go inwards that is the correct coordinate axis. So in this case if I take z axis which is like this this is correct. On the contrary if I had drawn the axis which was like this this is x, this y, and this z. This will be a wrong convention because (Refer Slide Time: 21:33) this axis now is not going to follow right hand rule. So whenever we draw the coordinate axis we must make sure that we draw y axis like and not like that because this axis does not follow the right hand convention let me define.

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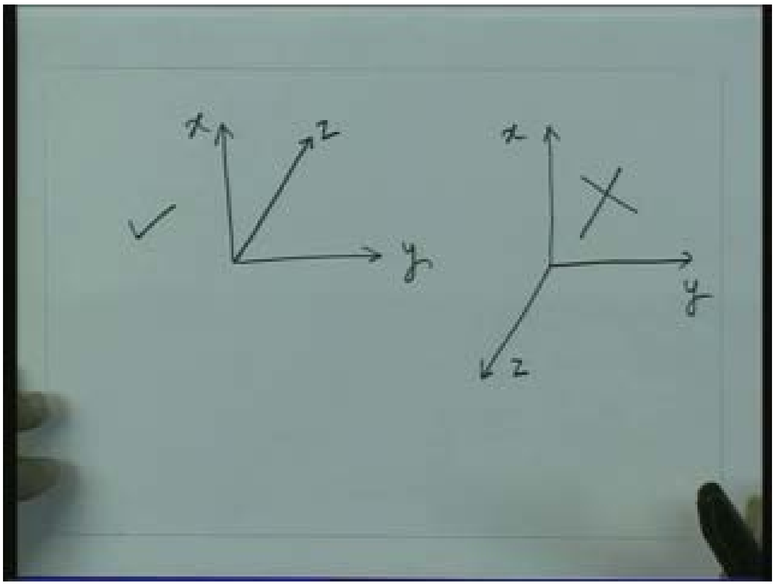

The third coordinate system which we have seen here is the spherical coordinate system. It has another special property and that is if you take a Cartesian coordinate system you can shift the origin anywhere in the space whereas when you define the spherical coordinate system this point origin is defined or summation from there; if you use the cylindrical coordinate system then the line is the point. So basically all distances are measured from that line (Refer Slide Time: 22:24).

So whenever we have a problem like antennas kind of problem where you have a source of energy which is sending away the electromagnetic waves and so on and this source is more like a localized point or a region in the space, this coordinate system is more appropriate.

As I mentioned if I take a structure which will like a coaxial cable or wave guide or a transmission line where energy is going to flow along the length of the structure there the cylindrical coordinate is more appropriate and in some general cases the Cartesian coordinate will be more appropriate; if I consider a closed structure like a closed box if you see like a resonator or cavity or something like this the Cartesian coordinate will be more appropriate. So with this understanding of coordinate system now we go to the basic definitions of vectors and their operators.

Firstly, when we have a vector as I said it is a set of three quantities which we call as components. So, any vector can be represented by some three components from a, b and c and depending upon which coordinate system I am using Cartesian or cylindrical or spherical they will have different meaning or they will represent the components in different directions. Mathematically this description of vector is enough that the vector is the set of p elements. However, when we go to the solution of the physical problem like electromagnetic waves we would like to visualize this vector in three dimensional space.

Now vector this set of three elements is an abstract thing. So, if I say there is an electrical field is is a very abstract concept, we do not know how to visualize the electrical field, same is true for magnetic field also. So what you have to do now you have to give some physical picture for this vector this abstract thing. So let us say if I have a vector which represented by three elements and we say these are three components of the vector the most commonly used convention for this is represent a vector like an arrow.

If I take an arrow which is having a head and a tail then I say a vector essentially is this arrow. So the arrow direction tells me the direction of the vector and the length of the arrow tells me the magnitude of the vector. So this is one of the conventions that if I look at the vector like an arrow in three dimensional space then it will have a length and the length will correspond to the magnitude of that vector and the arrow will indicate in which direction the vector is pointing.

This is the... if I look at this arrow sidewise, suppose the arrow is going away from you or coming towards you then you are looking at this arrow in north and then the same arrow if I see from this side it look like... the arrow is like that... so if I see the arrow from the back side it will look like that, if the arrow is coming towards you I will see the tip of the arrow so the arrow will appear something like this. So if I look at the vector and if I visualize that as an arrow then a arrow going away from you can be denoted by this (Refer Slide Time: 26:22) and an arrow coming towards you can be denoted by this. So this is going away, this is coming towards you.

So a vector its orientation if it is seeing a side on then it can be either a circle with a dot as this arrow comes towards you, circle with a cross that shows the arrow which is going away from you. As we now wrote the magnitude of the vector when we are seeing the arrow side on the length of this vector essentially indicated the magnitude of the vector. If I am looking at the arrow or the vector side on then how do I look at the arrow, I see only a point or a circle so how do I see the magnitude of the circle. So many times the convention followed the size of the circle will denote the magnitude of that particular vector. So smaller the size of the circle weaker or the less is the magnitude, if I make the circle size larger that represents the larger magnitude of the vector.

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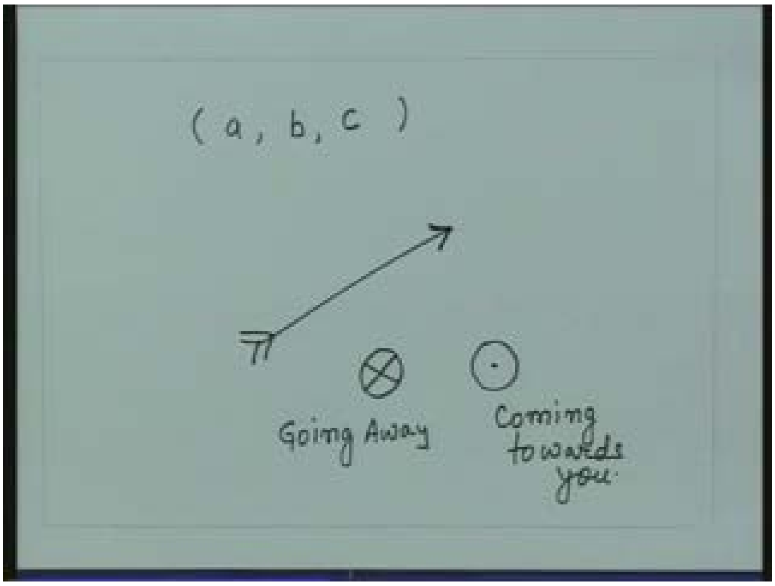

This convention we will uniformly follow and later on when we go for visualizing the three dimensional fields essentially we will see the electric and magnetic field as a distribution of this vector or these arrows in the three dimensional space. So this abstract quantities like electric and magnetic fields will have some physical means of visualizing and that is what essentially is this framework that we use this framework to visualize the abstract quantities in the three dimensional space.

Having understood this now then we can go to the basic operation of the vectors and let us say now I defined the vector as we said the three elements; so let us say let me describe all the vector operations in the Cartesian coordinate system. So my... a vector can be now represented by three components in the three directions the x direction, y direction, and the z direction.

So let us say a vector A which is denoted by A bar that is the x component x cap I will explain what it is, this is y component y cap plus z component z. The quantities which are denoted by caps: the x cap, y cap and z cap they are the unit vectors in three coordinator axis x, y and z. So if I imagine vector like arrow and if I oriented arrow in the direction of x axis if the length of this arrow is unity then that vector will be denoted by this quantity x cap.

Similarly, if I consider a vector of unit length which is oriented in the y direction that vector is denoted by this quantity y cap and same if I take a vector of unit amplitude which is oriented in the z direction that quantity is represented by z cap. So these quantities are called as unit vectors in the three directions the x, y and z direction and A x, A y and A z are the components of the vector A in the three directions xyz. So a general vector A in the three dimensional Cartesian coordinate system can be represented by the x components multiplied by unit vector in x direction plus the y component of the vector multiplied by the unit vector in y direction plus z component multiplied by the unit vector in the z direction.

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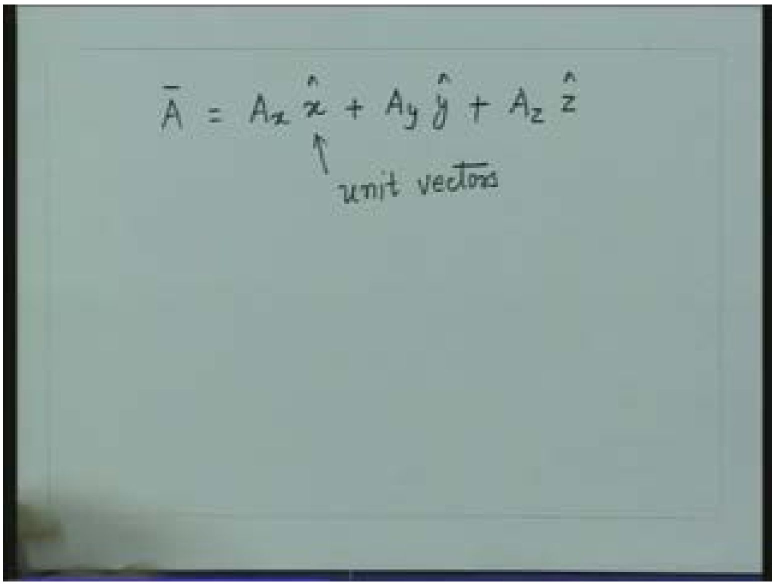

Now we can define certain operations on the vectors and let us say I have another vector B which is having component B x in x direction plus B y in y direction plus B z in z direction. The addition, subtraction of these vectors is ferrite A plus B, it is adding the components of this vector, if I subtract B from A it will be subtraction components y of two vectors. So the addition, subtraction operation is this is will be A x plus B x x plus A y plus B y into y A z plus B z into z and the same is true for the subtraction. So, instead of adding these two vectors so if I subtract then you will subtract B from component from corresponding A component. So the addition and subtraction of two vectors is component-wise addition or subtraction of the two vectors.

The other important operation which we have between these two vectors is the multiplication or the product operations and there are two product operations which are defined for the vector quantities: one is called is the scalar product and other one is called the vector product.

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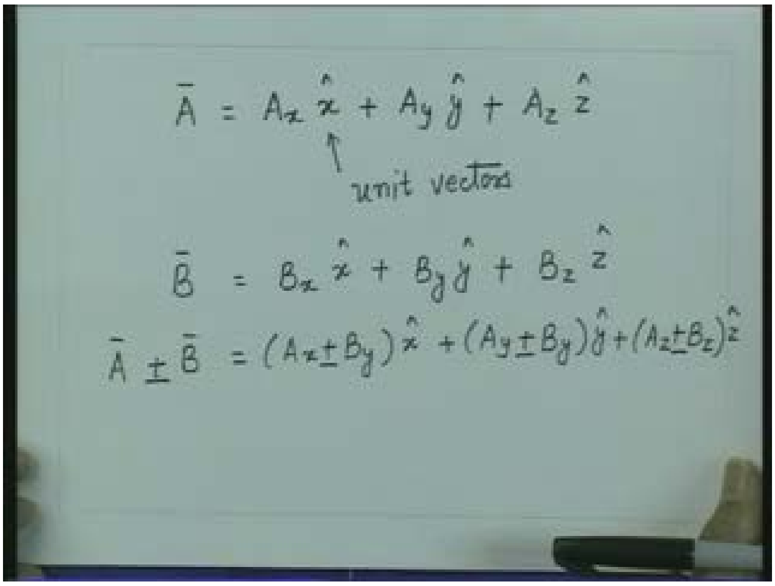

So the product which we defined is the scalar product. It is also called as dot product which is denoted by A dot B and that is defined as the component-wise multiplication of these two vectors (Refer Slide Time: 00:33:28) so this product is defined as A x multiplied by B x; A y multiplied by B y; Az multiplied by Bz sum of that so this will be A xB x plus A yB y plus A zB z. So the dot product of the two vectors is the quantity which is the scalar quantity. It is the sum of the product of the components of the two vectors. That is the reason this product we call as the scalar product of the two vectors.

The another product which we defined for the two vectors is what is called the vector product, it is also called the cross - product and that is defined as A cross B is equal to determinant of the unit vector x y z A x A y A z B x B y B z you can solve this determinant and you can get the x component which will be A y B x minus A z B y; the y component will be B x A z minus A x B z and the z component will be A x B y minus B x A y. So if we expand this determinant I will get a vector which will be the cross - product of these two vectors and since this quantity is the vector quantity we called that product as the vector product and it denoted by this cross operator and so this is required any times as the cross - product of two vectors.

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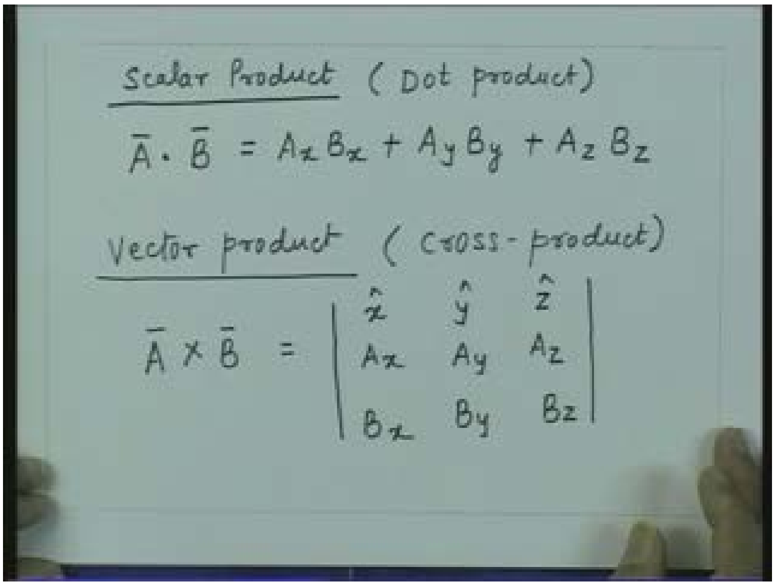

We can see immediately that in this is case if I change the order of this, if I make B dot A this product remains same, this product remains same, (Refer Slide Time: 36:07) this product remains same so the scalar product does not change if I interchange the order of the product so from here I see that A dot B is also equal to B dot A. However, that is not true for the vector product. So if I take this quantity A cross B that represents a vector but if I change the order the magnitude of the vector remains same but the direction of the vector reverses which we can see for interchange B to A so A comes here and B comes here, we can work out and see that now the quantities which we have for each of the components that quantity has become negative of the previous quantity. So, for every component a sign has been inverted if I interchange these two rows. So if I change the order of A to B I will get minus B cross A.

Now here again this quantity is the vector quantity represents a vector which is perpendicular to the plane containing these two vectors A and B. So if I imagine these two vectors A and B like the arrows and if I consider a plane which is passing through these two arrows then the cross - product vector will be a vector perpendicular to these two arrows or it will have perpendicular to the plane passing through these two arrows A and B.

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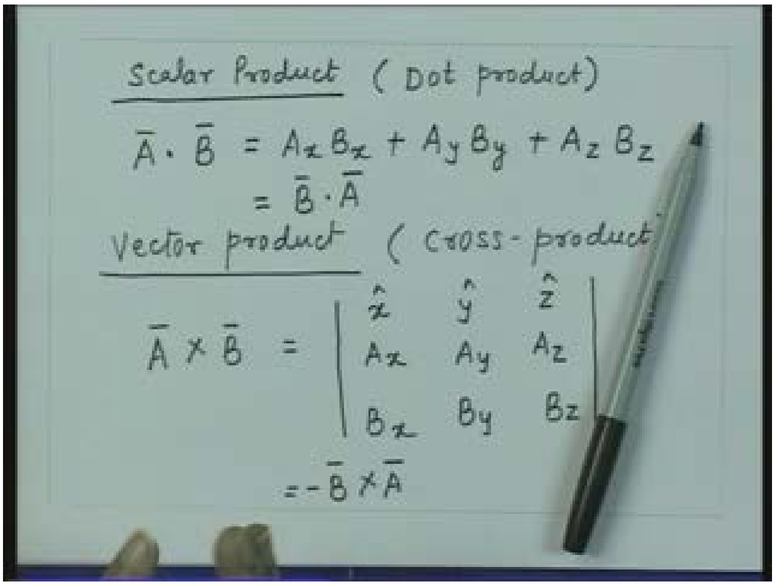

Question now again is that how do I know what is the direction of this arrow which is the cross – product. So again it is the, so by the same convention that if I go my fingers from A to B the direction of the thumb will represent the direction of this vector cross – product. If I interchange the sign B to A now my fingers will go from B to A so direction of the thumb will be opposite. So by interchanging A and B essentially following the same convention my direction of thumb will become opposite and that is what the direction of the vector will become opposite. So the magnitude of the vector will remain same but the orientation of vector will be in the opposite direction. So these are the two important operations on vectors which we will encounter when we go to analysis of the electromagnetic waves.

Then we require the operators on the vector which are the differential operators. So consider now a field which is a vector field that means at every location in the space you define this quantity which is a vector quantity. So I consider the space to go to any point if I measure this quantity. This quantity will have a magnitude at that location and this quantity will have an orientation if i imagine this vector like an arrow. Just to give you an example of this vector fields let us say I have a quantity like velocity distribution; let us say I want I have an air velocity in the medium.

So, if I go around me and in every location if I measure the velocity of the wind and if I find out in which direction the wind is flowing I know the direction of that wind so I know the strength of the wind movement, I also know the direction in which the wind is moving so these two quantities together I can put in the form of an arrow; the strength of the wind movement I can denote by the length of this vector, the direction in which the wind was flowing I can mark by the arrow, so at every location I have this quantity which is the velocity of air around me which can be denoted by this vector.

Similarly, if I have let us say flow of some liquid, if I go to various locations I will again have the quantity of liquid which is flowing at that point; also we will know the direction in which the liquid is flowing so again we can represent that quantity by an arrow at that location. So, if you have a quantity which can be represented by strength or magnitude and also it had a direction then you can call this quantity as the vector field. So electric field or magnetic field is a vector field. So, if I go in three dimensional space and if I measure the electric and magnetic fields they will have a different value in magnitude and also they will have different orientations. So if I now consider a vector field in three dimensional space then one can define certain operators for these vector fields.

Before getting into the vector fields let us say suppose I had a scalar field, suppose I have temperature variation around me this quantity is normal scalar quantity. But if I measure the variation of the temperature the variation of the temperature in different directions is defined; suppose I take temperature variation right about the surface of the earth as we go to higher and higher altitudes the temperature drops. If I move in the horizontal direction may be the temperature variation is not very much. But if I travel a distance of 100 kilometers on the surface of the earth the temperature will not vary significantly. But if I travel 100 kilometers above the surface of the earth the temperature variation will be significant, it would drop at least by 40, 50 degrees.

So that means though the quantity temperature is a scalar quantity its variation is a vector quantity, its variation depends upon the direction, it does not have a variation in the horizontal direction but it does have variation in the vertical direction. So if I have now a quantity scalar quantity which is the function of three dimensions basically this function is a scalar function of the three dimensions. But if I find the variation of this quantity this variation is a vector quantity. So we can define an operator a differential operator what is called the gradient operator which operates on the scalar field and outcome of this is a vector quantity. So this operator is what is called the gradient operator and that gives you gradient of a scalar field.

So I have a certain function f which is a scalar function of xyz. This quantity is scalar. The gradient defines the maximum rate of change of this function in three dimensional space. So if I find out the rate of change of the function in three directions three coordinate direction xyz then I can find out the direction in which the function is changing maximally; that vector of the rate of change of this function is what is called the gradient. So this is denoted by a differentiation of the scalar function f and this differentiation is in three dimensions. So therefore to represent this operation we define an operator what is called del operator, we define an operator called as del and denote it like this that is tau derivative in the direction x multiplied by unit vector x plus derivative in the direction y multiplied by unit vector y plus derivative in the direction z multiplied by unit vector.

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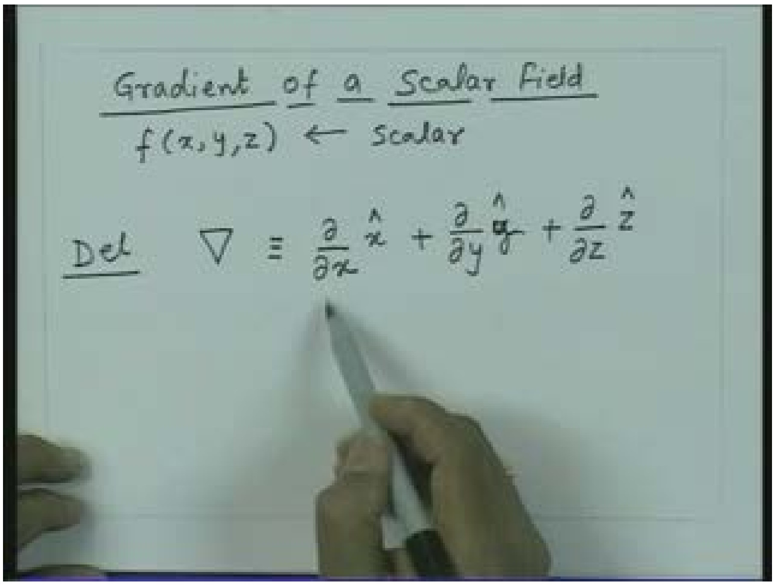

So this operator del essentially is the differential operator which can operate on the scalar field... and later we will see it can operate on the vector field also; but if it operates on the scalar field then that operation is called the gradient operation. So here your gradient is of a scalar function del of f. So if I take this function f and I take three derivatives this is df by dx in x direction plus df by dy in y direction plus df dz in z direction.

So if I take this scalar function and take its derivative with respect to x this quantity tells me now rate of change of this function; special rate of change of this function in the x direction. Similarly, this quantity df by dy tells me the rate of change of this function in the y direction and this quantity represents rate of change of this function in the z direction. So this quantity what is called the gradient of the scalar function f is a vector quantity and these are the components of this vector which represent the rate of change of this function along the three coordinate access x, y and z.

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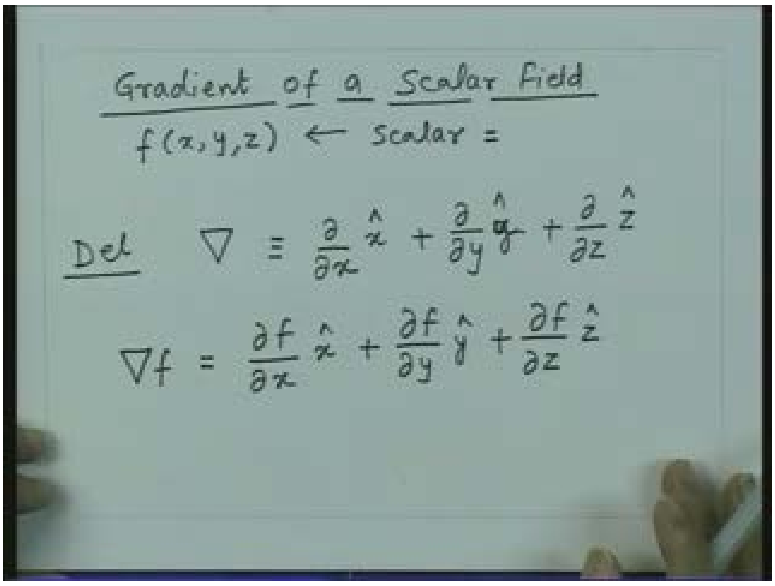

So, when we have a scalar function and if we want to find out the rate of change of scalar function which is a vector quantity that we can find out from by operating del on that scalar function. The expression we have written here is for the Cartesian coordinate system. Similar expression can be obtained for the cylindrical coordinate system and the spherical coordinate system.

Let us now say that I have a vector field and as we mentioned earlier the quantity like velocity or the the flow of liquid or the electric field or magnetic field these are the vector quantities; so if I have any of these quantities then I have a vector field. So let us say I have now the vector field. So there is a quantity vector f which is the function of x, y, z but it has components also in the direction xyz. So the function x, y, z it has a component F x x plus F y y plus F z.

So F x is the x component of this vector, F y is the y component of this vector and F z is the z component of this vector and each of this component is a scalar function of x, y and z. So all these quantities are scalar functions of x, y, z. Then we can define now the differential operator for this vector field and there are two operators which we can define: One is what is called the divergence of vector which is like a dot product of the del operator and the vector f it is del dot F and that is as we saw in case of the the dot product (Refer Slide Time: 49:57) it is the component wise product and the sum of all these products so this is dF x by dx plus dF y by dy plus dF z by z.

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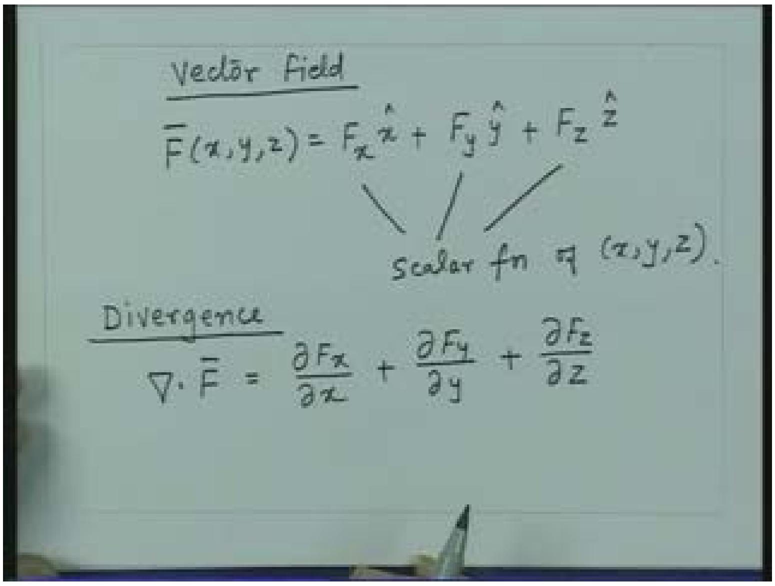

We will see the physical meaning of this little later. But this is like defining the scalar product of the del and this vector field F. As you have defined the cross - product you can define the cross - product again between the del and this vector F and that product what is called the curl the curl of vector F and that is the cross - product of the del and F. So it is del cross f which will be x y z and if you see here the way we wrote the cross - product it was the component (Refer Slide Time: 00:51:15) of this first vector and the component of the second vector.

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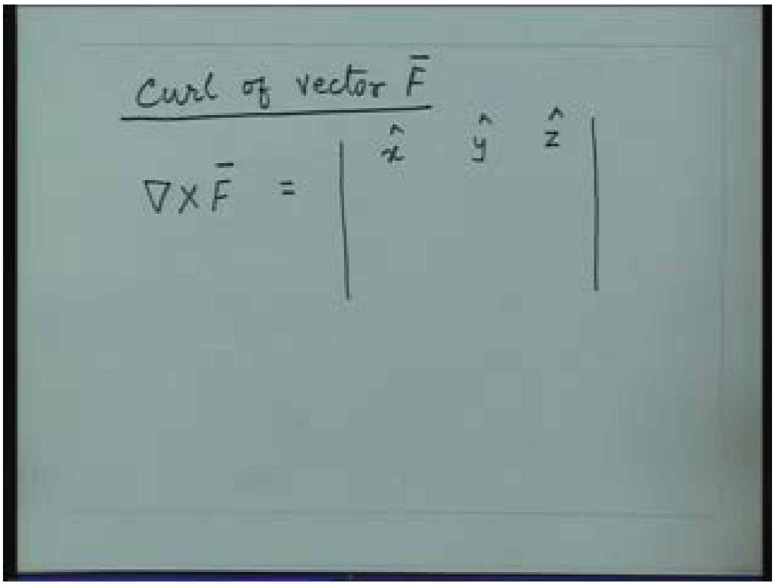

We have now got for the del, we treat it like a vector (Refer Slide Time: 00:51:25), its components are d by dx; d by dy; d by dz; you can write here d by dx; d by dy; d by dz; F x F y F z. So if we have a vector field then we can define these two operators called the divergence operator which is the dot product of the del operator and the vector F. If we take a cross - product then that product is what is called as curl of the vector F and which is the del operator a cross - product with F which is given by the determinant give like this. So the components can be written the x component will be d Fz by dy minus d Fy by dz and so on. You can expand this and we can write the component of this curl vector.

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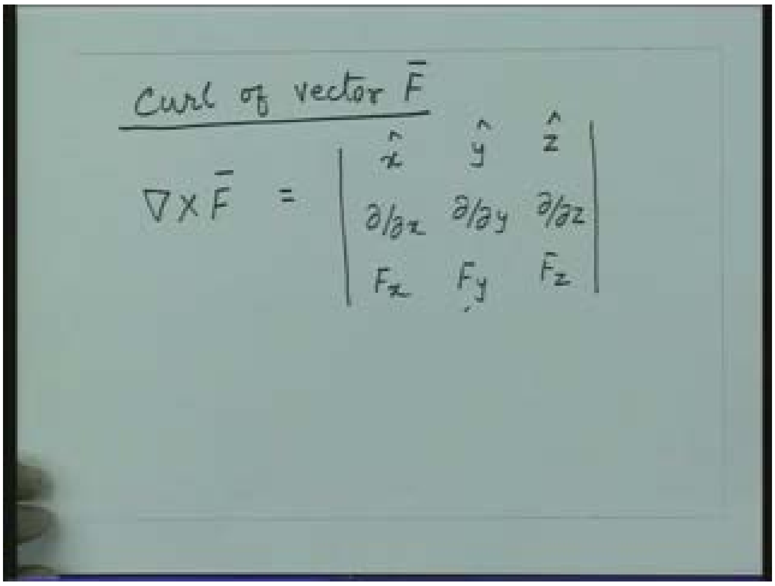

So curl of a vector is a vector quantity whereas the divergence of a vector is the scalar quantity.

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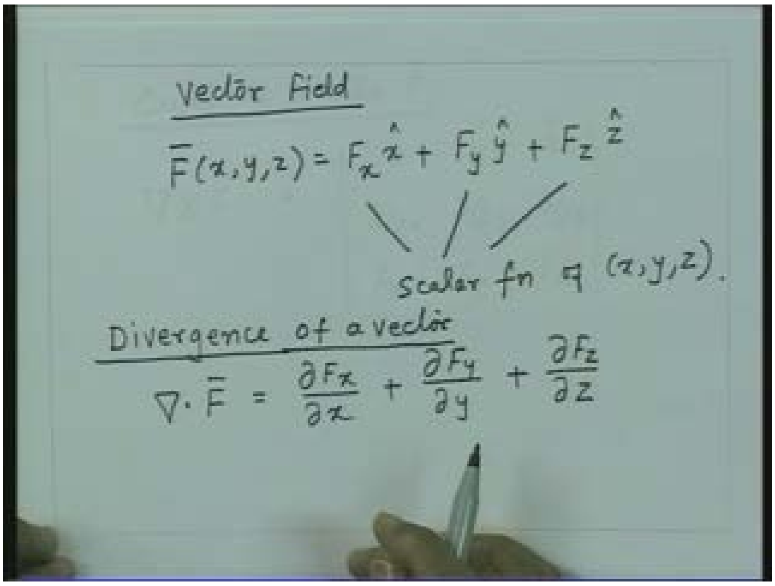

Next time when we meet we will see the physical interpretation of these quantities: the divergence and curl and once we get their physical field for these quantities of divergence in curl then we write the lot of the electromagnetics; that time it will become obvious that yes if you want to capture those physical effects then the appropriate concepts will be divergence and curls. So the formulation of electromagnetic problems can naturally follow in the direction of divergence and curls. So this gives you the basic framework now to define the vectors and define certain basic operations on the vectors in three dimensional space.