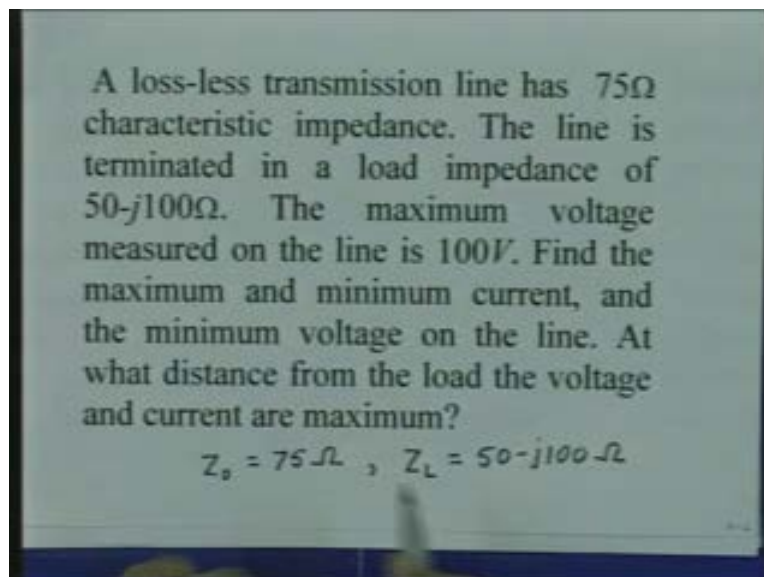


**Transmission Lines & E M. Waves**  
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**Lecture – 14**

In this lecture, we solve some problems of transmission lines based on the theory which you are developed so far. We will solve some problems analytically and later on we go to the solution of the problems by graphical means that is by using the Smith chart. Let us consider this problem here; a lossless transmission line has  $75\ \Omega$  characteristic impedance. The line is terminated in a load impedance of  $50 - j100\ \Omega$  the maximum voltage measured on the line is 100 volts. We have to find the maximum and minimum current and the minimum voltage on the line also we have to find out at what distance from the load, the voltage and current are maximum. This problem can be solved either analytically or graphically, we will solve here this problem analytically.

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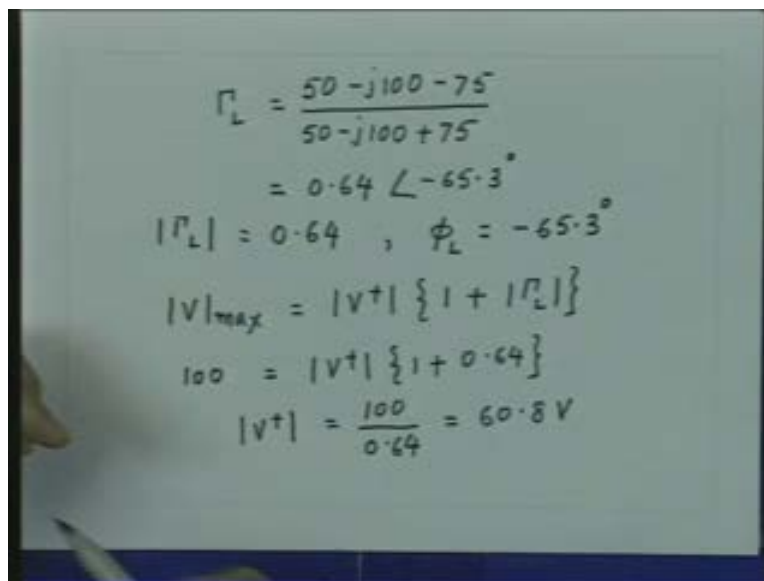


So what we are given here is that characteristic impedance which is  $Z_0$  that is given as  $75\ \Omega$ , the load impedance which is  $Z_L$  is given as  $50 - j100\ \Omega$  and magnitude of the

voltage maximum on the line  $V_{\max}$  is given as 100 volts. So  $\text{mod } V_{\max}$  is given as 100 volts using this information then we have to find out the other parameters like voltage and current minimum and distance of the voltage minimum from the load point. First I have been all this calculation is the calculation of the reflection coefficient. So now, I know the load impedance, I know the characteristic impedance. So from here I can write down the reflection coefficient at the load point  $\Gamma_L$  which is  $z_L - z_0$  divided by  $z_L + z_0$ , if I simplify at this complex expression, I will get the  $\Gamma_L$  that will be equal to “0.64” which an angle of minus “65.3” degrees.

So the magnitude of reflection coefficient  $\text{mod } \Gamma_L$  that is equal to “0.64” and the angle of the reflection coefficient  $\phi_L$  that is equal to minus “65.3” degrees then next step could be there once you know the magnitude of the reflection coefficient, since nothing is stored about the line we take liberty we chose the line to be lossless and therefore the magnitude of reflection coefficient remains same on every point on the line. So we know that the maximum voltage which you see on the line  $\text{mod } v_{\max}$  that is equal to  $\text{mod } v$  plus into  $1 + \text{mod } \Gamma_L$ , we are given this  $v_{\max}$  voltage which is 100 volts, we know now the modulus of reflection coefficient.

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$$\begin{aligned}\Gamma_L &= \frac{50 - j100 - 75}{50 - j100 + 75} \\ &= 0.64 \angle -65.3^\circ \\ |\Gamma_L| &= 0.64, \quad \phi_L = -65.3^\circ \\ |V|_{\max} &= |V^+| \{1 + |\Gamma_L|\} \\ 100 &= |V^+| \{1 + 0.64\} \\ |V^+| &= \frac{100}{0.64} = 60.8 \text{ V}\end{aligned}$$

So we can find out the magnitude of the incident wave which is  $V_{\text{max}}$  plus modulus. So from here, we can substitute that is 100 is equal to  $V_{\text{max}}$  plus into  $1 + |\Gamma|$  inverting this relation, we get  $V_{\text{max}}$  plus that is equal to 100 divided by  $1 + |\Gamma|$  that is equal to "60.8" volts. Since, we know the magnitude of reflection coefficient, we could calculate  $V_{\text{max}}$ , we can calculate  $I_{\text{max}}$  also because once we know the  $V_{\text{max}}$  plus quantity, we can calculate  $I_{\text{max}}$ . However, we know from our analysis that the maximum current seen on the transmission line is nothing but the maximum voltage divided by the characteristic impedance. So we know  $V_{\text{max}}$ , so we can divide this quantity by the characteristic impedance  $Z_0$  and we can find out directly the quantity which is the maximum current  $I_{\text{max}}$ .

So from here, I can get the maximum current seen on the line  $I_{\text{max}}$  that is  $V_{\text{max}}$  divided by  $Z_0$  that is equal to 100 volts divided by the characteristic impedance which is 75. So, that gives you the maximum current that is equal to "1.33" amperes. The  $I_{\text{min}}$  which we get on the line  $I_{\text{min}}$  that if we go back to the expressions of the minimum and maximum currents that will be  $V_{\text{max}}$  divided by  $Z_0$  into  $1 - |\Gamma|$ . We know this quantity  $V_{\text{max}}$  which is "60.8" the  $Z_0$  is 75 ohms. So this is "60.8" divided by 75,  $1 - |\Gamma|$  which is the modulus of the reflection coefficient. So from here I can get the value of  $I_{\text{min}}$  that is equal to "0.29" amperes.

So the maximum can which you see on this line is "1.33" amperes and the minimum current will we see on this line is "0.29" amperes. Once you know the minimum current on the line, we can find out what is the minimum voltage and that is  $V_{\text{min}}$  that is nothing but  $Z_0$  into  $I_{\text{min}}$ . So that is equal to 75 into "0.29", so that is equal to "21.88" volts, so this line has a maximum voltage of 100 volts, minimum voltage of "21.88" volts, maximum current of "1.33" amperes and minimum current of "0.29" amperes. Now we are also have to find out what is the location of the voltage and current maximum on the line for this we have to go to the basic relation of the 2 travelling waves and when the 2 travelling wave have a constructive interference that time we have a voltage maximum, when the 2 wave have a destructive interference, we have the voltage minimum and it has a same location we have the current maximum.

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Handwritten calculations on a whiteboard:

$$|I|_{\max} = \frac{|V|_{\max}}{Z_0} = \frac{100}{75} = 1.33 \text{ A}$$
$$|I|_{\min} = \frac{|V|}{Z_0} \{1 - |\Gamma_L|\}$$
$$= \frac{60.8}{75} \{1 - 0.64\}$$
$$|I|_{\min} = 0.29 \text{ A}$$
$$|V|_{\min} = Z_0 |I|_{\min} = 75 \times 0.29$$
$$= 21.88 \text{ V}$$

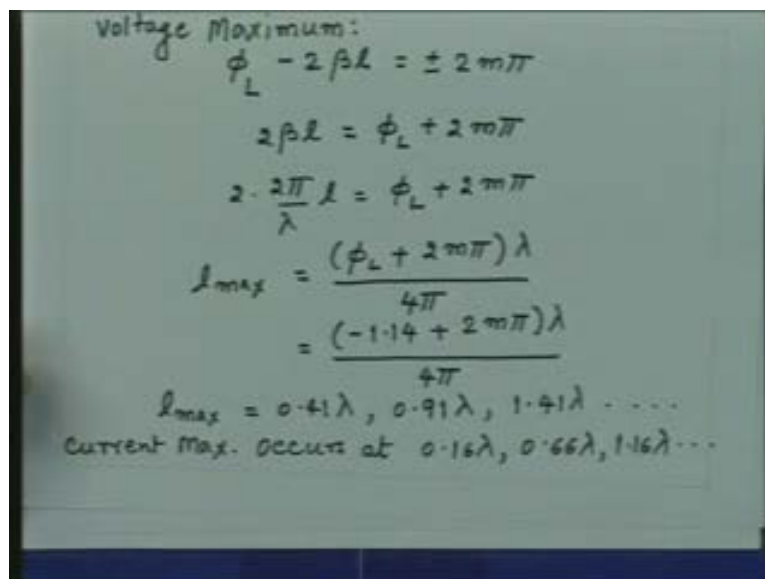
So we know that the phase difference between the 2 waves, the forward and the backward wave that is given as the load reflection coefficient phase minus  $2\beta l$ . So when this quantity  $\phi_l$  minus  $2\beta l$ , if it is even multiples of  $\phi$  that time the 2 wave which will have a constructive interference and I will have voltage maximum, when this is all multiples of  $\phi$  that time we have destructive interference and we have a voltage minimum. So in general when this quantity is equal to plus minus  $2m\phi$ , where  $m$  is an integer quantity. We will get the voltage maximum. So the condition for voltage maximum is when the phase of the reflection coefficient at the load minus  $2\beta l$  is equal to plus minus  $2m\phi$  that time the 2 waves have constructive interference and we get a voltage maximum.

So from here we can get  $2\beta l$  that is equal to  $\phi_l$  plus  $2m\phi$ . I have chosen here the sign positive because I want to get the lengths which are positive; we are measuring all the lengths now towards the generator,  $l$  equal to 0 at the load point. So all the lengths which are going to measure from the load point or the positive lengths. So I have chosen the sign in such a way that I get all the distances which are positive substituting now for  $\beta$  that is  $2\phi$  by  $\lambda$

and substituting for the phase of the reflection coefficient which you are calculated here which is “minus 65.3” degrees, we can get now the location of the voltage maximum. So let us say this is till  $\phi_1$  plus  $2m\phi$ .

So the length at which the maximum voltage appears, let us called that is  $l_{\max}$  that substituting into this will be  $l_{\max}$  will be equal to  $\phi_1$  plus  $2m\phi$  into the wavelength divided by  $4\phi$ . Substituting for  $\phi_1$  minus “65.3” degrees that is an radian’s, it is minus “1.14” radians. So this is equal to minus “1.14” radians. We can get now  $l_{\max}$  is equal to minus “1.14” plus  $2m\phi$  lambda divided by  $4\phi$ . Now the substituting different values for  $m$ , you put  $m$  equal to 1,  $m$  equal to 2 and so on. You get the  $l_{\max}$  or the location where the voltage is going to be maximum that is “0.41” lambda, “0.91” lambda, “0.41” lambda and so on. To find out the current maximum, the current maximum occurs where voltage is minimum that means when the phase 3 are destructive interference either we can go by that argument or we can say that the current maxima and voltage maxima are shifted by a distance of lambda by 4.

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Handwritten derivation for voltage and current maxima:

$$\begin{aligned} \text{Voltage Maximum:} \\ \phi_L - 2\beta l &= \pm 2m\pi \\ 2\beta l &= \phi_L + 2m\pi \\ 2 \cdot \frac{2\pi}{\lambda} l &= \phi_L + 2m\pi \\ l_{\max} &= \frac{(\phi_L + 2m\pi)\lambda}{4\pi} \\ &= \frac{(-1.14 + 2m\pi)\lambda}{4\pi} \\ l_{\max} &= 0.41\lambda, 0.91\lambda, 1.41\lambda, \dots \\ \text{Current Max. occurs at } &0.16\lambda, 0.66\lambda, 1.16\lambda, \dots \end{aligned}$$

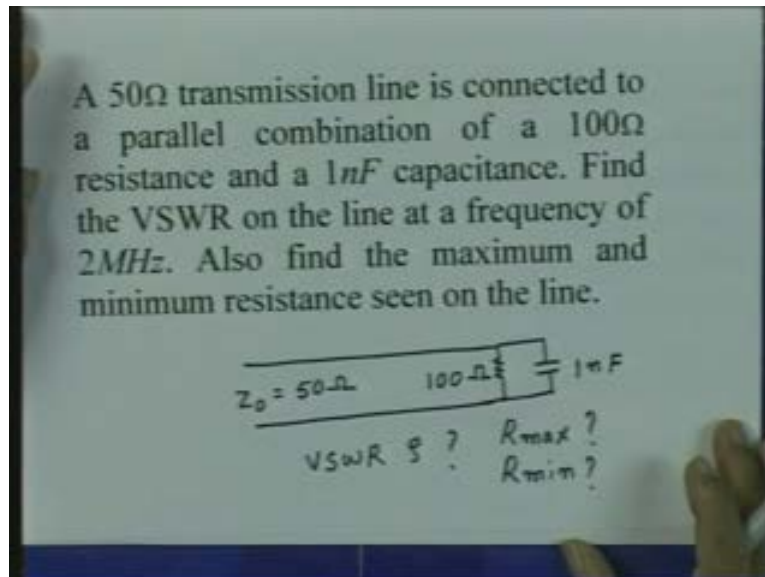
So if I add lambda by 4 or “0.25” lambda to all this value I will get the location of the current maximum. So I substituting into this I can get the location for the current maximum, current

maximum occurs at this values plus minus “0.25” lambda and when you are subtract “0.25” lambda from here I still get a length positive. So I have to choose even that value of the distance, so the location at means the current is maximum that will be at “0.16” lambda, “0.66” lambda, “1.66” lambda and so on. So you see using the basic equation which you are derive for a transmission lines if you systematically proceed find the reflection coefficient, go to the relationship of the voltages in current, write down the simple voltage expression in the current expression on transmission line, substituting the values of the reflection coefficient, its magnitude, its phase. We can calculate the maximum and minimum currents on transmission line. We can also find the location at which the voltage or currents maxima or they are minima.

So this is one of the problems which you will do you ask to solve in the real life because normally your line is terminated in an unknown impedance and we are supposed to find out and where the voltage will be maximum so on and so on. Let us consider another problem, let us say now we are having a transmission line which is having a characteristic impedance of 50 ohms and this line is connected to a parallel combination of 100 ohm resistance and 1 nano farad capacitance. In fact this is the very common problem whenever we are having a cable at higher frequencies and let us say, I want to connect these cable to some circuit whose output impedance is about 100 ohms is possible at that point we may have some straight capacitances or intensically the impedance might be having a capacitance.

So in variable you will see the output or input impedance of a circuit which will be a combinations something like this. So here the impedance in which the line is terminated is 100 ohms in parallel with 1 nano farad capacitance, we are supposed to find out what is the VSWR on the line at a frequency of 2 megahertz also find the maximum and minimum resistance seen on the line. So 2 things we have to find out, first thing we have to find out what is the VSWR because that is the measure of how much power is reflected on the line and also we are intersect in finding out what is the maximum and minimum resistance. We will see on the line again we proceed the same way. So the problem we are having is a line which is connected to a resistance of 100 ohms and capacitance of 1 nano farad, this is 1 nano farad, this is 100 ohms, the characteristic impedance of the line  $Z_0$  is given as 50 ohms.

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So we have to find out the quantities VSWR  $\rho$  and  $R_{max}$  and  $R_{min}$ . Again the first step in this calculation, first thing we have to now do is find out the complex impedance for this combination of the resistance and capacitance, once you know the impedance then you find out what is the reflection coefficient from the reflection coefficient, you find out the magnitude of the reflection coefficient and for the magnitude of the reflection coefficient then you can find out what is the voltage standing wave ratio VSWR  $\rho$ . So the impedance, load impedance  $Z_L$  is the parallel combination of  $R$  and  $C$ . So this is  $E_1$  is  $R$  divided by  $1 + j\omega RC$ , the frequency which is given is 2 megahertz.

So the  $\omega$  at the frequency is  $2\pi \times 2 \times 10^6$  radians or seconds. So  $\omega$  will be " $1.256$ " into  $10^7$  radians per second, substituting for  $\omega$  and  $R$   $C$   $R$  is 100 ohms and  $C$  is 1 nano farad, we can get 100 divided by  $1 + j "1.2566"$ . So  $\omega R C$  are substitute in the value of  $\omega$  and  $R$  and  $C$ , you get this quantity " $1.2566$ ". So that impedance will be 100 divided by " $1.6$ " with an angle " $51.5$ " degrees. So the load impedance, I can write either in the polar form which is " $62.3$ " angle minus " $51.5$ " degrees or in the real and imaginary part which is " $23.9$ " minus  $j "45.6"$  ohms. Once I know the load impedance now, I

can find out the reflection coefficient  $\Gamma_L$  which is  $Z_L - Z_0$  divided by  $Z_L + Z_0$ . So that is equal to “23.9” minus j “45.6” minus 50 divided by “23.9” minus j “45.6” plus 50.

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Handwritten calculations on a whiteboard:

$$Z_L = \frac{R}{1 + j\omega RC}$$

$$f = 2 \text{ MHz}$$

$$\omega = 2\pi \times 2 \times 10^6 \text{ rad/s}$$

$$= 1.256 \times 10^7 \text{ rad/s}$$

$$= \frac{100}{1 + j1.2566} = \frac{100}{1.6 \angle 51.5^\circ}$$

$$= 62.3 \angle -51.5^\circ = 23.9 - j45.6 \Omega$$

Reflection Coefficient

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{23.9 - j45.6 - 50}{23.9 - j45.6 + 50}$$

$$= 0.6 \angle -88.3^\circ$$

Now solving this, we get the reflection coefficient has “0.6” angle minus “88.3” degrees. So the magnitude of the reflection coefficient from here we can get and that is equal to “0.6”. So we get now from here the magnitude of the reflection coefficient mod  $\Gamma_L$  that is equal to “0.6”, once we get that then the VSWR  $\rho$  is  $1 + \text{mod } \Gamma_L$  divided by  $1 - \text{mod } \Gamma_L$  that is equal to  $1 + “0.6”$  divided by  $1 - “0.6”$  that is equal to 4. So for this combination of resistance in capacitance at 2 megahertz, we will get a VSWR of 4. Once we know the VSWR then finding out the maximum and minimum resistance is very straight forward we know the characteristic impedance of the line.

So the maximum impedance will be characteristic impedance multiplied by  $\rho$  and the minimum impedance will be characteristic impedance divided by  $\rho$ . So we get  $R_{\text{max}}$  on the line which is  $Z_0 \times \rho$  which is  $50 \times 4 = 200 \text{ ohms}$  and  $R_{\text{min}}$ , minimum resistance which will see on the line is  $Z_0$  divided by  $\rho$  that is equal to  $50$  divided by  $4$  that is “12.5”.



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$$\begin{aligned} |\Gamma_L| &= 0.6 \\ \text{VSWR } S &= \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 0.6}{1 - 0.6} = 4 \\ R_{\max} &= Z_0 S = 50 \times 4 = 200 \Omega \\ R_{\min} &= Z_0 / S = \frac{50}{4} = 12.5 \Omega \end{aligned}$$

So in this case, we were given some combination of **lambda** circuit with which the line is terminated and we were asked to find out what is maximum and minimum resistance on the line and also we were asked to find out, what is the VSWR which will see on the line and recall higher the value of VSWR, worse is the match. So this by VSWR is quite large, it is 4 that means in this case lot of power will be reflected from this terminating load. Let us now try to solve some of the problems by using the Smith chart that is the graphical means of solving the problem. Before, we get into the specific problems let us first try to identify some of the special points on the Smith chart.

So let us say, I have to identify some of the special points that is I am given the impedance  $50 + j 75$  ohms, you to mark this on the Smith chart or the impedance  $10 + j 0$  ohms or an impedance  $0 - j 80$  ohms or the reflection coefficient that is “0.3” angle 60 degrees or you to mark constant VSWR circle with  $\rho$  equal to “2.5” or we have to find out the  $R_{\min}$  point on the VSWR circle which is having value “1.5”. As we know all the impedances which we see on the Smith chart or normalized impedances, so the first step towards marking all this points impedance point on the Smith chart is to normalize these impedances.

So firstly, we take the characteristic impedance of the line let us say it is 50 ohms. So we have to normalize these values with 50 ohms. So if I normalize this impedance A which is 50 plus j 75 this will become 1 plus j 75 upon 50, so that will be "1.5". Now I go to the switch chart and first I identify a circle for which the resistive part is 1, we know this is the circle which passes through the center of the switch chart. So this circle is R equal to 1 circle then I go to the reactive part which is "1.5" so this circle which is a "1.5" circle.

So the intersection point of these 2 circle, this circle and this circle is the point A here which represents an impedance 50 plus j 75 ohms or a normalized term 1 plus j "1.5", if I take an impedance 10 plus j 0, again we normalize this impedance with 50 ohms. So this will be ".2" plus j 0, so immediately we notice that this is the resistive impedance we are talking about and the resistive impedance must lie on the horizontal axis in the complex gamma plane. So we go to circle which is R equal to ".2" circle and find this intersection on the horizontal axis.

So this point B represents an impedance which is ".2" plus j 0 in normalized terms are 10 ohms plus j 0 in the absolute terms. The third impedance which you want to mark on the switch chart is 0 minus j 80 ohms. Again we normalize this it is a purely reactive impedance and since it is the minus sign here, it is purely capacitive reactance, its normalized value will be 80 divided by 50 "1.6", since if the capacitive reactance we know it must lie on the lower half of the switch chart, switch chart impedance, the lower half of the switch chart represent the capacitive, reactance, impedances and the upper half represent the inductive impedances, since the real part is 0, we know that this point must lie on the outer most circle on the switch chart.

So if I go to the outer most circle which is this circle on this switch chart and find the constant reactance circle which is having value minus "1.6", I will get this point C. So this point C represents a pure reactance of minus j 80 ohms or a normalized term minus j "1.6". Now these were the marking of the impedance points on the switch chart. Suppose, somebody gives me the reflection coefficient is magnitude and its phase and ask me to mark that point on the switch chart. Let us say the magnitude of the reflection coefficient is "0.3" and the phase of the reflection coefficient is 60 degrees, what do we do? First we measure the length of the switch

chart here and that length is 1 because we know the switch chart in complex gamma plane has a radius equal to 1.

So this length now is from center to this point is unity. Now we can scale it to find out what should be the length for "0.3" and then the angle is 60 degrees which is measure from the real axis in the gamma plane. So which is this line, so from here we read the angle like normal convention that go in the anticlockwise direction by 60 degrees, take a radius vector whose length will be "0.3" normalize to this length and you will get this point D. So the point D represents a reflection coefficient whose magnitude is "0.3" and whose phase angle is 60 degrees. The another thing which you would like to get is draw a VSWR circle for rho equal to "2.5", how do we draw this circle. We know the rho corresponds to the maximum normalize resistance seen on the transmission line. So this constant VSWR circle for rho equal to "2.5" must pass through a point which is normalize "2.5" and reactance 0. So we just find out a circle which is "2.5", R equal to "2.5" and x equal to 0 that means it is this point lie on this line.

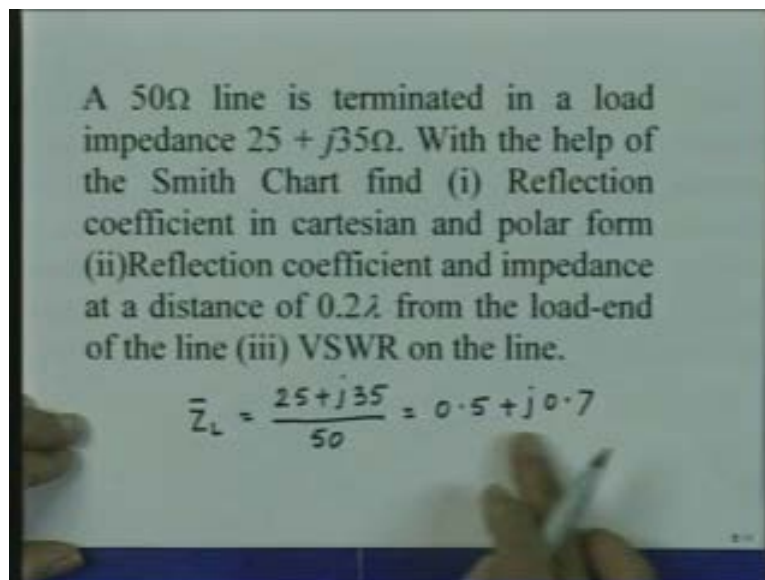
So this point represents here normalize or maximum of "2.5", if I draw a circle passing through this point with the center as the center of the switch chart, this circle than represent a constant VSWR circle for rho equal to "2.5". Next thing we want to find out A R min seen on a constant VSWR circle of "1.5", again first we draw the constant VSWR circle then on the circle you go to a location where R will be minimum and that will give me the value of R min. So first since the rho equal to "1.5" it has be the R max normalize, I find out the point which is R max this is "1.5" I first draw a circle passing through this point.

So this is the constant VSWR circle for rho equal to "1.5" then we know the left most point on this circle represents R min. So if I read this point here this value corresponds to the R min. So the R min value from here that will be equal to "0.75". So the normalize value of the minimum resistance on the line will be "0.75" if I multiply by a 50 ohms then I will get the minimum impedance on the line. So this why just a small exercise just to identify some points on the switch chart when the impedances are given and the reflection coefficient is given, when the VSWR is given and then find out some other values, with this understanding now of the basic switch chart, now we can go to a specific problem which we want to solve by using switch chart.

Let us say I have a fifty ohm line which is terminated in a load impedance of 25 plus j 35 ohms. With the help of the smith chart find reflection coefficient in Cartesian and polar form at the load point reflection coefficient an impedance at a distance of “0.2” lambda from the load end and the VSWR on the line.

So we have to find out 3 quantities now we are given the absolute load impedance, we are given the characteristic impedance and we have to find out the reflection coefficient at the load at a distance of “.2” lambda from the load and also the VSWR on the line. To do this first we have to normalize of the impedance, so we can calculate the  $Z_L$  normalize that is 25 plus j 35 divided by the characteristic impedance which is 50, so that will be “.5” plus j “.7”.

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So the normalize impedance which we have is “.5” plus j “.7”, we take a smith chart and mark this point of “.5” plus j “.7” since the reactive part is positive, this point must lie on the upper half of the smith chart. So first you find out a constant of circle which is “.5”, this value and then you find out a constant reactance circle which is having a value of plus “.7”. So this value this R is “.5” and this x is “.7”, this circle. So the intersection point of these 2 which is point A, this point now represents a normalize impedance of “.5” plus j “.7”. I can now read of

the value of the reflection coefficient, complex reflection coefficient by measuring this distance and normalizing that with the radius of smith chart that will give me the magnitude of reflection coefficient, I can measure the phase of this radius vector and that will give me the phase of the complex reflection coefficient.

So if I do that I get the quantity of  $\Gamma$  in the polar form that will be “0.52” angle 100 degrees, you can see from here, this is almost half of this radius. So the magnitude of the reflection coefficient is “.52”, this angle is close to 90 degree. So its actual value is 100 degree, so in polar form the reflection coefficient is “0.52” angle 100 degrees, I can either convert this into Cartesian form by using calculator or I can measure the projection of these on the horizontal and vertical axis to find out the reflection coefficient in the Cartesian form. So this same thing can be written as minus “0.09” plus j “0.512”. So these are the reflection coefficient in the polar form these reflection coefficient in the Cartesian form. The next thing you have to do is to find out the reflection coefficient at a distance of “0.2”  $\lambda$  from the load point.

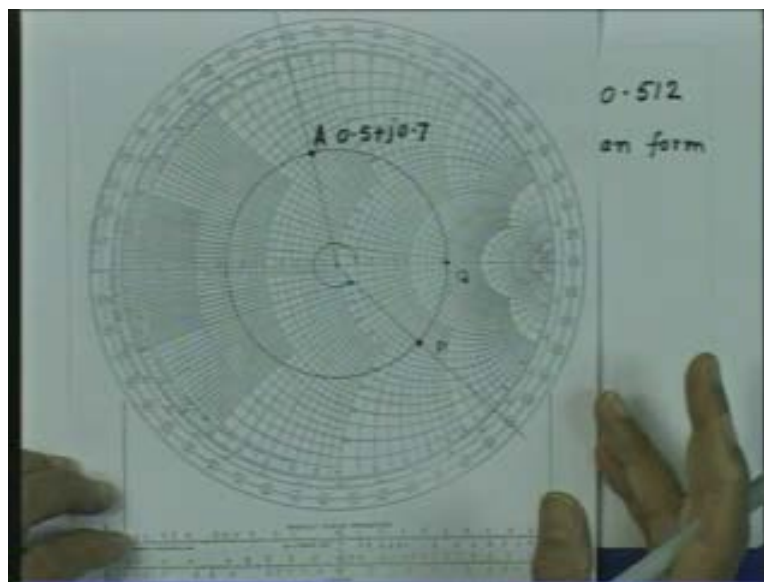
Now for this first thing we do is we draw a constant VSWR circle now passing through this point, this point A. As we move towards the generator on the transmission line, we move on this circle and we want to move by distance of “0.2”  $\lambda$ , since the entire one circumference is equal to  $\lambda$  by 2, I can find what is the angle correspond to “0.2”  $\lambda$ . So from this point if I move by “0.2  $\lambda$ ” I will reach to this point P and I can find out what is the value of reflection coefficient for this point. So I just move by an R which corresponds to “0.2”  $\lambda$  on this and then this magnitude of reflection coefficient will remain same because line is lossless, so this radius is same as this radius the phase of the reflection coefficient we read in the conventional way, so from the real axis we measure in the anticlockwise direction up to this radius vector.

So the magnitude of reflection coefficient is still “0.52” but the phase of the reflection coefficient is 316 degrees. So  $\Gamma$  at  $l$  equal to “0.2”  $\lambda$  will be equal to “0.52” with angle 316 degrees. We have also to find out what is the impedance now at a distance of “0.2”  $\lambda$ , this is the location which is that “0.2”  $\lambda$  from the load we have move towards the generator higher distance of “0.2”  $\lambda$ . So now if I read of the value of the complex impedance that is

find out the circle which is  $R$  equal to constant circle passing through this point and  $x$  equal to constant circle passing through this point, I will get the normalized impedance corresponding to this point.

So from here I can get the normalized impedance  $Z_{\text{bar}}$  that will be “1.4” minus  $j$  “1.37”. So multiplying by 50 ohms the impedance at that location will be 50 into  $Z_{\text{normalized}}$  that is equal to 70 ohms minus  $j$  “68.5”. So the impedance at this point is 70 minus  $j$  “68.5” ohms and the VSWR from this if I go to the right most point of this circle, this point is value, it has been the VSWR.

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So if we read carefully we can see this value lie between “3.1” and “3.2”, so its accurate value is “3.17” when we are doing graphical calculations that kind of error is acceptable. So if the point is lying somewhere between “3.1” and “3.2” roughly, we may say the value is “3.15”, the accurate value of this point is “3.17”. So we can read of this value and we will get a VSWR  $\rho$  that is equal to “3.17”, so in this problem we are given the load impedance we were asked to find out the reflection coefficient at the load at a distance of “.2”  $\lambda$ . The impedance at a distance

of “.2” lambda and also the VSWR on the line and see as we mention earlier that without doing any calculation, rigorous calculation, we could estimate all these values on the transmission line.

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$$\Gamma_L = 0.52 \angle 100^\circ = -0.09 + j0.512$$

Polar form          Cartesian form

$$\Gamma \text{ at } l = 0.2\lambda$$

$$= 0.52 \angle 316^\circ$$

$$\bar{Z} = 1.4 - j1.37$$

$$Z = 50\bar{Z} = 70 - j68.5 \Omega$$

$$\text{VSWR } S = 3.17$$

So the point which was made that or the use of switch chart makes the calculations of transmission line extremely simple, can be seen here if we are use the analytical expression for the impedance transformation we have to do with the complex calculation. In this case just by drawing now the constant VSWR circle passing through this point, we can just read out the values for the reflection coefficient impedance VSWR and so on. Let us take further problem, let us say now I have a line which is terminated in a normalize admittance of “.2” minus j “.5”. So up till now we have use the switch chart like the impedance switch chart, let us now consider a problem where we are dealing with the admittances, find the location of the voltage maximum from the load end also find the reflection coefficient normalized admittance and normalized impedance at a distance of “.12” lambda from the load.

So in this case we are given the admittance which is directly a normalized admittance. So firstly what we are given is y which is “.18”. Let us take the smith chart at the moment use the switch chart as the admittance switch chart. So now switch chart is the admittance one, you take the

point which is “0.2” minus “0.5”. So find out the circle which is “.2” constant g circle find out the circle which is constant b circle which is minus “.5”, the intersection point of 2 gives you this point here which is  $y_l$  bar normalize admittance at the load point.

Now we are ask to find out the location of the voltage maximum from the load and also find the reflection coefficient, normalize admittance and normalize impedance at a distance of “1.2”  $\lambda$  from the load. So first thing to find out the voltage maximum since, we are using now the admittance chart to voltage maximum does not lie on this point because if you recall the  $y$  or the complex  $\gamma$  is has been rotated by 180 degrees. So this point now represent the voltage maximum and this point represents the voltage minimum. So if I move now towards the generator by a distance till I reach to this point that is the length at which I will get the voltage maximum.

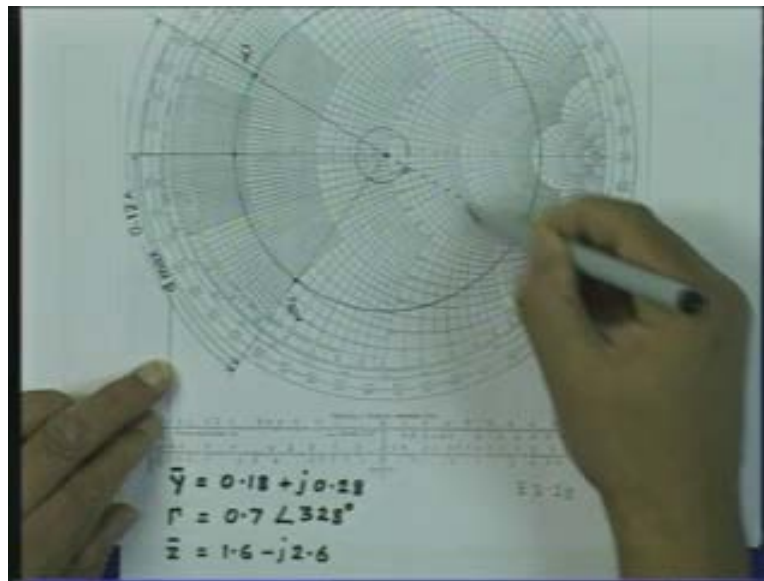
So first thing from this normalize admittance, I draw as constant VSWR circle move on this circle by a distance till I reach to this point where the voltage is maximum. So this distance from here to here that gives you the distance of the voltage maximum from the load point. The next thing we are ask to find out is the reflection coefficient at the normalize admittance at a distance of “0.12”  $\lambda$ . So I move on this constant VSWR circle by a distance of “0.12”  $\lambda$  in the clockwise direction because we are moving towards generator. So various to this point which is  $y$  bar if I read of the value here, I get the value of normalize admittance at a distance of “1.2”  $\lambda$  from the load point, find out the constant conductance circle passing through this, a constant acceptance circle passing through this and I get the value which is normalize value which is “0.18” plus  $j$  “0.28”.

So the admittance at a distance of “0.12”  $\lambda$  from the load is “0.18” plus  $j$  “0.28” to get the reflection coefficient either I can calculate from admittance itself but we are also it have to find out the normalize impedance at this distance. We know from the transmission line property that the normalize impedance inverse itself at a distance of  $\lambda/4$  but if we invert the normalize impedance that will become normalize admittance and vice versa that means if I take a point on the constant VSWR circle and rotate on this circle or move on this circle by  $\lambda/4$ , I will get a value which will be inverted value of  $y$  bar which is nothing but  $Z$  bar.



So taken diagonally opposite point on the constant VSWR circle gives me the value of  $Z$  bar and in this point now the smith chart becomes the impedance smith chart ,to see interesting things we started on the switch chart as a admittance switch chart mark this point which was the load admittance do a constant VSWR circle, moved on this to find out the admittance at a distance of “0.12” lambda and just taking now a diagonally opposite point on this, I get the quantity which is normalize impedance and beyond this point I can start reading the switch chart like an impedance switch chart.

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So if I read the value for the normalize impedance at this point that value will be “1.6” minus j “2.6”, so constant R circle which is passing through this is “1.6” the constant reactive circle which is passing through this is minus “2.6”, so this point here is “1.6” minus j “2.6”. Once I get that then the reflection coefficient now since the impedance switch chart, the real gamma x is on the right side I can measure now the magnitude and phase on the reflection coefficient. So the radius of this circle gives me the reflection coefficient magnitude, the phase measure from the real axis in the conventional way up to this radius vector gives me the phase of the reflection coefficient at a distance of “0.12” lambda. So if I take that from here I will get the reflection coefficient which is “0.7” magnitudes and the angle will be 328 degrees.

So this angle is 328 degrees in this length normalize to the radius of the smith chart that gives you the magnitude of the reflection coefficient and that is "0.7". So this problem clearly demonstrate the effectiveness of the smith chart for switching over from the admittances to impedances and vice versa. So even if some quantity that given as impedance or admittance as the need be, you may smith from the impedance to admittance just by taking the diagonally opposite point on a constant VSWR circle and treat the smith chart as the other smith chart. So in this case, we started with the admittance to get diagonally opposite point here and the smith chart becomes impedance smith chart, at some other point if you want to go back to the admittance again wherever we are take a diagonally opposite point you will get the normalized admittances and there onwards I can start using the smith chart as the admittance smith chart just to ask the question at the load point to what will be the normalized impedance. We know the normalized admittance which is given by this point.

So this value we know, if you want to find out now the normalized impedance just simply find out what is the diagonally opposite point on this. So if I extend this radius vector, I will get a point somewhere here and if I read off that value, I will get the normalized impedance at the load point. So we can find now the use of the smith chart or one more thing and that is whatever number we are having here, it inverts itself to get this number. So we have you admittance become impedance and the impedance becomes admittance but essentially what it is doing you know, it is inverting a complex number that is for it is doing. So the smith chart can be use to inverting any complex number.

You take the complex number which you want to invert treat it like a normalized admittance or impedance mark that point draw a circle pointing to that take a diagonally opposite point and you will get the inverse of the complex number. Now we will solves more problem by using smith chart like finding out the unknown impedance doing the matching problem by quarter wave transformer and the single step matching. So in the next lecture, we will take the impedance measurement problem and also the impedance matching problem by using the smith chart