

**Transmission Lines & E M. Waves**  
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**Lecture – 13**

Up till now, we discussed the lossless or low loss transmission lines. We studied the characteristics of lossless transmission lines and also saw various applications of lossless transmission lines. In practice, however as the frequency increases, the loss increases and the line becomes lossy. Today, we will see briefly the characteristic of a lossy line, of course if the line is very lossy then, it is not very efficient medium for transferring power from one point to another.

So, when we say a line is lossy in practice, it is not very lossy but it is moderately lossy. So first we will see very briefly if the line was very lossy, how the characteristic impedance and the propagation constant of a transmission line will change and then, we will go to the moderately lossy transmission lines. So, if I consider a very lossy line which we can define when the resistance is much much greater than the inductive reactance and the conductance is much much greater than the capacitive reactance. We can call that line as a very lossy line compare to what we are define for a low loss transmission line where the reactance's, they are much larger compare to the resistance component.

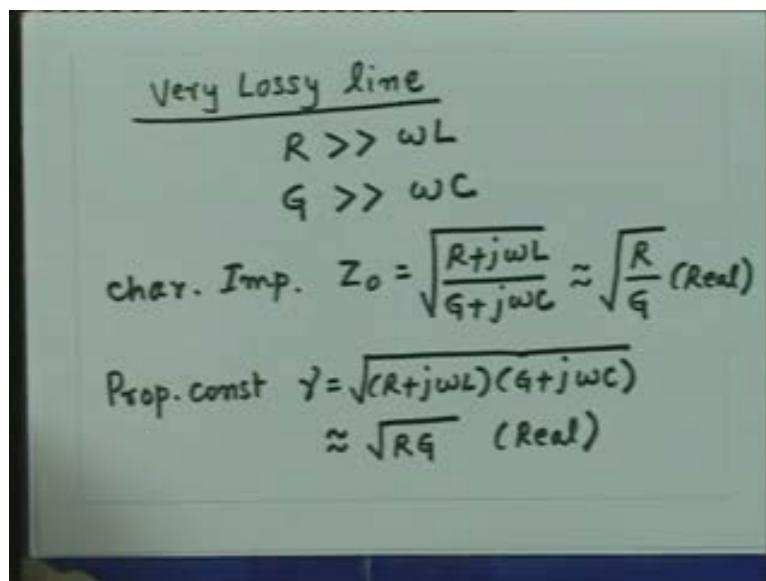
So for a very lossy line, we can say that if  $R$  is much much greater than  $\omega L$  and  $G$  is much much greater than  $\omega C$ , we can call this line a very lossy line. For this line than the characteristic impedance,  $Z_0$  will be square root of  $R$  plus  $j\omega L$  divided by  $G$  plus  $j\omega C$  and now,  $R$  is much much greater than  $\omega L$ ,  $G$  is much much greater than  $\omega C$ . So this is approximately square root of  $R$  by  $G$  which is a real. Similarly, the propagation constant  $\gamma$ ,  $\gamma$  which is square root of  $R$  plus  $j\omega L$  into  $G$  plus  $j\omega C$  that is approximately equal to square root of  $R$  into  $G$ , again is a real part.

So first into note here is that if the line is very lossy then the characteristic impedance is real but the propagation constant also is real. Of course, when the line was lossless the characteristic impedance was real. So the realness of characteristic impedance does not tell

you whether line is lossy or line is lossless. In both the extreme cases, when the loss is very small or the when the loss is very large the characteristic impedance turns out to be  $L$ .

So looking at the characteristic impedance, we will not be able to judge whether line is a very lossy line or the line is a lossless line. However, if I look at the propagation constant for a lossless line in the propagation constant  $\gamma$  was purely imaginary, the  $\alpha$  was equal to 0 and we had  $\gamma$  equal to  $j\beta$ . However, for a very lossy line the  $\gamma$  becomes the real quantity that means it has only  $\alpha$  and  $\beta$  is equal to 0, what that means is now there is no phase variation in space, for whatever voltage or current variation we have on this line and if there is no phase variation on the transmission line then it does not represent the way of phenomenon.

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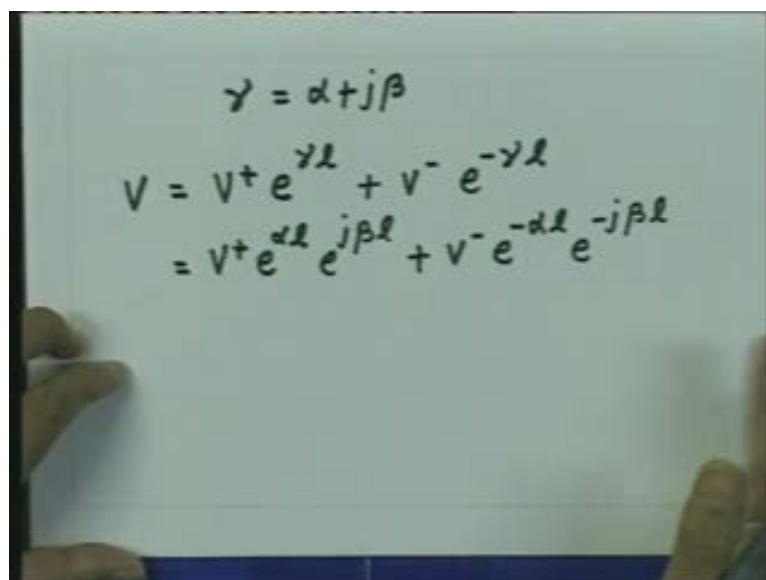
Very Lossy line  
 $R \gg \omega L$   
 $G \gg \omega C$   
char. Imp.  $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \approx \sqrt{\frac{R}{G}} \text{ (Real)}$   
Prop. const  $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} \approx \sqrt{RG} \text{ (Real)}$

So essentially in this case, when the line is very lossy this structure is not even representing a medium which is carrying voltage or current waves. So we have a voltage and current variation on this structure but these variation does not represent the way of phenomena. So you have a voltage which varies now, exponentially with the attenuation constant which is given by square root of  $RG$  and there is no phase variation, so there is no travelling wave which is set up on this structure. So obviously, we are not interested in this phenomena we are investigating the way of phenomena on transmission line.

So we are essentially considering the case where this condition is not really prevalent, what we have is R and G, they are comparable to these 2 quantities. So R is comparable to  $\omega L$ , G is comparable to  $\omega C$  and that line then we can call as the moderately lossy line. Even in that case however that standing wave of characteristic on transmission line change and we will see, how the standing wave of patterns will get modified if there was a significance loss on transmission line. So what we have now considering that just the waves are there on transmission line and  $\gamma$  is a general complex quantity.

So  $\alpha$  is not negligible compare to  $\beta$ , so we have in general the propagation constant  $\gamma$  which is  $\alpha + j\beta$ . So the traveling wave exponentially decays with an attenuation constant  $\alpha$  and it has a phase constant which is  $\beta$ . Once I get this, I can go back to my original voltage equation which I derived for a general line and then, ask in this condition, how the standing waves will get modified on the transmission line. So if I write down now the voltage on the transmission line which is again  $V$  plus  $e$  to the power  $\gamma$  into  $l$  plus  $V$  minus  $e$  to the power minus  $\gamma$  into  $l$  and if I substitute for  $\gamma$   $\alpha + j\beta$  this is  $V$  plus  $e$  to the power  $\alpha l$ ,  $e$  to the power  $j\beta l$  plus  $V$  minus  $e$  to the power minus  $\alpha l$ ,  $e$  to the power minus  $j\beta l$ .

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$$\gamma = \alpha + j\beta$$

$$V = V^+ e^{\gamma l} + V^- e^{-\gamma l}$$

$$= V^+ e^{\alpha l} e^{j\beta l} + V^- e^{-\alpha l} e^{-j\beta l}$$

So if I look at the first from here, it is telling me that as I move towards the generator, this way which is the forward travelling wave, the wave going towards the load away from the

generator exponentially goes towards the generator while in other words, as I move away from the generator, this wave exponentially decays. Similarly, this wave exponentially decays towards the generator or exponentially grows towards the load. Depending upon the value of  $V_{\text{minus}}$  or  $V_{\text{plus}}$  are the ratio of  $V_{\text{minus}}$  and  $V_{\text{plus}}$  which is related to a reflection coefficient at the load, you may have a certain amplitude of the reflected wave compare to the incident wave at the load point then, the 2 way of travelling in the opposite directions grow or decay exponentially as we move towards the generator.

So at every point now if I look at the super position of the waves first of all if there was certain ratio of  $V_{\text{minus}}$  and  $V_{\text{plus}}$  amplitude in the lossless case, this ratio was same at every point on line. However that is not true now because the amplitude of this wave is exponentially changing with  $e$  to the power minus  $\alpha l$ , amplitude of this term is changing with  $e$  to the power plus  $\alpha l$ . So ratio of the amplitude of reflected in incident wave now become a function of location and the transmission line. So let us say, if I draw these travelling waves on the transmission line, so this is my load, the  $l$  is measure towards the generator. They having some impedance  $Z_l$  and from here, I know the reflection coefficient, I can calculate at the load point and if I now plot the incident wave, the incident wave exponentially decays as I move towards the load.

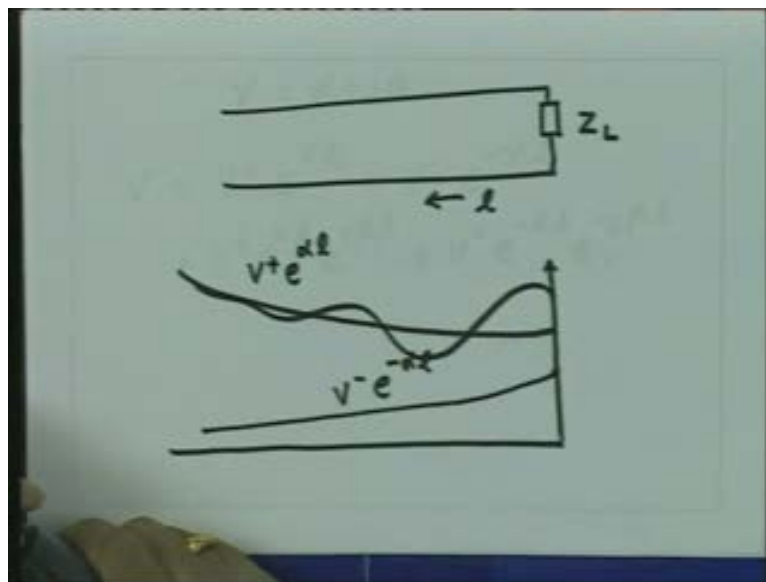
So if I plot the magnitude of the incident voltage wave that wave will be exponentially dying down like that. So this represents  $V_{\text{plus}}$   $e$  to the power  $\alpha l$  depending upon the load value, I have a certain reflection coefficient at the load. So the value of the reflected wave will let us say be something this at the load point. This now exponentially dies down, as I go towards the generator. So this is  $V_{\text{minus}}$   $e$  to the power minus  $\alpha l$  and then, total voltage will be super position of these 2 at every point. So here the voltage will be this plus this in case something here and I can add similarly and the phase is changing.

So as I move now on the transmission line towards the generator, the standing wave amplitude does not remain constant because this wave is becoming weaker and weaker, reflected wave. The incident wave is becoming stronger and stronger towards generator that means the reflection coefficient which is the ratio of these 2 is becoming smaller and smaller and if that is becoming smaller and smaller, the wave is appearing more and more like a travelling wave rather than a standing wave. So, if I look at the standing wave pattern on this

line, it will be something like this, it will start like that, like that and then the reflection is becoming weaker and weaker, so slowly it will merge with this line.

So, if I move close to the load I see a standing wave but as I move towards the generator the standing wave becomes weaker and weaker and it merges with this line which essentially represents a forward travelling wave. So that means even if you are having a very large mismatch at the load end, as you move towards the generator, the mismatch becomes weaker or the matching improves or you see the impedance from this side which is very close to the characteristic impedance because you are seeing only the forward travelling wave from the generator. So one way of that I wonder that if I have a lossy line irrespective of what is the load impedance, I will always see a match from the generator side but this is not a very good situation because although we are seeing match from this end, the power which is supplied by the generator is not delivered to the load. In that substantial amount of power has been lost in the transmission line.

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So in this case saying that I got a match at the input does not have much meaning because now, even if the load is match at the input, the power is not delivered to the load, power is lost in the transmission line. But, if you are interested only in getting a match towards the end of this line that is towards the generator and I am not too much worried about the efficiency of power transfer, I simply want the reflection should not come and hit back the generator, may

be that is a very very good technique. We can introduce deliberately a lossy transmission line between a mismatch load and a generator and even if the full power is not transfer to the load at least generator does not seen any reflection.

So its characteristics do not get modify, so many times in practical work a deliberately lossy piece of transmission line is introduce. So that even if in experiment, you connect some arbitrary loads to this and even if you have very strong reflections at least these reflections will not go and damage the generator. So here the purpose suddenly is not maximum power transfer. The purpose is to protect the generator from any unwanted reflections coming from the load. So in that case definitely a lossy line can be used for avoiding the reflections coming back and hitting the generator, nevertheless if you are having now a general transmission line, the standing way of or going to go like this.

So in general then, we can write down now the reflection coefficient. So the reflection coefficient now at any location  $l$ ,  $\gamma_l$  that will be  $V^- \text{ upon } V^+ e^{-2\alpha l}$  and as we have seen earlier at  $l = 0$ , the reflection coefficient is the reflection coefficient at the load end. So  $V^- \text{ upon } V^+$  is the reflection coefficient at the load end. So we know  $V^- \text{ upon } V^+$  that is equal to  $\gamma_l$  that is equal to  $z_l \text{ minus } z_0 \text{ divided by } z_l \text{ plus } z_0$ .

So the magnitude of reflection coefficient is  $\gamma_l e^{-2\alpha l}$  or more  $\gamma_l e^{-2\alpha l}$  and the phase of the reflection coefficient is  $\text{minus } 2\beta l$ . So if I go now to the complex plane and ask, as I move towards the generator from the load point, what kind of curve will be traced on the complex  $\gamma$  plane or if I know the reflection coefficient at the load end which is this. As I move towards the generator, how the reflection coefficient will change, one thing is immediately clear if I substitute for  $V^- \text{ upon } V^+$  is  $\gamma_l$ , this reflection coefficient at any location  $l$  will be equal to  $\gamma_l$ .

Let me write down this is magnitude of this and the phase of this will write explicitly  $e^{-2\alpha l}$ ,  $e^{-j\theta_l \text{ minus } 2\beta l}$ , where  $\theta_l$  is the phase of the reflection coefficient at the load  $l$ ,  $|\gamma_l|$  is the magnitude of the reflection coefficient at the load  $l$ . So the total phase at distance  $l$  is  $\theta_l \text{ minus } 2\beta l$  and the amplitude of the reflection coefficient at location  $l$  is  $|\gamma_l| e^{-2\alpha l}$ . So as we move towards the generator  $l$  is positive. So the amplitude of the reflection coefficient goes on systematically reducing.

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$$\Gamma(l) = \frac{V^-}{V^+} e^{-2\alpha l} e^{-j2\beta l}$$
$$\frac{V^-}{V^+} = \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$
$$\Gamma(l) = |\Gamma_L| e^{-2\alpha l} e^{j(\theta_L - 2\beta l)}$$

So this one as we have seen earlier in the complex gamma plane, the variation of phase gives me a rotation in the gamma plane because the angle is reducing as I am move towards generator but that time lossless line, this quantity was constant because this was not there. So you should trace a curve which was the circle. However, now since we are having this term the radius of this circle is reducing continuously that means this curve now, essentially draw a spiral on the reflection coefficient plane.

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So if I, if I take the Smith chart which is the complex gamma plane and let us say initially, the point was something here that was a reflection coefficient magnitude. As I move on the transmission line towards the generator, I do not get a circle but I will get a curve which we look more like that because the radius of this is reducing continuously because of the loss of the transmission line. So the point as we move towards the generator comes closer and closer to the center of the Smith chart and as we have seen earlier, the point is closer to the Smith chart. Let represents and impedance closer to the characteristic impedance or better match conditions that is what we have seen earlier. As we have, as we move towards the generator you get the reduction in the reflection coefficient and therefore the impedance appearance is more and more match.

So one think, one can note here is that on a lossy transmission line, the point moves on a spiral as we move along the transmission line. The quantity which we have use for measuring the contribution of the reflected wave, all the voltage standing wave ratio which was the ratio of the maximum voltage to the minimum voltage on transmission line. You will see now that this quantity is not a very meaningful quantity because I cannot really uniquely define this quantity on the transmission line. If I look at now the standing wave pattern which is this pattern on the transmission line, if I get this voltage maximum and this voltage minimum, I will get one ratio, if I take this voltage maximum and this voltage minimum, I will get another value that means the VSWR is not uniquely define on transmission line. In fact, the VSWR is loss a meaning because it is no more a characteristic of the load, it also has become a function of the transmission line characteristic as a result now, the VSWR is a function of length along the transmission line.

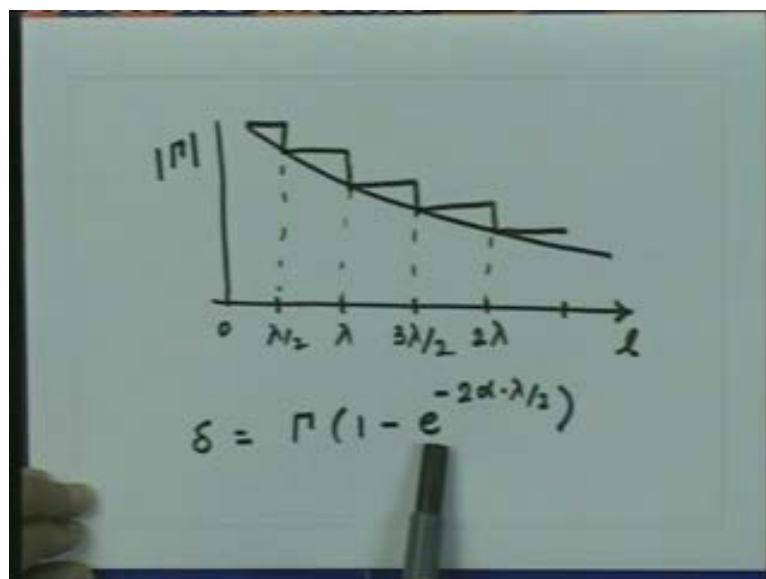
So in case of a moderately lossy line, the VSWR is not a very meaningful parameter, if the line was low loss, I can still say one what value I get from this and what value I will get from this. I can take may be mean of these 2 quantities and that is the VSWR which is in this region of the transmission line. If I was somewhere here I will give another value of VSWR and so on. So in general when we are having a moderately lossy transmission line, the VSWR does not serve any purpose. So we have to really go to the quantity which is the reflection coefficient quantity also you will note that now since, there is a loss and this function is no more a sinusoidal function, the separation between the 2 minima is not exactly equal to  $\lambda/2$ . If I get the minimum point here and if I get the next minimum here, this separation is different then half wavelength which we get just from the phase relationship.



However, just as I mention these are the extreme cases where the loss is very large in practice most of the transmission line have a loss which is reasonably small. So we can still make an approximation that separation between the 2 minima is almost  $\lambda/2$  and the VSWR is more or less same, at least zone wise on the transmission line. One can then ask if I have a moderately lossy transmission line, how do I make use of the switch chart, can I make use of the switch chart for impedance and other calculations then, it says yes and no. Strictly speaking, if you want to do the impedance calculation by using switch chart you have to draw the spiral unless, you draw this spiral you will not get the correct variation of the reflection coefficient or the impedance variation on the transmission line.

However, if the loss is small we can make this spiral like a circle and apply a correction of a every  $\lambda/2$  to the radius of the circle essentially what then we are saying is that ideally that amplitude would have reduced for the reflection coefficient which will go something like that. This is the mode of gamma at the function of  $l$ . So reflection coefficient value will reduce as I move towards the generator and this variation essentially was giving me the spiral, what we say is let us divide this transmission line in sections of  $\lambda/2$ . So this is say 0 and this is  $\lambda/2$  this is  $\lambda$  this is  $3\lambda/2$  and so on and let us make an approximation this to this curve, a staircase approximation for every  $\lambda/2$ .

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So I say that for this distance reflection coefficient is almost this then this  $\lambda/2$  it changes to a value which is this then,  $3\lambda/2$  the like that  $2\lambda$  is like that and so on. So what we have done is we have now done a staircase approximation to the actual curve of the magnitude of the reflection coefficient. So, we are approximating now saying that between length 0 and  $\lambda/2$ , the reflection coefficient value is this, then after  $\lambda/2$  you apply a correction which in the value to this, you change the value to this.

So every  $\lambda/2$  you apply a correction to the magnitude of the reflection coefficient or in other words, what we are saying is on the switch chart, let us see you take this point first you draw a circle, for  $1\lambda/2$ . If you move a distance more than  $\lambda/2$  after  $1\lambda/2$  moment, find out what is the change in the magnitude of the reflection coefficient make a correction and then, move on the second circle here for next  $\lambda/2$ , if you have to move further on transmission line I can make a correction to this, draw a next circle here and so on.

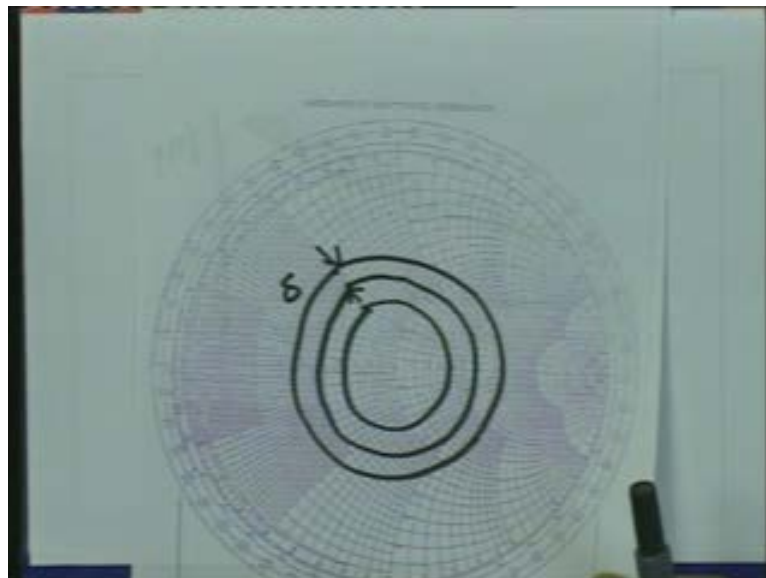
So every distance of  $\lambda/2$  on transmission line, I make a correction to the magnitude of the reflection coefficient and then, assume that for the next distance of  $\lambda/2$ , the reflection coefficient remains constant, its magnitude remains constant. So this correction, if I call these are the correction, let us say this quantity is some  $\delta$ . This  $\delta$  will be to this quantity  $\delta$  will be nothing but a reflection coefficient at the location  $1 - e^{-2\alpha\lambda/2}$ . So at any location on transmission line the reflection coefficient will be  $\gamma$  after a distance of  $\lambda/2$  the reflection coefficient would change to  $\gamma e^{-2\alpha\lambda/2}$ . So the difference in  $2$ , these 2 quantities is nothing but this correction.

So if we are doing calculation on transmission lines which has a length much larger than wavelengths, then every  $\lambda/2$  **uq** of following correction to the magnitude of the reflection coefficient and then, essentially solve the problem. As I said earlier, if you want to have very accurate analysis then ideally, you have to really draw this spiral on the switch chart. Of course, nowadays with the help of the computer, you can even draw this spiral on the switch chart and can do the accurate calculation.

However, if you are not using a computing tool then maybe the approximate step which I am mention here will be very useful in analyzing the impedance transformation characteristic on

a lossy line. The analytical calculation of the impedances on a lossy line they are as general as we discussed earlier. So those impedance transformation relationship using the hyperbolic cosines and sin, they are applicable for the calculation of the impedances on transmission line. So, with this minor modification to the analysis, the smith chart can be use for moderately lossy transmission lines and as we saw, if the line is very lossy than its losses the meaning of transmission because you does not have a wave phenomenon.

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So that line than represent to only some kind of a resistive network on which the power dies down as we move on the transmission line. Now let us move to the important aspect of the analysis of transmission line and that is the measurement of characteristic impedance of transmission line, up till now we are said data transmission line is there, it has characteristic impedance which is given to you a priority and then we proceeded and did the calculations on transmission lines. One can ask, if somebody has given you a transmission line, how would you first find out, what is the characteristic impedance and what is the propagation constant on transmission line.

So as I mention earlier transmission line is characterized by the primary constants  $l_g$  and  $c$  but normally for calculations of transmission line, we generally do not estimate this primary parameters. The secondary parameters of the characteristic impedance and the complex propagation constant are enough and therefore for any unknown line, we would like to

estimate this secondary constant  $z_0$  and  $\gamma$ . So the characteristic impedance and the complex propagation constant. Let us say now, that I have a technique for measuring the input impedance which would be same as what we have discussed earlier for measuring the unknown impedances.

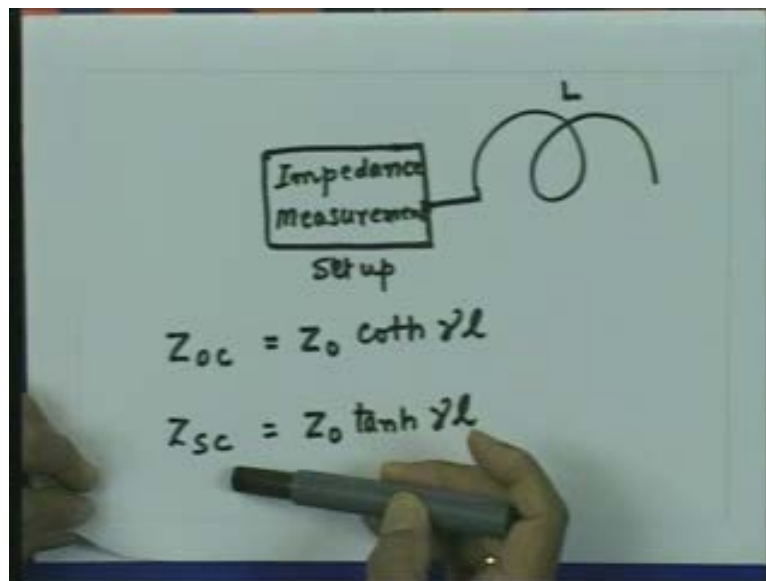
So let us say, I have a set up which can measure an unknown impedance at a unknown frequency. Let us say, now I want to measure the characteristic impedance of a line which in general is a moderately lossy line or low loss line. This measurement now can be done by conducting what is called a short circuit and open circuit test of a section of a transmission line. So first what we do, we take some length of transmission line. Let us say, I have some length of transmission line which is  $l$ , length is  $l$ . I take one end of a transmission line and connect to this set up which measures the impedance. So this is the impedance measurement set up.

So if I connect the end of this transmission line to this impedance measurement set up. It can measure the input impedance of this line, now what we do we conduct the 2 test, I measure the input impedance of this length of line by making the other end of the line short circuit and open circuit. So I measure 2 impedances input impedances, one with the open circuit condition at the other end of the line and one with the short circuit condition at the other end of the line. So I get 2 impedances, let us say let them denote by  $Z_{oc}$  and  $Z_{sc}$ .

So  $Z_{oc}$  is the input impedance measure at the input end of the transmission line when the other end was open circuited. As we know, this will be equal to the characteristic impedance  $\coth \gamma l$ . Similarly, the input impedance when the other end of the line is short circuited will be given as  $z_0 \tanh \gamma l$ . So I got a physical length of the line, I can measure these impedances under the short circuit and open circuit conditions.

Now it will be immediately clear to you that if I take product of these 2 equations, these 2 will cancel and I will get a quantity which is  $z_0^2$ . So once I conduct the short circuit and open circuit test, the calculation of characteristic impedance is straight forward. So let us say, I multiplied these 2 and I get now, the characteristic impedance of the line  $z_0$  that will be equal to square root of  $Z_{oc}$  and  $Z_{sc}$ .

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$$\text{Char. Imp. } Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}}$$
$$\frac{\tanh \gamma l}{\coth \gamma l} = \frac{Z_{sc}}{Z_{oc}}$$
$$\tanh^2 \gamma l = \frac{Z_{sc}}{Z_{oc}}$$
$$\tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

So the characteristic impedance is geometric mean of the input impedance open circuited and input impedance of short circuited line. From the same 2 equations, if I take ratio of these 2, the  $Z_0$  will cancel and I may estimate the value of the propagation constant  $\gamma$ . So let us say short take a ratio of this upon  $\coth \gamma l$  that will be equal to  $Z_{sc}$  divided by  $Z_{oc}$  and  $\coth \gamma l$  is upon  $\tanh \gamma l$ . So this quantity  $\tanh^2 \gamma l$  that will be  $Z_{sc}$  divided by  $Z_{oc}$  or  $\tanh \gamma l$  is equal to square root of  $Z_{sc}$  by  $Z_{oc}$ .

Now to get the value of gamma, I can first expand the hyperbolic tan and write them in the exponential form and then they solve for the value of gamma l. So if I write down explicitly that will be so the tan hyperbolic gamma l that is equal to e to the power gamma l minus e to the power minus gamma l divided by e to the power gamma l plus e to the power minus gamma l that is the quantity which is square root of Z<sub>sc</sub> upon Z<sub>oc</sub>. Just for the clarity, let me just call this quantity as some complex quantity A. So the tan hyperbolic gamma l which is equal to this is quantity and since, in general Z<sub>sc</sub> and Z<sub>oc</sub> are complex A is a complex quantity. I can invert this relation I can take e to the power minus gamma l common.

So this term will become e to the power 2 gamma l and I can invert this from there, I can get e to the power 2 gamma l that will be equal to 1 plus upon 1 minus A. Let us write this quantity explicitly as the magnitude and phase term. So let us say, it has a magnitude R and it has a phase theta. So e to the power 2 gamma l which is e to the power 2 into alpha plus j beta l that is equal to R e to the power j theta.

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$$\tanh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} = A$$

$$\cdot e^{2\gamma l} = \frac{1+A}{1-A} = R e^{j\theta} \text{ (say)}$$

$$e^{2(\alpha + j\beta)l} = R e^{j\theta}$$

So separating out now the phase and amplitude term, here essentially I get alpha is equal to 1 upon 2 l, ln of R that is equal to 1 upon 2 l ln of 1 plus A upon 1 minus A. So from this relation if I take the real part of that e to the power 2 alpha into l, l is the length of the line which is now in capital l. So this quantity e to the power 2 alpha into capital l that will be equal to R. So now substituting for this R in this, I get the quantity alpha which is given by

that. This has to be quantity which is magnitude because  $R$  is the magnitude of  $1 + A$ ,  $1 - A$ .

So the attenuation constant can be calculated once you know, this value  $A$  which is related to the open circuit and the short circuit impedances. So we get the  $Z_{sc}$  and  $Z_{oc}$  square root of the ratio of these 2 will give me this complex quantity  $A$ . From this  $A$ , I can get this value  $R$  and from here then substituting into this, I get the attenuation constant of the transmission line. Equating the imaginary part or the phase part of the system, we get the propagation constant  $\beta$  that is equal to  $1 \pm j\theta$  that is the phase of this term. However, now when I am equating the phase of this 2 term, there is a ambiguity of  $2\pi$  parts, multiple of  $2\pi$  parts. For the value which we get for phase here is always modular  $2\pi$  that means when I calculate the value of  $\beta$  this could be  $e^{j\theta} + e^{-j\theta}$ .

So I do not have a unique answer for  $\beta$  because of this ambiguity of multiples of  $2\pi$ . So in general I will get the value of  $\beta$  which is  $\theta \pm 2m\pi$ , where  $m$  is a integer quantity. So if I substitute the value of  $\theta$  that is from here which is the angle of  $1 + A$  upon  $1 - A$  then, I can calculate the value of  $\beta$  but I will have a uncertainty that what is the value of  $m$ , how many modular  $2\pi$ , how many multiples of  $2\pi$  have to be removed from here are added to get the right value of  $\beta$ .

So from the short circuit and open circuit test, the calculation of attenuation constant is straight forward but when I go to the calculation of the phase constant, there is a ambiguity because of the phase which is always modular  $2\pi$ . One may say that if I take a length of the line  $l$  which is less than  $\lambda/2$  then, I know for sure that this  $m$  is equal to 0. So I do not have multiples of  $2\pi$  then I have a unique value of  $\beta$  and there is a correct value of  $\beta$ . However, if I take a length of the line which is less than  $\lambda/2$  especially at higher frequencies, this length of the line is very small and since attenuation is very small on transmission line, the  $\alpha$  generally is very small.

So for a small length of transmission line, the loss is not very significant as a result when you try to calculate the value of  $\alpha$ , you do not get a very accurate number for  $\alpha$  because over the small length of transmission line which is less than  $\lambda/2$ , the line behave more or less like lossless line. So the estimation of the attenuation constant become unreliable if I take a small length of line which is less than  $\lambda/2$ . On the other hand to improve

the reliability of  $\alpha$  if I take a long length of the cable, then certainly I have now many more periods of the wavelength on this length of the cable or transmission line.

So  $\alpha$  becomes reliable but then I have to resolve, this modular  $2\pi$  problem in the measurement of the phase. So given this set up that we are measuring only the short circuit and open circuit impedance, we cannot resolve the ambiguity in the phase measurement or we cannot measure in the phase constant of transmission line reliable. So what people normally do in practice, they do the measurement of transmission line at 2 frequencies. First they do the measurement of the  $Z_{oc}$  and  $Z_{sc}$  and from there, they get the value of  $\beta$  which is ambiguous with  $2m\pi$ , then we change the frequency slowly. So that I get now again the same value of the phase variation and as a result now, the number of cycle which we have in the transmission line or just change by 1.

So by changing slowly the frequency on transmission line, if I make sure that now only one cycle changes taken place, the  $m$  is change from  $m$  to  $m + 1$ . I get the value of  $\beta$  and then from here now I can subtract in the  $2\pi$ , I can find out what would be the correct value of the propagation constant  $\beta$ . So by carrying out the multiple frequency observation or 2 frequency observations of  $Z_{oc}$  and  $Z_{sc}$ , I can find out the phase constant accurately. So in practice whenever we do the measurement of the characteristic impedance that time, we have to do the measurement at 2 frequencies and then from there, we can get correct value of the estimate of  $\beta$ .

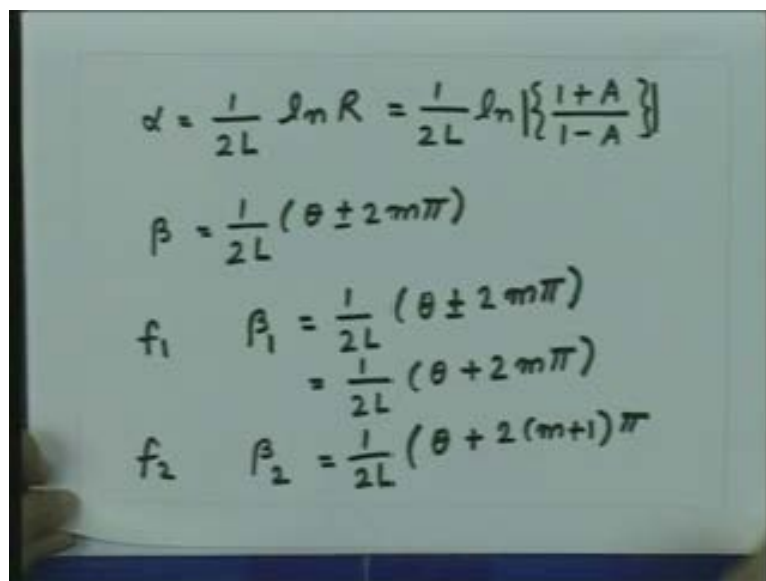
Let us now consider 2 frequencies  $F_1$  and  $F_2$  on which we carry the measurement of  $Z_{oc}$  and  $Z_{sc}$ . Let us say at frequency  $F_1$ , I get some value of  $Z_{oc}$  and  $Z_{sc}$  on from there I get a phase constant  $\beta$  and let us called that quantity as  $\beta_1$  that is equal to  $\frac{1}{2l}(\theta - 2m\pi)$ . Let us take the plus sign here. So this is  $\frac{1}{2l}(\theta + 2m\pi)$ , now what we do is this slowly change the frequency and we go to a frequency where the  $Z_{oc}$  and  $Z_{sc}$  value again become same. Assuming that the length of the line is very large by changing the frequency by a small amount, the attenuation constant  $\alpha$  does not change significantly.

So what we have now essentially saying is, if I increase the frequency by a small amount the number of wavelengths which are set up on the length of the transmission line, they are change and if they are change by 1 cycle or  $\frac{1}{2}\lambda$  then, the loss does not change



significantly and that is the reason I get the same value of  $Z_{oc}$  and  $Z_{sc}$ . So another frequency  $f_2$ , I will get a propagation constant  $\beta_2$ , the value of all will come same because by changing this frequency by small amount, the attenuation constant is not change but since your accommodated now, one cycle of the wave on the transmission line, this quantity help becomes  $m$  plus 1.

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$$\alpha = \frac{1}{2L} \ln R = \frac{1}{2L} \ln \left\{ \frac{1+A}{1-A} \right\}$$

$$\beta = \frac{1}{2L} (\theta \pm 2m\pi)$$

$$f_1 \quad \beta_1 = \frac{1}{2L} (\theta \pm 2m\pi)$$

$$= \frac{1}{2L} (\theta + 2m\pi)$$

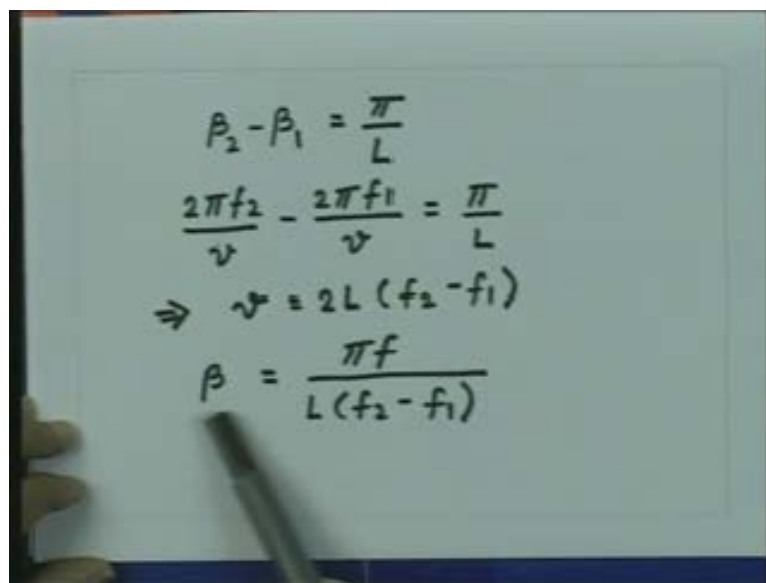
$$f_2 \quad \beta_2 = \frac{1}{2L} (\theta + 2(m+1)\pi)$$

So the phase constant  $\beta_2$  will be  $\theta + 2(m+1)\pi$ . From here now, I can take different of these 2 quantities and from here this term will cancel out and what I will get is  $\beta_2 - \beta_1$  that will be equal to  $\pi$  upon  $l$ . Now we know  $\beta_2$  and  $\beta_1$ , they are related to the velocity for the wavelength on the transmission line. So this we can write  $2\pi$  into  $f_2$  divided by velocity minus  $2\pi f_1$  upon the velocity that is equal to  $\pi$  upon  $l$  from here then, we can get the value of the velocity that is equal to  $2l$  into  $f_2 - f_1$  and the phase constant now  $\beta$  will be equal to  $\pi$  into  $f$  divided by  $l$  into  $f_2 - f_1$ .

So what you have done, you did the measurement at 2 frequencies which are closely space in such a way that by changing the frequency from  $f_1$  to  $f_2$ , the number of cycles are section of the line or increase by 1. Then from there we calculate the value of the velocity on the structure and must we get the velocity, then we can find out the value of propagation constant because from velocity we can find out the wavelength and that  $2\pi$  by  $l$  wavelength gives me the phase constant of the transmission line. But one could wonder at this point that why

are we doing the estimation of the velocity on the transmission line, does not the wave of travel with the velocity of light on the transmission line or if you know the value of beta cannot you find out what is lambda m from there, we can find out what is the velocity. Infact the problem is exactly opposite, the problem is depending upon the structure the velocity of the wave changes frequency is the one which is very secret but depending upon the propagation characteristic velocity changes or the wavelength changes and therefore the phase constant changes.

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$$\begin{aligned}\beta_2 - \beta_1 &= \frac{\pi}{L} \\ \frac{2\pi f_2}{v} - \frac{2\pi f_1}{v} &= \frac{\pi}{L} \\ \Rightarrow v &= 2L(f_2 - f_1) \\ \beta &= \frac{\pi f}{L(f_2 - f_1)}\end{aligned}$$

So it is not that we know the wavelength from the velocity and we are trying to find out beta in fact, beta is the quantity which is the most unknown quantity. So for a given structure we first estimate the value of beta then  $2\pi$  divided by beta gives me a number that we call as a wavelength and that wavelength multiplied by frequency that gives me, what is called the velocity of the wave on this structure. So in practice, the measurement of the attenuation constant in the phase constant have to go to these steps but this is the method which you mention if the short circuit or open circuit measurement test, this is the most widely use test in practice for measuring the characteristic impedance and propagation constant of the line. Of course there are certain practical difficulties that whenever we try to put the open circuit or short circuit at higher frequencies, it is difficult to put a very good open or short circuit to the end of the transmission line. Even if you short the 2 conductors of a transmission line there will be always a some conductance at the end, if you leave the 2 end, 2 conductor open

at the end of the transmission line, there will be a some fringing capacitance at the end of the transmission line.

So to very higher frequency realizing an open circuit and short circuit is not the straight forward. So people make extra effort to develop these modules, what are called the short circuit modules and open circuit modules which can be connected to the end of the line to realize a good open or short circuit. As I mention again that getting an ideal open circuit or a short circuit is not very easy as we go to higher frequency. So let me summarize what we have done today, we discuss very briefly the characteristic of the lossy transmission line. We also saw how do we make use of a Smith chart, when the line is moderately lossy and we also saw a practical method for estimating the characteristic impedance and the complex propagation constant of a transmission line.