**Transmission Lines & E M. Waves**
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**Lecture - 11**

We are discussing applications of transmission lines. We saw the application of transmission line or measuring the unknown impedance at higher frequencies. As pointed out a higher frequency the measurement of phase is not very reliable. So one you would like to estimate the phase indirectly, without doing the direct measurement of the phase, say as we saw by using transmission lines just by measuring the magnitudes of the voltages on transmission lines, we can find the voltage standing waves, their parameters and from that we can indirectly estimate the phase of the unknown impedance. The another application which we have investigating, e is the transmission line as circuits elements.

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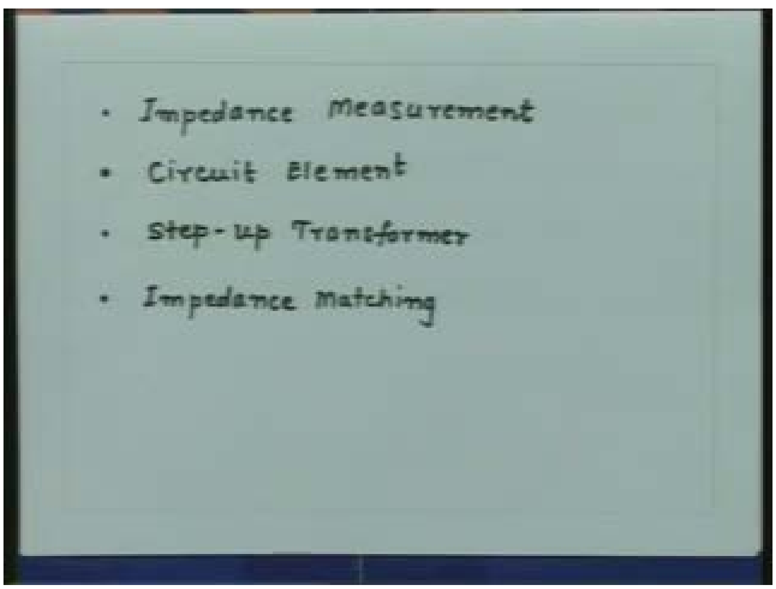


We saw that if you take a section of a transmission line which is either open circuit or short circuit then, these sections of transmission lines can be use as a reactive element in the higher frequency circuits. So last time we saw the characteristics of the short circuit and open circuit section of transmission lines and you also saw that these sections, if the length is multiples of lambda by 4 then these section can be used as a resonance circuit in the higher frequencies. So essentially by taking a length of a transmission line which is either lambda by 4 or lambda by 2, we can realize a parallel or serial resonant circuit. We saw that the if the input impedance of this line is close to 0 then the circuit behave like a series resonant circuit, if the input impedance of the line is close to infinity then the line behaves like a parallel resonance circuit and once the circuit behave like a resonant circuit than the natural question to ask is what is the quality factor of the resonant circuit  and that is the thing essentially we are investigating now, let if I consider this section of a transmission line as a resonance circuit then what is the quality factor of this resonance circuit.

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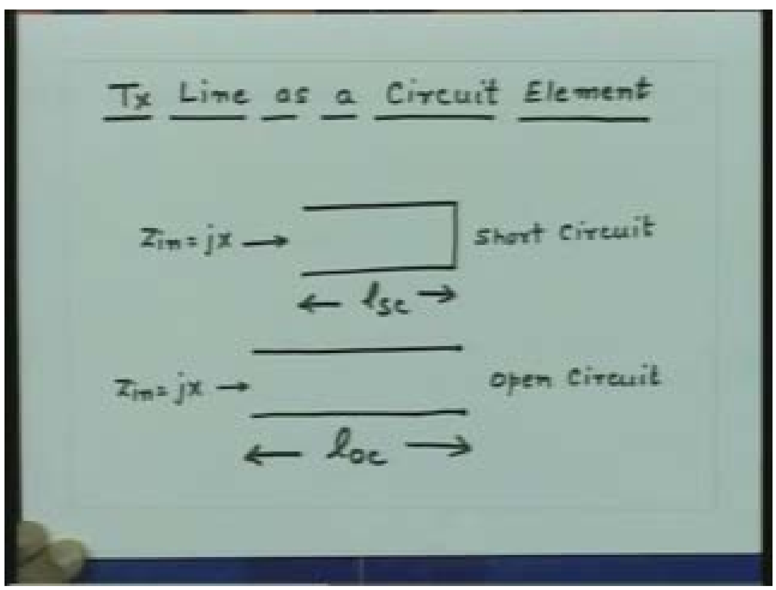


As mentioned earlier, there whenever we talk about resonant circuit the quality factor is related to the losses in the resonant circuit. However, up till now we have consider the line to be lossless. So by definition for a lossless line the quality factor is always infinitive. Now since we are investigating the quality factor no matter, how small the loss is in the transmission line. We have to account for that loss then and then only we will be able to get the correct answer for the quality factor.

So we investigate now, the input impedance of the transmission line, when the length is multiples of lambda by 4 and from there then we find out the quality factor of the resonance sections of transmission line have been said that let us consider, now a specific case that the length of the line is lambda by 4 either the line can be open circuited or short circuited. So the length which you call last time is LSC which is the length of a short circuited line, this length is now lambda by 4. Similarly, LOC is also lambda by 4, so since the length is lambda by 4 and as you have seen then the impedance, normalize impedance inverse itself every distance of lambda by 4. This open circuit will appear like a short circuit in the line, the short circuit will appear like a open circuit of the distance of lambda by 4.

So this line, if the length is lambda by 4, will appear like a parallel resonance circuit whereas this if the length is lambda by 4, will appear like a series resonance circuit because the impedance will be very close to 0. So now you specifically we just consider the sections of transmission line which are having length of lambda by 4 and find out this input impedance and then, go to the calculation of the quality factor. In general, since we are considering loss now, the input impedance of the open or short circuited line has to be written in the total propagation constant gamma and not only the phase constant.

So if you go back to your original impedance relationship, if you have derive as substitute z equal to either 0 or infinity depending upon whether a short circuit or open circuit, I get the input impedance. So this is the input impedance for a line of length L, when the line has loss. So let us call for a open circuit deadline, let us say this is denoted by Zoc. So that is equal to Z 0 cot hyperbolic of gamma L and for Z sc the short circuited line that will be z 0 tan hyperbolic of gamma l, where if you remember gamma is equal to alpha plus j beta, the line is low offline.

So alpha is much much smaller than Beta, as we have consider earlier. When the line is Loc, the propagation cot and now is alpha plus j beta and it is not only j beta is still assume that the characteristic impedance that this line is almost real. So, although we have set now the propagation constant is complex it is not purely imaginary, the z 0 be still consider to be almost here. So now, what again we are saying is consider any of these cases, the open circuit line or a short circuit line and under the assumption that alpha is much much less than beta right and the length is either multiples of lambda by 4 or any arbitrary length find out under these approximation, what will be the input impedance. We know if you consider a short circuit line and if the length is lambda by 4, if you appeal ideal like open circuit.

Similarly, if you take an open circuited line and take a length lambda by 4, the input impedance will be like a short circuit. However, in the presence of the loss that will not be true, so the impedance will neither be infinitive nor it will be 0 and that is what we are trying to calculate now that if, the loss is present what will be the input impedance of these 2 lines. So let us consider these Z sc and substitute for gamma alpha plus j beta, so from here I can get Z sc that is equal to X 0 tan hyperbolic of alpha plus j beta into l. I can expand this, so that will be z 0 tan hyperbolic alpha l plus j tan beta l divided by 1 plus tan hyperbolic alpha l j tan beta l. So, in general for a length of line, the input impedance, the short circuited line, the input impedance essentially given by this.

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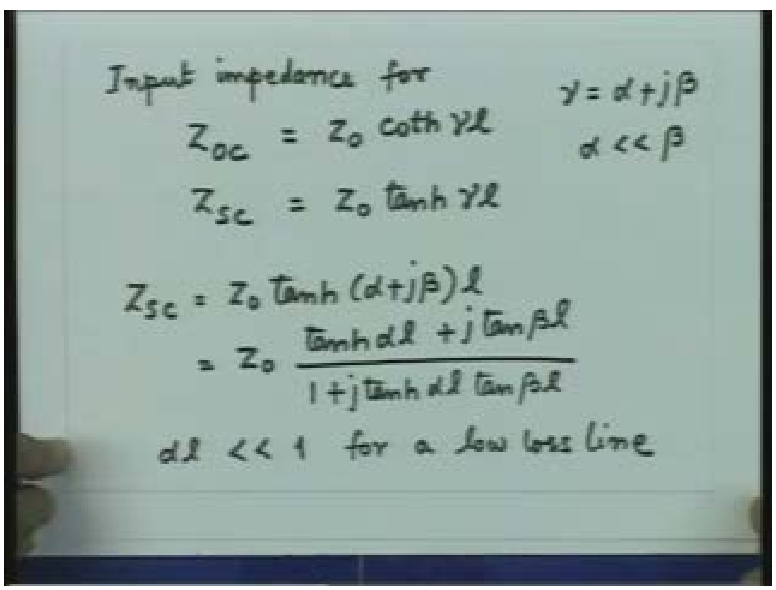


Now since alpha is small for a length of lambda by 4 section of transmission line or a length which is smaller than lambda by 2, this quantity alpha l is much much smaller than 1. So we have alpha l much much smaller than 1, for a low loss line. Once, I get this then I can make an approximation that tan hyperbolic alpha l will be approximately equal to alpha l, if I substitute that then the input impedance of a short circuited line, Z sc will be approximately equal to alpha l plus j tan beta l divided by 1 plus j alpha l tan beta l.

Now, if the line is short circuited and if the length is lambda by 4, this l is equal to lambda by 4. So for l equal to lambda by 4, beta l will be 2 phi by lambda into lambda by 4 that is equal to phi by 2. If I substitute now, the length of the line equal to lambda by 4, this quantity will be infinity this quantity will be infinity. So I can take this tan beta l common, no infinity. So what will I see that Z sc will be equal to if I take j tan beta are common, this will be alpha l divided by j tan beta l plus 1 divided by 1 upon j tan beta l plus alpha l, substituting now for beta l equal to phi by 2 that means tan beta l is infinity, this quantity will be 0, this quantity will be 0.

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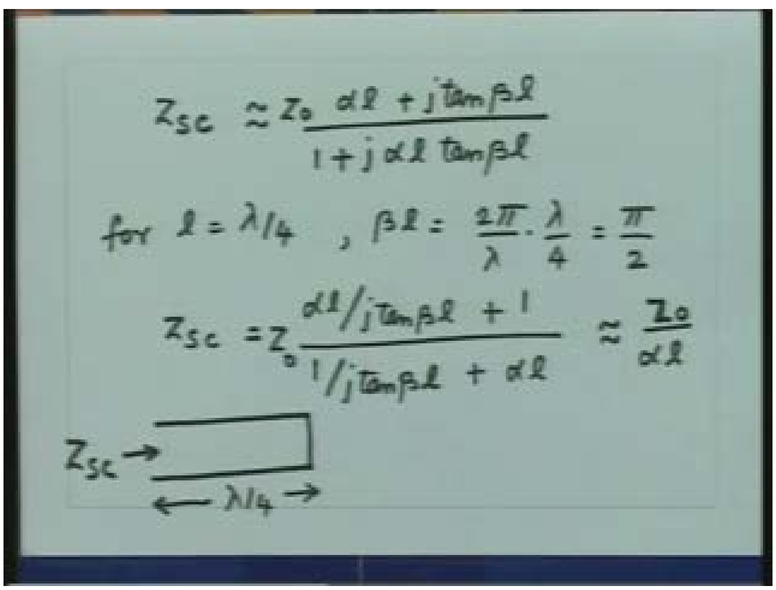


So I will get Z sc is approximately equal to 1 upon alpha l. So the input impedance of a short circuited line,  if the length is lambda by 4 is a short circuit, this length is lambda by 4, ideally for this line it should have appear open circuit at the input, these impedance Z sc. Now I do not see open circuit but I see an impedance which is very large because alpha l is very small quantity for a low loss line but this impedance is not infinity. So short circuited section of a transmission line of length lambda by 4 does not in practice appear open circuit but it has some impedance measure at the input terminal which is essentially this.

Similarly, if I done the similar exercise for the Z oc and it did the expansion and all those similar things by done then, the input impedance of open circuited line would appear to be alpha l multiply by z not anyway here the you have to put z not here. So this quantity should be multiplied by z not, this is z not, this is z not. So doing the same exercise for the open circuit deadline this is lambda by 4, this is open circuit. so the impedance which we measure here, here is now Z oc.

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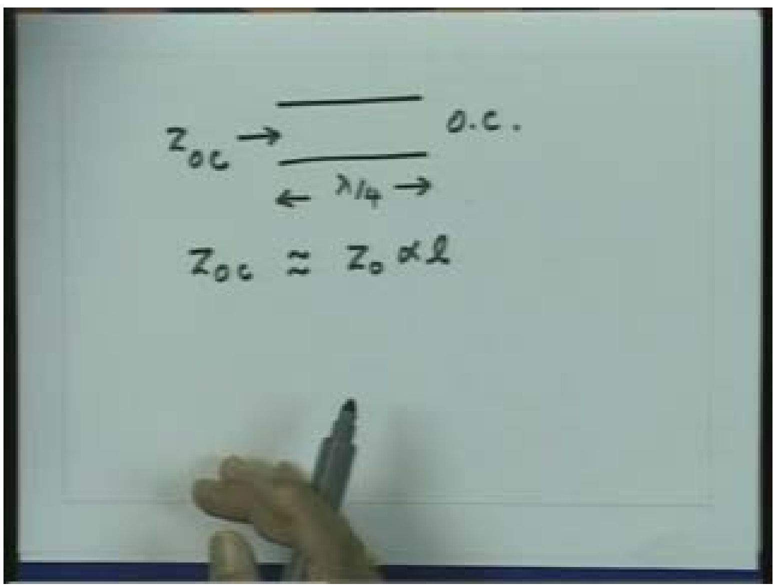


So by doing the same exercise as we have done for the short circuit deadline the Z oc will be approximately equal to into alpha l. So for a short circuital section of a transmission line, the input impedance is z 0 divided by alpha l for a open circuited section of a transmission line, the impedance is z 0 multiplied by alpha l. So what we now note here is that the input impedance of a resonance section of a transmission line is ideally infinitive or 0 depending upon the resonance circuit is a series circuit or a parallel resonance circuit. However, in practice Vc, large impedance for a parallel resonant circuit and small impedance for a series resonance circuit, must you get this either I can use this information that if I very now the frequency around the resonance frequency, how these impedance changes from there I can get the impedance variation at the function of frequency or what is called frequency response of your circuit and from where I can calculate the quality factor of the circuit.

So quality factor can be calculated in 2 ways, one is from the frequency response of the circuit. Secondly, it can be calculated from the basic definition of the quality factor. So let us see what is the quality factor by definition. So the quality factor Q is defined 2 phi multiplied by the energy stored in the circuit divided by the energy loss per cycle, loss per cycle. If the resonance frequency is f not, the energy loss per cycle is there are f not cycle per second. So I can write down the same expression for the quality factor as 2 phi into the resonance frequency of the circuit multiplied by the energy stored in the circuit divided by the energy loss per second which is nothing but the power loss in the circuit. So this is divided by the power loss in circuit.

So to calculate the quality factor, if I find out what is the energy stored in the section of a transmission line. If I calculate what is the power loss in the transmission line then I can find out what is the quality factor of that section of a transmission line. Alternatively, as we mention that the quality factor is related to the frequency response. So, if I have a plot the variation of current or voltage, when a voltage or current sources applied to the input section of a transmission line and measure the 3 db bandwidth of the response, the center frequency divided by the 3 db bandwidth of the frequency response give you by the quality factor.

So if I have a frequency response of the circuit which is for this is my impedence at the function of frequency, I get to a frequency response which will look typically like that depending upon whether I am using series or parallel resonance circuit, this will be the voltage response of the current response but at resonant frequency I get the maximum response and other deviate from the resonant frequency, the response drops. So I can measure the 3 db frequency for this response where the amplitude reduces to 1 over root 2 of its maximum value, if I take the amplitude response.

So this is um the delta f which is the 3 db bandwidth then, the quality factor for the circuit is equal to f naught divided by delta f. So, now the quality factor can be calculated from the impedance calculation because the impedance can give me the frequency response. Alternatively, take a section of a transmission line find the voltage distribution on the transmission line and the current distribution on transmission line, find out what is the energy stored, find out what is the power loss any transmission line and use definition to calculate the quality factor. Let us go to this first method which is more fundamental method of calculating the quality factor for transmission line.

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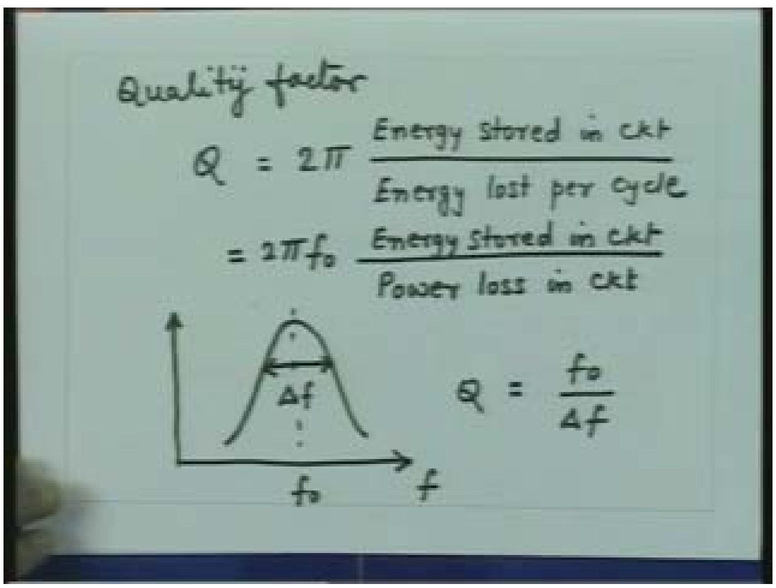


So let us say I consider specifically a section of a transmission line which is of lambda by 4 length and let us say the line is short circuited. So let us take a specific case here, if the line is short circuited this length is lambda by 4 and since, I am not talking about the resonance frequency, the length cannot be lambda by 4 at all frequencies. So let us say this is having a specific frequency lambda 0 by 4, where lambda 0 corresponds to the wavelength of the resonant frequency. So this length is lambda 0 by 4, so the resonant frequency of the circuit f 0 is nothing but the velocity of the wave divided by the length lambda 0.

Now, we can get the voltage and current expression on the transmission line with the load short circuited and when the load is short circuit, the reflection coefficient at the load point is minus 1 we have seen earlier. So reflection coefficient load gamma l that is z l minus z 0 divided by z l plus z 0 and z l is now short circuit, so z l is 0. So that is equal to minus z 0, upon z 0 that is equal to minus 1. So the reflection coefficient for short circuit load is minus 1 which is saying that the amplitude of reflection coefficient is 1 and it has a phase change of 180 degrees.

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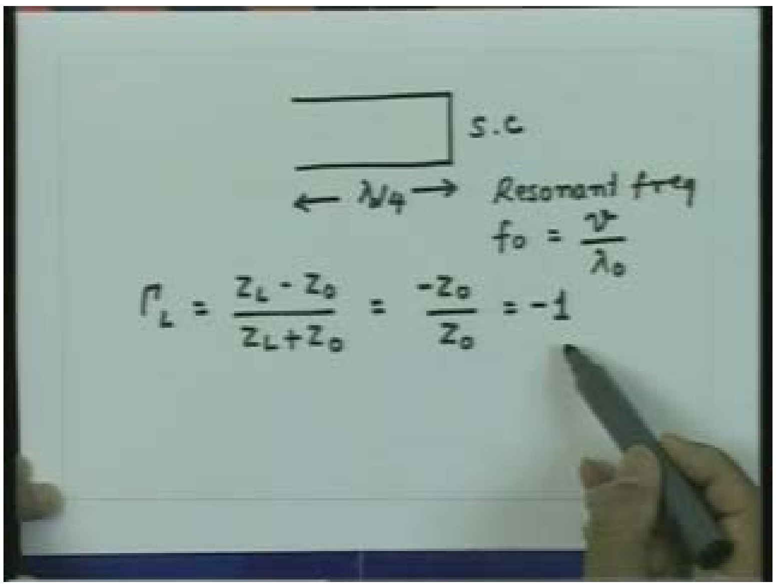


We will make use of this information later when we go to the application of transmission line for the voltage or current stating of transformers. But at this point, once I know the reflection coefficient I can write down the voltage and current equation and I can find out, what is the variation of the voltage and current on the transmission line. Going to the basic equations of the voltage and current, the voltage now on the transmission line can be given as v plus e to the power j beta l plus v minus e to the power minus j beta l but for reflection coefficient of minus 1 at the load, this quantity gamma l is nothing but v minus divided by v plus.

So, if I substitute for v minus is equal to minus v plus the voltage on the line is essentially given by this. I can combine this, I can take v plus common, this is e to the power j beta l minus e to the power minus j beta l which is nothing but j 2 into sign of beta l. So the voltage on transmission line can be j 2 v plus sin of beta l, since there is nothing very special about this parameter v plus I can combined 2 v plus to define another quantity which is v 0. So the voltage magnitude on the transmission line is magnitude of v 0, some voltage multiplied by sin of beta l. Similarly, for the current if I substitute for the reflection coefficient equal to minus 1, the current on the transmission line will be given by this which again by combining these 2 terms, I will get a v 0 divided by z 0 multiplied by cosine of beta l.

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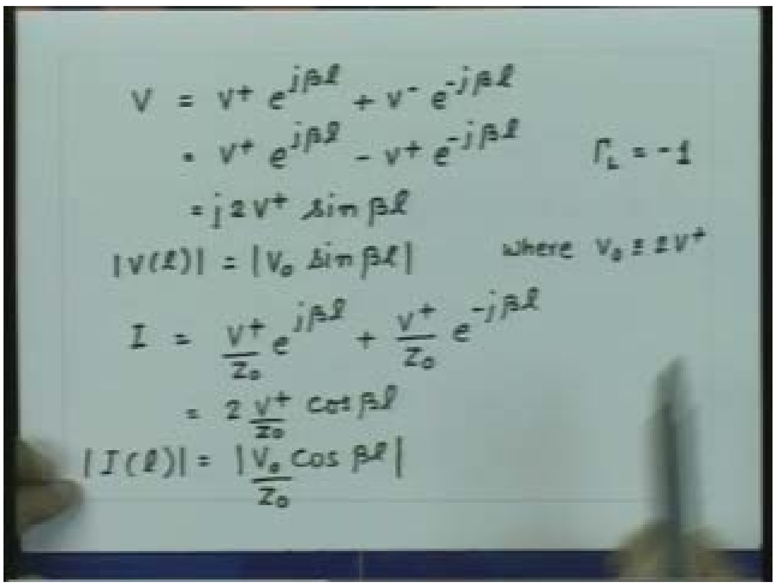


So if I plot now this voltages on the line, say this is now, lambda 0 by 4 line, this is short circuit. So the voltage is 0 at this point because its variation is the sign of beta l. So at l equal to 0 which is said, if the load the voltage is 0 and when I go to a distance of lambda by 4, this quantity beta l becomes phi by 2 that is quantity 1. So I see the voltage which is v 0, so I get a voltage variation which is a 1 cycle variation from lower point to the input point. So this voltage here is v 0 and this is, this point is 0. So this is the variation of magnitude of magnitude of voltage on the transmission line. Similarly, if I plot the current the current has a variation which is cosine variation.

So it l equal to 0, this quantity is v 0 up on z naught and when l is lambda by 4 that time this quantity will be 0. So I will get a current variation which will go like that, so this is mode of v plus divided, this length is lambda 0 bar 4. So I know now on the section of the transmission line, the voltage and current variation and once, I know the voltage and current variation then I can find out what is the capacity when inductive energy stored at different locations on transmission lines. So if I take a small section of transmission line, the capacitance of the small section of transmission line is c multiplied by the length of the line bl, as we discussed in the very first lecture that for a small section of a transmission line, we define the capacitance as the capacitance per unit length multiplied by the length of the section of the transmission line.

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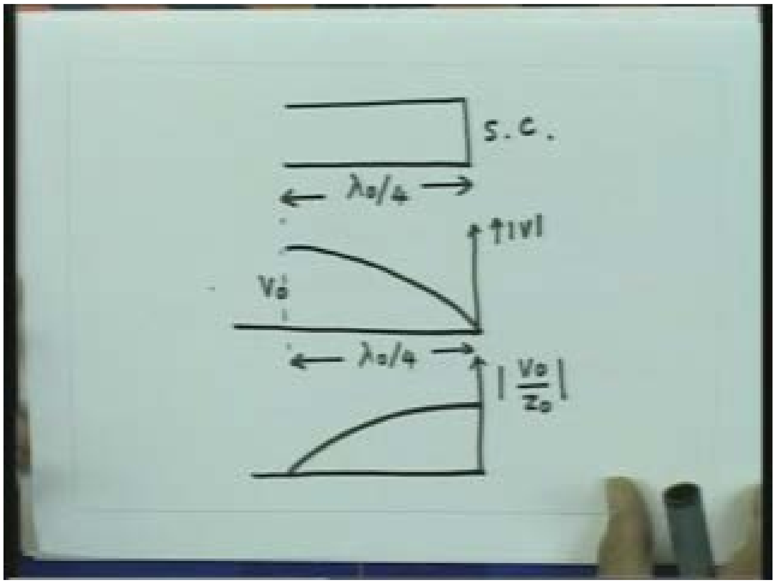


So for a section of the transmission line of length dl, the capacitance is c into dl. So the capacitive energy stored in the section of a transmission line is half c into dl into the voltage square at their location. Similarly, I can get the inductive energy look it in the infinitive section of a transmission line. So once I get that the total energy stored in the transmission line can be written as, so this is the total energy stored, let us called that u that will be equal to half c, total energy will be the integrated version over the length of the line.

So this is from 0 to lambda 0 bar 4 mod of v into l square into dl plus half l integral 0 to lambda, so again it is lambda 0 by 4 mod I of l square into dl, substituting for voltage and current, this is sign of beta l this is cos of beta l, this is half into c integral 0 to lambda 0 by 4, mod of v 0, sin beta l square dl plus half l integral 0 to lambda 0 by 4, v 0 upon z not cos of beta l square into dl. The integrals are very straight forward, you can calculate the value of these integrals and that will be equal to 1 upon 4, c v naught square into lambda 0 by 4 plus 1 upon 4, L v naught square upon z naught square into lambda 0 by 4. Now since z naught, we know is square root of L upon c, the quantity L upon z naught square, this quantity is nothing but c. So, what you find is that these 2 terms are equal, what does means is that the energy stored in this resonant circuit is equally distributed into the capacity energy and the inductive energy.

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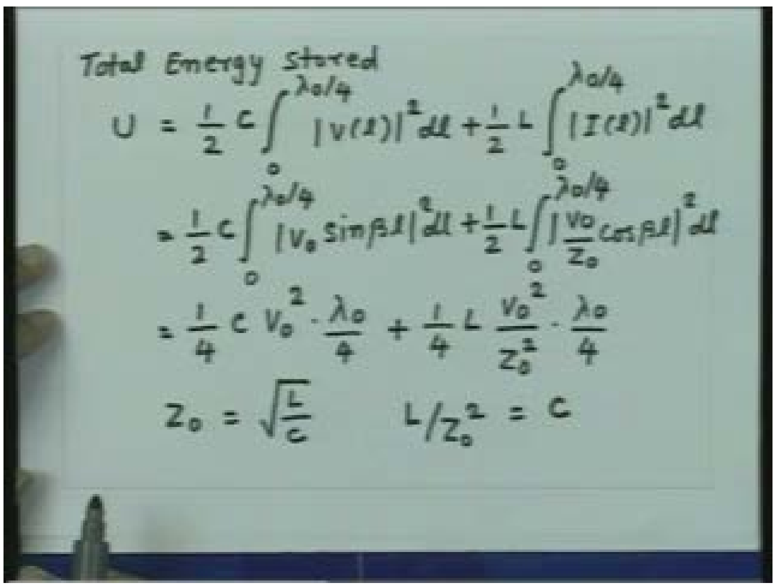


So this step is represented the energy stored in the inductance of the line, this term shows the energy stored in the capacitance of the line and from here, we see these 2 quantities are equal that means the energy stored in the inductance and capacitance of line or equal. If I substitute for this then the total energy stored will be nothing but twice of this term, what twice of this term because these 2 terms are equal. So I get the total energy stored now u that is equal to half into c v naught square to lambda divide by 4, lambda 0 divided by 4. This is the total energy stored in the transmission line. For calculation of the quality factor now we require, the power loss in the transmission line. One way of doing that is again, you go to the primary parameters that the resistance and the conductance of transmission line and get the voltage and current on the transmission line again integrated to the transmission line, to find out the I square R losses, but we can do something different here to get the loss and that is if I take a section of its transmission line here, the energy sources is connected to the line here, see if I find out that what is power loss at this location, this power loss will be nothing but the power loss in this line because the line is short circuited to the other end that means the load is not consuming any power, see by at all that any power is supply to this.

This power will be only equal to the loss which are taken place inside the section of this line. So without going into the primary constant of transmission line just by calculating the input impedance of this line, we can find out what will be the equivalent resistance to be the power is supplied and from there we can calculate the loss of power in this section of transmission line. Precisely, this is what we do now, so we say that since the line is short circuited though input impedance which will seen here will be Z 0 divided by alpha L.

So the input impedance of this line, you are seen earlier this is Z sc is approximately equal to Z 0 divided by alpha into l, where l is equal to lambda 0 by 4. So, now the power loss in the transmission line, say we call it P loss that will be equal to V naught square but that is the voltage which you are seeing at the input terminals here. So it is as you I am having a voltage source of V naught here which is supplying power to a resistance whose value is this. So, I get the power loss which is V naught square divided by the R which is Z naught divided by alpha l.

So this is equal to here v 0 square divided by z naught into alpha into L which is equal to lambda 0 by 4 in this case, this is lambda 0 by 4. Once, I know the energy stored in the circuit at the power loss in the circuit then, I can get in the quality factor Q that is equal to 2 phi into f 0 into energy stored in the circuit. So this is half c, V naught square lambda 0 by 4, V 0 square upon Z naught alpha into lambda 0 bar 4.

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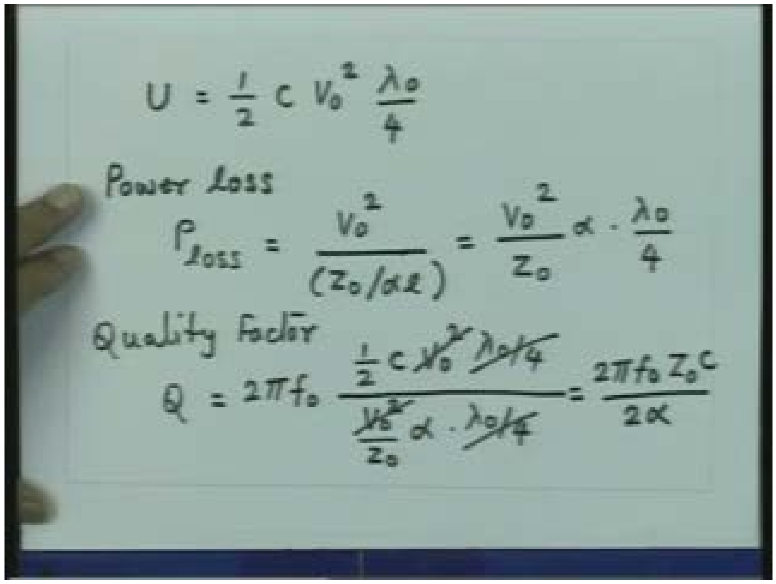


So this quantity cancels V 0 square cancels, so essentially we get this equal to 2 phi f 0, z 0 C divided by 2 alpha. Now we can do some small manipulation because you know this Z 0 C square root divided by C if I write this quantity here Z 0 into C, so Z 0 into C will be square root of L upon C into C that is equal to square root of LC and the quantity here 2 phi into f naught is nothing but omega. So the numerator now in the quality factor Q is omega square root LC divided by 2 alpha, where omega is nothing but 2 phi into the frequency. Now if you were called this quantity omega square root LC is nothing but the phase constant of the transmission line beta.

So this quality factor Q is beta divided by 2 alpha, same expression you can obtain as I mentioned by finding out the frequency response of the section of a transmission lines. So if I measure the impedance and measure the variation of the impedance as a function of frequency, I can find out the cdb bandwidth of the circuit and then from their using the definition that the quality factor is the resonant frequency divide by the 3 db bandwidth, I will get the same expression for the quality factor. So now the quality factor is now related to the phase and the amplitude constant or the phase and attenuation constant of the transmission line and for a low loss line, since beta is much much larger than alpha, this quantity is very large.

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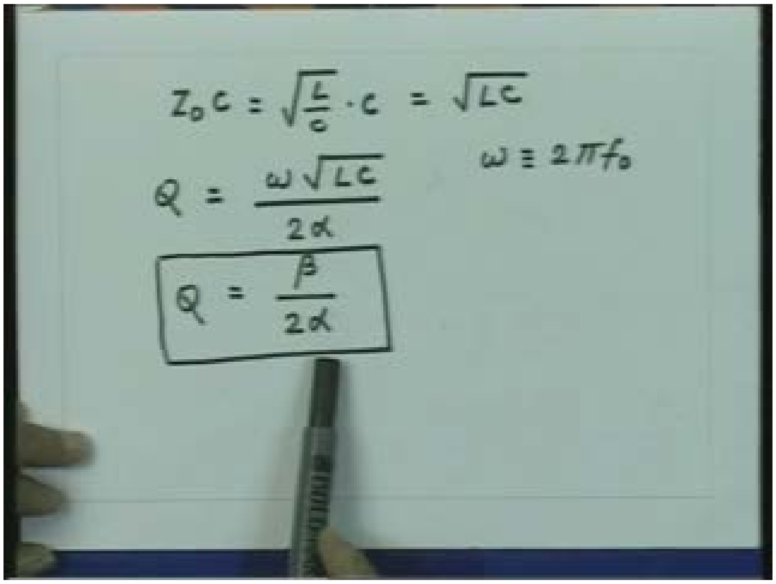


So for low loss line beta is much much greater than alpha, so Q is much much greater than 1, on thus a very useful property that when you go to high frequency, a section of a transmission line can really give me a very high quality factor circuit and since, as you mention the quality factor is relatively to the 3 db bandwidth higher the quality factor, smaller is the 3 db bandwidth or more tune in the circuit is. So the frequency selectivity of a circuit is very good, if the quality factor of the circuit is very large.

So for a low loss section of a transmission lines in the quality factor is large and typically, you can get of few 100 mega, the quality factor could be as higher as few 100 or 1000, you can really get a very good frequency selectivity by the sections of transmission lines used as a resonant circuit that is the reason when we go to the high frequencies, the sections of transmission lines serve very good purpose for realization of the resonant circuits. So this is one of the very important application of transmission line that is in the high frequency circuit. The next application of the transmission line is the voltage or current stepping up transformer.

So third application which we have here is voltage or current, step up transformer. Let us again consider a quarter wavelength section of a transmission line which is shorted at one end and open circuited other. So let us say, I have a line and let me draw little big a picture here. So this is short circuited, this end of the line is open circuited and this length is lambda by 4. Let us say, by some means, some voltage is induce on some point on this section of transmission line.

So let us say at some location here some voltage is induced, if I consider a voltage source connected to this at this location, it will see as if there are 2 sections of transmission line connected in parallel, this section and this section. Now since, the standing way of still or not set up and just voltage just getting induce inside a transmission line, this voltage does not see any other impedance but the characteristic impedance of the line. So as soon as the voltage tries to get induce inside this structure is sends 2 traveling waves on both sides of the cannon transmission line because it will see as if, the energy supply to the characteristic impedance on this 2 transmission line.

So essentially 2 equal amplitude waves or send from this location. This way of travels up to this point, when it reaches here, this waves see the short circuit that means you see the reflection coefficient of minus 1. So this wave here is reflected completely from here with a reflection coefficient of minus 1 that is magnitude is unity and the phase is 180 degrees. So here, I have a reflection coefficient  gamma that is equal to minus 1, say it is equal to 1 angle prime. So this way which was traveling here right words after reflection undergo the phase change of phi, this way now traverse all the way up to open circuit, I am considering only this way, say it wave comes here it get reflected from the short circuit point travels all the way up to the open circuit point. At the open circuit again the again the reflection coefficient is plus 1, its magnitude is 1 is angle is 0.

So again the entire wave from here get reflected with no phase change and it reaches to this location. So by the time this wave has travel a round trip from here to here, it has traveled a distance of lambda by 2 and has gone in additional phase change of phi because of this reflection co-efficient here. Now a distance travel of lambda by 2 corresponds to a phase change of phi plus a phase change of phi which is taking place because of the reflection coefficient here.

So this wave which travels a round trip and reaches here has under gone a phase change of 2 phi that means the wave which is getting induce at that this location, C is now a phase which is of the wave which is reflected, same as the wave which is getting excited. In other words, it is some kind of a positive feedback which is going on now. The induce voltage and the voltage which is gone on these after round drift, they add up. So the 2 voltages together now start travelling on this, when the again come after round trip, it is again in phase.

So the induce voltage again add through this voltage, so the voltage essentially starts growing on the transmission line, exactly same thing happens to this wave also. This wave will go like that after reflection from here, it will not undergo any phase change but the entire r will be reflected. It will travel a distance of lambda by 2, this will against your reflection coefficient of minus 1 here. So this wave also will have a regenerative process, so this wave will grow in a amplitude, this wave will grow in amplitude exactly same wave and the standing wave of this line will grow because the 2 waves are equal in amplitude and they are going exactly the same way.

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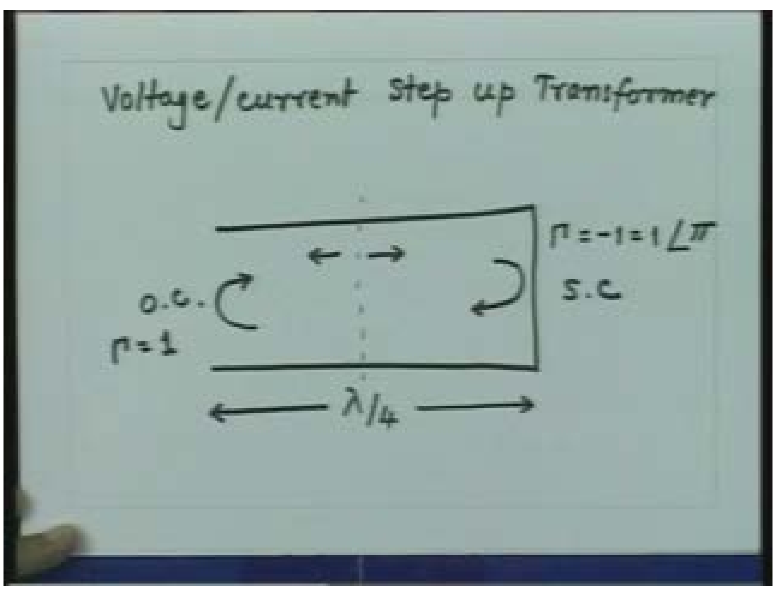


So we will develop the fully standing way of this line because of the small induce voltage at this location, one way wonder than how for this going of the wave will go on. In fact if there are no losses in the transmission line, this growth of the wave will grow up to infinitive. So the voltage will go on growing for a infinite amplitudes because that is no controlling parameter on the transmission line.

So even if a small voltage is induce on a transmission line was some stay coupling or something and if the section of transmission line is resonant then, the voltage which you see on this transmission line is now much much larger compare to the induce volt because of this regenerative feedback. So this process, now can be use for your advantage that the voltage which you measure on transmission line could be much larger compare to the coupling voltage at this location or in other words, this resonant section of a line can be use as the voltage stepping of transformer. Precisely, this application we are talking about take a voltage source which is a small source which is connect to this resonant section of a line and if I measure the voltage on the open circuit of the line, this voltage much much larger compare to the coupling voltage on the line.

So if there is no loss of the transmission lint ultimately in the steady state, the voltage will reach to infinity between the terminals of the line. However, in practice it does not happen because when the voltage and current starts going on the transmission line, the ohmic losses also go on increasing and when the loss in the line because of the resistance and the conductance becomes equal to the energy supplied by the coupling source at that point that is the energy balance now and the growth of the standing wave on this section stops. But before this stage reaches the voltage as in currents already reach to a substantially large value.

So even in practice, when the losses of the transmission line are small one may generate very large voltage is in currents, voltage here and the current here, by a small coupling source connected to the section of a transmission line. So this application is the useful application whenever, we want to step up the voltages and currents at higher frequencies, one can show that the voltage which is going to grow that is related to the quality factor of the transmission line.
So again the higher quality factor means low losses on transmission line and that will give me the higher voltage on the turbulence of the transmission line. The phenomena which can be use for your advantage, for steeping up the voltages are current could be harmful also in many cases. Consider an application, where a small energy source gets unwillingly coupled to the section of

transmission line and if the section of transmission line is of resonant length then, the voltages which will develop on this line will be much larger which your electronic circuit can handle.

So any small coupling of the energy to a resonant  session of transmission line, may unknowingly develop the voltages which will be much larger compare to what a circuit can handle and it is possible that your electronic and circuit can get damage because there are very large voltages which are developed here. So while doing the design of the high frequency circuit you should be kept in mind that no forces get coupled to the sections of transmission line, especially if there of resonant length. Otherwise you will unknowingly, you will be developing very large voltages at some locations right in the circuit which might be a harmful to the electronics which are employed with high frequency.

So the phenomena of stating a voltage as in current could act as a later end, when unknowing voltage sources or current source get coupled to the section of whereas in transmission line. These are certain applications now which are for designing the higher frequency circuits and as I mention earlier, the sections of transmission lines are use for high frequency circuit designs.

So whenever, we have high frequency circuit design especially a micro of frequencies, we rarely see the components like capacitances and inductances, what we see are the active devices like transistor effects and the reactive components are completely realize by the sections of transmission lines. So most of the microwave circuits will appear as some arbitrary sections of transmission lines connected here and there across the active devices. So the sections of transmission line play a very important role in the design of the higher frequency circuits.