

Transmission Lines and E.M. Waves
Prof R.K. Shevgaonkar
Department of Electrical Engineering
Indian Institute of Technology Bombay

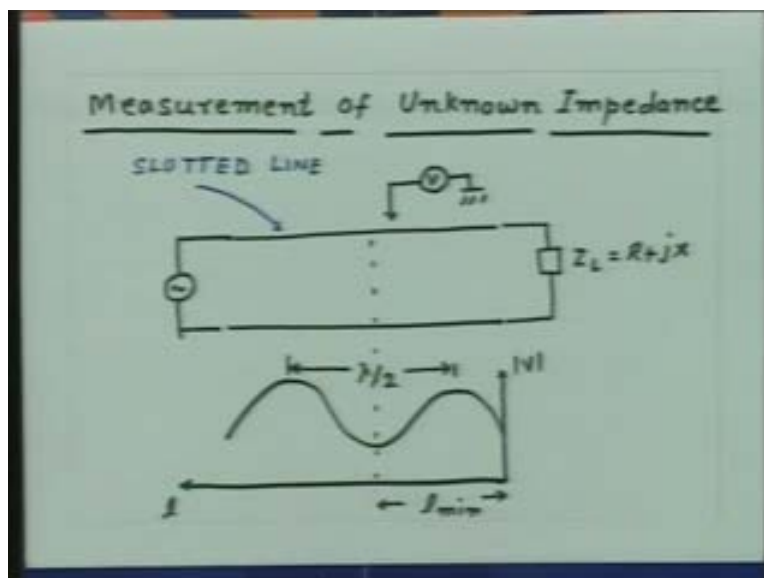
Lecture-10

Welcome, till now we discuss the analysis of Transmission Lines we derived the transmission line equations, also study the power flow on Transmission Lines voltage and current wave characteristics on Transmission Lines and later we developed the graphical tool for carrying out the analysis of Transmission Lines.

Now we discuss the applications of Transmission Lines in high frequency circuits. Although originally Transmission Lines were proposed for efficiently transferring power from one point to another people have found many other applications for transmission line sections. When you go to high frequencies in fact many of the reactive circuit elements are replaced by transmission line sections. So now onwards we discuss some of the important applications of Transmission Lines.

The first application of Transmission Lines is in the measurement of high frequency impedances.

(Refer Slide Time: 01:51 min)



If you recall I had mentioned that when you go to high frequencies the measurement of phase is a very tedious task you can measure the amplitude of a signal rather reliably but the measurement of phase is rather unreliable. We have to measure the complex voltage and complex currents for measurement of complex impedances and since the measurement of phase is not that simple the measurement of complex impedance becomes very difficult at high frequencies.

However we have seen from Transmission Line that the phase which is the temporal phase between the voltage and current gets translated into the spatial phase in the form of standing wave patterns. Therefore we can estimate the temporal phase between the voltage and current by indirect means that is by measuring the standing wave patterns on the Transmission Lines. Precisely that is what we use for measuring the unknown impedances at high frequencies using the Transmission Lines.

For this purpose we use a Transmission Line which is of special type called a slotted Transmission Line. In fact this Transmission Line has a group cut along the length of the Transmission Line so that we can measure the voltage variation along the Transmission Line. Again when we are doing the voltage measurement we are measuring only the magnitude of the voltage along the Transmission Line. So there is a probe which is a voltage probe which lies along the Transmission Lines and measure the magnitude variation of the voltage along the Transmission Line. So this probe measures the voltage along the Transmission Line this line is the slotted transmission line.

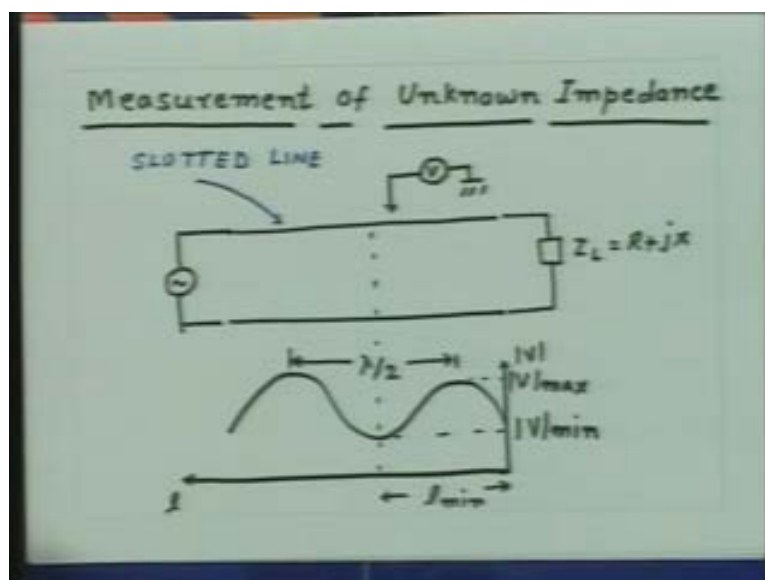
Now for the measurement of the unknown impedance we connect the voltage source at one end of the slotted transmission line and connect the unknown impedance which we want to measure to the other end of the transmission line. In general the voltage standing wave can be created on the Transmission Line and by moving the voltage probe along the Transmission Line we can measure the magnitude of the voltage. If you plot the variation of magnitude of voltage along the Transmission Line we get this voltage standing wave pattern.

Now as we have seen earlier that the distance between two voltage maxima or two voltage minima is equal to $\frac{\lambda}{2}$ so by measuring the separation between the two maxima point or two minima point on the Transmission Line we can estimate the value of the wavelength. Once we get the wavelength we can find the value of β which is 2π divide by wavelength.

So the first step in measurement of impedance is you find out the voltage standing wave pattern identify the location of the voltage maxima or voltage minima, find the distance between the two adjacent voltage minima or voltage maxima, find the value of $\frac{\lambda}{2}$ and from here you find the phase constant β .

The second step would be to find out the location of the voltage maximum on the standing wave. So let us say if I measure from the load end the voltage minimum occurs at a distance l_{\min} from the load point, also we can measure the value of the maximum voltage and the minimum voltage on the line. So I can get mod $|V|_{\max}$ I can get $|V|_{\min}$ on the Transmission Line.

(Refer Slide Time: 05:48 min)

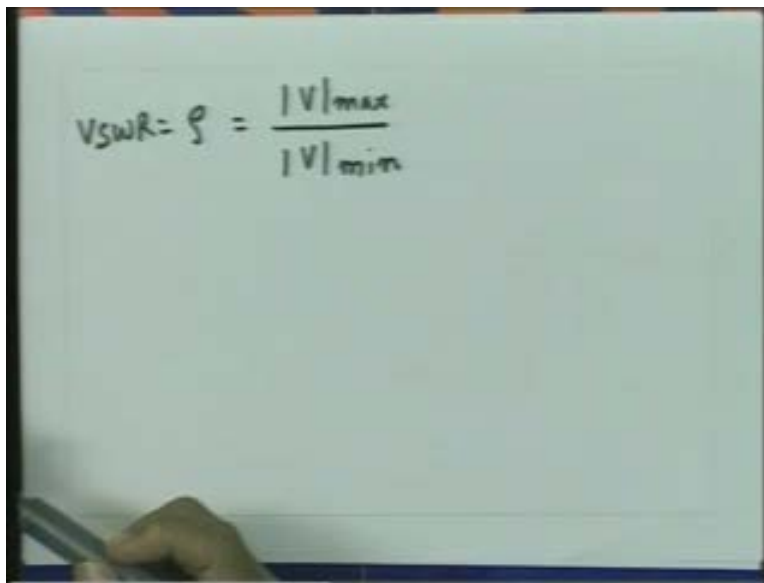


So now I have measured the propagation constant or the phase constant β on Transmission Line, measured the maximum and minimum voltage on Transmission Line and also the location of the voltage minimum on Transmission Line.

Once I do this measurements then I can find out the parameter which are required for the impedance calculations. The first quantity which you calculate is the VSWR for this load.

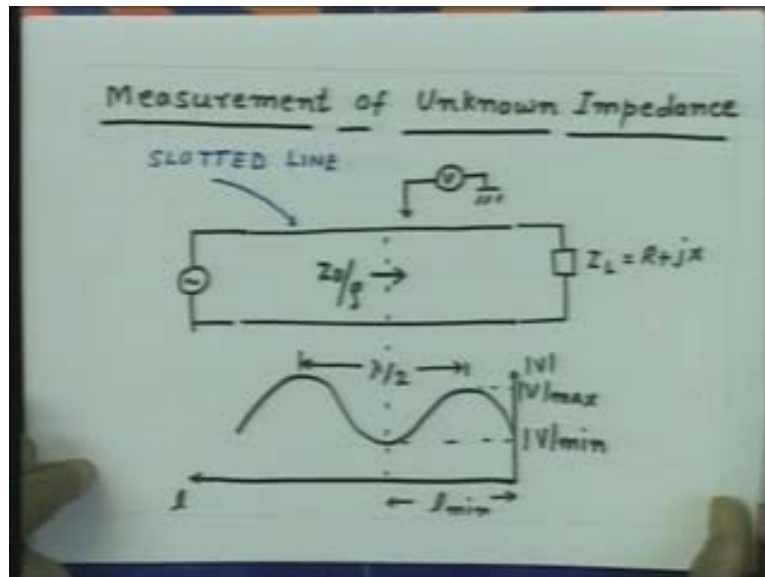
As you have seen the VSWR the ρ is $\frac{|V|_{\max}}{|V|_{\min}}$. Once you do the measurement of maximum and minimum voltage we can calculate this quantity ρ .

(Refer Slide Time: 6:40 min)

A photograph of a whiteboard with the formula for VSWR written in black marker. The formula is $VSWR = \rho = \frac{|V|_{\max}}{|V|_{\min}}$. A hand is visible at the bottom left, holding a pen.
$$VSWR = \rho = \frac{|V|_{\max}}{|V|_{\min}}$$

Once you get the VSWR ρ we now know the maximum and minimum impedance which one can see on Transmission Line. So at a location where the voltage is minimum that is this location here now the impedance will be the minimum impedance and that will be R_{\min} and as we have seen earlier $R_{\min} = Z_0$ divide by the VSWR so I know the impedance at this location here that is Z_0 divide by the VSWR.

(Refer Slide Time: 7:17 min)



So from here now I calculate the quantity R_{min} which is the characteristic impedance divide by the VSWR ρ . Once I know the impedance at this location then the calculation of unknown impedance is simply an impedance transformation problem. As we had said if we know the impedance at any point on Transmission Line we can find the impedance at any other point on Transmission Line by using the impedance transformation relation.

So in this case we know the value of the impedance at this location which is $\frac{Z_0}{\rho}$ once I

know this I know the distance of this thing from the load which is l_{min} . So I can transform this impedance by a distance l_{min} away from the generator so that the impedance is equal to Z_L so in fact the Z_L unknown impedance is nothing but the transformed version of $\frac{Z_0}{\rho}$

by a distance l_{min} away from the generator. So now the $Z_L = Z_0$ where you apply a transformation relation so here is $R_{min} \cos \beta$ in to the distance and keep in mind here we are having a distance away from the generator here we are transforming these

impedances away from the generator so the distance is negative so you have l_{\min} which is negative in this case so this will be $\left\{ \frac{R_{\min} \cos(-\beta l_{\min}) + jZ_0 \sin(-\beta l_{\min})}{Z_0 \cos(-\beta l_{\min}) + jR_{\min} \sin(-\beta l_{\min})} \right\}$.

(Refer Slide Time: 9:45 min)

$$V_{SWR} = \rho = \frac{|V|_{\max}}{|V|_{\min}}$$

$$R_{\min} = Z_0 / \rho$$

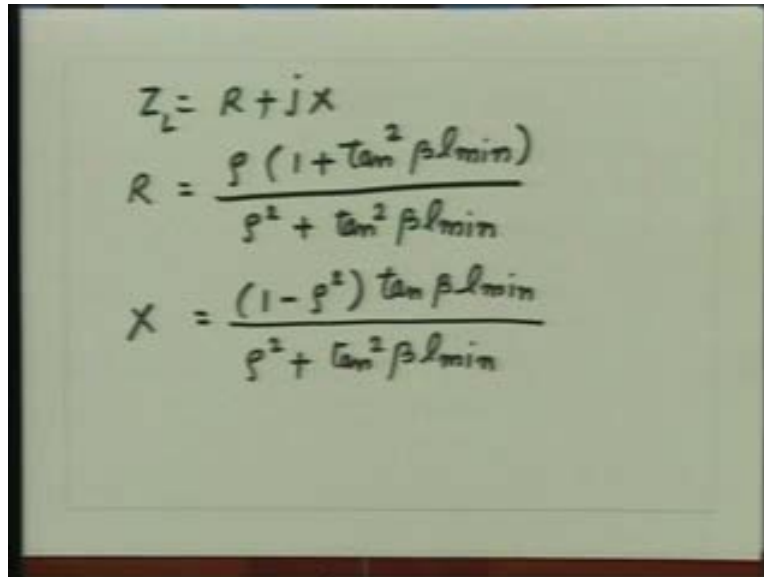
$$Z_L = Z_0 \left\{ \frac{R_{\min} \cos(-\beta l_{\min}) + jZ_0 \sin(-\beta l_{\min})}{Z_0 \cos(-\beta l_{\min}) + jR_{\min} \sin(-\beta l_{\min})} \right\}$$

So now the unknown impedance is the transformed version of R_{\min} away from the generator so the distance is negative so the l_{\min} is negative. Once you get this finding out Z_L is a matter of simply separate out the real and imaginary parts. So if I separate out the real and imaginary parts I get explicitly the value of the resistance and the reactance as I

get the unknown impedance Z_L is $R + jx$ where R will be equal to $\frac{\rho (1 + \tan^2 \beta l_{\min})}{\rho^2 + \tan^2 \beta l_{\min}}$ and

the reactive part $x = \frac{(1 - \rho^2) \tan \beta l_{\min}}{\rho^2 + \tan^2 \beta l_{\min}}$.

(Refer Slide Time: 11:10 min)


$$Z_L = R + jX$$
$$R = \frac{\rho^2 (1 + \tan^2 \beta l_{\min})}{\rho^2 + \tan^2 \beta l_{\min}}$$
$$X = \frac{(1 - \rho^2) \tan \beta l_{\min}}{\rho^2 + \tan^2 \beta l_{\min}}$$

This is nothing but the real and imaginary part of this by rationalizing this function we can separate out the real and imaginary parts. So the real part gives me the resistive value of the unknown impedance the imaginary part gives me the reactive value of the unknown impedance. So by transforming the impedance R_{\min} to the location of the unknown impedance we get the complex impedance. So once you measure the VSWR of the line and the location of the minimum then finding out the unknown impedance is extremely straight forward.

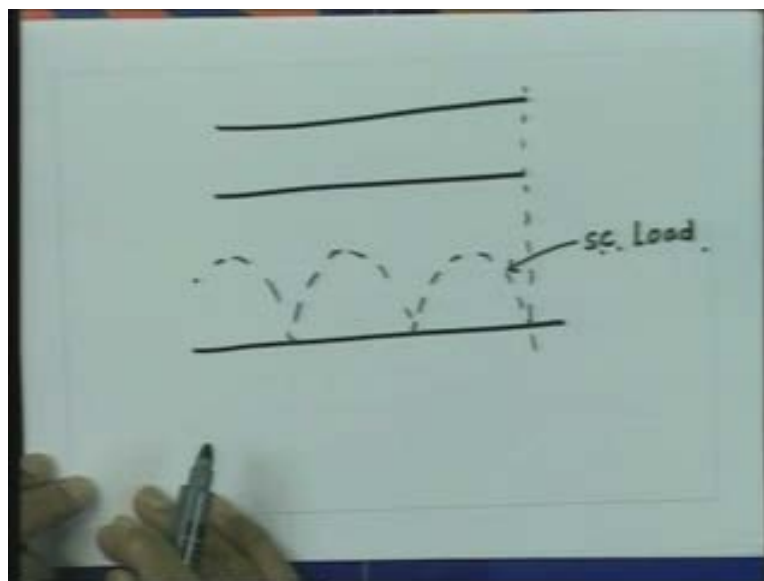
However when we go into the practice we will see that while connecting the unknown impedance to this slotted line section we use some connectors and other things and because of this the location of the impedance is not that accurately known. So measurement of l_{\min} sometime become unreliable in some sense so if I knew l_{\min} very precisely then of course the transformation is very accurate and we can find out what the unknown impedance is but if l_{\min} is not known then you will have a error in the estimation of the unknown impedance so normally what people do is to define the location of the load.

First you replace the load by a short circuit, do the measurements on the Transmission Line find out the location of the voltage minimum with the line terminated in a short circuit. Then you replace the short circuit by the load and now find out the standing wave pattern with a load. So we have two standing wave patterns measured on Transmission Line, one is with the line terminated in a short circuit and other one is the line terminated in the actual load which you want to measure.

So in first case when let us say this is the Transmission Line and this is the plane at which I can connect either the load or the unknown impedance.

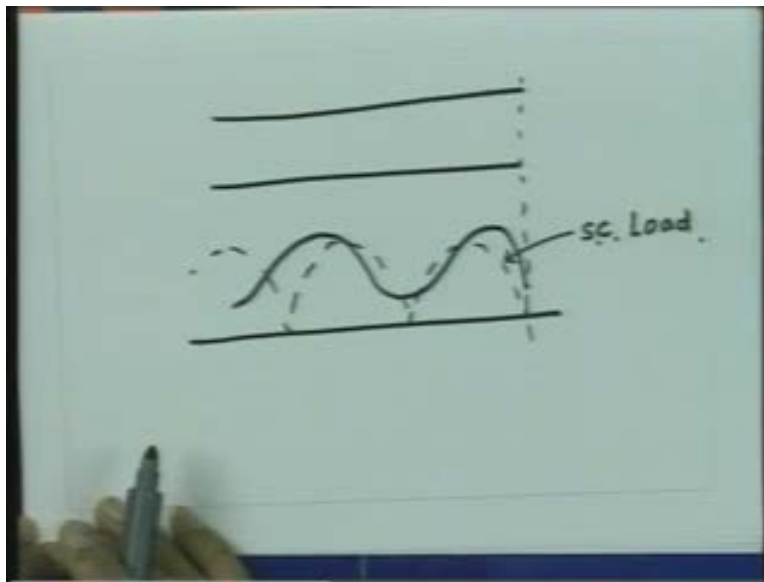
When I connect the short circuit at this location the standing wave pattern will have a voltage minimum at this location and since this is ideal short circuit the VSWR for this will be infinity the minimum voltage will be zero. So if I plot a standing wave pattern in this case the standing wave pattern will look like this so the voltage is minimum there will be maximum voltage so the ratio of maximum to minimum voltage is infinity so you have VSWR infinity, this is the voltage standing wave pattern for the short circuit conditions so this is the short circuit load.

(Refer Slide Time: 14:10 min)



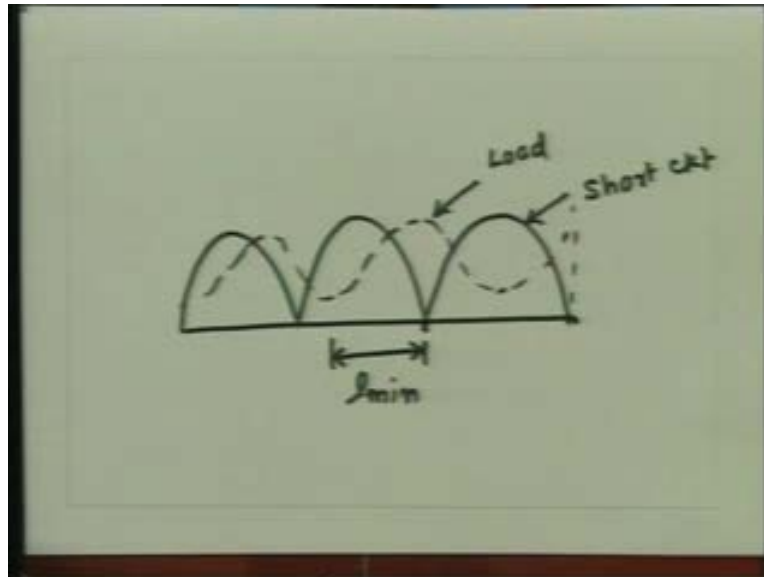
Now I replace this load by the actual load on the Transmission Line. Two things will happen the voltage standing wave pattern in general will be shifted with respect to the short circuited standing wave pattern and also the VSWR is not infinity so V_{\min} voltage will not be zero. So in general we might get a voltage standing wave pattern which will look something like this. So now I may not have a maxima and minima of the short circuited pattern and this pattern at the same location.

(Refer Slide Time: 14:55 min)



So I have two patterns here. This is for the load, this for short circuit and now this is the location at which the load is either connected or the short circuit impedance is connected. As we know that the location which is at this location has been repeated at every $\lambda/2$ distance so whether I do the measurement at this point or I do the measurement at this point or this point they are identical because the impedance characteristic repeats every $\lambda/2$ distance.

(Refer Slide Time: 16:03 min)



So now once I find out the minimum point for the Transmission Line when it is terminated in the short circuit I know precisely the location of this point because separation between this point and this point is $\lambda/2$. So instead of doing the measurement from this point I can do the measurement from this point which is very well identified on the slotted transmission line. So I can do the measurement of this location from the minimum which I see from the standing wave of the unknown load so essentially distance from here to here distance is nothing but l_{\min} .

Now without worrying about what I am connecting to the Transmission Line as long as I can connect a short circuit impedance at a location where the unknown impedance is connected I can precisely find out the value of the l_{\min} . And once I know l_{\min} then I can go back and substitute into that formula to find out what is the unknown impedance.

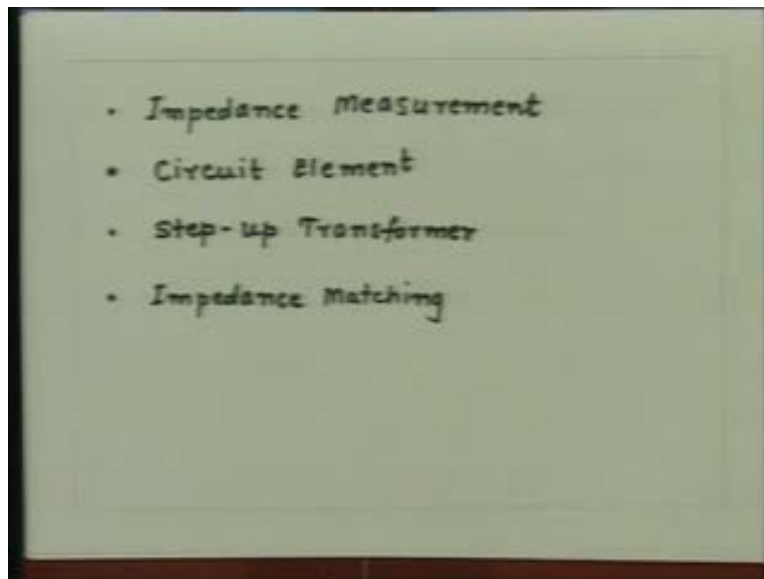
So in practice whenever we are doing the measurement of unknown impedances we have to do the measurement of standing wave pattern twice in which first by short circuiting the line and later by connecting the unknown impedance at that location. And then by doing the measurement of l_{\min} from the relative pattern of the short circuited standing

wave patterns and the unknown impedance standing wave patterns we estimate the value of I_{\min} .

So this technique for impedance measurement at high frequency is very useful and in fact if you go to frequencies like microwave frequencies without the measurement something like this on the slotted line one will not be able to measure the unknown impedances. So at high frequency the Transmission Lines come out very handy for measurement of the unknown impedance.

The second application which you have for the Transmission Line is the circuit element use of transmission line sections as a circuit element.

(Refer Slide Time: 18:21 min)

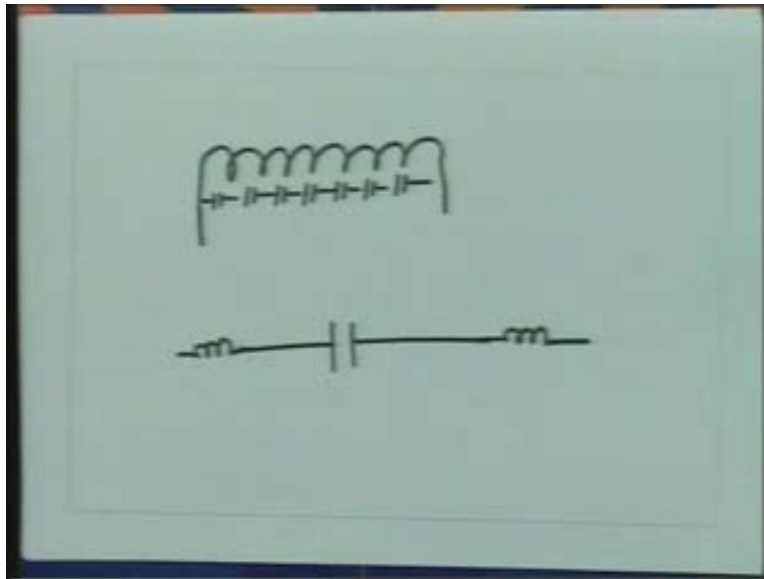


As we increase the frequency as we have seen earlier that distributed elements starts showing its effect. So let us take an example suppose I wind an inductance to be used at very high frequencies. Let us say I take a coil or I take a wire and make a coil out of it to make an inductance so let us say I make an inductance to be used at very high frequencies. I can calculate the value of inductance by a relation which is available.

However what we do not take into consideration is the capacitance between the windings of this inductor so you also have the capacitances which are distributed capacitances but in terms of the inductances. As the frequency increases the effect of these capacitances are also start showing and one can show that for typical inductance which you wind the resonant frequency lies for the distributed capacitances and the inductance which you want typically in the range of about hundred to two hundred mega hertz. That means if I wind an inductance beyond about three 300MHz the inductance will not behave like an inductance it will rather behave like a capacitance because we have already crossed the resonant frequencies.

So now the effect of capacitance is more dominant compared to the inductance. Similarly suppose I make a capacitance at low frequencies that behave like a capacitance but as I increase this frequency we have this leads which have inductance and again as the frequency increases this inductance effect starts dominating so beyond certain frequencies the capacitance does not behave like a capacitance it rather starts behaving like an inductance. So if we make an inductor at low frequency we can reliably design the inductor but when you go to high frequencies this reliability is very poor because the inductor might start behaving like a capacitance, similarly if I have capacitance to operate at low frequency it works reliably but when i increase the frequency the lead inductance of the capacitances start dominating and its possible that the capacitance will appear like an inductance.

(Refer Slide Time: 21:01 min)



So realizing a reactive element at high frequency is not that easy because we do not have a reliable circuit element which will be guaranteed used like an inductance or like a capacitance. At the same time when the frequency is increasing the wavelength is becoming smaller and smaller and as we have seen earlier that if you consider a short circuit or open circuit line the input impedance of this line behaves like a reactance. So when the frequency increases and the wavelength becomes smaller the size of the **trans** section of Transmission Line which can give you impedance which is reactive, that becomes more and more physically realizable.

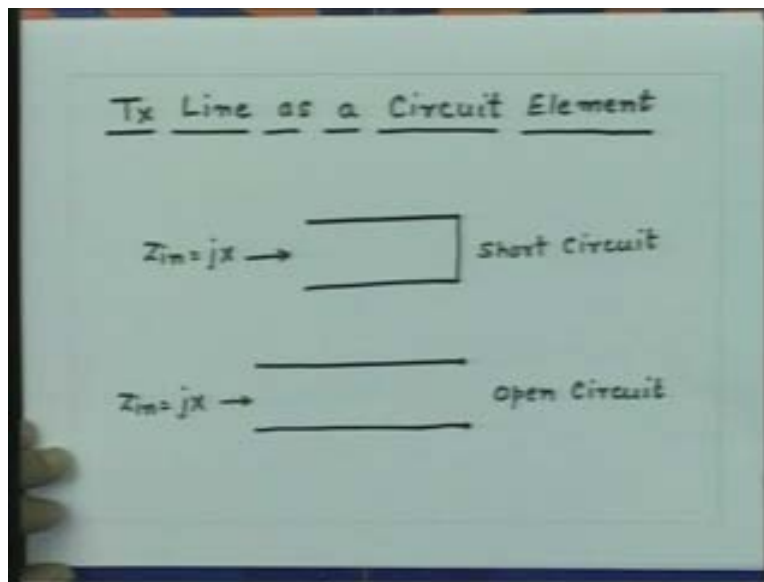
Now we having two things working in the favor of Transmission Line, the lumped circuit elements are becoming more and more difficult to be realized at high frequencies and realization of reactances by using Transmission Line section is becoming easier because the wavelength is becoming smaller and smaller so sections of transmission lines are becoming more physically realizable their size is more accommodate..

So these two factors essentially support the use of Transmission Line section for realizing the reactive elements in the high frequency circuits. And in fact if you go to a typical high

frequency circuit of few hundreds of mega hertz or few giga hertz most of the reactances in the circuits are replaced by the sections of transmission lines.

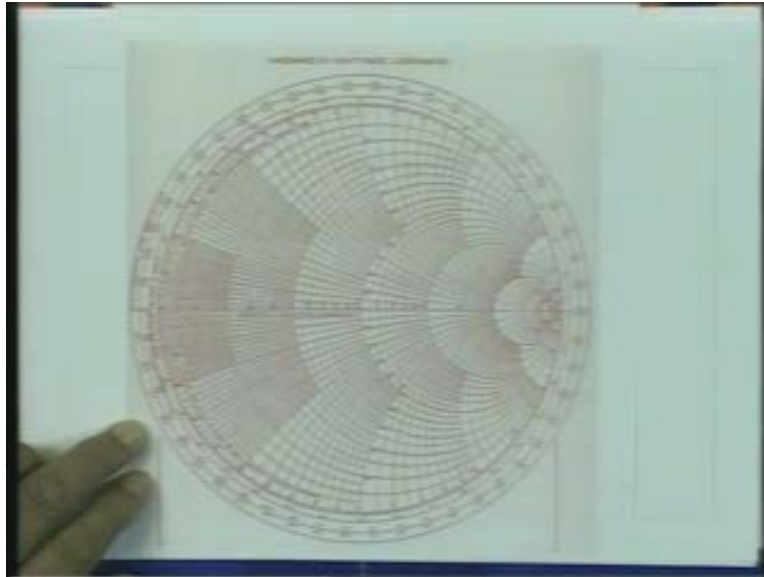
Let us see how the transmission line sections can be used as a reactive element. So let us consider section of a Transmission Line let us take this section of a Transmission Line which can be either short circuited or open circuited.

(Refer Slide Time: 23:01 min)



Now if I go to the smith chart and if I consider this smith chart as the impedance smith chart I know that this point here the right most point represents the open circuit on the Transmission Line and this point here represent the short circuit on the Transmission Line. So for a open circuit or short circuit load on the Transmission Line as I move on the Transmission Line towards the generator I move on the outer most circle of this smith chart and if you recall for the outermost circle on the smith chart R is identically zero so it represents only pure reactive impedances on the outermost circle of the smith chart.

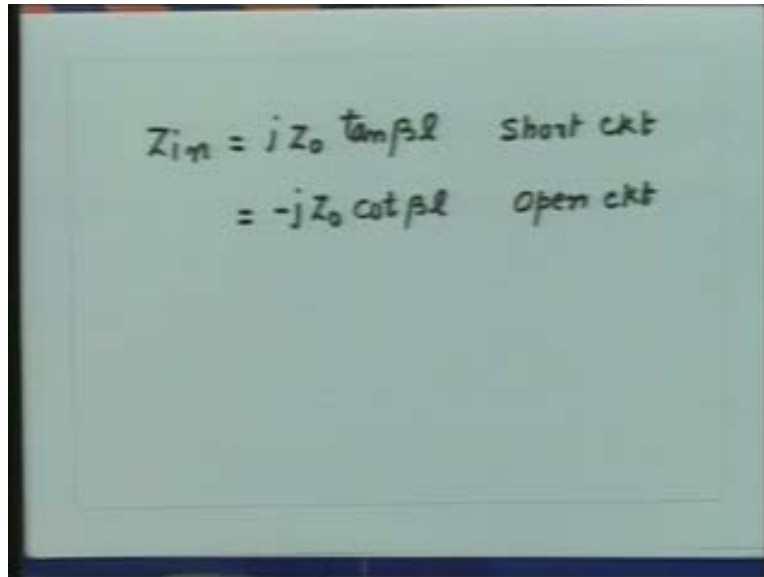
(Refer Slide Time: 23:59 min)



So I can get different reactances as the input for a load of short circuit or open circuit by using different lengths of Transmission Line. Precisely what the idea is that you take a Transmission Line terminate this Transmission Line into a short circuit or open circuit the input impedance if you measure for these sections of transmission line will ideally be reactive impedances so by changing the length of the sections of the transmission line I can realize a reactance between these terminals. So wherever I want to connect a reactive element I simply connect a section of a transmission line at that location and this section of transmission line will appear like an appropriate reactance at that location. Let us elaborate on this idea and ask what would be the lengths of sections of transmission lines so that we can realize the particular reactances on the Transmission Line.

If we do analytically then the input impedance of a section of a transmission line Z_{in} will be $jZ_0 \tan \beta l$ when the line is short circuited and will be equal to minus $-jZ_0 \cot \beta l$ when the line is open circuited.

(Refer Slide Time: 25:50 min)



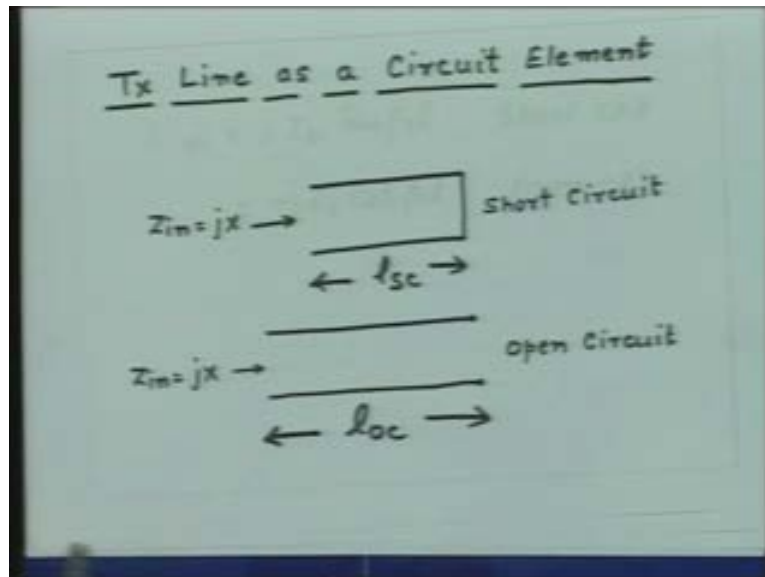
The image shows a handwritten note on a light blue background. It contains two equations for input impedance Z_{in} of a transmission line of length l with characteristic impedance Z_0 and propagation constant β . The first equation is $Z_{in} = jZ_0 \tan \beta l$ for a short circuit (Short ckt). The second equation is $Z_{in} = -jZ_0 \cot \beta l$ for an open circuit (Open ckt).

$$Z_{in} = jZ_0 \tan \beta l \quad \text{Short ckt}$$
$$= -jZ_0 \cot \beta l \quad \text{Open ckt}$$

Getting this is very straight forward in the impedance transformation relationship if I put load impedance zero for the short circuit case I get the Z_{in} which is the transform impedance over a length l towards the generator which will be the input impedance, if I take the open circuited line then I put load impedance equal to infinity then over a distance l towards the generator I get the input impedance which will be given by that.

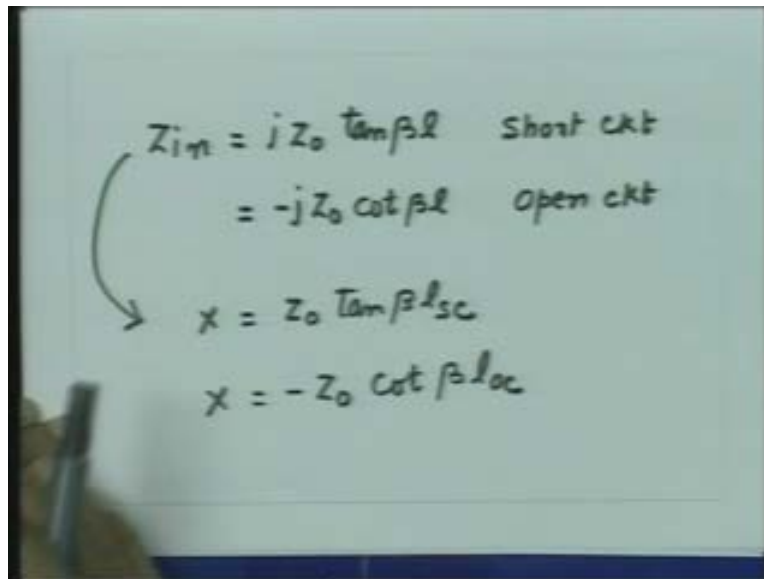
Now the problem for realization of a given reactance is just to find out what is the length of this section of a transmission line which will realize that reactance in which we are interested in. Let us say the short circuited line and open circuited line are denoted by l_{sc} and l_{oc} .

(Refer Slide Time: 26:55 min)



And let us say I want to realize certain reactance x so for the short circuit case from here x will be equal to $Z_0 \tan \beta l_{sc}$ for short circuit case and x will be equal to $-Z_0 \cot \beta l_{oc}$. I can invert this relationship to find out the value of l_{sc} because the phase constant β on the transmission line section is known. So if I know the frequency of operation if I know the velocity of the wave on the Transmission Line then I can find out what is the value of λ . Once I know the value of λ I can find out what is the phase constant, once I know the value of β I can invert this relation and find out what is the value of the length of these sections of transmission line if the line is short circuited or if the line is open circuited.

(Refer Slide Time: 27:52 min)



Handwritten notes on a whiteboard:

$$Z_{in} = jZ_0 \tan \beta l \quad \text{Short ckt}$$
$$= -jZ_0 \cot \beta l \quad \text{Open ckt}$$

A curved arrow points from the first two equations to the following two equations:

$$X = Z_0 \tan \beta l_{sc}$$
$$X = -Z_0 \cot \beta l_{oc}$$

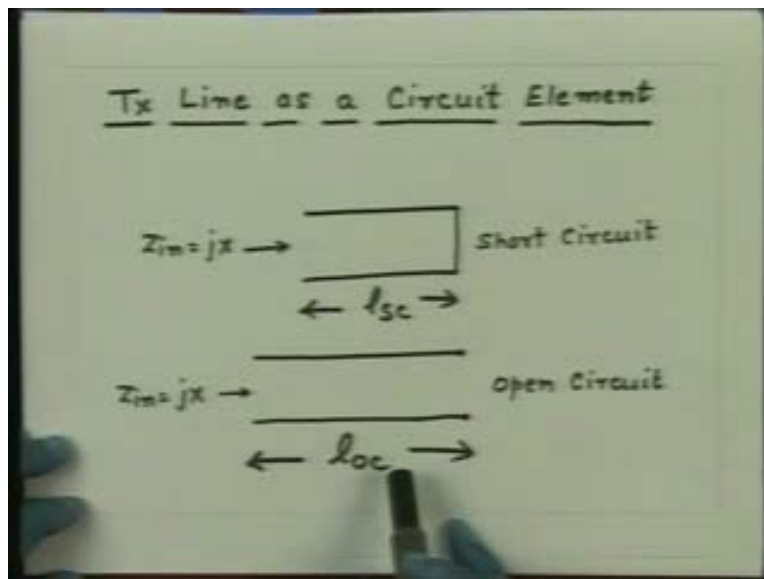
Now by using the sections of the transmission lines I can realize an unknown reactance on the Transmission Line. Now for doing this calculations also the use of smith chart comes out very handy but before I get into that let us see that this function which we are having here the tan function or the cot functions. For the variation of βl_{sc} from 0 to 2π any of these functions vary from minus infinity to infinity so that means if I change the value of l_{sc} or l_{oc} transmitted lines then I can realize any arbitrary value of the reactance there is absolutely no limit so any arbitrary value of reactance between $-\infty$ to $+\infty$ can be realized either by using short circuited line or by using open circuited line. so the choice which one to be used whether short circuit line to be used or open circuit line to be used will depend upon the system in which you want to employ this circuit element.

Let us take a simple example suppose I have a parallel wire transmission line then connecting two ends of Transmission Line is rather very easy. So I can short the end of the Transmission Line so in those structure parallel wire structure where connecting the two ends of wire is easy we prefer to use the short circuited end of a Transmission Line.

However if I consider a line like a something on printed board so there is a ground plane on one side and the line on other side for short circuit in this line we have to drill a hole into the printed circuit board so realizing a short circuited line in that configuration is rather difficult so we prefer to use an open circuited line. So there may be situations where the short circuited Transmission line will be preferred or the open circuited lines will be preferred. And as we saw irrespective of whether we used short circuited lines or open circuited lines we will be able to realize all the reactances from $-\infty$ to $+\infty$, that means we can realize any capacitive or inductive reactance by using an open circuited or a short circuited section of a transmission line.

Now, how do we find out the lengths for getting the open circuited or short circuited section of a Transmission Line? Firstly we will note here to calculate that length now the problem is opposite of impedance calculation what we are saying is generator is again on the left side this is the load end

(Refer Slide Time: 30:54 min)

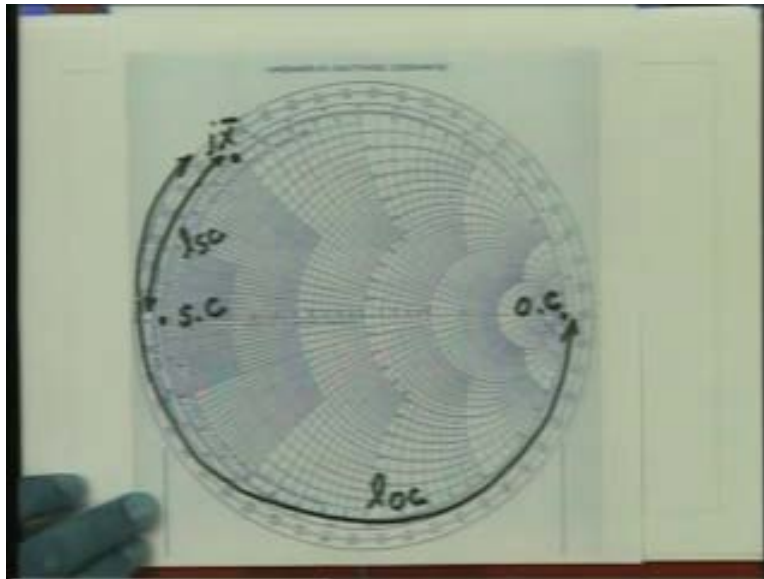


I know the value of the reactances at this location I know the value of impedance at this location which is either short circuit or open circuit and I want to find out what should be

this distance. So the impedances are known and the transformation length is to be found out. So the problem is on the smith chart if I can mark this impedance x and ask a question how much should I move from the generator so that I reach to a location short circuit or open circuit.

So now the problem here is after we mark the reactance on the smith chart find the length so that we reach to the short circuit point or open circuit point.

(Refer Slide Time: 31:52 min)



Let us say I want to find out some reactance x which lies somewhere here. Note again this point is lying on the outermost circle of the smith chart so this is the reactance which is which is jx which we want to find out.

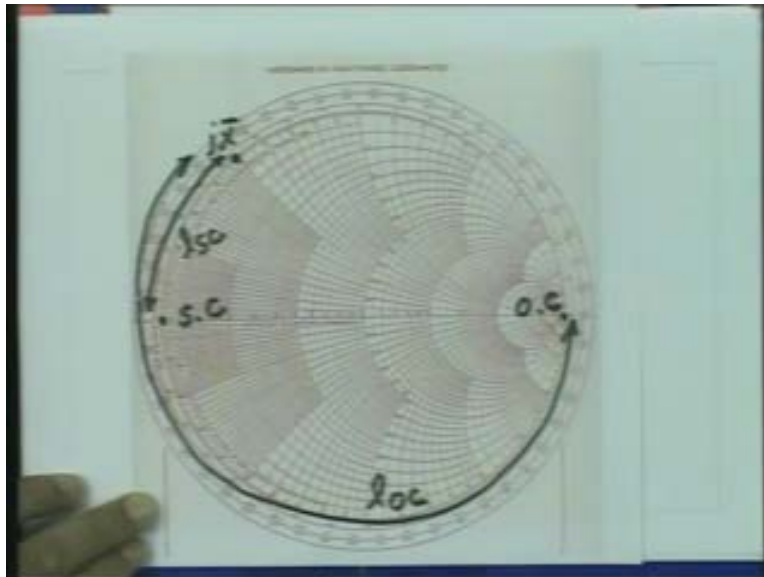
Now as we have seen if you are treating this chart as the impedance chart say the point is lying on the upper half we want to realize the reactance which is the inductive reactance, Notice here that though the reactance is inductive we are not realizing an inductance either to low frequencies we realize an inductance and a capacitance. Here we are directly

realizing the reactances so this is the quantity which is the inductive reactance we want to realize at a particular location at a given frequency.

Now if I move from this reactance point away from the generator such that I reach to a short circuit point or open circuit point. So if I move away from the generator and that is in the anticlockwise direction because on the smith chart clockwise movements are towards the generator. Now we want to move away from the generator from this reactance we want to move away from the generator till I reach to the short circuit point, away from the generator till I reach to the open circuit point. So if I move away from the generator in anticlockwise direction till I reach to the short circuit point which is this point and this point is open circuit so if I find out this length which corresponds to the length of a Transmission Line l_{sc} , similarly if I move up to the open circuit point in the anticlockwise direction that is the length which will give me l_{oc} . So to realize this reactance x here if I take this arc which represents l_{sc} and if I take an arc which is from this point all the way up to this location will represents l_{oc} .

So let me repeat how do we find out the length of the short circuited or open circuited lines for realizing a reactance take the value of a reactance mark it on the smith chart. Remember on the smith chart we always have the values which are normalized so this reactance here is a normalized quantity. So let me denote that specifically as \bar{x} so the first step is whatever reactance you want to realize convert them into normalized reactance mark the normalized reactance on the smith chart, move in the anticlockwise direction to this point the left most point on smith chart gives me the length which is the length of the short circuited line l_{sc} . Then I can find out the l_{oc} either by adding $\lambda/4$ to l_{sc} because we know that these two points are separated by a distance of $\lambda/4$ or I can measure this length all the way from here to here which will give me the open circuited length of the transmission line section.

(Refer Slide Time: 35:49 min)



So depending upon the choice of the circuit we can either have the section of line which is open circuited or short circuited and we can realize that for any arbitrary inductance or capacitance on the Transmission Line.

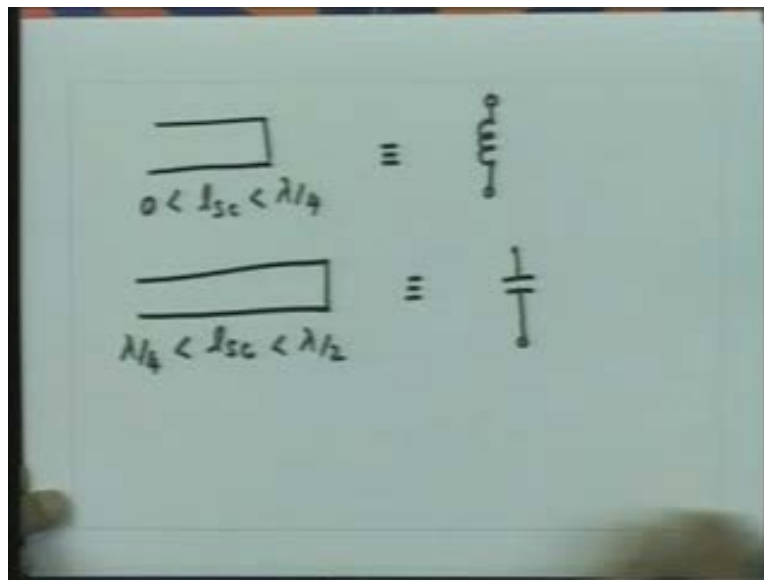
Now if I look at this carefully as we have already seen that any length of a Transmission Line can behave like a capacitance or inductance because depending upon the length I can get reactive element which will be like a capacitance or which will be like an inductance. So let us say suppose I take a short circuited line so at the load impedance it is somewhere here, now if I move towards the generator that means in the clockwise direction up to a length of $\lambda/4$ the input impedance will be an inductive impedance beyond $\lambda/4$ two $\lambda/2$ the impedance seen at the terminals of the line will be a capacitive impedance. So a short circuited line if the length lies between zero and $\lambda/4$ it will behave like inductance, if length lies between $\lambda/4$ and $\lambda/2$ it will behave like a capacitance and so on.

Similarly if I consider a line which is open circuit then the movement towards the generator for first $\lambda/4$ the reactance will lie in this so a open circuited line of length from

zero to $\lambda/4$ will behave like a capacitance, from $\lambda/4$ to $\lambda/2$ it will behave like an inductance and so on.

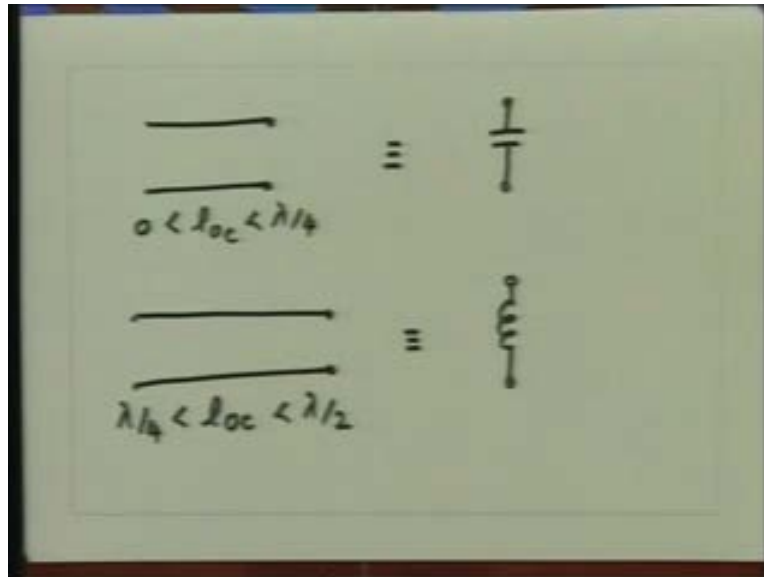
Now depending upon the length of the transmission line I can get a inductive behavior or I can get a capacitive behavior, I can write this thing rather explicitly and that is I have a situation this is a short circuited line where l_{sc} is between $\lambda/4$ and zero. As we saw this line is a short circuited line so between $\lambda/4$ and zero this is equivalent to an inductance. If I take a short circuited line whose length is l_{sc} lies between $\lambda/2$ and $\lambda/4$ then from here now the length is this region so that will represent a capacitance so this line will appear like a capacitance.

(Refer Slide Time: 39:13 min)



Similarly if an open circuited line l_{oc} is between $\lambda/4$ and zero then this is equal to a capacitance and if the length is between $\lambda/4$ and $\lambda/2$ then this is equivalent to an inductance.

(Refer Slide Time: 39:55 min)

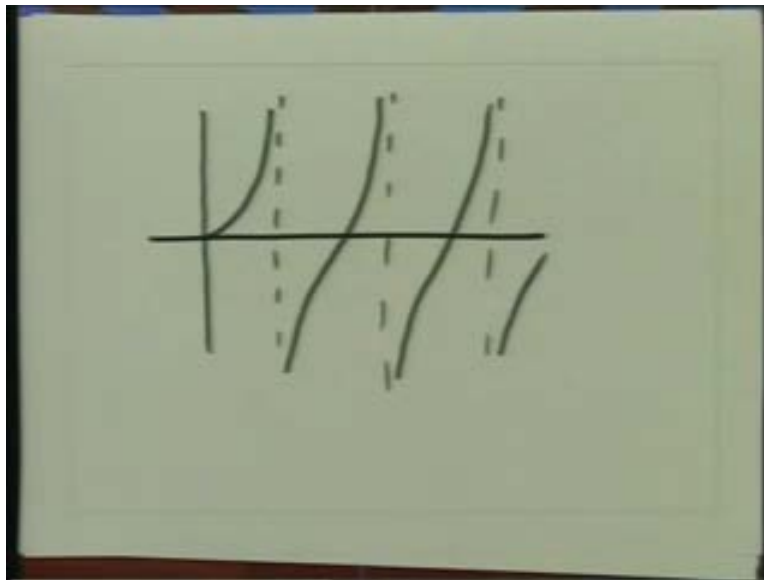


Now any section of a transmission line of size from zero to $\lambda/2$ can realize any arbitrary value of capacitance and inductance at a given frequency. Let me again mention that we are not realizing an inductance and capacitance we are realizing the inductive or capacitive reactance and equivalently we can say that the reactance is having that capacitance or that inductance.

Now once you are having a value of capacitance or inductance take a case like this up to $\lambda/4$ this behave like a capacitance from $\lambda/4$ to $\lambda/2$ this start behaving like an inductance so that at $\lambda/4$ the behavior of this line changes from capacitance to inductance so at that location at $\lambda/4$ you have a characteristic which is more like a resonant line characteristic. so if take a **lc** resonant circuit then at resonant frequency on one side of which you have inductive behavior and other side of which you have capacitive behavior so if I go to a section of a line which is $\lambda/4$ or $\lambda/2$ then at that frequency the line will behave like a resonant circuit. So section of line may not be used only like a reactive element but it is possible to use the section of a line like a resonant **lc** circuit also.

Now we see that suppose we take a line around the length of $\lambda/4$ or $\lambda/2$ then they will have a behavior which is similar to I_{sc} resonant circuits. Let us consider a line which is a short circuited line and now ask if I change the frequency for keeping the length of the line fixed if I change the frequency how the behavior of the reactance is going to change. So as I change the frequency for a given length of the line the impedance seen between the terminals of the line is going to change so I get a characteristic for any length typically a length of the line which will look something like that like this, like this and so on

(Refer Slide Time: 42:55 min)



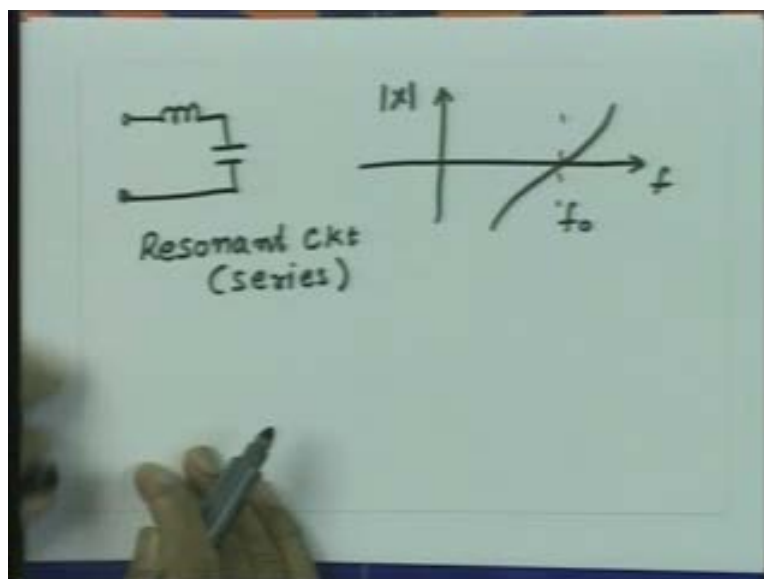
So if I take a line let us say a short circuited line and the length is equal to $\lambda/2$ since this is short circuit so the impedance will be equal to zero. Then the impedance will change and when I reach to a point around $\lambda/4$ that time this line will appear open circuit at this location so the impedance will become equal to infinity. One side of that frequency will be **capacitive** the other side will be negative. When the frequency is such that this length is zero or $\lambda/2$ or λ I get the impedance which will be here or here or here, where the impedance seen between the input terminals is zero. However if I go to the frequency for which the length is equal to $\lambda/4$ then the impedance seen between the terminals of the line

is infinity because this will appear like a open circuit so I will see a location somewhere here. So as I change the frequency for a given length of a line when the length of the line becomes zero or $\lambda/2$ or λ then I see the impedance which is here or here or here.

If I go to the frequency for which the line length becomes $\lambda/4$ or $3\lambda/4$ or $5\lambda/4$ four then I will see this point, this point, this point. That means in a frequency range close to this, this characteristics is very similar to a **LC** series resonant circuit, where as if I am in the vicinity of this frequency then the behavior of the Transmission Line is similar to a parallel resonant circuit because if I take a characteristic of series and parallel resonant circuits we know for a series circuit the impedance variation for this will be something like that this is the resonant frequency of your circuit f_0 .

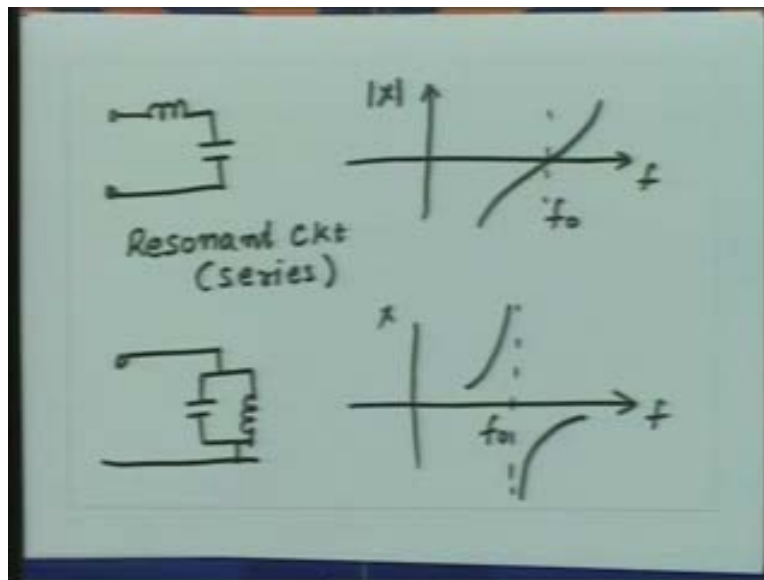
So this is the frequency I am plotting this you have to plot x so you are plotting negative values so if I plot the reactance as a function of frequency for this circuit at resonance the reactance is zero, as we increase the frequency this becomes larger so you see the inductive reactance and if you decrease the frequency this reactance is larger so we see capacitive reactance. So a behavior crossing through zero is a characteristic of a series resonant circuit so this circuit is a resonant circuit series.

(Refer Slide Time: 46:15 min)



On the other hand if I consider a circuit which is a parallel resonant circuit then the frequency characteristic for this this is ∞ this will be because at the resonant frequency the reactance will become infinity just like an open circuit and as I go away from the frequency on one side you see the inductive effect the other side you see capacitive effect this frequency is the f_0 . So a line can behave like a series resonant circuit if I go to a frequency where the frequency characteristic is like this or it can behave like a parallel resonant circuit if I go to a frequency around which the frequency characteristic is like that.

(Refer Slide Time: 47:20 min)



So for a given length of a Transmission Line those frequencies for which the length of the line is close to $\lambda/2$ the line will behave like a series resonant circuit. On the other hand for those frequencies for which the length of the line is around $\lambda/4$ this line will behave like a parallel resonant circuit, this is for short circuited line and exactly opposite will happen for the open circuited case that for an open circuited line length of the line around $\lambda/4$ the line will behave like a parallel resonant circuit because input impedance will be infinity.

So in this case when the line length is $\lambda/4$ this will appear short circuit so it will behave like a series resonant circuit and if the length of the line is $\lambda/2$ then the line will behave like parallel resonant circuit. So depending upon whether I am using a short circuit or open circuit I can realize a series or parallel resonant circuit at different frequencies.

So invariably when we go to high frequencies the line can be realized for getting reactive elements into the circuit the line can be used for realizing the series or parallel resonant circuit into a high frequency circuit. Whenever we are having a resonant circuit the immediate question one can ask is what the quality factor of the resonant circuit is because the resonant circuit is always characterized by its quality factor. If you recall from the very basic definition of the quality factor the quality factor is related to the losses into the circuit higher the loss smaller is the quality factor. So whenever we have reactive elements like a inductor or capacitance the losses into these elements characterize the quality factor.

In this case however when we are doing a Transmission Line we have treated this Transmission Line as a Loss-Less Transmission Line. So by definition the quality factor for these resonant circuits is infinity because there is no loss. However in practice there is always a small loss in Transmission Line. Till now we have treated the low loss transmission line as Loss-Less Transmission Line because we were not really interested in the small loss.

However, now if we treat the line as the Loss-Less Transmission Line. We will not get the answer for the quality factor because the quality factor will be always infinity. So in this case when we are interested in finding out the quality factor of the resonant circuit no matter how small the loss of the Transmission Line is, we have to include the loss of the Transmission Line and then we can calculate the quality factor.

So we will continue the analysis of the resonant transmission lines and we will calculate the quality factor of the Transmission Line and then also we will see that these resonant

sections of transmission lines can be used for other applications that are for stepping up the voltages or currents in the high frequency circuits.

So the sections of transmission lines can also be used as the voltage or current stepping up transformers apart from its use as a reactive circuit elements and the resonant circuit elements.

Thank you.