Digital Communication
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Lecture - 06
Channels and their Models (Part – 1)

Hello, everyone. In this class we will discuss ((Refer Time: 00:55)) types of channels and channel models that we will come across in this course. In different phases of this course we will come across define types of channels; a most importantly different types of channel models we will discuss those models. And, we will also discuss what are the typical physical channels in reality that these models try to model. So, let us first discuss what type what are the different types of physical channels that through which communication takes place in reality and then we will discuss their models. So, what are the different types of channels we know of through which communication takes place in today's world.

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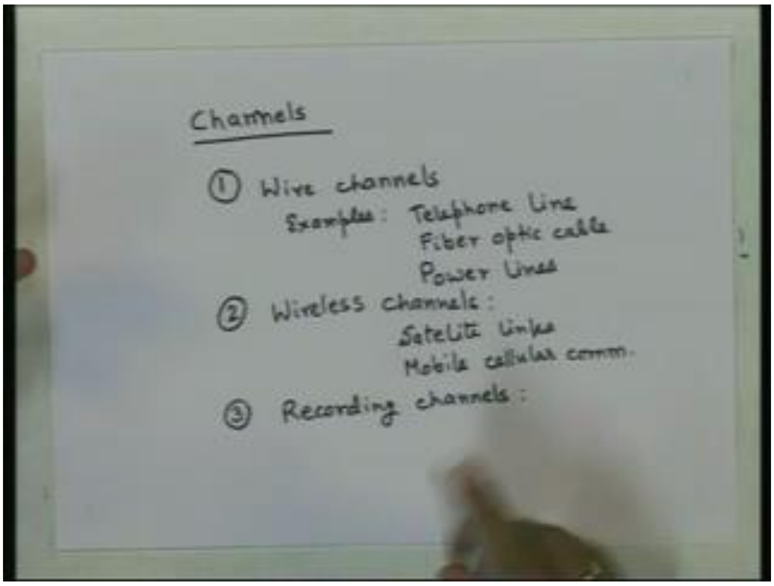

So, the different types of channel that we see; one is wire channels. Examples this is the most common and probably the oldest channels through which we have done communication. And, the most common example is telephone, telephone line, then fiber optic cables in todays world; third we have today digital communication also takes place through power communication lines. So, power lines. So, the of the wires that are used to take electrical power to different sides from the from the power station. Today we also use those lines to all to communicate. So, they are used for multiple purpose; the power the transferring power electrical power is the primary use of those lines. But at the same time lot of bandwidth is unused in those lines; and that bandwidth can be used to communicate through those lines.

Then, there is wireless channel again this is very common today. For example, satellite links, mobile cellular communication, uses wireless channels. So, here there is no wire connecting the transmitter and the receiver; the signal goes from the transmitter to the receiver wirelessly through space. And, there may be air there may not be air in the medium through which the signal goes. And, there are other channels which most of the times we do not think as channels but they can be considered as channels through which communication happens. For example, recording channels.

So, what are these; when you record something let us a say video or audio on a cassette or C D or magnetic discs; what we are doing is we are recording the signal on the media. So, that we can play back in future and so the signal is going in time from past to future; when you are recording we are it is equivalent to transmitting something and the channel is the recording media and the time. And, then when you play back it is like we are receiving the signal. So, in the mean time when the signal passes through time some corruption may take place in the signal. For example, on a C D there may be a scratch someone may put a scratch on the C D intentionally or unintentionally. And, when we play it back we may not hear the signal or we may not get the signal exactly as it was recorded. So, this is just like a channel just like a wireless channel or wire channel; it has it will have different types of different properties than the wire or wireless channels. But nevertheless it is a channel through which communication is taking place.

And, there are other things which can be considered as channels though we do not usually consider them as communication at all but they can be considered as channels. For example, we are writing on I am writing on this paper and this is being recorded and transmitted and you are seeing this image in the mean time lot distortion may take place; when I am writing I might my hand writing may not be legible even if it is legible may be that while recording or while editing or while transmission some corruption the signal that I wanted to convey takes place.

And, so this whole thing can be considered as a channel. So, this is just like recording channel except that there are may be added components in this channel; I may also do mistake while writing itself. So, I have some idea which I want to convey but while I am writing I may does I may write erroneously. So, I myself can be considered as a channel.

So, the idea I have in my mind that I want to convey is what needs to be transmitted. But what is received on this paper when I am writing is possibly different from what I wanted to write. So, this itself can be considered as a channel; in particular we can consider a typist typing on a paper and so let us say we have a typist.

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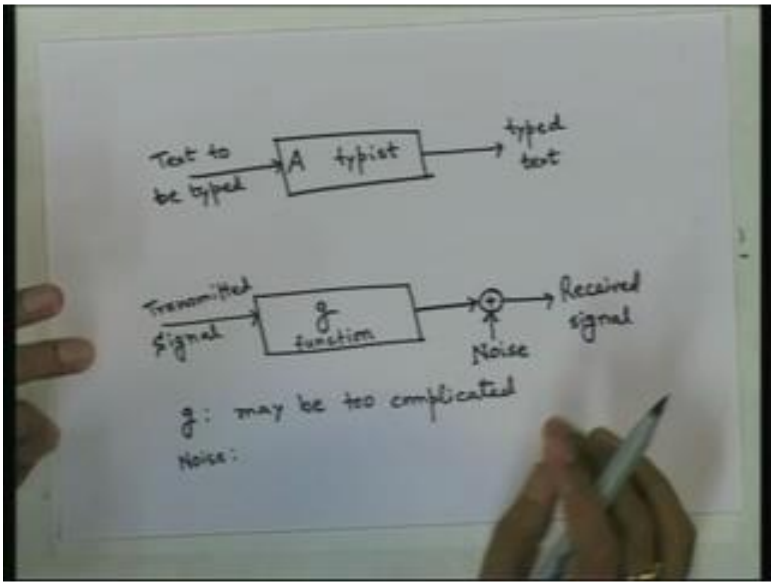

And, that typist wants to type something. So, he has some text to be typed and what he actually types may not be exactly what he wanted to type. So, the typed text is what is received on the paper. So, typist may introduce errors in the typed text and that can be; so the typist himself can be considered as a channel through which the text is going through. So, this is also a channel. So, channels we know have also all the typical channels we come across in reality have some amount of randomness; otherwise if there is no randomness in the channel our job has communication engineers will be much simpler.

So, for example if we knew that the typist has that the error that the typist a particular typist makes are known up ready; we know beforehand what kind of errors, what errors the typist exactly makes. Then, the person who is using the typed text he can actually know which locations or which letters are typed wrongly. And, so he can correct and we can considered that there is this the job is quite simple. And, even in the telephone line; if we now what kind of distortion the signal is going through. Then, at the receiver we can possibly invert the distortion because we know exactly what distortion took place when the signal was coming through the channel. And, so we can possibly recover the signal from the received signal; whereas if that the distortion that takes place when the signal goes through the channel; if the distortion has some random part then we have no way of knowing exactly what distortion took place; we may have know the distribution or some statistical properties of the distortion. But if we do not know what exactly the distortion was we will not be able to recover the signal back fully.

So, the difficulty comes because of randomness of the channels. And, most of the channels we most of the channels we use are can be considered to be of this type; we are transmitting some signal and the channel distorts the signal in a deterministic way by a function g; let us call it g, it is the function this is deterministic. And, this may be known at the receiver may not be known at the receiver or may be possible to estimate at the receiver. So, there are different situations on that. So, whether g is known g is unknown or g can be estimated may be with some estimation error.

So, whatever it is transmitted signal goes through some deterministic function; and then a noise is added and this noise is random then we receive the signal. So, it is because of this noise that so much theory for communication engineering is required. If there no noise then the only job for the receiver is to invert this function. And, if the inverse exits there is nothing else required; we can invert this function and put it in the receiver. And, then we will get exactly what was transmitted; whereas if the there is a random noise then we will not know exactly what the noise was when the signal was transmitted. And, so we cannot really subtract the noise from the received signal.

So, this g now the function g in reality may be too complicated and this noise itself; the noise also so we are considering additive noise this case even though the noise always may not be additive; there this is the simplest model we have. And, there is value for simple models for channels instead of considering the most complicated channel with most realistic channel in its all complexity; that has some benefits. So, we will considered now a additive noise; and we will assume that we know the statistics of the noise.

So, for g as well as for noise will consider simple models, different models some are simpler than the others; and why you do we considered simple models for channels? So, there are various reasons one is that it may be too difficult to consider a real channel to start with. Because it is too complicated and we do not know how to go about analyzing the system and designing signals, designing techniques for communicating through the channel if a consider too complicated a channel; whereas, if we start with simple channel we are able to analyze the channel well. And, we can develop techniques for communication through that simple channels; then that will also possibly give a some intuition about what kind of techniques will work even for the real channel; which is actually much more complicated than the simple model we are using. So, analyzing simple models can give us intuition about complex real systems in this case channels.

The techniques develop for simple channels may work well for even complex channels, complex real channels. And, solving any problem for a simpler channel by itself may be a step towards solving the problem for a more complicated channel. So, we will know what kind of techniques can be used to solve the problem for the complex channel; we will get some idea about it if we solve the problem for the simple model first. So, for all these reasons it is important to start with simple channel models and see. And, analyze those channels analyze what how much information can be transmitted through such channels; and also analyze how information can be transmitted through such channels. So, we will now consider different types of channel models.

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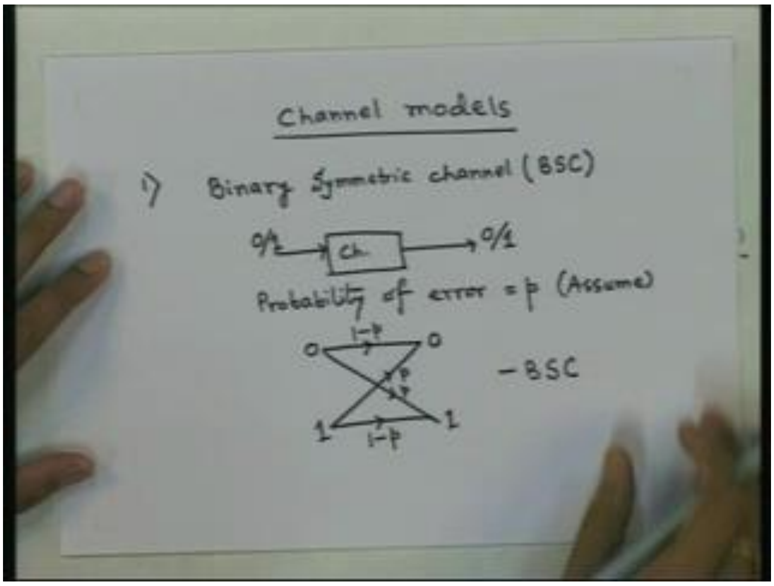

So, simplest channel model that one can consider is binary symmetric channel, abbreviated commonly as BSC; the channel model is the following if transmit either 0 or 1; this is the channel you transmit either 0 or 1 you will also receive 0 or 1. So, you whenever you transmit a bit you receive the same bit with certain probability; and you will receive the opposite bit with 1 minus that probability. So, if we assume that the probability of error is p. Then, we can represent this channel model with this famous butterfly diagram; the left hand side represents the transmitted bit, the right hand side represents the received bit. And, if we transmit 0 what is the probability that we will receive 0 itself; it is 1 minus the probability of error. Because the probability of error is 1 for the both bits; if we transmit 0 the probability of error is p, if we transmit 1 then also the probability of error is p. And, that is why the channel is called binary symmetric channel.

So, the probability of error is p. So, we will have p, we will write p here, we will write p here also to denote that if transmit 1 the probability that we will receive 0 is p; if we transmit 0 we will the probability that we will receive **1** is also p. And, if we transmit 0 the probability that we will receive 0 itself is 1 minus p; and if we transmit 1 the probability that we will receive 1 is also 1 minus p. So, this is binary symmetric channel; I would like to emphasize again that this is binary. Because both the input that is both the transmitted signal as well as the received signal is binary; you transmit 0 or 1 and you receive 0 or 1. And, symmetric the channel is symmetric because probability of error p is independent of whether we transmit 0 or 1. So, one can consider next generalization of this channel that is binary channel not necessarily symmetric. So, this is binary symmetric channel.

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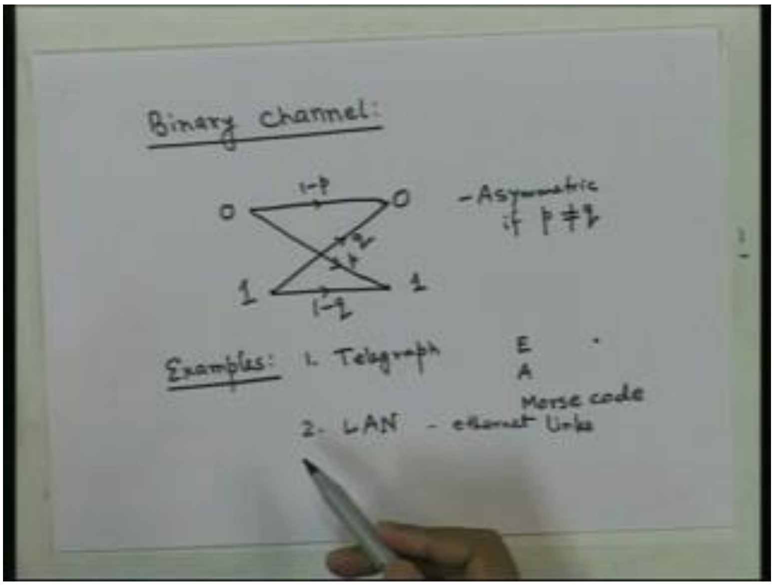

And, a binary channel in general will be may not be symmetric. So, we will transmit 0 or 1 and we will receive 0 or 1. But 0 will be received with error probability let us say p. So, this probability the probability of correct reception of 0 is 1 minus p and 1 may be received as 0 with probability q; and as 1 is itself with probability 1 minus q. So, this the general binary channel. So, this is asymmetric if p is not equal to q. So, what are the examples of binary channels most of the time they are binary symmetric channels. So, examples an old example may be telegraphic, telegraph channel. So, an old system for telegraph was to transmit English letters using dashes and dots.

So, every English letter will be represented by a sequence of dash and dot. For example, E will be represented by a single dot. Similarly, different letters A will be represented by something some sequence of dash and dot; just like 1 and 0. So, and so that is call Morse code. So, this is Morse code and there will be a an instrument through which these dots and dashes will be generated by placing a switch for a longer or shorter duration. So, the signal, the switch will be connected for either for a short duration or a longer duration and that will convey to the receiver whether this is the dash or dot.

So, it is like transmitting 0 and 1. So, dots and dashes are basically can be considered as different names for 0 and 1. And, so this communication can be considered binary because it is happening in terms of dots and dashes. In present day we have local area network, internet cables and binary signal is always going from one computer to other. So, that also Ethernet links and so these are some common examples. So, if we have for a binary channel if the probability of error is small it is good for us; if it is not small it is not so good; is it always true that if the probability of error should be smaller the better?

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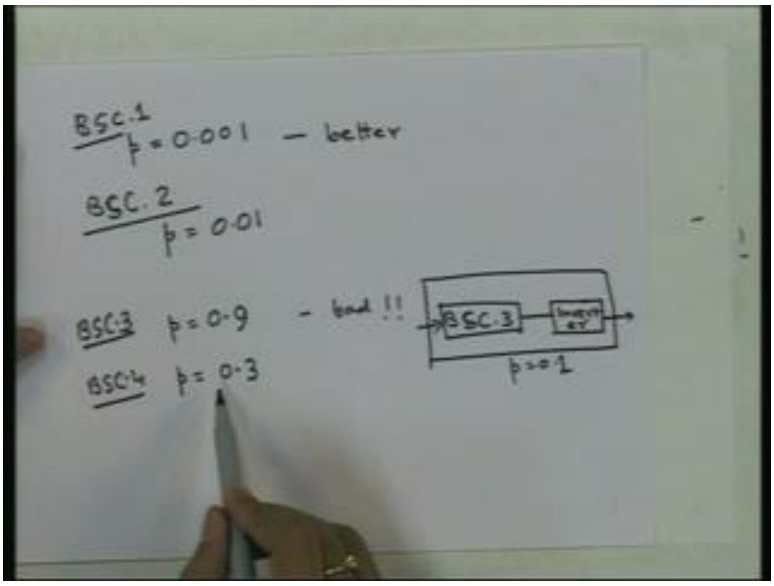

For example, can we say that if the consider binary symmetric channel consider p equal to 0.001 and there is another channel. So, this is binary symmetric channel 1 and binary symmetric channel 2 has p equal to 0 .01. Then, we know that this is better because this has less probability of error out of 1000 bits 1 bit on average 1 bit goes wrong. And, here out of 100 bits on average 1 bit goes wrong. Now, can we also say this p is probability of error as high as 0.09, 0.9 is worse than p equal to say 0.3.

So, out of 10 bits 3 bits go wrong here for this channel and for this channel. So, this is BSC 3, this is BSC 4; for this channel out of 10 bits 9 bits go wrong. So, can we say that this is bad, this is worse than this we cannot say that. In fact, this is better than this channel why? Because we can always invert the received bits in this channel; we know that the probability of error it is 0.9 for this channel. So, what we can do is at the receiver we can simply have an inverter which will pick the bit that was received. And, then for that channel what will be the what will be the probability of error? So, we have BSC 3, the channel BSC 3 and then we will implement 1 inverter. And, this whole channel we have probability of error 0.1; 1 minus 0.9. So, this channel will better than this.

So, this channel itself is more useful than this channel; because by using a simple inverter at the receiver we are able to convert it to a channel for which the probability of error is only 0.1. Whereas, the probability of error of this 0.3 and it cannot be improved by doing some invertible operation at the receiver like inverting the bit; this is the invertible operation. Because you can if you know the inverts of the bit you know the bit itself; you can invert again and then that will give you the this bit, the original bit.

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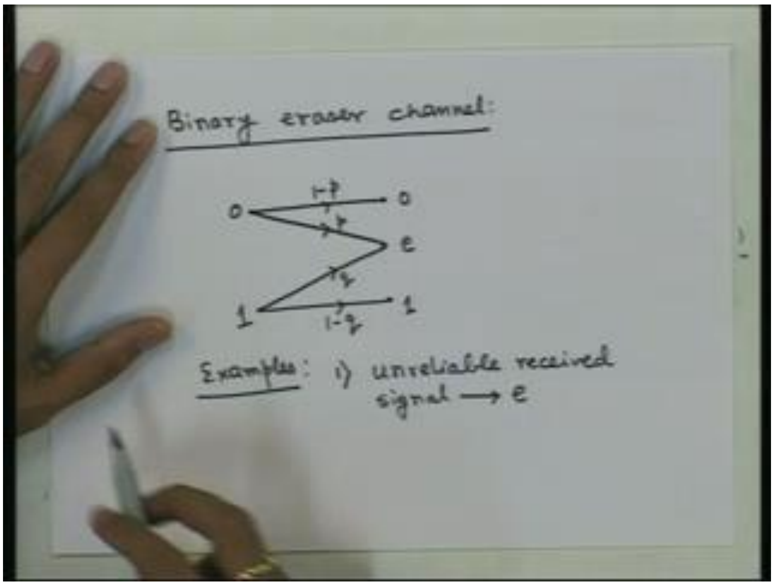

So, another channel which is common in digital communication is binary eraser channel. So, in this channel the bits are not received erroneously but bits are received either correctly or they are corrupted. And, the receiver detects that the bit is corrupted. So, when we say the bit is corrupted it is like receiving some value which is neither 0 nor 1; we are saying the bit is corrupted it is some it is. So, we have received a corrupted bit. So, either if we do not know whether 0 was transmitted or 1 was transmitted in that bit time. So, we receive if we transmit 0 or 1 we receive either 0 or 1 respectively or we receive a corrupted bit which is neither 0 nor 1.

So, we will call that e or sometimes x. So, e to denote that eraser; so it is erased. So, if we transmit it 0 there is probability that the bit will not be received correctly but it will be corrupted when it goes through the channel. And, the receiver will know that the bit is corrupted. So, then we will say that the bit is erased. So, we have no information about what was transmitted.

And, similarly even if we transmit 1 the bit may be erased. So, there is some probability.

So, this eraser probability may be again same for these 2 or may be different. So, we have some probability p, some probability q and 1 minus p, 1 minus q. So, this is call binary eraser channel; this has this comes up in different scenarios. For example, it we are transmitting bits and at the receiver suppose we are transmitting plus 1 for 1 and minus 1 volt for 0 or plus 5 volt for 1 minus 5 for 0. And, at the receiver we see whether the voltage level is above 0 or below 0; if the voltage level is above 0 we assume that plus 5 was transmitted if it is below 0 we assume that minus 5 transmitted. So, it is like but even if we transmit plus 5 there is possibility that the received level voltage level is below 0.

So, then we will have error but we can we can have a system where the receiver sees; if the voltage is level near 0; if it is near 0, if the received voltage level is near 0 it says well I have received the bit with which is very unreliable receive bit is received voltage level is near 0. So, that means I am not sure whether plus 5 volt was transmitted or minus 5 volt was transmitted. So, I will say that I will not decide on that based on that value but we will say that the bit was erased; we will assume nothing about that bit we will say the bit was completely corrupted. Whereas, if the value is away from 0 then based on whether it is positive or negative we will decide on 0 or 1. So, then we will have a channel of this type, binary eraser channel. So, one is if we decide unreliable received signal as e; if we represent unreliable received signal as eraser then we will get the channel of this type. Secondly there are some coding techniques; channel coding techniques where you first use a code we will see we will possibly discuss this in little more details in some other part of the course.

So, there are some coding techniques; where first a code is used there is a bit sequence that is generated based what information that is to be transmitted. And, then to save bandwidth some bits from that bit sequence are dropped are not actually transmitted. So, we transmit after puncturing some bits; it is called puncturing. So, you generate a bit sequence based on what message needs to be transmitted and then we punctured that sequence in some fixed locations; and then we transmit that punctured bit sequence.

So, we are not transmitting all the bits of that sequence. Then, when you receive what we do is we insert those puncture positions. But what can we insert they are not transmitted and as a result we have not transmitted any values in those locations. So, what is filled in is the symbol e that is eraser we assume that what was received was eraser; it was equivalent, it is equivalent to receiving a corrupted bit and then we decode the message from that. So, that is when we do puncture when we punctured a code then the channel can be equivalently set to be binary eraser channel. And, it can be also error and eraser channel if the transmitted bits themselves are possibly receive erroneously receive to it error. Then, there is possibility that those bits are received wrongly. So, then the channel is error and eraser channel. So, this previously we are considered binary symmetric channel, binary channel in general. And, now we are considering binary eraser channel; we can also consider combination of these 2 and call it binary error and eraser channel.

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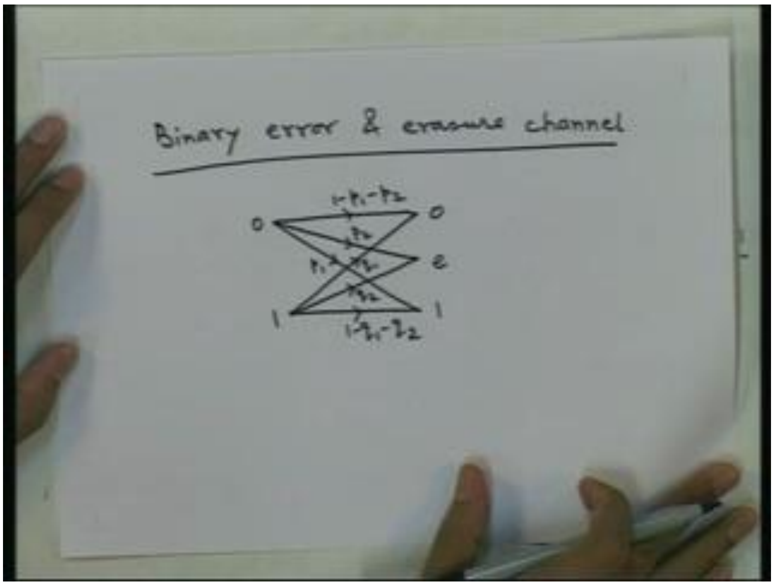

So, what is binary error and eraser channel it will have 0 and 1; so it this 0, 1 and e; there will be some probabilities associated with each of these edges, this transitions. So, this may be p 1, this is may p 2; and this may be 1 minus p, 1 minus p 2 and this may be q 1, q 2 and 1 minus q1, 1 minus q 2. So, this is binary error and eraser channel. So, this channel is also relevant is some applications in some techniques. Now, generalization of all these channels is discrete memory less channel; what is discrete memory less channel?

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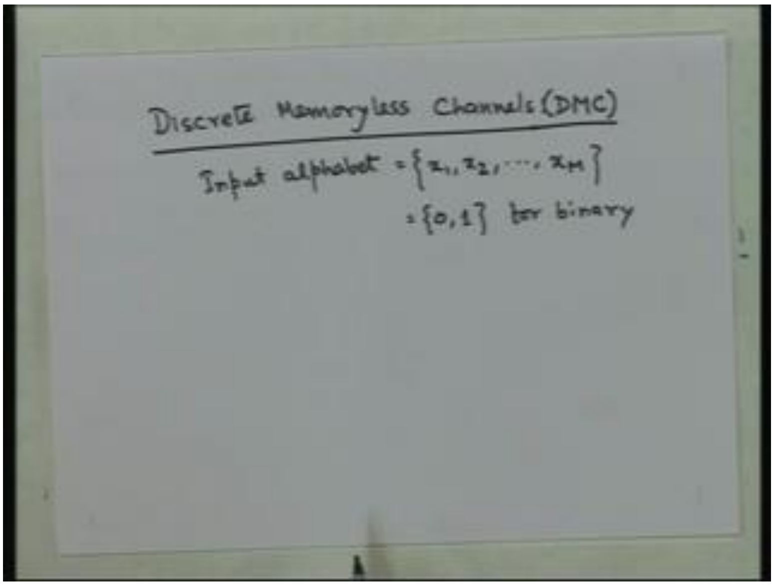

Discrete memory less channel commonly abbreviated as DMC; we will first discuss what is discrete channel and then we will discuss why it is called memory less? A discrete memory less channel has input alphabet a set. So, that is one of these elements is transmitted say x 1, x 2 till x m. So, it was like 0, 1 for binary symmetric channel for binary. So, this is the generalization instead of saying the channel is binary you are saying we will not transmit only 0 or 1 but we will transmit may be 0, 1, 2, 3, 4, 5 out of these 6 symbols. So, then is a 6 ((Refer Time: 38:51)) channel instead of it is instead of binary channel it is 6 ((Refer Time: 38:56)) channel. So, we can have sub generalizations of the channels of the binary symmetric channel.

So, in general for example if we have a consider the channel the that the typist channel that the typist is the channel. So, he has some idea he has some text in mind which he wants to type but he is prone to error. So, he makes error while typing. So, what is the input to the channel? The input to the channel is the letters of the English alphabet. So, there are 26 of then if you do not consider lower case, upper case, numerals, punctuations and all these; we can simply say it is 26 letters. And, the received symbol itself is set up 26 letters may be including space it is 27. So, we have an input alphabet of x 1 to x m. Now, these x 1 to x m will be then a, b, c, d till z and then blank space; there are 27 characters.

So, the input the alphabet is had either m possible symbols that could be transmitted. Similarly, the received alphabet is another set of symbols y 1, y 2 till y n. Now, you may wonder why this set is not same as this; but you should also get the answer immediately from what we have done so far. We have already considered binary eraser channel. And, what is binary eraser channel here we have input alphabet 0, 1; the output alphabet is 0, e and 1 there are 3 letters. So, the number output letters may be different from number input letters. So, we can have in general a different set of received symbols than the transmit alphabet. And, so a discrete memory less channel will be specified by input alphabet, received alphabet and the conditional probability distribution which gives which is the P the P probability of p y j given xi; it will denote the probability that y the received signal, this x is transmitted, y is received; this is the random variable, x this is the random variable y; x takes values from this alphabet, y takes values from this alphabet. And, p y j given x j denotes the probability of y equal to y j given x equal to xi.

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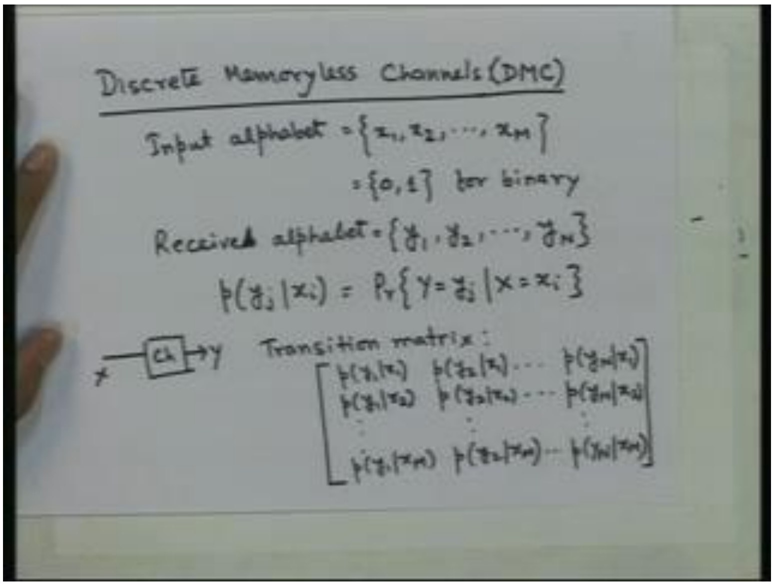

So, these 3 together will specify a discrete memory less channel. And, usually this these probabilities remember there is a transition; these are transition probabilities transition from xi 2 y j. Now, this transition probabilities that there are how may transition probabilities m times n. And, usually these transition probabilities are written as matrix.

And, this probability is the ij th element of that matrix; and that matrix is called the probability transition matrix. So, the transition matrix is what; it will have p y 1 given x 1 then p y 2 given x 1, p y n given x 1; and then we will have p y 1 given x 2, p y 2 given x 2, p y n given x 2. So, this is probability transition matrix. So, the discrete memory les is channel specified by the input alphabet, the output alphabet and transition matrix because this gives us the transition probabilities.

Now, why is it called memory less; it is called memory less because no channel will be actually used only once. Then, the what is use of that channel if you can only transmit once? So, all channels are used for sequential transmission sequential use one after another use of the channel to transmit more and more data.

So, this channel will be used many times. So, when we have we transmit 1 symbol then we transmit another symbol; what happens to the present symbol when it goes thought the channel does not depend on what happened to the previous symbol. So, what or what was received at the previous symbol interval. So, when you use the channel we use the channel last time or before; the channel changes symbols in a certain way. And, all those what happen to those symbols does not matter to what is going happen to the present symbol. And, that is that means the channel does not have the any memory; it does not remember its past action.

So, that is why the channel is called discrete memory less channel. It is discrete because the input alphabet is a set of finite number of elements, the output alphabet is also set of finite number of elements. So, it is discrete and it is memory less because it does not remember its past action. So, what happens to the present symbol does not depend on what happened to the previous past symbols. Now, this is the simplest discrete channel but there can be now different other aspects introduced in the channel which can make the channel more complicated.

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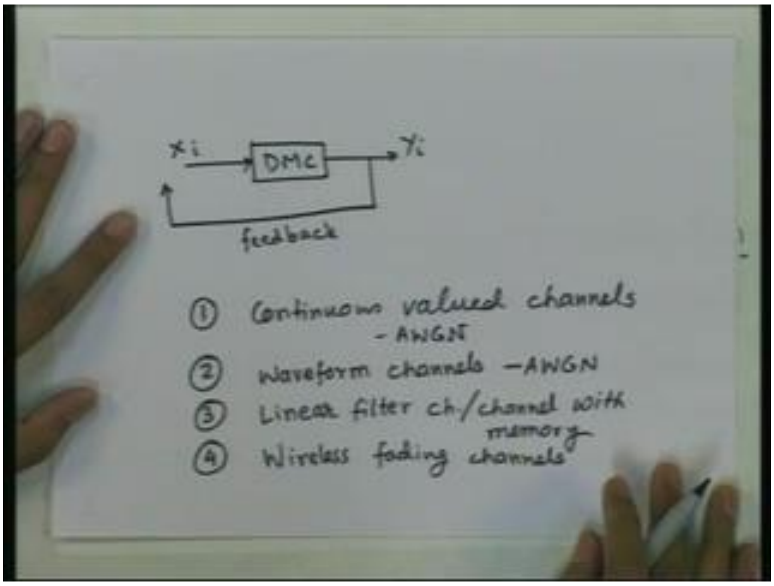

For example, we can now say that we not only have a discrete memory less channel. So, if you consider the i th use of the channel we can say that we are transmitting xi and y i is received for xi. Then, if we say there is feedback, so that means what is received is also available to the transmitter while transmitting the next symbol. So, when the transmitter is deciding what to transmit at the I plus 1 th symbol it has already known what was received at the receiver till the last symbol. So, it has all the information about what receiver received till the last symbol; when it is transmitting the I plus 1th symbol.

So, that is feedback. Now, with feedback is a question whether the channel you can transfer in more information if there is a feedback or it does not matter; can we transferring more information that is very important question. And, information theory which was answered by Shannon himself in the negative. So, he said you to prove that with feedback you cannot transmit more information than what you can transmit if you do not have feedback. So, channel with feedback.

Now, one can also say that the channel has memory. Now, if the channel memory then again that is a generalization. So, then can you possibly transmit more information; that is again a question. Now, we will consider various other channels; we will consider different other channels in the next class. But let us now summarize what all channel models we have discussed. And, we will also summarize what are the other channel models we are going to discuss in the next class.

So, in the next class we are going to discuss first of all or so far we have discussed only discrete channel; that is input alphabet and the output alphabet both require discrete that is finite set. Now, in the next class we are going to discuss channels for which the input as well as the output is infinite set it is continuous value. So, we will as you we will consider continues valued channel; in particular we will consider AWGN additive white Gaussian noise channel which we will be considering very much in the rest of the course; 2 continuous valued as well as continuous time are also called waveform channels. Because we transmit waveforms through such channels; again we will consider AWGN continuous time AWGN channels.

Then, we will consider linear filter channels where there is no memory; we will discuss in detail; linear filter channel or channel with memory and then we will consider wireless fading channels.

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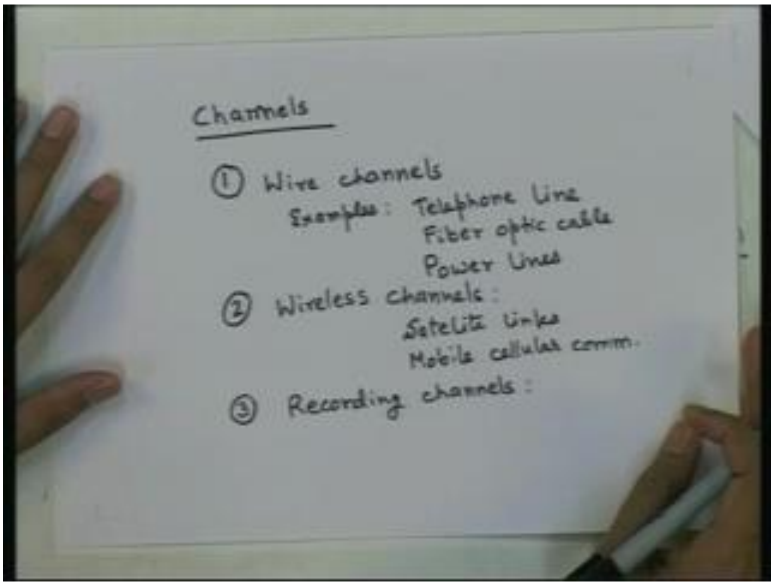

So, in this class what are we discussed? We have discussed so far different type of physical channels that we have come across in digital communication; we have come across wire channels for example, telephone line, fiber optic cable, power lines. So, which also communication can take place in the extra bandwidth that they have. Then, wireless channels for example satellite links, mobile cellular communication, T V, radio all these channels are wireless channels. Then, recording channels have magnetic media like tape or magnetic discs channels or magnetic disc or C D or even computer hard drive, hard disc they can be considered as recording channels; where you where the transmission is equivalent to whether the transmit transmission is basically writing in that media and reception is reading from that media.

And, in between writing and reading some distortion may take place in the channel. Then, we have considered the other channels which are little abstract they are not channels in the physical sense that we usually considered but they can be considered as channels in their abstraction. So, for example we have considered the typist who wants to type some text but he possibly types with some errors. And, as a result the typed text may not exactly same as what you wanted to type. And, so we can say that the text has gone through a channel to the typed text. And, then we said that all these channels that we have considered are have a general type; they are of this type that you are transmitting something it goes through a deterministic function; and then some random noise is added to the received signal.

Now, this is deterministic but this itself in some cases may have some random quantity like wireless channel we will consider in the next class. So, g and as well as the noise the additive noise statistics of the additive noise they themselves may be very complicated but we want to start with some simple models. So, we start with simple models. And, develop techniques for communication through the through such simple channel models.
So, we have considered binary channel, binary symmetric channel and binary channel in general. And, we then have considered binary eraser channel and in general next we have considered discrete memory less channel which is the generalization of all these channels. So, in the next class we will consider we will discuss some more channel models.

Thank you.