Digital Communication
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Lecture - 25
Equalizers

In this class, we will discuss equalizers which are used at the receiver to cancel the effect of channel and ISI introduced by the channel. To motivate the use of equalizers first recall let us recall we have done so far. We have discussed digital modulation techniques where, at the receiver we have assumed that that the successive the consecutive symbols do not interfere with each other. And we have also derived the condition for no inter-symbol interference. We will just revise what we did and from there we will start the new topic on equalizers. So, if we use a transmit pulse gt, gt is either: the baseband pulse or the passband pulse.

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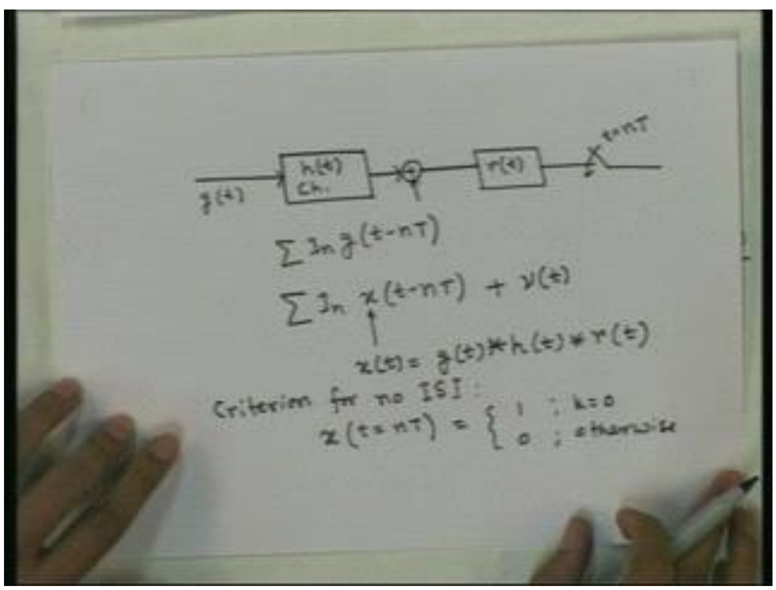

Once it goes through the channel with the impulse response ht channel filter and of course, there is noise. And then, at the receiver we implement some matched filter receiver. Now, this is the let us say this is baseband pulse and there is the received filter at the output of which we sample every t seconds. So, this is sampled at multiples of T seconds. Now, we are transmitting not only gt, but shifted versions of gt after multiplying by the information carrying values.

So, what we are transmitting is actually summation some information carrying values then gt minus Nt. But, when this signal goes through the channel and then it goes through the received filter what we receive is summation In then x t minus Nt where x t is now this plus of course, there is some the noise also goes through the filter the received filter and at output of sample that you have some noise. But the signal here is this plus some noise signal. And what is xt, xt is the convolution of gt ht and the received filter r t.

So, when you sample this signal every T seconds we will get not only the at the nT second that is at the nt sample, we will not only get In we may also get some contribution of the other symbols. If the xt is not chosen properly that is if gt and rt are not chosen properly to have xt satisfying certain conditions.

So, we have seen that for ISI free reception we need this to be 1 only for k equal to 0 and 0 otherwise. So, that the all the shifted pulses the xt is the pulse resulting from gt ht and rt and after gt goes through all these filters it becomes xt here. And xt should have zero values at the multiples of T seconds except at the 0 itself. Because at let us say t equal to T the next sample is taken here and that sample should be contributed by only the next symbol the previous symbol should not contribute anything to that sample.

From there comes the restriction that t equal to T at t equal to T the value of x should be 0. And similarly, at all other times multiple of T. So, this is called the Nyquist criteria and we have discussed already this criteria before in depth and we have seen, what it means in the frequency domain also. Now, it may not be always possible to enforce this criteria in practice for various reasons.

So, 1 such reason is that the channel itself may change with time and as a result it may be difficult to difficult to adaptively design gt based on what the channel is. So, if the channel changes slowly or fast with time the gt for ISI free transmission needs to change with time. And that requires difficult dynamic design of the pulse gt. So, that involves first of all the channel needs to be estimated at the receiver well. And then that information needs to be transmitted back to the transmitter and the transmitter needs to design gt and use that gt for transmission.

That is a difficult thing to do in general. So, often what is done is if the channel is changing with time you may not consider channel you simply consider rt and design gt accordingly and work with some sort of optimal system. It may not be exactly ISI free. Then we try to remove the ISI in other manners which we will discuss in this class. So, let us say let us first formulate what will happen if we do not have ISI free transmission. What is the exact form of ISI at the receiver.

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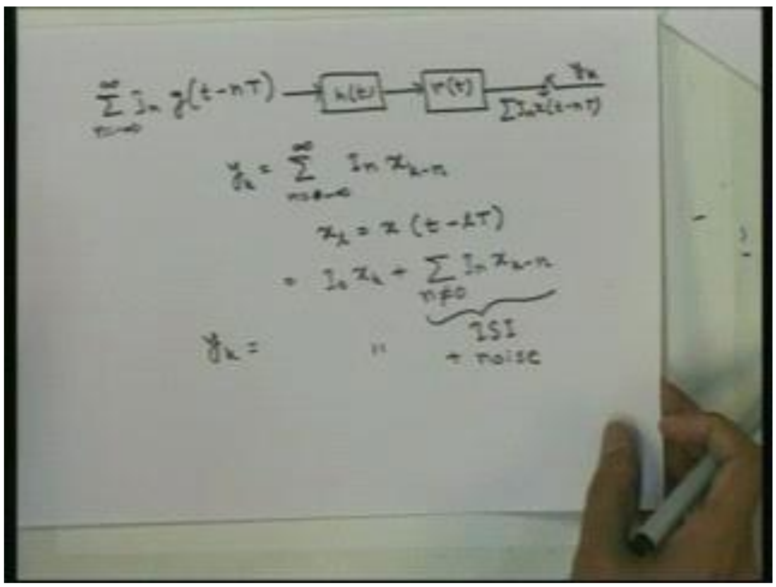

If we have if we transmit summation In gt minus nT n equal to minus infinity to infinity this is transmitted through ht then it goes through rt we are not drawing the noise here, but noise is there. We are not considering what will happen to the noise at this movement. So, here what we have is summation In xt minus nT and then we have sampling. So, these samples let us call them x let us call them yk the k th sample is yk then what is yk yk is this signal sampled at kt. So, that will be summation n equal to 0 to n equal to minus infinity to infinity and In xk minus n. Where this is nothing, but so xl is nothing, but x sampled at lt th time lt seconds.

So, this is what we will have at the output of the sampler this has contribution from all the symbols. And ideally we would like to have only I naught some non-zero value and all the other all the other components here should be 0. So, that the 0 th symbol is not interfered by the other symbols which came before or which will come after the zero th symbol. So, this is basically I 0 xk plus n naught equal to 0 In xk minus n. So, this is the ISI part. So, this actually yk will be this plus noise. And this actually looks like this formula looks like as if xk the discrete time signal xk is going through a filter and this is the convolution.

So, the equivalent model for this whole system this from here if you consider In is being and the after modulation it is going through ht rt and then sampling. Then the discrete time signal time we are getting here looks like convolution of the sequence In with some other discrete time filter impulse response.

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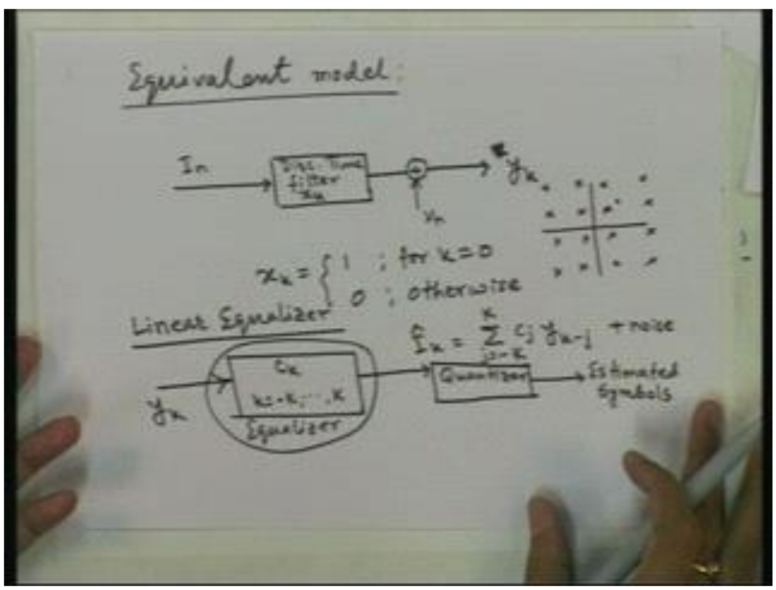

So, the equivalent model of the system becomes In and there is a discrete time filter with impulse response xk and there is some noise discrete noise. And here we have yk and our job at receiver will be to cancel the effect of this filtering and recover what was transmitted.

So, ideally we would like xk to be 1 only for k equal to 0 and 0 otherwise; that means, the discrete time filter should have delta as the impulse response. But it may not be possible to have such an impulse response here because, that will be difficult to design because gt will require will be required to satisfy some conditions depending on the present state of the channel. And design of gt may not be so, easy to do satisfying this condition.

So, after having this, we even if we have some non-ideal filter here having arbitrary impulse response how to get this In sequence back from yk, is what we would like to investigate. So, what do we do from this is that we in general we call a system through which this yk will be passed at the receiver. So, this is what we are considering now is called linear equalizer where the way we recovered we tried to recover In from yk is by passing yk through a discrete time filter.

So, it does some linear operation on yk. It combines some yk's by the impulse response of the filter by doing convolution. So, after passing yk through the filter and let us say this filter has impulse response nonzero Ck from the range minus k to k and then suppose we receive here Ik hat. We want this Ik hat to be same as In, but it may not always be possible what may happen is that there may be still some ISI remaining or there will be certainly the noise added to this some function of the noise which is the noise itself.

So, some different noise and this Ik is nothing, but j equal to minus k to k cj y k minus j plus noise. And what is done after this is this is passed through the quantizer basically the detector for the constellation. So, what does this quantizer do this is an estimate of the value transmitted the value transmitted in In the value of In is an estimate of that, but In was actually a value peak from a constellation. It may be if it is real value it may it will be possibly from a PAM constellation or if complex valued it may be a 2 dimensional constellation from where it was chosen.

So, the quantizer will pick the nearest constellation point from Ik. So, if you had sixteen qm constellation if it is was a complex constellation. This Ik hat may be a point somewhere here and this quantizer will chose the nearest point; it will quantize to the nearest constellation point. That is what the quantizer does. And this is the estimated symbols. These are the estimated information symbols.

Now, the question is how to design this system. This is the equalizer this is the linear equalizer and we want to design this system and how do we this. So, there are different ways of the designing this system depending on what criteria you impose on these symbols. What is the criteria for the performance of this equalizer. When do you say that, 1 equalizer is better than another. So, based on that criteria we have to design this equalizer. And there are very standard 2 techniques 2 criteria's used very much in practice 1 is called peak distortion criteria.

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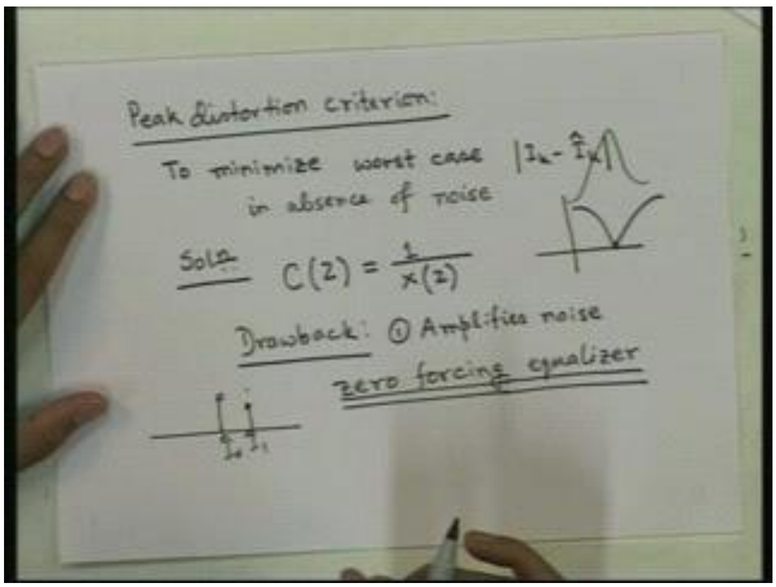

Peak distortion criteria and what it tries to do is to minimize the worst case error that is the absolute value of Ik minus Ik hat. So, we will not consider the noise part this noise is random we will not bring that in this criteria. What we say is that, this is the estimate of the transmitted symbol In Ik and we would like to see what is the maximum difference Ik hat and Ik can have that will depend on ck. And we would like to design ck so that, that maximum difference between Ik and Ik hat is minimized.

So, it will be maximum Ik hat minus Ik for a given selection of the equalizer will be maximum for certain choice of the symbols input symbols In there is I 0 I 1 I 2 and so on. So, this sequence when this sequence has a certain value then this Ik hat minus Ik will be maximized and that worst case difference we want to minimize. So, that is the peak distortion criteria. So, this we want to minimize this in absence of noise. So, what is the solution if we reach if we take this criterion?

If you take this criterion it can be shown that the optimum solution for the equalizer for the equalizer filter which minimizes this quantity is that filter whose Z transform is the inverse of the equivalent channel filter. That is we have seen that this is this yk depends on In in a way which is equivalent to passing In through the through a filter discrete time filter xk. When this impulse response xk has a Z transform and that is the; that is the Z transform of the impulse response. From which you can get frequency domain characterization of the filter frequency response of the filter and what we want to take here is the inverse of this filter.

That is 1 by xz, xz is the z transform of xk and cz which is the z transform of ck will be chosen. So, that it is 1 by xz. And this is also intuitively a good selection good choice for the equalizer filter because what it is saying is that what it is trying to do is to invert this filter. Here we are putting a filter we are passing yk through a filter which is inverse of this filter. So that, we get back this sequence In at the output except for the noise. This noise also will go through the same filter and at the output of it there will be some other sequence of noise. And at least the signal part will be recovered fully there will not be any inter symbol interference left between the output samples.

So, that is the inverse filter solution for the linear equalizer. And though this completely removes ISI it has 1 draw back, the draw back is that it amplifies noise. It amplifies noise in some bands wherever this discrete time filter has very high attenuation. So, that if you consider the frequency response of the discrete time filter xt xk. That is if you consider the frequency response of this filter xz then if it has let us say some very high attenuation here then the cz will have very high game in this frequency because, it is just the inverse filter of this filter.

So, it will have something like this and as a result the noise which is introduced here and which is going through that equalizer filter will be amplified very much in this part wherever this filter xZ has very high attenuation. And that and that amplification too much amplification of the noise in some bands actually creates problem because the overall noise itself increases because of that. So, it amplifies noise otherwise it cancels ISI completely. So, the contribution of neighboring symbol say if you plot I naught here this is I naught this is let us say I 1 and so on. At the output of it you get this plus some noise there is no contribution of I naught in I 1 hat.

So, if you estimate this I naught I 1 hat there is no contribution of I naught on I 1 hat and vice versa. So, there is 0 interference; from the neighboring symbols. This is referred as, 0 forcing. So, that is why this equalizer is also sometimes referred as 0 forcing equalizer which means that, by this equalizer there is 0 inter symbol interference.

So, completely eliminates interference between neighboring symbols, but it has the draw the back that it amplifies noise in high attenuation frequencies; at the frequencies where the xz has high attenuation. Now, another criterion which is also very commonly used is the MSC criteria, that is minimum mean square error criteria.

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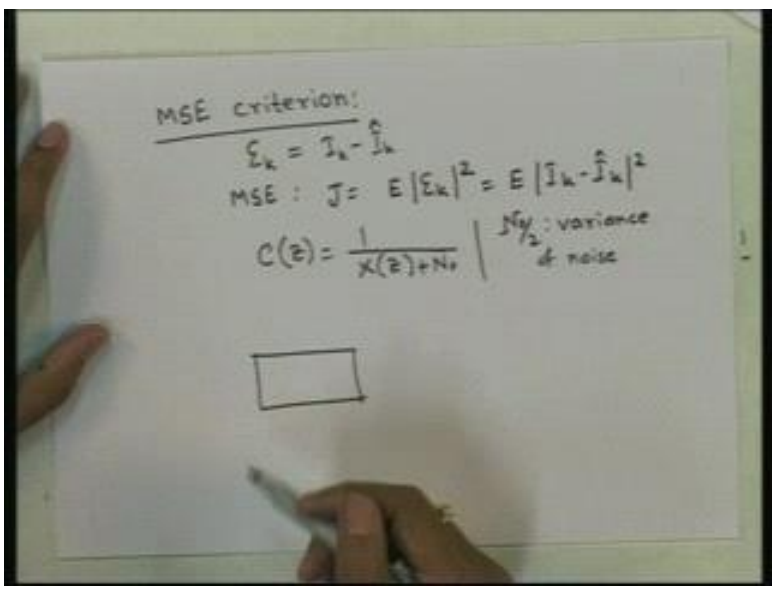

What this says is that instead of instead of neglecting noise while designing the equalizer filter which we did in this case if you remember we actually did not consider noise at all while designing the system. We said they we want to minimize the worst case error in a absence of noise. So, we did not consider noise at all. And that ultimately resulted in a solution which amplifies noise too much in some bands. So, if we want to avoid the problem we actually need to consider noise as well as interference together and design an optimum system.

So, what criteria should we take for that? A very simple criteria is minimum mean square error criteria. Which says, that if this is the error for the k th symbol that is Ik minus Ik hat is Ek in the peak distortion criteria we wanted to minimize the absolute value of this maximum absolute value of Ek. But now, we want to minimize the mean square error which is which is the expected value of Ek square. So, this is the square error square error Ek mod Ek square is the square error and because this is no more random there this is no more deterministic there is some noise. We have to take expectation of this and we want to minimize this error which is nothing, but Ik minus Ik hat square.

So, this is the mean square error and we want to minimize this quantity. And the solution which comes if we want to minimize this quantity while designing the equalizer filter is a slight variation of the 0 forcing equalizer it is given by 1 by xz plus N naught. Where, N naught is the N naught by 2 is variance of noise that is criterion noise or equivalently N naught by 2 is the power spectral density. So, this is the filter which minimizes the mean square error between the transmitted symbols and the received symbols.

Now, both these filters are a single linear filter both these equalizers they are basically a single linear equalizer. Which is designed based on different criteria's: one is designed by taking peak distortion criteria the other is designed by taking the mean square error criteria. And we get 2 different solutions if we design based on 2 different criteria's, but this is not the only way 1 can design an equalizer itself. Another equalizer which is very commonly used another structure that is commonly used is decision feedback equalizer.

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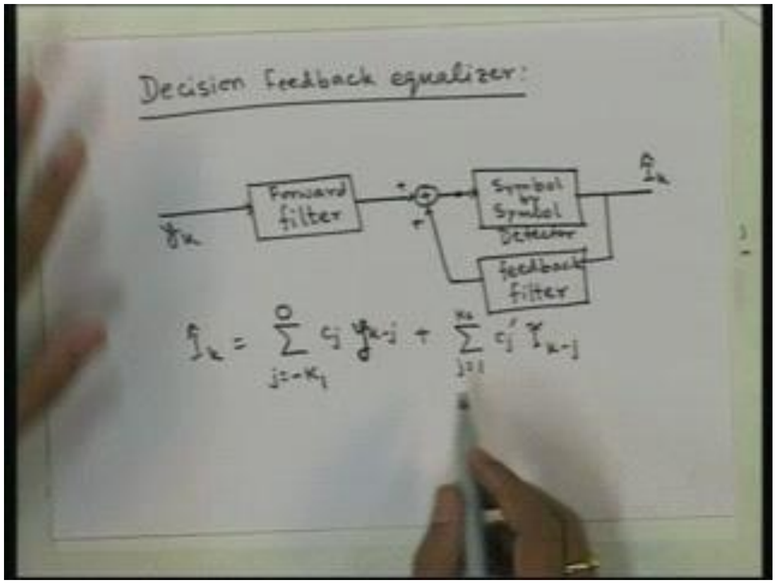

So, what is the philosophy of this? The philosophy of this is that when we consider a linear equalizer here after we have received some symbols here. And we have also decided after quantization what was transmitted from the constellation point then even though there is some interference here from those symbols which we have already received. We are not using these received values these values which we have already detected here to cancel the interference from the future symbols which are coming by this points.

So, in other words, when we have received here we have already decided the values for I naught I 1 I 2 and suppose we are receiving the future symbols here the interference caused by I naught I 1 I 2 is known to us because we probably have estimated this xk, this filter. So, we will be able to determine what is the contribution of I naught I 1 I 2 on yk for the future symbols. And then we can cancel the effect of I naught I 1 I 2 from the future values of the samples that we are passing to the passing to this equalizer. And as a result we will be getting a better estimate of this symbols the In’s here itself.

So, possibly we will need a smaller length filter because this k itself is also a parameter which determines the complexity of the equalizer. We would like to have as small K as possible. So, if we cancel the effect of previously detected symbols from the incoming signal then it may be possible to equalize this stream with a smaller length filter here. Whose job will be now to cancel the effect of future symbols from the past symbols only? So, which is basically not really the effect of future symbols on the past symbols but which is taking help of the future symbol to detect the previous symbols because, the previous symbols will have some effect on the future symbols.

So, we can still use a filter here even after canceling the effect of previous symbols from the future samples. We can still use a filter here so that, we can cancel the effect of future symbols cancel the we can use the information that is available in the future symbols to get a better estimate of the previous symbols. So, we will have 2 filters: 1 is the feed forward filter which is the forward filter this is and then we will have of course, this detector symbol by symbol detector. And then, we will have 1 we will have the estimates of the symbols Ik the information symbols.

After getting these values what we said is that now the samples we are receiving here have some effect of this symbols which we have already detected. So, we will derive the effect of this on this signal and; that means, which is basically saying that we will pass this Ik hat through the filter xk itself. Because, this In sequence actually pass through xk to give us this stream this signal. So, we will pass after detecting In we will pass it through the same filter and cancel it from the received signal.

So, how to design this filter exactly how to design this filters this filter also exactly we will consider now. But, what we want to put here is that this origin the fundamental structure of the equalizer is that from this signal we will pass it through a filter and then subtract it from this signal. So, we are receiving yk here we are passing it through a forward filter, but at the end of it we are also removing the effect from from the previously detected samples.

So, we will design these 2 filters together so that, the error here is minimized. That is, we get a Ik hat which are almost same as Ik. So, what is the Ik hat here Ik hat is j equal to minus K 1 2 0 cj. So, if you consider the signal here this is some this is the quantization is also happening here, but we will not consider that. We will take this signal this signal is sum of 2 signals it is negative, but you can say that this filter impulse response itself is negated. So, that this is plus itself.

So, this becomes cj prime I tilde k minus j. So, this is we can assume that this is going through this and this is yk yk minus j and this is passing through this. So, the 2 filter outputs are taken and they are added to get Ik hat. Now, again how to design these 2 filters is the question and again we can have 2 different criteria's for designing the filters.

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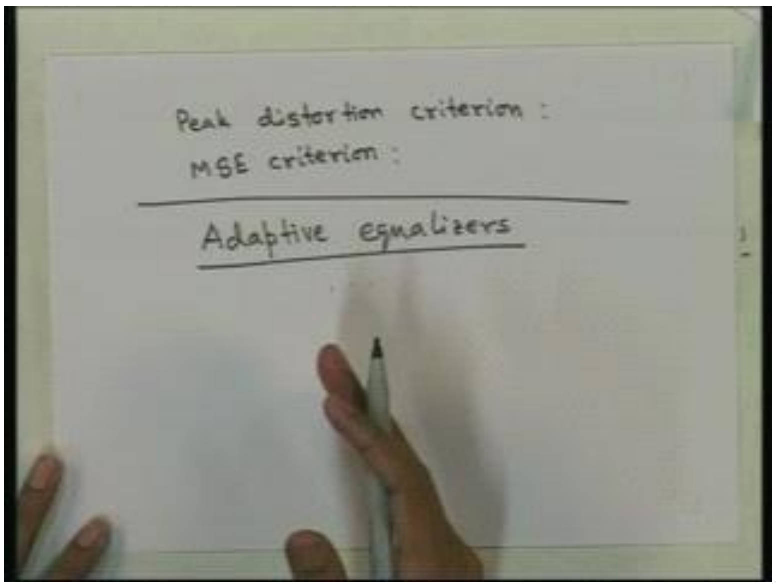

One is peak distortion criteria which will give us 1 solution and another is MSE criteria mean square error criteria; which will give us yet another solution. So, what we are saying is that we will have these 2 filters where the decision previous decisions are feedback to the decision process of the future symbols. And there is the forward filter as before, but hopefully we will with need a smaller length filter here because, we have canceled the previously detected symbols from the future symbols.

So, there is less ISI in the signal because there is some more cancellation going on even after this. So, we can afford to leave some ISI here which will be canceled by this contribution. So, this is the decision feedback this is the previous decisions are feedback to the decision process of the future symbols. So, that is why this filter is called the decision feedback equalizer. Now, as we said these 2 filters now needs to be designed. Need to be designed based on some criterion and just like feed forward filter we have 2 different criteria's in this case also: one is peak distortion criteria the other is MSE criteria that is mean square error criteria.

So, by these two different criterions we will get 2 different solutions for the 2 filters. Now, this these are some techniques for equalizer design there are many other techniques and there are many other variations of the same ideas which we will not discuss in this course. There is also another important concept and that is to design the filters adaptively. So, as we said the channel in real life may actually change with time and we as a result may like to design the filters adaptively as the channel changes we will like to change our equalizer filter also.

So, there comes the adaptive equalization. And adaptive equalization we will not discuss in detail, but we would like to just mention that there is something called adaptive equalizer. And again we can have different criterion different criteria's for designing adaptive equalizers different criteria's will give us different types of solutions. And there are various ways of designing adaptive equalizers. Some are more complex than others some use 1 criteria some use another and so on.

There is all together a different approach to estimating the transmitted information stream than designing equalizers and trying to cancel the equalizers. So, the or the whole problem itself can be can be formulated as that of a sequence estimation.

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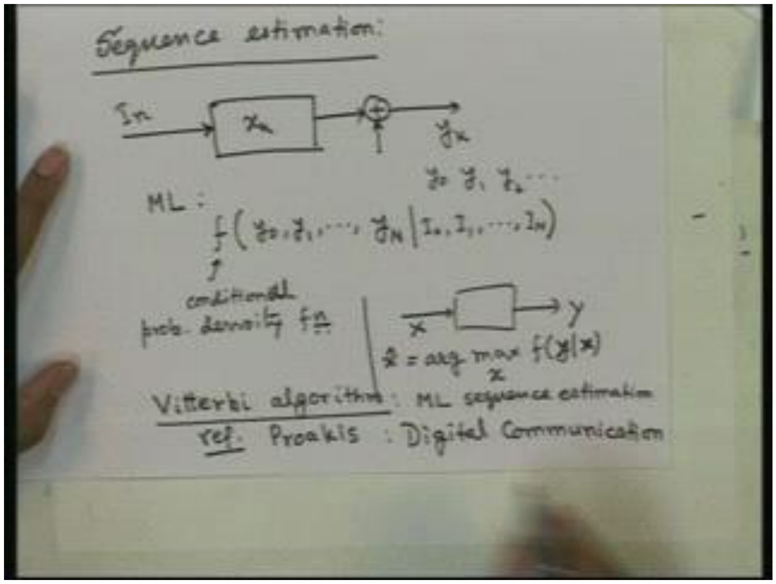

So, what is that as you said we have a sequence In that is equivalently passed through a linear filter Xk, then there is some noise. What if we know this filter and we know the characteristics of the noise and we know what we receive of course, that we will always know. We are receiving this sequence and we want to estimate In from yk that is the original problem.

And in the equalizer design we have considered a simple structure for the processing we do after this and that is we said we will either use a linear filter here or then do quantization. Or we will have a decision feedback equalizer which is again done by implementing 2 filters 1 is forward filter another is feedback filter. But we have not address the problem with its originality.

And that is we want to estimate In from yk and we want to first say what is the criterion for estimating. When do you say some estimator is better than another way of estimating and 1 can say 1 very useful criteria is ML estimation which we have used for symbol by symbol detection for ISI feed transmission. When there is no ISI when you do symbol by symbol detection we have used ML estimation for Gaussian noise case. We have seen that that is the equivalent to picking up the constellation point which is nearest to the received symbol received signal. That can also be the similar formulation can also be setup for sequence estimation.

We want to basically choose a sequence In after we have received a sequence yk that is y naught y1 y2 and so on. We would like to choose that In that sequence of In which is most likely. So, we can compute the probability for which this receiving this is most likely; that means, compute the density function conditional, conditional density probability density function of y naught y 1. Let us say we receive till N, N may be 1000 may be 1 million very large very long sequence.

We observe a long sequence and then ask the question for what sequence of transmitted symbols that is I naught I 1 IN for what sequence of transmitted symbols this density function value is maximum? So, for what transmitted sequence this is the probability of this not really probability the probability density at this value of the sequence is maximum that is the maximum likelihood estimation.

We know because originally we in another context of detecting symbols we said that you are transmitting 1 random variable x and we are receiving 1 random variable y. Maximum likelihood estimate of x is the value of x which maximizes f y given x. The density of x given y given x evaluated at the received value. So, we want to maximize this that is the maximum likelihood estimate. So, that same concept is used here in terms of sequence we have say we transmitted 1 N length sequence and we have received 1 N length. And we want to have the ML estimate of the transmitted sequence from the received sequence.

So, we are saying our ML estimate is that transmitted sequence for which the density function; conditional density function of this sequence given that transmitted sequence is maximum, that is the maximum likelihood estimate. And in general it is very difficult to perform this ML estimate because this is a very high dimension problem. If you let us we say we are possibly transmitting hours after hours hour after hour and as a result we will this N will be very large.

So, how do we do this? This is too complicated a function that, there are so many variables says 1 million or a 10 to the power 100 variables. So, how do we choose that so many variables from a very long sequence. And also can we wait to receive all 1 million symbols before we decide even the first symbol that is what this says we should do. We should receive all the N symbols and then only we can decide all these together including I naught.

So, to decide what was transmitted at the 0 th symbol interval we need to wait for all these N symbols to be received. And that is too much delay to be afforded in practice, but it can be shown that there are iterative ways of doing this. So, in other words, you may not you will not need to wait all wait to receive all the capital N symbols to decide on even the first symbol. So, there is iterative ways of doing it.

So, where after receiving hundred symbols you will able to estimate let us say 5 symbols 5 transmitted symbols. Then after receiving 150 you will able to decide above 50 symbols. So, there will be some lag there will be some fixed delay, but you will be able to estimate more and more symbols as you receive more and more.

You do not have to wait to receive the whole block of symbols before you decide even the first symbol. So, that famous algorithm which can be used to do that is called the Vitterbi algorithm for ML sequence estimation. If you want to know more about Vitterbi algorithm for ML sequence estimation please look at the book by Proakis; reference is Proakis Digital Communication. And this Vitterbi algorithm the same concept is not only useful here, but it is also useful in decoding of a family of codes called convolutional codes.

It is also it can also be used for block codes, but it is specially famous for convolutional codes where basically something like convolution happens the something which is happening here in the channel and the whole system equivalent system. There are coding methods where you pass the bits the digital signal through a digital filter at the encoder at the transmitter and you then transmit. And after receiving after receiving the sequence you can estimate the transmitted sequence by using Vitterbi algorithm.

The problem because the problem is again same you have some transmitted sequence which is passed through a a filter and you receive the sequence and you want to do ML estimation of the transmitted sequence from the received sequence. So, again you do not want to wait for a very long time before you even detect the first symbol. So, there needs to they there is requirement for a an algorithm which can give you an estimate of the first symbol after receiving a reasonably good number of symbols. And then as you receive more and more bits you would like to estimate more and more transmitted bits.

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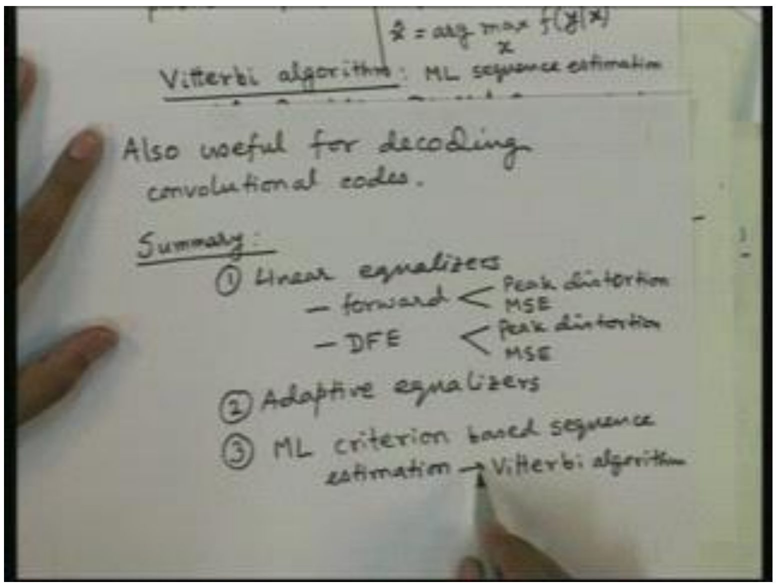

So, there again Vitterbi algorithm is used so, that class of codes are called convolutional codes. So, we have discussed in this class equalizers linear equalizers. We considered feed forward equalizers and decision feedback equalizer commonly abbreviated as DFE. And in each case we have considered 2 criteria's for designing the equalizers: one is peak distortion criteria and the other is MSE criteria. And then, we have mentioned briefly about first adaptive equalization where, you adaptively change the equalizer based on the way the channel is changing.

So, that is adaptive equalizer. And third altogether a different approach for estimating the input sequence from the output sequence by according to a an overall criteria. And ML criterion is so popular, ML criterion based sequence estimation. And 1 very popular method for this is Vitterbi algorithm. Not only this, Vitterbi algorithm for any of the topics please refer to Proakis Digital communication.

Thank you.