# Digital Communication
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## Lecture - 21
## Digital Modulation Techniques (Part –10)

We have been for many classes now; in this class also we will continue discussing the same topic. So, far we have covered all the memory less modulation schemes we wanted to discuss in this course. And we have also qualitatively discussed the relative benefits of different modulation schemes and their decoding techniques.

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We have discussed also the genetic decoding technique for any modulation any signal set and before going to the modulation schemes with memory. In this class and in the next class we will discuss some other topics related to memory less modulation schemes themselves. One most important thing is the analysis of probability of error for a given signal set that also we will discuss before we go memory modulation schemes with memory. In this class we will start with what is called eye diagram where we can see on an oscilloscope.

How much interference is there between, how much inter symbol interference is there in a qualitative way. So, we will now start discussing eye diagram. We have seen that, for a pulse amplitude modulation system, biz band pulse amplitude modulation system. The there is first a pulse PT which is fixed at the transmitter side then it goes to the channel and then at last it goes through the goes through the receiver filter at the receiver.

And when the PT the pulse PT goes through all these filters first the channel filter and then the receiver filter it undergoes some change due to convolution with the channel impulse response and the receiver filter impulse response.

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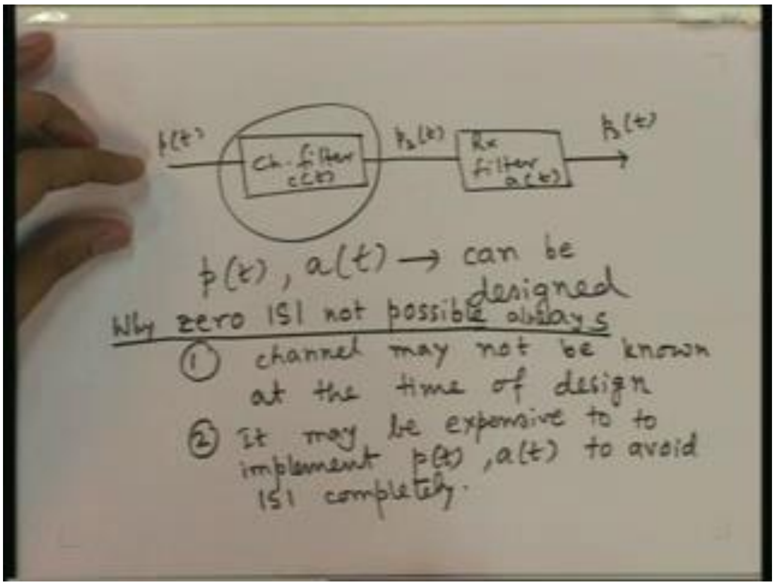

$p(t), a(t) \rightarrow$ can be designed

Why zero ISI not possible always

① channel may not be known at the time of design

② It may be expensive to to implement $p(t), a(t)$ to avoid ISI completely.

So that, so the diagram we had is this. This is the channel filter, we are not considering the noise which would have come here after channel filter and then it goes through the received filter. This is CT this is a t. So, we call this P2 t and this as P3 t. Now, we have also discussed the condition to have 0 ISI. What is the condition on P 3 to have 0 ISI that was given by Nyquist criteria? And now, the problem is that we cannot always satisfy Nyquist criteria. We cannot all this first of all the channel is not in our control, channel is not in our control it is given to us, only thing we can change is p t and a t.

So, p t and a t can be designed, but not the channel. Now, of course, we will always try to design p t and a t such that the pulse P3 t which is the convolution of p t with c t and a t satisfies Nyquist criteria that will avoid ISI completely. But, it is not always possible to avoid ISI completely, that is it is not always possible to design a t and p t in a such way that the pulse P3 t will satisfy the Nyquist criteria. Because, first of all the channel may not be known at the time of design of the filter.

So, channel first reason is that channel. So, this is why 0 ISI not possible always. That is; because, first of all the channel may not be known at the time of design. Secondly it may be too expensive to implement p t and a t which will avoid ISI completely. So, it may be expensive too expensive to implement p t, a t to avoid ISI completely. So, we can design p t and a t to. So, which will avoid ISI completely, but it is, it may be that the design p t and a t are too difficult to implement. In that in that situation we would like go for a suboptimal p t and a t which will not which will have some ISI, but which will not hopefully result in too much ISI.

So, now once we have p t and a t how do we know in practice how much inter symbol inference is there. First of all the channel may change. So, as channel is changing is there a way of observing, how the ISI is at any time like channel may change for example, for wireless channel which under which is continuously changing if we go in a car or in a train. If it moves very fast, if we very fast the environment around is changing and as a result the channel. Wireless channel is changing.

So, in that situation can we observe how much ISI my communication system is facing?

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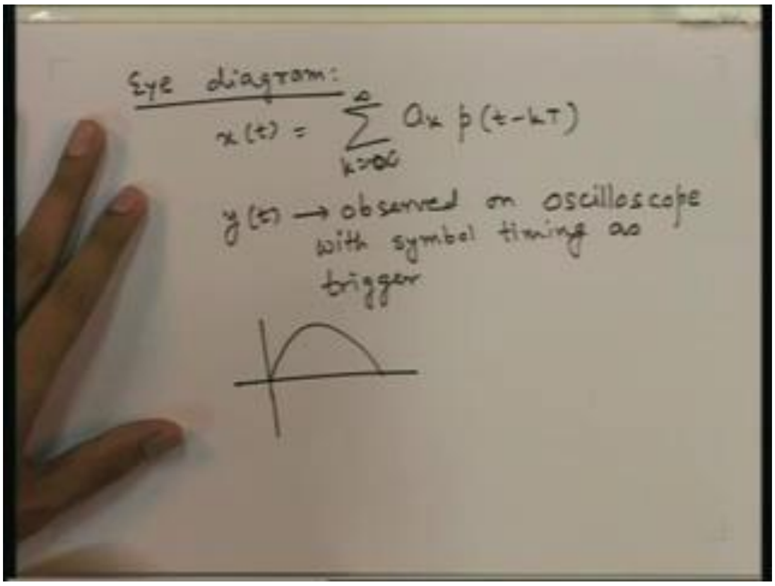

Eye diagram:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k p(t-kT)$$

$y(t) \rightarrow$ observed on oscilloscope with symbol timing as trigger

So, that can be so, Eye diagram is a solution to that. Eye diagram is a graphical illustration of ISI. How do we get that? It is a graphical illustration which will give us some impression about how much ISI is there. So, how do we get ISI? Simply, taking the output of the received filter. So, we will not transmit only p t we will transmit the in our summation a i, p t minus I t their summation that is the many symbols we were transmitting symbols after symbols, symbol after symbol and we will receive some signal here. So, that is y t noise will also be there.

Now, we simply try to see y t on an oscilloscope with symbol timing as trigger. So, we take we will have some transmitted signal x t this is ak, p t minus kT, k equal to 0 to infinity or minus infinity to infinity. And then we will have y t which is output of the receive filter and then we will pass that output we will observe that output on oscilloscope. So, this is observed on oscilloscope with so, you should have some synchronization to view anything which will make sense. So, oscilloscope with symbol timing as trigger.

So, we will take the signal we will generate a trigger based on from that which we will synchronize with the symbol timing. So, once we have the trigger which will synchronize with the symbol timing, we can give that on x and the Yt in the Y signal of the oscilloscope then we will see 1 or few symbols on the oscilloscope. So, how will it look like? Let us say, we have the binary PAM system, if you have a binary PAM system. Suppose, the pulse is after is goes through the channel and the receiver filter suppose the pulse is like this say like this. Then we will observe a on the oscilloscope we will observe a signal like this.

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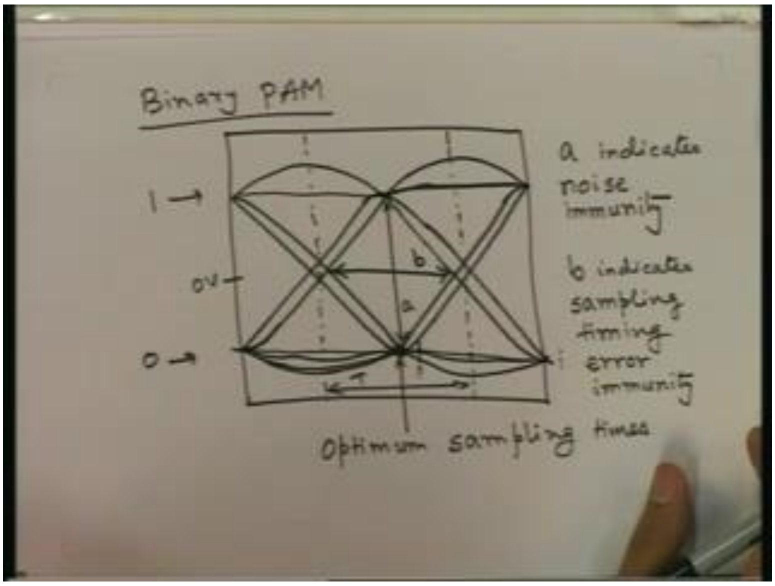

We will have this kind of view on the oscilloscope. Why is that because, suppose the amplitude here is 0 that is amplitude here is minus and this is plus then we have a minus pulse here, but it is going to plus pulse here. So, this is the symbol 1 symbol interval starts from here and ends here. So, this is 1 symbol interval and if it is 1 here, it will not be a rectangular pulse like this, but it will be something like this. And if it is going from negative to positive it will follow this, because it is negative it will have some inertia and it will go like this and go here, but even if it is 0 in the next symbol interval it will go slowly to 0 here.

So, if for example, here also it came from 1 to 0 it would have come this way and then, it would go this way and then again it will go back to negative. But if it was say 1 here also and 1 here and in the previous symbol also it was 1 then it would have come this way like it has come here. If it is 1 here this symbol and in this symbol also then it will go this way and it will go to if it is minus in both the symbol intervals then it will go here this way and then it will go up if the symbol here is positive that is if it is 1. So, then it will traverse different routes to go to different states either plus or minus remember for 0 we will we assume therefore, 0 will transmit this level for 1 we will transmit this level then it will go like this.

So, if it is 0 for continuously 2 symbols or more it will stay here, here, in this on this line, but not exactly on this line it will it will do something like this and if it wants to go back it will then go this way. But if it comes this way and then immediately goes back it will traverse a different route here. So, as a result the line here will not be a single line it will be thick line because many symbols are plotted at a time here and a different symbol the lines here are different. So, we will have the thick line we will in between lines possibly here. And here also you can see that, this is 1 line, this is another line there may be other lines here.

So, this is a type of plot we will see on the oscilloscope for binary PAM. And why is it called eye diagram? Because, this looks eye and this what is the use of this diagram? The use is that; now let us see, we how we do the detection at the receiver. How we do is we know that this is output of the match filter that is the received filter which will, which is the match filter is the optimum for received filter and we will then sample. So, these are optimum sampling times, optimum sampling times because this is unperturbed by the neighboring symbols. You can see that, here it is always a point or more or less almost a point.

So, if you sample somewhere else, is the values was minus here that is 0 bit then its value will go up if you sample somewhere else. So, this is the optimum point to sample here and so on. So, this is this need not be the exact symbol interval, but this length is the symbol time T ((it)) whether you consider it is starting from here to here or here to here this is the, this is T. Now, how do we know how much ISI is there what is the immunity to noise? The immunity to noise is given by this gap because as you can see that if these are large then, that is better. Because then noise cannot bring this level to somewhere here. So, that it will be negative. So, this is the 0 line 0 line.

So, this gap is most important for noise immunity. So, this a the length a indicates noise immunity. Then this 1 this gap indicates symbol timing error immunity. Suppose, instead of sampling at these instances at the receiver we sample here, here and then here and so on. That is there is a offset our sampling clock is not completely synchronized with the symbol timing. So, we are sampling at the wrong instances which are deviated from the optimum sampling instances. But how much error would we get due to that. That depends on this b.

Because as you can see, if this is b, if the error is b by 2 here, it comes here, the sampling here you will not be able to judge at all anything about the bits because they will be near 0 volt all the time here. So, more this gap, the better for sampling time error immunity. So, b indicates sampling timing error indicates immunity to do sampling timing error. Sampling timing error immunity. So, even if there is some missynchronization of the sampling clock and the symbol timing. If the b is large we will not have too much problem. So, ((we)) give some some quantitative is some quantitative measure of how much, how immune the system is to sampling timing error whereas, a indicates the immunity to noise.

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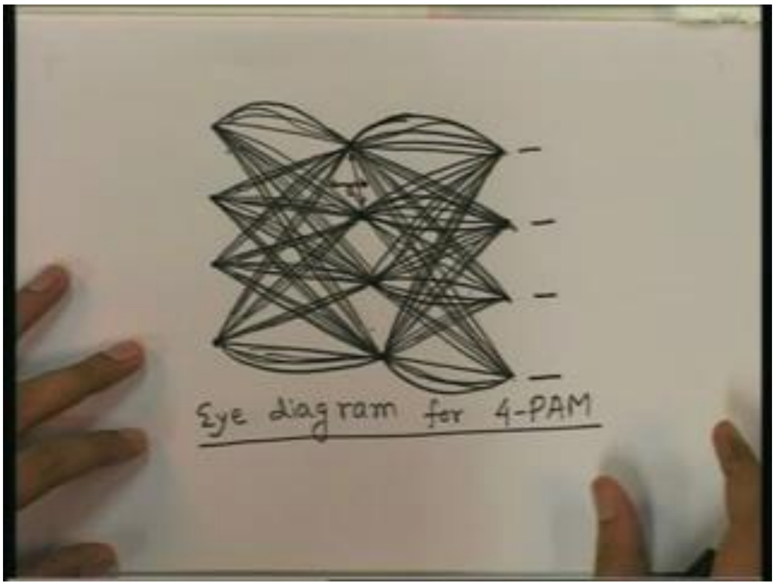

How will? So, this is for binary PAM, how will the signal look like for 4 PM. So, then the signal will look like this, we will have more lines there are 4 levels. This is 1, this another, this is another, this is another. So, there will 4 levels, but there can be transition from any level to any other level or any level to any level from 1 symbol interval to another symbol interval. So, it may go from here to here, here to here, here to here or here to here. Similarly, it can go to anywhere from here, anywhere from here and anywhere from here.

So, and again just like binary you will have many lines for each transition because this 1 if the previous symbol was also at this level it will take a different path here if it came from different level here it will come again from it will go again in a different way and. so on. So, for binary there will be for PAM also similarly there will be many lines from any state, any level to any level in the next interval. So, this will be the eye diagram for 4 PAM. And again here, this is the, this indicates the noise immunity and this indicates the this indicates the immunity to sampling time offset. So, we have discussed what is eye diagram? and we have seen, how eye diagram looks like eye diagrams look like eyes anyway. And we have seen, particularly how eye diagram looks like for binary PAM and for PAM postulations and for binary PAM and 4 PAM signaling and also we have seen, how from the eye diagram we can infer how much immunity is there against noise and also against sampling timing error.

Now, we will discuss another concept another topic which we discussed before also. But now we will can see, the implications or rather relevance of those topics now that is different kind of channel models. Previously we have discussed different kind of channel models. We have discussed for example, discrete channel where you have a discrete set of possible transmitted symbols and also discrete set of possible received symbols and they are there are different transition probabilities for the for a particular channel. If we for example, binary symmetric channel is the most simple example for that 0 to 1 there is probability 0 to 0 there some probability of transition and so on. And in general we can have M-ary discrete channel. And also we have discussed different discrete time real channels meaning; by what is transmitted is a real number, what you receive is also a real number.

But it is on discrete time. So, and the channel is also of discrete time input, output and the channel both are all are discrete time. Now, all the practical channels most of the practical channels are analog channels meaning by we transmit some continuous time signal through the channel we receive some continuous time signal from the channel such channels are called waveform channels also. But, what is then what is the relevance of discrete time channel or even discrete channel. So, though there are implicitly discrete time channels or discrete channels, those channel models are also useful for even continuous time channels.

So, let us see some examples,

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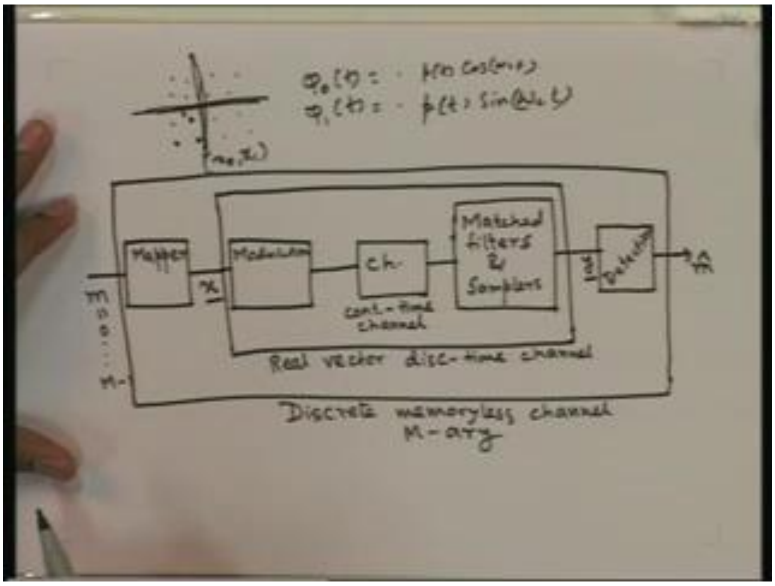

So, we have seen that a typical continuous time communication system continuous time, but digital communication system is of this type. You have some message which will take value from 0 to m minus 1. And then there is a mapper, which will translate m to a vector in some N dimensional space. So, you have a mapper which gives x this is the vector. So, this x may be x 0, x 1 (( )) and. So, until xm minus 1 they are vectors. So, for 2 dimensional constellations like QA MP SK this will be a 2 dimensional vector. So, these may be 1 of the points from. So, of the 1 of the points from here 16 (( )) constellation.

So, for 0 we will suppose select this and 1 for 1, we will select this and so on. That is job of the mapper for the bits for the integers it will give you a vector. And then the actual modulation which is basically which will convert this vector into an waveform. How

We have discussed this already for QAM signal. For example, you will have 2 basis vectors phi naught t and phi 1 t 1 is cosine omega CT time something and the other is sin omega CT time something. So, p t cos omega CT then, p t sine omega CT there is come scalar here. So, and once we get vector a like this say x naught, x 1 then we multiply x naught with this x 1 with this and add those signals then we will get 1 signal which is to be transmitted.

So, that is the job of the modulation block. It will take this vector; it will take the linear combination appropriately and then give a continuous time signal as output then it goes through channel. Then we do demodulation which is basically matched filter bands. So, matched filters and samplers so, what we are drawing is basically a matched filter receiver. So, this will give us output the received vector the estimate of the vector which was transmitted here some point say here. And then we have the deduction we have to we discuss before that this will involve picking of the point from constellation which is nearest to these vector.

So, this is a vector y, this is a vector x and here the signals have continuous time. Here it is discrete time. So, this will give an estimate of the message m. Now, this is the overall block diagram of the communication system. And this channel is continuous time channel. Now, if we consider this whole thing as a channel why do we need to do that?

We need to do that for several reasons. One is that suppose these systems are already designed by someone we cannot change these systems. We can only play with the other things. Then we cannot design these things. So, we have to consider this whole block as a black box.

Then we need a characterization of how this block behaves. So, that is like a channel to us when I am going to design the system outside here and on this side here all I can know about this is the overall response of this part as a black box. So, this is like a channel to me and we need a model for this channel. I do not require the channel model for this because I am not going to use this channel I am going to consider this whole block as the channel. So, now what kind of channel is this? This is now a discrete time channel, but it is a vectored channel we have also discussed a vector real discrete time channel. It is a it is taking a real vector as input and real vector it is giving out as output.

So, this is a real vector discrete time channel. We can still design the constellations this is only modulation and this is basically, demodulation of matched filters and samplers to get y. So, this is the space is given that it is suppose dimension is 2 then, this 2 dimensional plane is given. How you select the points is still up to us it does not require any change inside this block. So, we can still design the constellation, but we have to design is on this 2 dimensional plane because here there are only 2 basis vectors for this case. So, we can still take we need not take 15 QAM we can take 8 PSK we can take any other constellations we want which will be which will have less probability of error ultimately. So, we can still design these things and we can design M also M also can be changed.

M is not inside. So, we can consider this channel and there are. So, another reason for taking this as the channel is that we can modularize the design face. First design the modulation matched filters sampler separately and then because, it is easier to design anything in a modular way design first this part in the best way you can, then design the other part in the best way you can and. so on.

So, first designs the modulation match filter etcetera all the analog parts first and then, consider the whole block once you have designed as a real vector channel. And then consider it is the then design the constellation. Keeping in mind, all the constants you have on the average energy number of bits that is to be transmitted that is rate and. so on. So, this is now once you have designed this also detection mapper all these. Now, this whole thing can also be considered for same reasons as a single channel. This can be considered as a single channel. Now, what kind of channel is this? This is now, a discrete channel it is not only discrete time, but its it takes as input also some discrete values.

So, it is now a discrete memoryless if there is no ISI if there is some ISI it will have some memory. So, let us consider that we have removed ISI completely then discrete memoryless channel and M-ary there are M possible values. Now, we have discussed the models of these channels before. Now, why you do we do this? Combine these and consider it as a single channel again we have the same results that these may be designed already and we would like to design all the outside blocks. For example, now we may like to design error correcting coding block here and decoding block here. There may be there will be still some error from here to here, this m estimate of m need not be same as this original M.

So, sometimes there will be some error and we would like to avoid or we will like to be able to correct some of the errors and for that reason we may need to do some error correcting codes and the transmitter side and some error correcting the decoding of the error correcting code at the receiver side. So, here we will have encoder here we will have decoder. And the models we have discussed before are for the real vector channel we have discussed these models.

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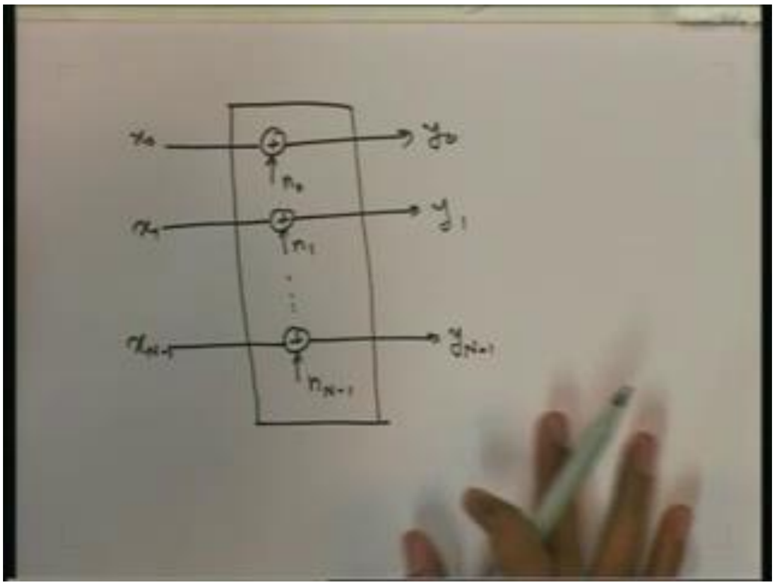

There is some noise component here, these noise processes are assumed to be independent, Gaussian, 0 mean Gaussian. This these are the inputs, this is the channel, this is a vector channel. So, this y 0 y 1 yn minus 1. x 0 x 1 xn minus one. So, this is such a channel, this channel because here we are giving x vector and here we are receiving y vector. So, x vector and y vector. And we have also discussed before that component y is y 1 is x 1 plus n 1 and. so on. Noise is being added. So, there is a noise vector which is being added to the transmitted vector and then received vector is the result of that.

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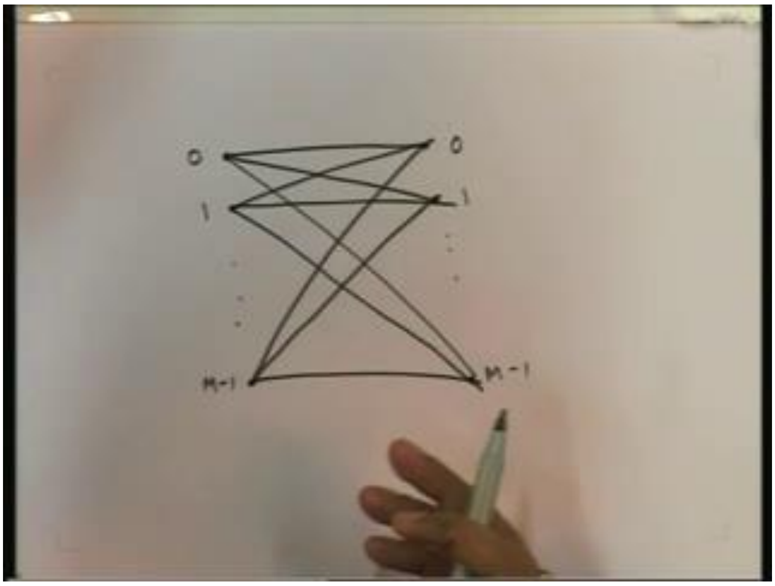

Then we have also discussed the discrete memoryless channel model, M-ary discrete memoryless channel model. Where there are m possible values transmitted values and m possible received values. 01 m minus 1,0 1 minus 1.And there is some transition probability. So, there is some probability and that we have to that can be possibly computed if these can be if it can be computed we can have its probabilities. So, we have seen how in the light of modulation and demodulation we have discussed. How discrete memoryless channel and real vector discrete time channel are relevant channel models even for continuous time channels.

If we consider the channel along with some transmitter blocks and receiver blocks as a single channel. Now, we discussed 1 block here which we have not discussed in detail. This is a mapper, which maps m a an integer to a vector here. Similarly, the there is some detection is off course, the minimum distance detection. So, this mapper does not really matter that we have to see.

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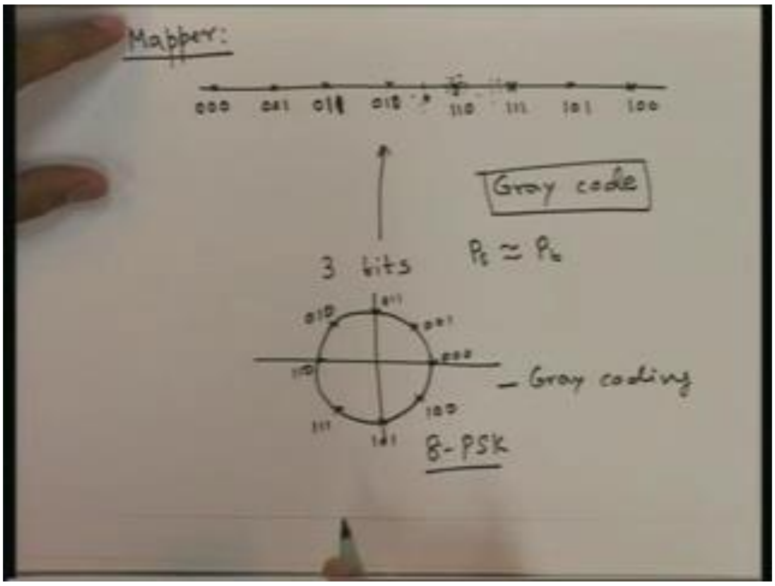

So, when we do the mapping we are actually mapping some bit sequences to a vector. So, for PAM signal set for example, so we are discussing mapper. For PAM signal set we have this 0 there are some points here. Say 8 PM we want to transmit 3 bits. Then 3 bits to so, all 3 bits combinations have to be mapped to 1 of these points that is the job of the mapper. So, how do we do that? We can do it anyway, but what is the purpose for doing in any particular way. So, what should we look for what we should for is that, when at the receiver we make an error in detection of the estimating the point which was transmitted. If we make an error there should not be too many errors in the bits.

So, if we have some bits here, some 3 bits here, some 3 bits here instead of say we transmitted this and instead of receiving this we receive this, then the number of bits that are different between these 2 should be minimal. If they are more then there will be more bits of error in a single symbol error. Now, it is more likely to detect when you transmit a particular symbol, it is more likely to detect when there is an error to detect either of these neighbours, that is; those are the most likely points which can be confused with this point if the received point is somewhere in between.

It is very unlikely there is some probability on 0 probability, but they are very small probability that if we transmit this the received point will be somewhere here, near the other points other than the neighbours. Most of the points will be near this itself, but sometimes they will be near this and even very with very less probability there will be with other points. So, when there is an error most of the errors are because the points that are detected are the neighboring points 1 of the neighboring points of the transmitted point. So, we should try to see that the difference of bits between the neighboring points is minimized there should be very little difference between the neighboring points.

So, this is 1 particular way doing it just this is called gray coding. Take 001 here, then there should be 1 bit difference between these 2 between any 2 consecutive points then there will be always only most of the times there will be only bit error when there is a symbol error. So, there is 1 bit difference here, then there is 1 bit difference here and then, here there is 1 bit difference here, this is 11 this is 10. So, this is different from the usual encoding usually we have 000001 then 010 and 011, but here we have said 011 and 010. Here, difference of bits between any 2 neighboring points is only 1 bit at a time only 1 bit is changing.

From here again we have this 0. 010, 110, 111, 101, 100. You can see that at any step there is only difference of 1 bit. So, this is called gray code this is a gray code is a way of mapping bit sequences into vector points. So, as a result in this case what will happen is that if there is some error most of the times the error will 1 symbol error will result in 1 bit error. So, the probability of symbol error will be same as almost same as probability of bit error for this kind of mapping. Now, for PSK also similar thing can be done; obviously, because we can there is some sequence in this. So, we have eight PSK. We can have 0 0 0 0 0 1 0 1 1 0 1 0 then 1 1 0 1 1 1 1 0 1 1 0 0. Again, most of the times if we transmit this we will receive 1 of these points if there is some error most of the times we will receive this then most sometimes we will receive this and probability of receiving other points will be very less. So, again this is the gray coding for 8 PSK signal set.

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Then let us see another what about QAM. Suppose, we have 16 QAM. This is not sequential. So, this is quite nice the even with even though this is not sequential all the neighboring points can be mapped in such way that all the neighboring points have only difference of 1 bit. Suppose we have these points we can assign bits this way. So, 16 QAM. So, there will be 4 bits to each point 0000, 0001, 0011, 0010. Then go this way 0010 then 0110 then 0111, 0101, 0100, then again this way, 1100, 1101, 1111, 1110, 10 0 1, 0111, 1001. This is the gray code for 16 QAM constellation. Again here, you can see that not we have all assigned the bit sequences in this order.

But so, we know that horizontally if we go to neighboring points from here, here or here then there will 1 bit difference here to here, this bit is different second bit is different here to here, first bit is different. But what about vertically neighboring points. Suppose you have this, go vertically up there is only 1 bit different the third bit come down there is only 1 bit different fourth bit. So, vertically also you can see that any 2 neighboring points have only 1 bit different. So, that is again nice because that will ensure that whenever there is some error there will be only 1 bit error. So, 1 symbol error will most of the times result in only 1 bit of error because most of the times the error will come because if you transmit this most of the times you will detect a neighboring point instead of that point.

So, these 4 are the neighboring points and there will be only 1 bit difference between these points. Even if you go to the diagonal point there will be only 2 bits different because they will only be different from here to here and here to here again another bit. So again, 2 bits different so this is a very nice mapping because, all the nearest points have only 1 bit different and next nearest points have 2 bits different and so on. So, that is a very nice property to have. So, this is gray coding. So, we have discussed now after discussing eye diagram we have discussed different the relevance of different channel models specially only if this M-ary discrete memoryless channel and also real vector discrete time channel.

So, those channel how they are useful those how those models are useful even for continuous time channels even in a communication digital communication system with continuous time channel. And then we have discussed some discussed, what criteria we should use for designing a mapper how do we map bit sequences to points in the vector space. So, criteria we said for that is to minimize the probability of bit error. So, there will be sometimes symbol error that does not depend on the mapper. But once the symbol is detected erroneously can we, keep the number of bits that are in error minimum.

So, that is the purpose of designing mapper properly and the best known mapper is the gray code a gray encoder. And there it is ensured that for PAM, PSK and also for 2 dimensional QAM signals it is ensured that the neighboring points have bit sequences which are different only in 1 bit. So, as a result most of the times symbol 1 symbol error will mean 1 bit error. So, instead of for example, here instead of receiving this point if we transmitted this point and detected this point at the receiver instead of these bits sequence we will take this bit sequence. So, there will be only 1 bit error the last bit. So, this is about mapper.

Now, we have discussed before qualitatively about probability of error how probability of error will change, we have said that probability of error will change if you decrease the distance between points. Because then, there will be more chance of getting confused between the neighboring points if they are very near by and noise takes the transmitted point near the other points, then we will be confused more easily and detect and commit error more frequently. And as a result the probability of error will be more if you have the distance between the points less. And also we have said that if you want to.

So, there is some (( )) of between the bit rate you can transmit the probability of error you can achieve and the average energy you are transmitting. If you want to get 1 better the others 1 of the others have to be sacrificed. For example, if you want to transmit less energy you have to either you have to decrease the number of bits or you have to increase the probability of error because if you want to in decrease the average energy you have to put all the points near origin bring all the points near origin. But if you bring all the points near origin, the distance between them will decrease if you keep the number of points same.

So, the probability of error will increase. But if you don't decrease the distance between them you have to decrease the number of points and you have to take less number of points and as a result your transmission rate will be less. If you had 16 points before now if you have 4 points instead of transmitting 4 bits per symbol you are transmitting now 2 bits per symbol. As a result we will be transmitting less bit rate than before. So, energy can be minimized at the compromising bit rate. Then we have also seen, that energy can be minimized reduced by compromising the probability of error

Similarly, if you want to do you know if you want to say transmit more bits then we have to add more points. So, either you add more points around far away from origin as a result you increase the energy, but keeping the distance between the point same to keep the probability of error same. So, if you want to keep probability of error same, but it will increase the bit rate then you have to increase the average transmitted energy. But, if you want to keep the probability of error same, but if you want to keep the energy also same, but increase the number of bits then if you increase if you keep the probability of error if you reduce the probability of error then with the same energy you can transmit more bits.

So, there are different tradeoffs you can improve 1 of them sacrificing other. But what about for FSK. For FSK you do not need to sacrifice on energy to increase number of points. You dont need to sacrifice on probability of error also because as you add more and more dimensions the distance between the points do not change. So, the probability of error will not be increased much. But what we are sacrificing there what you are compromising there is the band width which wouldn't change if you keep the dimension fixed.

But for FSK PPM all these orthogonal modulation schemes you are increasing the dimension to accommodate more bits. So, as a result we are just compromising on band width. So, there is some tradeoff between all these 4 bit rate, energy, average energy transmitted probability of error and the band width.

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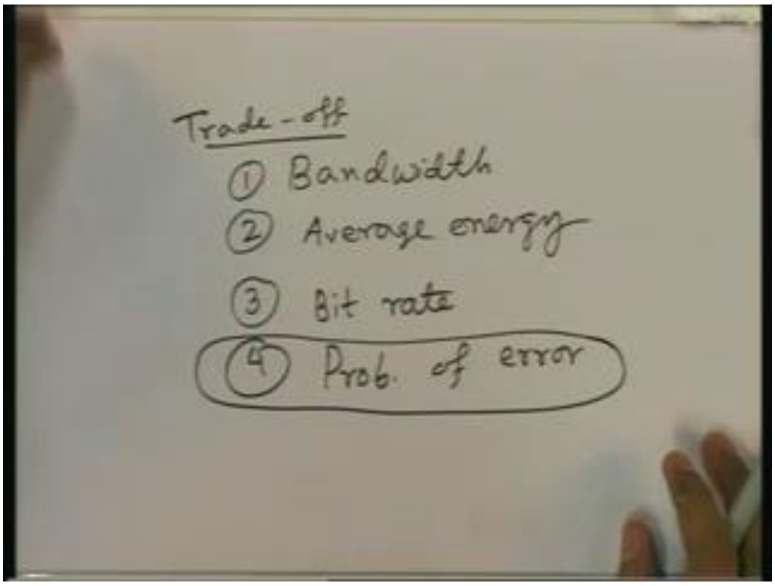

Trade-off

① Bandwidth

② Average energy

③ Bit rate

④ Prob. of error

So, there is some tradeoff between bandwidth, average, energy bit rate and probability of error. We will discuss this probability of error aspect in the next class. We will see for different modulation schemes how to compute the probability of error or how to get an estimate or a lower bound or an upper bound of the probability of error. Now, all the topics which we have covered in this class can be again found in the all common digital modulation books including.

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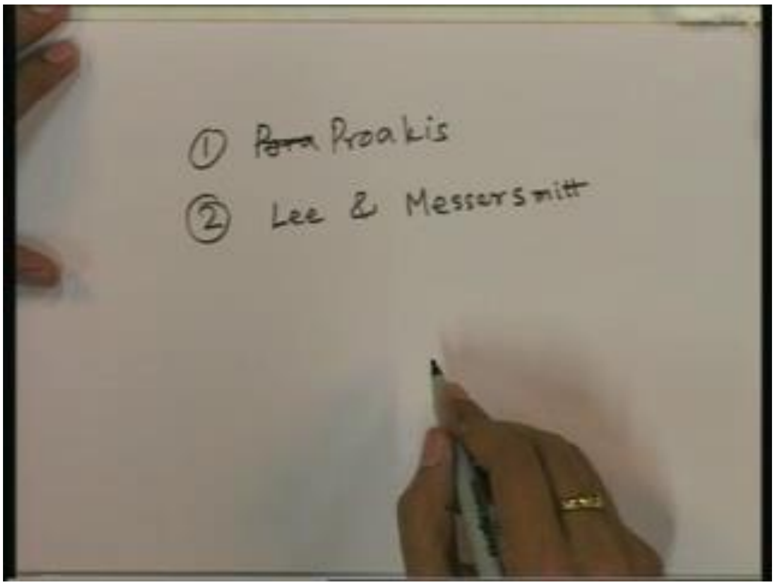

① ~~Pra~~ Proakis

② Lee & Messersmitt

Proakis or the eye diagrams can be started from Proakis or even better from Lee and Messers mi mitt digital communication by Lee and Messersmitt or Proakis. So, in the next class we will discuss probability of error calculation for different signal set.

Thank you.