

**Digital Communication**  
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**Lecture - 20**  
**Digital Modulation Techniques (Part – 9)**

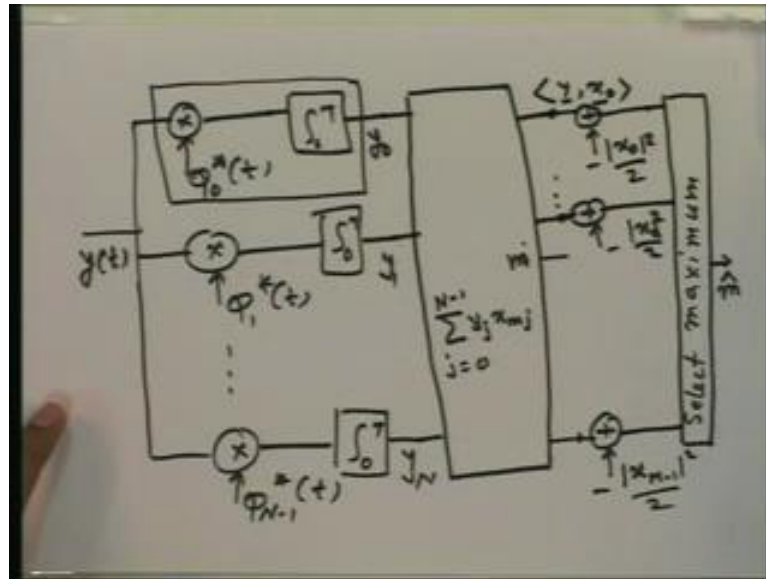
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We have been discussing digital modulation techniques for some classes. We have so far discussed PAM, PPM, PSK, QAM, FSK all these modulation techniques. And we have also discussed what is an orthogonal signal set we have seen that FSK, PPM signal sets are orthogonal. We have also discussed, what a bi-orthogonal signal set is and we can get a bi-orthogonal signal set also from the FSK PPM signal set but, also including their negative signals.

Then in the last class, we have discussed receiver structures, how to demodulate for a given signal set. So, we have seen that if we are given a set of signals that are used by the transmitter. How to do the receive reception? How to do the demodulation at the receiver? We have not assumed any particular type of modulation there, we have assumed that the signal set that is being used is known to us, but it is not decided by us. We have seen how to demodulate for the signal set and there we have discussed we have seen that using this block diagram by computing correlation.

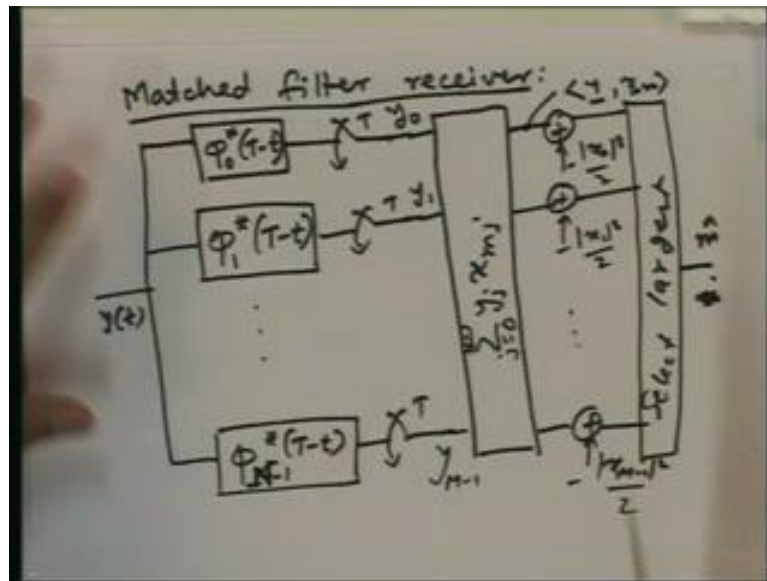
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First by multiplying by the correlation with the basis signals by multiplying and then integrating, we can find the correlation of the received signals with the basis signals. And then, we can see we can find the correlation of this vector with different signals vectors  $x_m$  by performing this operation for each  $m$ . Then by adding this we have seen that these are here we will get the quantities which we wanted to compare to decide on  $m$ .

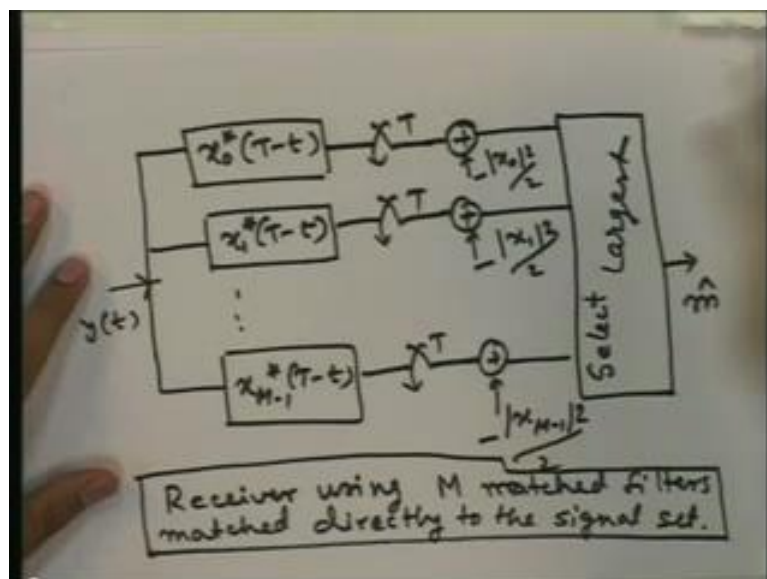
So, here the output is this correlation plus minus mod  $x$  naught square by two and so on and then we see that the maximum of them that will give us an estimate of  $m$ . So, which 1 is maximum that number  $m$  will give will be estimate at the receiver. So, we can get the message estimate of the message this way. We have also seen that these blocks this block and this block all these blocks can be implemented using matched filters matched filters matched to the basis vector basis signals like this.

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Using a matched filter and the sampler you can again get these quantities and then we can do the same operations here. So, this is a matched filter receiver, where the same operation is done using matched filters. Then we have also seen that matched filters using matched filters we can also demodulate differently by using matched filters matched directly to the signal set; instead of the basis signals.

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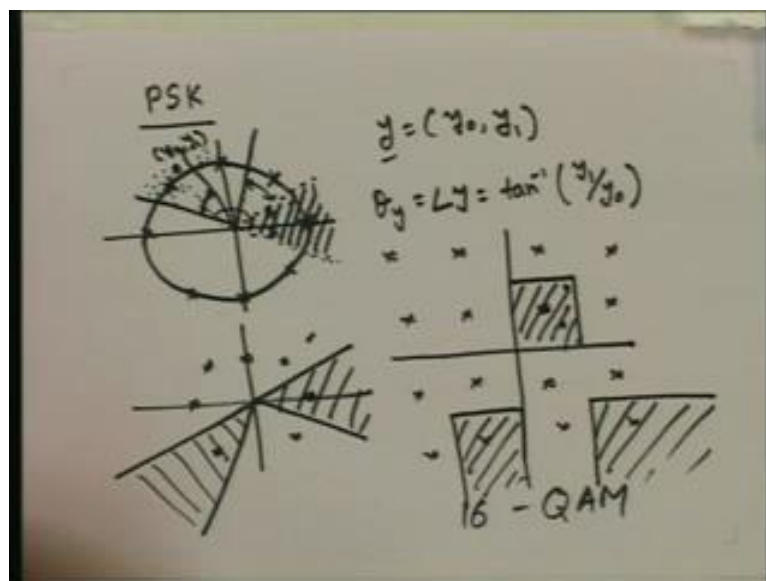
So, here we have seen that we can use these matched filters matched to the signal set, different signals and then sampling we will get now we can compare directly this. So, we

have also derived and shown that this we will do the same operation as we wanted to do. Using a correlation receiver or matched filter receiver using matched filters match to basis signals. So, now we will see that for see will see in particular what is what will be the receiver structure for different kind of modulations. So, first of all for PAM QAM signals we can with the dimension is either 1 or 2 for PAM it is 1, for QAM it is 2.

So, for PAM we will simply need it is 1 dimensional. So, we will need only 1 matched filter and then we will find in which section the receipt the point lies. Then for QAM again we will need two matched filters because we have dimension two number of points may be 16, 64, 32 anything. So, having matched filter match to all the signals will be too much complexity at the receiver because then, we will need 64 matched filters 32 matched filters.

So, for that reason it is better to implement the matched filter receiver structure in this form; match to the basis signals for QAM here. So, then we will need only two matched filters. Now for PSK again the dimension is 2. So, we can use the same matched filter structure with 2 basis signals and then here instead of doing this correlation and then finding this. For PSK signal particularly we can do it in a slightly different way and easier way.

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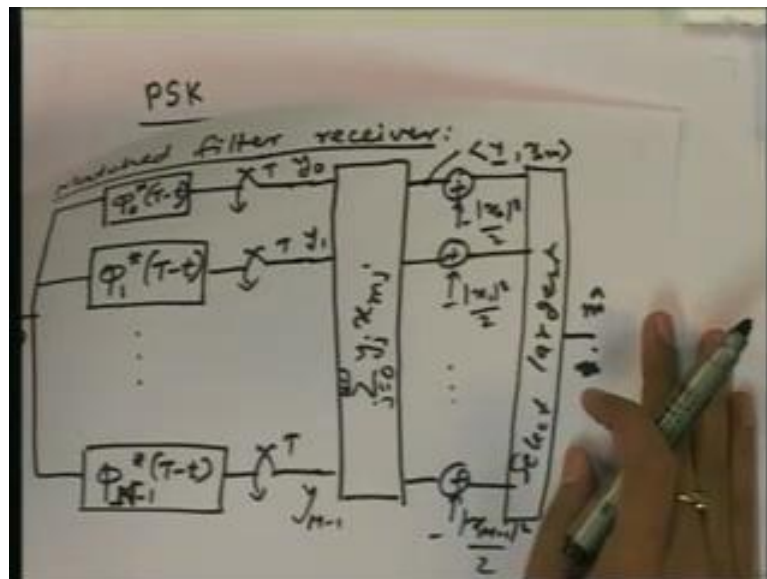


So, for PSK we have this transmitted signal set, suppose, 8 PSK. Now, when the lowest score out this signal if we transmit for example, this we will receive somewhere here;

some 1 of this points probably nearby. If the noise is too much at that point of time then it may go to near here also then we will decode wrongly because, it will come nearer to another point. But anyway, we cannot do anything about that because we cannot remove the noise completely. So, what we need to find out is to get to select the nearest point in the constellation.

Suppose we have received this we want to choose this. So, how do you select as you said, we can compute the distance. And then, we can show that, comparing the distance and picking the minimum is equivalent to finding the correlation with each and then adding something and then comparing those values and then taking the largest.

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We have taken we have seen that here, we are selecting the largest actually it is equivalent to comparing the distance from each point in the constellation. So, instead of even doing that; instead of taking the distance or instead of taking the correlation and then equivalently finding the distance and comparing. We can simply find the phase of this point and see which is the nearest point here in terms of phase meaning by the phase of this here is this then, this phase is near nearest to which phase of which signal here, which point here that we have to see.

So, if we receive  $y$  equal to  $y$  naught  $y_1$  a point here this  $y$  naught  $y_1$ . We can compute the phase  $\theta$  or angle  $\theta$  by taking  $\tan^{-1} y_1 / y_{naught}$ . And then, we can find in which interval it lies if it lie in the phases in this range it is this point because, then all

these points all these points this is the nearest point. For all these points in this interval this is the nearest point. So, it depends on the phase of the signal if the phase of the signal is in this range then, it is this point if the phase of the signal is in this range then, this is the nearest point and so on.

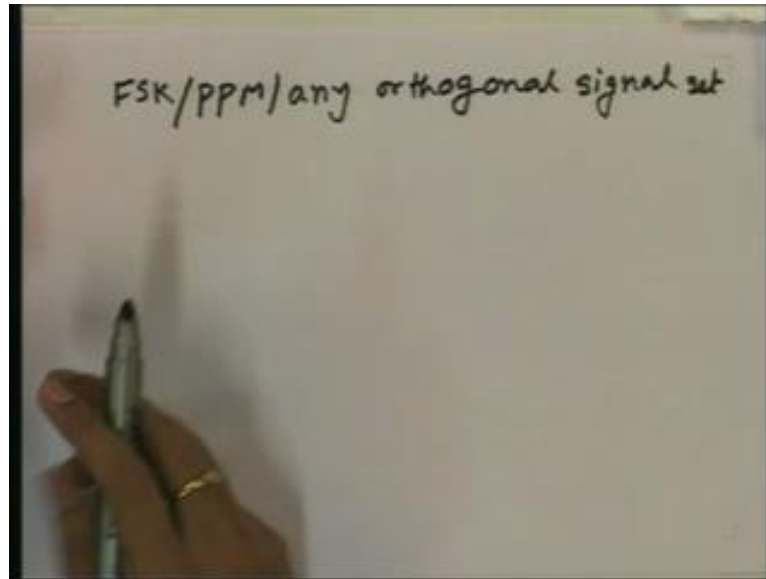
If the phase of the signal is in this range then this is the nearest point and phase of the signal is in this range then it is in this; this is the nearest point. So, this phase is now we can compare in which range it falls whether it falls in this or whether it falls in this and so on. So, here we will see that the phase of this is in between this. So, we will know that this is the 1 which was transmitted. Now, these regions are called the decision regions.

So, for PSK we have this point this points then, if the received point is in this region then, we will decide in favor of this point we will decide that this is the 1 which was transmitted. So, this region is called the decision region of this point. Similarly, for this point this region is called the decision region of this point. So, by finding the phase and then comparing this phase with these decision regions; phase of this we can find out which point is nearest.

So, this is simpler than finding the correlation and all just find the phase and see in which regions it falls. So, talking about decision regions what are the shapes of the decision regions for example, QAM. Say if we have 16 QAM constellations what is the decision region? What are the decision regions? This is the 16 QAM constellations. What is the decision region for this, this is the decision region.

We can see that if the received point is in this region then this point is the nearest to that point this is further from here. So, this is nearest even if it is here this is the nearest point so, all these points are nearest to this. Similar decision region of this point is this infinite region decision region of this point is this infinite region. There are 3 kinds of shapes of decision regions for 16 QAM 1 is: square closed, 1 is semi infinite in 1 direction, here it is semi infinite in both directions. So, now for PSK we have seen that this can be done this way finding the phase.

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Now, if we take the FSK or PPM modulation now we know that. So, we are considering now FSK or PPM or any orthogonal signal set. So, for any orthogonal signal set like FSK and PPM we know that the number of point is same as the number of the same as the dimension of the signal space. So, that for the particular case we can say that these 2 receiver structures are of approximately same complexity, see here also we will need  $n$  number of. So, for orthogonal signal set  $m$  is equal to  $n$ .

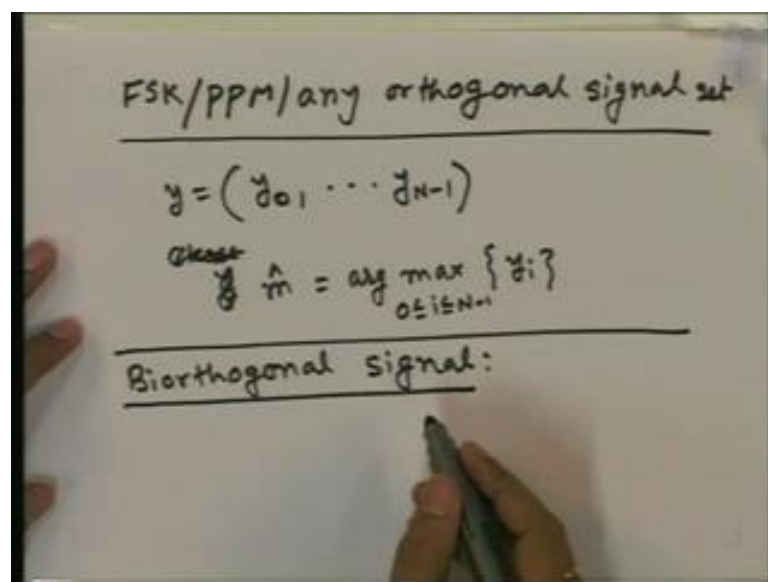
So, here and here we will have we require the same number of matched filters, but here we need to do all these. But, here we will see that these  $x_{n,j}$  will be 1 will be non zero only for 1 particular  $j$ . So, as a result this is a very simple operation. So, we can say equivalently that this itself this structure itself will do for FSK; this itself is of the best complexity. Just take the matched filters match to each signal because there orthogonal there they are the basis of the signal space anyway.

So, take find the components in each direction that the along each signal orthogonal signal and then this also need not be done now. Because, for FSK and PPM or for any orthogonal modulation technique we have seen, we have discussed that the energy of each signal is same is  $E$ . So, this  $x_{n,j}^2$  is same as  $E$  and so on. So, they are all same we are adding the same quantity to each and then comparing.

So, this need not be done, whichever is largest here will be here also this 1 will be largest among these because we are adding same quantity to all of them; because all these are same. So, for FSK we can say that we can simply we can simply take this, but we did need not have this additions. So, it is it is a simpler structure now how do you verify that it will be really near nearest to that. Suppose, this is largest can we say that this will be the nearest yes, we can because this we have proved that this actually does the optimum. And this is equivalent to finding the nearest point in terms of the distance.

So, if you just choose that largest here these will give us the minimum distance point this will tell us which signal is a nearest to the receipt signal.

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So, for FSK we have concluded that we can we can detect this way that first find this vector that is this vector this is  $y_0$  and  $y_1$  and so on. And then take the component which is largest. Choose take  $y$  take the  $m$  maximum of  $y_i$  take the argument of that  $\arg \max$ , this is our estimate of  $m$ . What does this mean? This means that this is the argument of maximum  $y_i$ . So, take which 1 is the maximum what is the argument of that if,  $y_4$  is the maximum of all these then 4 is the argument of that  $y_4$ . So, that 4 is the  $m$  hat.

So, this is our estimate of the message that was transmitted. So, this is quite simple this is this is simpler than finding over the inner product and all such things because, those actually will result in having the same result; we will get the same result using that. Now,



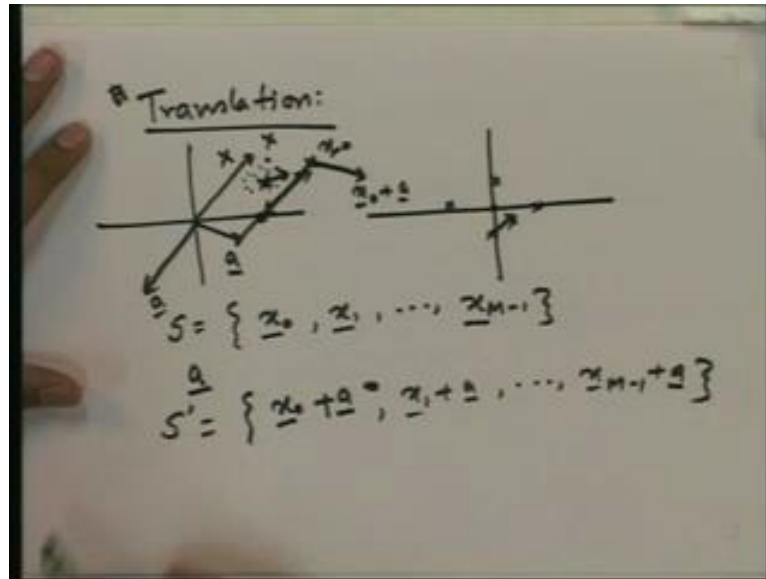
we leave it, I leave it as an exercise for you to find out what would be the deduction scheme for bi-orthogonal signal.

So, we will discuss it later, but please give it a try and then we will discuss it later. So, we have seen the receiver structure in general for any arbitrary signal set and we have also discussed some special cases of signal sets and what will be the receiver structure simpler receiver structure for that kind of modulation techniques. Now, we will discuss another kind of signal set which is also important and interesting, but before doing that we need to discuss some general aspects of signal design.

So, 1 is that is this we will discuss now about equivalence of signal sets. So, if you are given 2 signal sets. Sometimes 2 signal sets may be equivalent in terms of probability of error or in terms of the performance of the signal set or in terms of how much arability it gives in the communication. So, if both the signal sets give us same probability of error, but 1 signal set for example, suppose if it gives it uses less energy. Then obviously, we would like to take that signal set because we will be transmitting less energy to get the same probability of error using probably the same bandwidth.

So, whenever possible we would like to get the same probability of error with the minimum energy required. So, in that context we need to find out when two signal sets are equivalent. So, we will not find in general when two signals are equivalent, but we will discuss some types of transformations, which keep the signal sets equivalent. So, if we have 1 signal set and we do some transformation on that will that signal set remain equivalent to the original signal set.

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So, there are some kinds of transformation which keep the signal set equivalent to the original signal set. So, 1 of them is translation. So, suppose you have a qm constellation like this suppose, the amplitudes we use are this 3 points or let us say we have 4 points. They are using translating 2 phase using this signal set. So, this basis may be this may be  $\cos \omega_c t$  this may be  $\cos \omega_c t$  scaled by something.

But, now we can see that these signal set can be easily brought here by translating each point by the same number we can bring this point to 0. And then, this signal set will look like, this signal set will look like this and now this, the center is here. So, we can intuitively feel that the energy of each signal is reduced here we can see that because they brought nearer to 0.

So, as a result the average energy transmitted using this signal set will be less than, average energy that is used to transmit this signal set. So, this we can feel also that this performance of the signal set here and here will be same. How do we see that? You can see that if this signal is transmitted here and it is corrupted by noise it will go here. So, probability that the noise is so much that it will bring the point to nearer to other points. That will be same as if this is the noise deviates from here brings this point here the same noise will deviates this point to here. So, if this comes near this, this will also point will come here for the same noise.

So, probability of error here will be same as probability of error here now will also see in a different way how to feel this. So, for that let us suppose that this is the signal set we are using  $x_1, x_2, \dots, x_{n-1}$ . And suppose, we have this as any vector in the same signal space. So, these are the vectors vector representation of the signals that are transmitted. So, meaning by we have taken an orthonormal basis of the span of the signal space and then, with respect to that orthonormal basis we have expressed each signal as a vector of  $n$  dimension.

Then we are taking another vector in the same signal space and then suppose we construct this signal set we take  $x_1 - a$ . Now what is this, this vector suppose  $x_1$  is this and  $a$  is this. What is  $x_1 - a$ ? It is this; that means, from origin if you draw is this. So, we are we are basically subtracting  $a$  from this means we are bringing the point here the negative direction negative of  $a$ .

So, if we add this point will go here this is this is  $x_1$  if this is  $x_1$  this is  $x_1 + a$ . So, we can shift it in this direction we can take  $a$  here  $a$  to this vector then this will come here like this. So, by adding a vector adding a suitable vector we can shift a point to anywhere we want. So, now we take a single vector and shift all the points by that same vector. So, all the points will be shifted in the same direction parallel.

So, those new points will be  $x_1 + a, x_2 + a$  and so on till  $x_{m-1} + a$ . Now, our claim is that if we use this signal set which is the shifted version translated version of the original signal set; it will result in the same probability of error as the original signal set  $s$ . How do we see that? Suppose, we are transmitting this signal set the transmitter is using this signal set and receiver has received  $y$ .

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$$\begin{aligned} \underline{y} &= (y_0, y_1, \dots, y_{M-1}) \\ &= \underline{x}_m + \underline{n} \\ &= \underbrace{x_m + a}_{\text{transmitted}} + \underbrace{n}_{\text{noise}} \\ \underline{y} - \underline{a} &= x_m + n \end{aligned}$$

It has also extracted using matched filter or correlation receiver it has extracted the vector  $y$ . This is also same as  $x_m$  that was transmitted plus the vector  $n$ . So, this is what it has received after match filtering. So, now this  $x_m$  is this  $x_m$  let us say we are using this. So, we are not it will not receive  $x_m$ , but  $x_m$  plus if we transmitted this noise will be  $x_m$  plus  $a$  if we transmitted, noise will be added to that. So, we will receive here  $x_m$  plus  $a$  then plus  $n$  this is the transmitted vector and this is the noise. So, this is the noise and this is transmitted. So, we have received this.

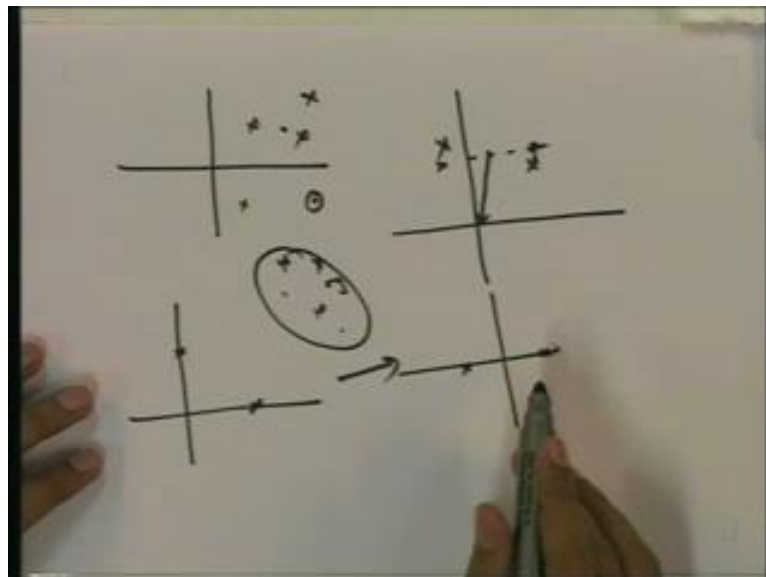
Now, what can the receiver know receiver do? Receiver knows this vector  $a$  because, this is a fix vector and it can simply subtract  $a$  from  $y$  it can subtract  $a$  from  $y$  and then get  $x_m$  plus  $n$ . So, if we transmit this signal set and if we then receive this that is this plus noise. Then we can simply subtract  $a$  at the receiver from the received vector and then we can assume as if, this signal set itself was used by the transmitter and we have received this. So, this is the it is the same situation as if we transmitted this. So, only thing we need to do is just subtract  $a$  and then we can decode we can do the demodulation just similarly, as you would have done here.

So, we will have the same structure to do this here we have received this  $y$  which are these. So, before comparing what will do is we will just subtract from each this  $a$  and then, we will go ahead with the same thing assuming that, this  $x$   $n$   $j$ 's and all are same. So, we have basically used this signal set we will assume. So, the receiver can do the

reception, do the demodulation exactly similarly with just a minor modification and the performance will be also same because here the same noise is being added.

So, it is as if this itself was transmitted and this is this noise is added. So, we will have the same probability of error. So, we have discussed that, the translation keeps thus keeps a signal set equivalent meaning by in terms of probability of error; the probability of error remains same even if you translate a signal set. Now, this is not the only transformation which keeps a signal set equivalent rotation also keeps a signal set equivalent.

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Suppose that, we have rotation in higher dimensions also can be defined, but we will not going to that we will just discuss in two dimension. Suppose we have say if we this have this 4 points we are probably using this. Then if we rotate this set of points around any point then also the signal set will remain equivalent. Why? Suppose, we are using we are rotating the whole set of points around this point around this point; we are rotating the whole set. Then at the receiver what can I do we can rotate it back to this position.

We will receive some vectors somehow we transmit this. So, we will rotate it we will get it some vectors here. So, some points we will have here once you rotate it and then we will if we transmit this we will rotate we will receive something here and then we will rotate back here. So, we will get something here and then we will decode. So, rotation

will not also affect the probability of error direction also can be done very similar to direction of this signal set and the probability of error also will not change.

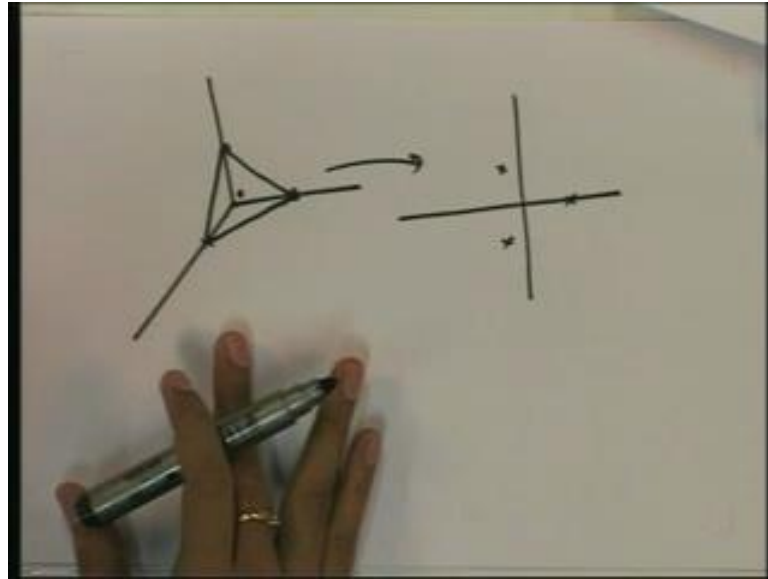
So, we have seen that transition and rotation do not change the performance of the signal set that is, probability of error of the signal set. And as a result, we say that the signal set remains equivalent after transition or rotation. So, let us now see, some interesting consequence of this concept. Suppose we are given a binary signal set say 2 point: if it is 1 it transmits this, if it is 0 it transmits this. Now, someone using this signal set, I can say that I will not use this signal set I will instead translate it and then rotate it appropriately. So, that the signals are here.

So, I will rotate the signal set around this point the midpoint here and to make it horizontal and then shift it to bring this point here. So, it will first go to horizontal location then we will translate in this way. So, we will bring the points here. Then it is same as doing PAM. So, instead of doing this why not I do this; this is a PAM modulation we are familiar with this we know how to demodulate it is very simple.

So, we see that any if you are given any two points in the constellation you want to transmit we can also equivalently transmit the translated and rotated version of the signal set. So, similarly if we say use binary FSK. So, orthogonal FSK it will be like this. So, I can say this is also equivalent to translating and rotating to bring it to this type.

So, performance wise reliability or probability of error wise this signal set will be same as this. But, this will probably use this will use more bandwidth this dimension 2 this is dimension 1 and decoding complexity is something we have to again see for probably this things will not matter for small number of points. But, suppose now, we are we take this is 2 dimension take 3 dimensional FSK.

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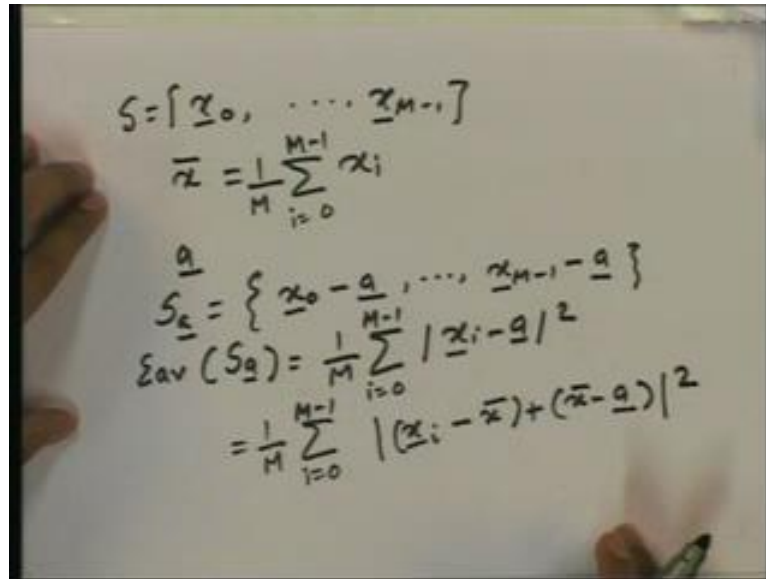


Take 3 dimensional FSK now, by translating and rotating we can now bring this in 2 dimensions. So, this is 3 dimensions we can translate the signal set and rotate so that, the midpoint of the if you join this 3 points there will be a plane. And there will be triangle on that plane joining these 3 points take the midpoint of that and then centroid of that and then bring that centroid to 0. And rotate the plane so that, it lies on the xy plane only so then, we will get signal set like this.

So, it is also equivalent to a 2 dimensional signal set, 3 dimensional FSK orthogonal signal set is equivalent to a 2 dimensional signal set with 3 points. This kind of signal set is called the simplex signal set. Now, how do we know how much transition is required. So, that we will see, we know that given a signal set we can translate it to anywhere. And it will that the probability of error will not change for that reason because of translation. So, now, but different translation different translated versions will have different average energy.

So, we would like to choose that particular translated version which has the minimum average energy. We do not transmit want to transmit more power. So, we have to find out, out of all the translated versions which is the minimum energy version.

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The image shows a whiteboard with handwritten mathematical derivations. The first line defines a signal set  $S = \{x_0, \dots, x_{M-1}\}$ . The second line defines the centroid  $\bar{x} = \frac{1}{M} \sum_{i=0}^{M-1} x_i$ . The third line defines a translated signal set  $S_a = \{x_0 - a, \dots, x_{M-1} - a\}$ . The fourth line defines the average energy of  $S_a$  as  $E_{av}(S_a) = \frac{1}{M} \sum_{i=0}^{M-1} |x_i - a|^2$ . The fifth line shows the expansion of this energy:  $= \frac{1}{M} \sum_{i=0}^{M-1} |(x_i - \bar{x}) + (\bar{x} - a)|^2$ .

Now, suppose we have this signal set  $x_0$  to  $x_{M-1}$  and suppose the centroid of these points is  $\bar{x}$ , so we call it  $\bar{x}$ . Centroid is nothing, but average of all of them. So, this now, take any vector  $a$  and translate this whole signal set by minus  $a$  or  $a$ . So, what do we get we call it this is  $S_a$ ; the original signal set and this translated version we call  $S_a$  is a subscript  $a$ . This is defined as  $x_i - a$  for  $i = 0$  to  $M-1$ . This is the signal set we are using suppose.

Now, we know that these 2 signal sets are equivalent in the sense that they will give us the same probability of error and they could be received; for this can be implemented in terms of receiver for this just with a minor modification. Now, we will compare the energy of these 2. So, what is the average energy of this signal set the average of this signal set  $S_a$  is  $\frac{1}{M} \sum_{i=0}^{M-1} |x_i - a|^2$ .

Now, this will do a little modification we will subtract here  $x_i - \bar{x}$  then, plus  $\bar{x} - a$  whole square. Now, what is this we know that length square this is basically energy, but energy is in terms of vector. The energy is the length of the vector square and that is same as the inner product of the vector is with itself. So, we take the inner product of this with itself. So, what is it, it is  $\frac{1}{M}$  and then inner product of this with itself we can break now, in 2 parts this times inner product of this with this and so on.



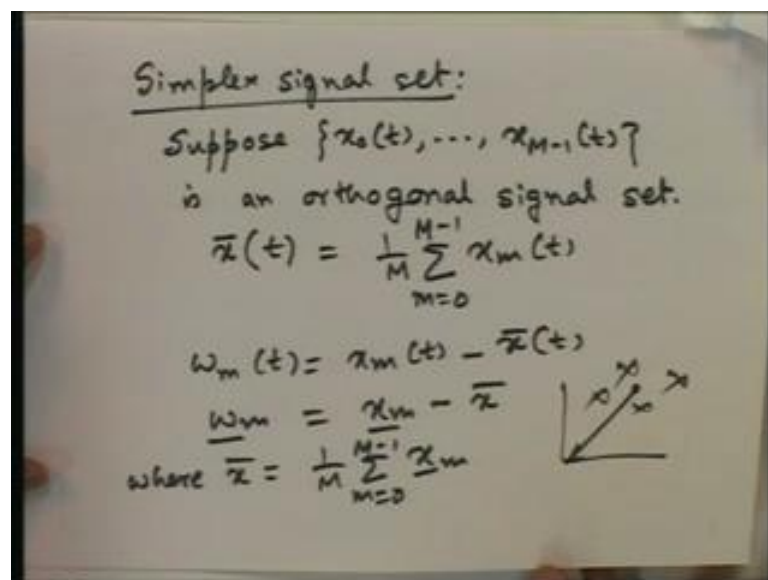


among all the translated versions this is the minimum that can be achieved and this is equality only when  $x$  is equal to  $\bar{x}$  because only then this will be 0. So, the average energy will be this. So,  $Sx$  has the minimum energy average energy among all the translated versions of  $x$  of  $S$ . So, we conclude that  $Sx$ , this translated version has the minimum average energy.

So, we have seen that if we translate a signal set by any vector the probability of error does not change, but the average energy changes. And among all the translated versions of a signal set the 1 which is translated such that the average of the average of the signals. That is the centroid of the signals is the origin when we translate such that the centroid comes to 0 then that has the minimum average energy.

So, now we will see an example of that we have just now seen before discussing this we have seen that, if we have an orthogonal signal of dimension 2 we can bring it to by translation and rotation we can bring it to a signal on 1 dimension. Similarly, 3 dimensional orthogonal signals set can be brought by translation to a two dimensional signal set.

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So, now we will generalize that concept and that kind of signal set is called the simplex signal, simplex signal set. So, what we do is simple suppose we have this is an orthogonal signal set. Then just like we did for the 2 dimensional and 3 dimensional orthogonal signal set, we will do the same thing here in general. We will first find out the

average of all this that is the centroid of all these signals this is  $\frac{1}{M}$  and then we will basically construct another signal set that is the translated signal set; where we will subtract this average from each.

So, that is let us call that  $w_m$  that is  $x_m - \bar{x}$ . That is now in terms of vector we can say that the vector representation of this will be  $w_m$  that will be  $x_m - \bar{x}$ . These are the vector representation of these signals with respect to some orthonormal basis. Where  $\bar{x}$  because  $\bar{x}$  is this if we represent in terms of the vector, vector of this will be again this linear combination of the vectors of this. So,  $\bar{x}$  is  $\frac{1}{M}$ .

So, what have we done in terms of translation, just like we did for 2 dimensional 3 dimensional sets. We have taken first the average of all these signals. So, we have computed the centroid now we are subtracting the centroid from each to get a new signal set. So, now, this signal set will have centroid 0 because, we have brought the centroid to zero. So, if we had some signals here we will compute the centroid and then bring it to zero. So, we will take this vector shift it to by this vector, we have done that.

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Properties:

$$\begin{aligned}
 |w_m|^2 &= |x_m - \bar{x}|^2 \\
 &= \langle (x_m - \bar{x}), (x_m - \bar{x}) \rangle \\
 &= |x_m|^2 + |\bar{x}|^2 \\
 &\quad - \langle x_m, \bar{x} \rangle - \langle \bar{x}, x_m \rangle \\
 &= \epsilon + \frac{\epsilon}{M} - 2 \cdot \frac{\epsilon}{M} \\
 &= \epsilon - \frac{\epsilon}{M} = \epsilon \left(1 - \frac{1}{M}\right) \\
 &< \epsilon
 \end{aligned}$$

Now we will see its properties the properties of this signal set. First of all we claim that, we claim that the energy of the signal each signal still remains same orthogonal signal set also had energy of each signal set  $\epsilon$ . But now, we will see that all the signals still have the same energy. So, let us see energy of  $w_m$ . What is  $w_m$ ?  $w_m$  we have just now seen

is  $x_m$  minus  $\bar{x}$  and then we take the energy of this. Now this is nothing, but the nothing, but the inner product of this with itself.

So, we take inner product of  $x_m$  minus  $\bar{x}$  with itself and this we can again now do distributivity use distributivity and use inner product for  $x_m$  with  $x_m$ . That will be  $\|x_m\|^2$  energy of  $x_m$  then  $x$  with  $x$  then minus  $x_m x$  minus  $x x_m$  both of these are same because, we are considering real vectors. And what is this inner product? This inner product this magnitude is  $x_m$  we have taken of energy  $e$ .

The orthogonal the origin orthogonal signal set we have assumed that to have energy  $e$ . So, this is  $e$ . So, what is the energy of  $\bar{x}$  energy let us see what is  $\bar{x}$ ,  $\bar{x}$  is this. So, what is the energy of this? It is just the  $e$  by  $m$  because energy of this is nothing but, inner product of this with itself. And when we when we do that we do the inner product we can do distributivity we can do distributivity like this.

So, if we have  $x_1$  by  $M$   $x_2$   $\dots$   $x_m$  minus  $\bar{x}$  with  $x_1$  by  $M$   $x_2$   $\dots$   $x_m$  plus  $x_m$  minus  $\bar{x}$ . If we take this inner product  $1$  by  $M$  square will come and then inner product of each combination  $x_1$   $x_2$   $\dots$   $x_m$  all these inner products we will take. But, this signal set was orthogonal. So, inner product of  $x_1$  with  $x_2$  will be  $0$   $x_1$  with  $x_3$  will be  $0$  and so on. So, we will have only  $x_1$  with  $x_1$   $x_2$  with  $x_2$   $\dots$   $x_m$  with  $x_m$  and those only those inner products.

So, we will have from  $i = 1$  to  $M$  inner product of  $x_i$  with  $x_i$  itself, all the cross inner products will be  $0$ . So, this inner product we know this is the energy of  $x_i$ . So, it is  $e$ . So,  $1$  by  $M$  square there are  $M$  number of them. So,  $M$  times  $e$  so this is  $e$  by  $M$ . So, we have this inner product this as  $e$  by  $M$ . Now what is this is  $2$  times this these  $2$  are same.

So, minus  $2$  times  $x_m$  with  $\bar{x}$ , now if you take  $x_m$  with  $\bar{x}$  inner product  $\bar{x}$  is this we take a particular  $x_m$  and take inner product  $x_m$  with all the other components other than  $M$  those inner products will be  $0$ . Only for  $M$  it will be  $e$  inner product of  $x_m$  with  $x_m$  itself is energy of it so,  $e$  then  $e$  by  $M$ .

So, we will have this inner product as  $e$  by  $M$ . So, this is  $e$  plus  $e$  by  $M$  minus  $2e$  by  $M$ . So,  $e$  minus  $e$  by  $M$  this is nothing  $1$  minus  $1$  by  $M$ . Now, this as you can see this is less than  $e$  this is something interesting, it shows that all the signal points in the simplex

signal set are same. But they are less than the energy of the orthogonal signal set, which was  $e$ . And for the simplex signal set the energy of each point is each signal is  $e$  times  $1$  minus  $1$  by  $M$ .

So, the energy of each signal has reduced, but the distance between the points remains same because, all the points are translated by the same vector. So, the distance between them has not changed and we have also discussed before that the probability of error does not change at all. So, we have seen that energy of the simplex signal set is less than the original orthogonal signal set though the probability of error for both signal sets are same.

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$$\begin{aligned} \textcircled{2} \quad & \langle \underline{w}_m, \underline{w}_n \rangle \\ &= \langle \underline{x}_m - \bar{x}, \underline{x}_n - \bar{x} \rangle \\ &= -\frac{1}{M} \text{ : constant} \\ \textcircled{3} \quad & d(\underline{w}_m, \underline{w}_n) = d(\underline{x}_m, \underline{x}_n) \end{aligned}$$

Now, second what is the inner product of any two signals in the simplex signal set? For orthogonal signal set it was 0, but now they are not orthogonal and we can compute this. And this, once you compute you will see that this is minus 1 by  $M$  and it is independent by  $m$  and  $n$ . So, it is also constant this is constant and so all the any pair of signals from the signal set simplex signal set have the same inner product; that is correlation and also distance between any two points remain same,

Distance between  $w_m$  and  $w_n$  is same as distance between  $x_m$  and  $x_n$  because, we have translated both the points by the same vector. They have come to a different location parallelly. So, the distances between the pairs of points have not changed when we translated the whole orthogonal signal set to bring its centroid to 0.

So, we have discussed in this class that by translation and rotation of a signal set the probability of error does not change and as a result the signal sets are called, said to be equivalent. They have the same performance in every aspect except for average energy transmitted which changes when translation is or rotation is done. So, using this principle we have seen how to, bring an orthogonal signal set to centroid 0 and thereby, reduce its average transmitted energy.

Among all the translated versions, that will give the minimum average energy other signal set. So that, signal set when we obtain from orthogonal signal set is called the simplex signal set. From the next class onwards we will we have finished this memory less modulation techniques. Next class we will discuss modulation techniques with memory.

Thank you.