Digital Communication

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Lecture - 19
Digital Modulation Techniques (Part - 8)

We have been discussing Digital Modulation Techniques for some classes. So, far we have discussed some digital modulation techniques like: PAM, QAM, PSK, FSK and also PPN. Among them we have seen that FSK and PPM are orthogonal modulation techniques satisfy some conditions. For example: for FSK the step frequency should be multiple of 1 by 2t and similarly, for PPM also the shifted pulses should not overlap with each other. So, once those conditions are satisfied then, FSK and PPM are orthogonal modulation.

Now, we have also covered some basic linear algebra. Which will be which has come to use and which we will come to use even later. In this, class we will start with another modulation technique called biorthogonal modulation and then. This is actually a class of modulation techniques PPM and FSK were orthogonal modulation. So, that is orthogonal and we will have another similar technique called biorthogonal modulation.

Afterwards, we will go into decode demodulation of modulated signals for different kind of modulation. So, let us start with biorthogonal modulation. Biorthogonal signals we know what is an orthogonal signal set? And suppose, phi naught t phi 1 t till phi N minus 1 t are orthonormal.

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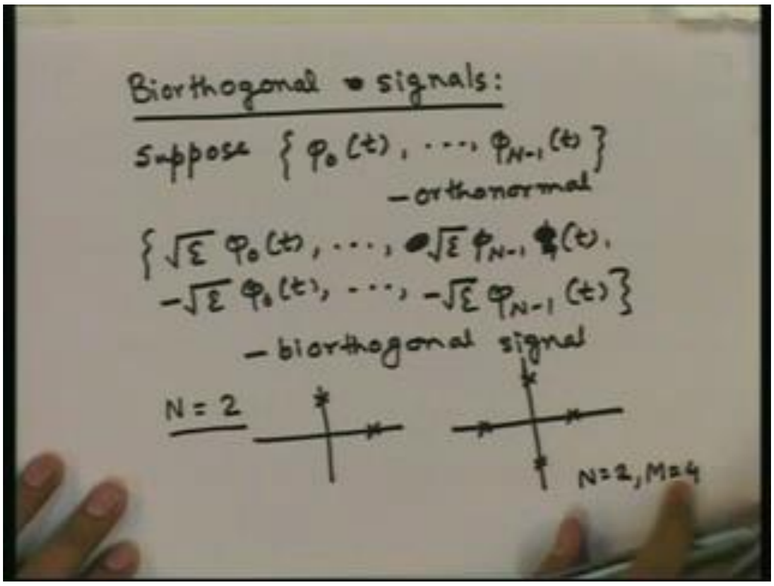

For example: they may be frequency shifted versions of the same pulse or they may be time shifted versions like in the case of PPM of the same pulse. So, that they are orthogonal and then, length is the magnitude of each is scaled in such a way that they have unit length. Then, for the orthogonal modulation we have seen that, if you want to transmit energy E signals. Then, we transmit these signals these signals phi naught to phi N minus 1, but multiplied by root over E to get energy E.

But for biorthogonal signals we take all these signals and also we add all the negatives of these signals. So, we add minus root E phi naught t minus root E phi N minus 1 t. So, this is biorthogonal signal set. This is biorthogonal we can see that an orthonormal basis for this signal set is also this phi naught 2 phi N minus 1t. Because, all these other signals also can be expressed as just scaling of each 1 of them.

So, the dimension of the signal space is still N because, the dimension is actually defined as the number of elements in a basis set. So, this is the basis set orthonormal basis set and N is the number of number of functions there so, N is the dimension. So, we can plot all these signals as points in the N dimensional space. Let us, see some examples: suppose N is 2.

Then for orthogonal signaling in dimension 2 we have points like this: 2 points this is root E0 and this is 0 root E. Now, if we want biorthogonal signaling in dimension 2 we will add their negatives also, this 1 and this 1 number of points will be 4. That is 2 times

N similarly, if we want so, this is for N equal to 2 and M equal to 4. Similarly, if we have N equal to 3 that is 3 dimension.

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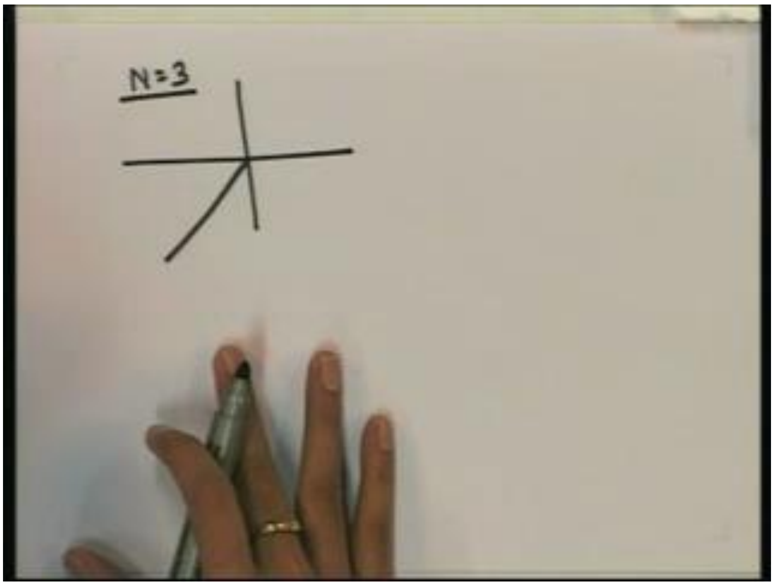

Then, the constellation will look like, so we will draw only the positive axis and the negative axis we will draw with dotted lines. So, this is N equal to 3. This is this in the negative sides. And for orthogonal signaling in dimension 3 we have points like this. We have discussed that in the case of FSK and PPN. Now, we will add minus of each of these points.

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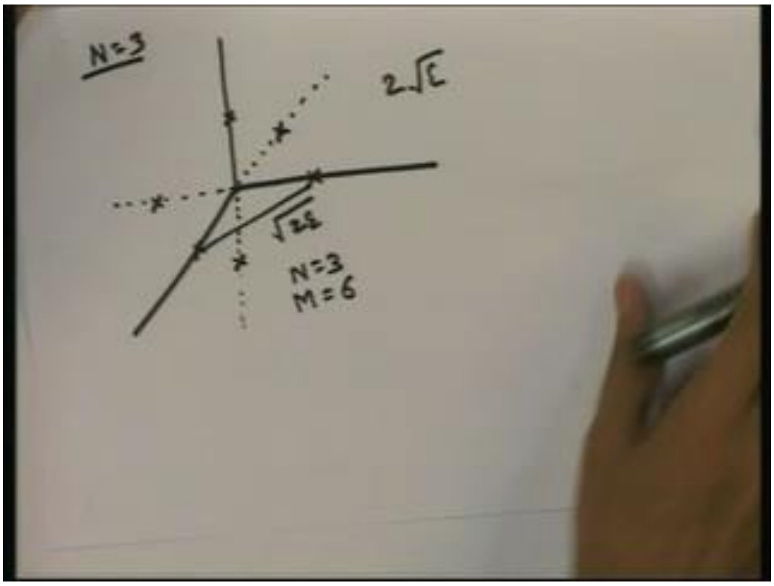

So, this is N equal to 3; number of points equal to 6. This biorthogonal signaling in dimension 3. Similarly, we can have in any dimension and so, we will see some properties of this signal set. 1 it is still constant energy each 1 has the same energy E. Second they are not this is not distance uniform meaning by if you take 1 point in the signal set. Distance to all other points are not same distance to all other points are not same. For example: if you take this point from here the distance to this point is root over 2 E; we saw that in case of orthogonal signaling.

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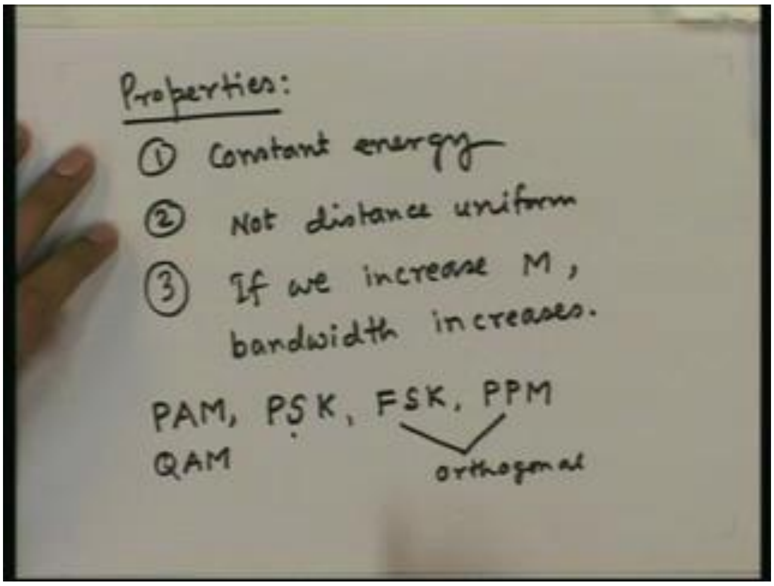

But from distance from here to here is just 2 times root E. So, this distance is 2 times root E whereas, this distance is root over 2 E. So, distance from this point to all 2 of these 2 points are not same; is not distance uniform. And here also like, orthogonal if you want to increase the number of points if, we increase M the bandwidth increases. Because, the dimension of the signal space will increase has to be increased.

Because, that is M by 2 and to increase the dimension you have to increase bandwidth. Because, that can be done either by doing FSK. If, you want to increase the bandwidth increase the number of dimensions you can do that by take picking up different frequencies or different time to pick up different time instances. The time shifts in PPM; we have to take a narrower pulse and that will increase the bandwidth.

See in either way, it is always true that to increase the dimension of the signal space you have to increase bandwidth. So, now we have seen several modulation techniques. We have seen PAM Pulse amplitude modulation; PSK Phase shift key. So, this is amplitude this is phase; the then frequency FSK. Then, QAM is can be considered as combined PAM and PSK or can be is a combined PAM of 2 different carriers.

So, we also have QAM and then, we have PPM Pulse position modulation; Pulse position modulation is used particularly, was used for ultra wideband applications. Like: detecting underground things. So, pulse position modulation is to be used for such ultra wideband applications. Now, we have seen that this FSK and PPM are orthogonal. Then, PSK is constant energy, FSK is constant energy, PPM is also constant energy these need not be.

Now, we will see we have discussed how to demodulate PAM signal in detail in match filtering. We will see that, the same kind of technique can be used for detecting all the other modulation also. So, we will discuss now Demodulation. So suppose, that now we will not assume that this is a particular kind of modulation.

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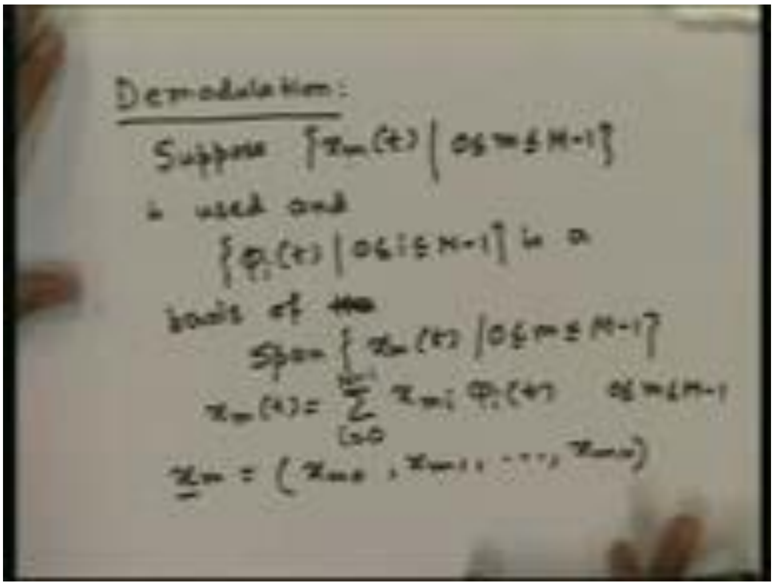

We will take any modulation that is we will take any set of signals, that is that are used in transmitter and then, we will see at the receiver how we should demodulate the signal? That is how we should estimate which message was transmitted? Which M was transmitted? So, we will not assume anything any property of the transmitted signals like: FSK, PPM or QAM we will not assume anything regarding that.

Suppose, we have xm t 0 M this is the signal set and this is a basis of the signal space; that is basis of span. So, we are given that the transmitter uses these signals to transmit message M and if we take the subspace that is generated by these signals. That is the span of these signals we will have this is a basis of orthonormal basis of this subspace.

So, we know that every xm can be expressed as a linear combination of this phi i’s. So, dimension of the signal space is n and these are the basis vectors. So, we know that xm t can be expressed as, a linear combination of phi i t. This is for all possible n. So, now we will construct with these coefficients, we will construct the vectors xm instead of taking as a function. We will express this, we will represent this signal as a vector xm with components this xm i.

So, xm0, xm1, xmN this is in the N dimension this vector is in the N dimensional vector space. So, this these are actually these are the points of the constellation of the signal set. We actually, draw these points in the N dimensional space and we call that as the constellation of the signal set. Now, suppose that a particular xm was transmitted. We will see, how to detect? How to estimate which m was transmitted?

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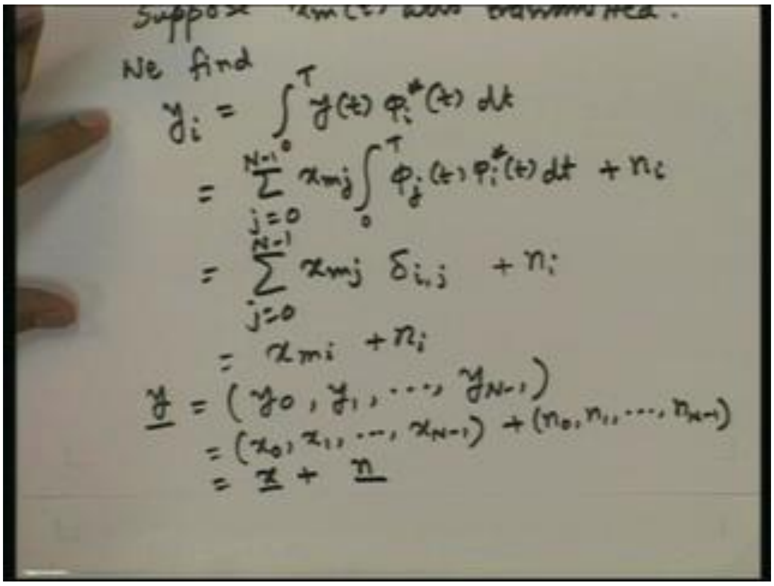

So, suppose xm t was transmitted. We will find this correlation y we will call yi; this is the correlation or the inner product of yt and phi t this is the inner product. And we have seen that, this inner product can be computed using a match filter. So, this can be written as now yt is nothing, but xm t plus nt. So, xm t plus nt; nt we will separate for the xm t part we can write this way: j equal to 0 to N minus 1 xmi integration 0 to T phi i phi jt. This is j phi jt phi i t dt plus the integration of the noise after multiplying by this.

So, we will say we wills assume that that is ni. Now, what is this integration? So, this is obtained by taking all the xm part here and xm is the summation j equal to 0 to N minus 1 xmj phi jt; we have seen that here. So, taking the integration inside the summation we get this. Now, what is this integration? We know that inner product of phi j and phi i is 0 if j not equal to i. So, this is non 0 and this is equal to 1 only if i equal to j otherwise, it is 0. So, it is the delta function this integration is delta i,j. So, that is if i equal to j then delta of that is 1. If they are not equal then, it is 0 plus ni. So, this is summation over j i is a fixed number. So, for only for j equal to i this will be 1 for all others it will be 0 and when it you multiply 0 with this you will get 0 there also. This sum this product also will be 0.

So, you will get only 1 term here which will be for j equal to i and that will be xmi because, j is equal to i for that case. So, x mi plus ni. Now, what is this? Xmi is the i'th component of xm. So, we are actually trying to find out this xm estimate xm. If we find xm we know what is xm t also. So, we know what is m. If we know xm so, this is the component how to find now this component? If we do this operations received signal multiply with this and take integration basically, finding the inner product of y t and phi i t then, we will get this.

So, this is except for the noise it is actually the i'th component of the transmitted signal. So, this is an estimate of this component because, it is corrupted only by noise. If, the noise is small this will give us approximately same as x mi. So, now if we take y we construct with these components a vector y0, y1, yN minus 1 with this components. Then, we see that this is an estimate of the vector x plus some noise vector x naught plus n naught, x1 plus n1 dot. So, those noise components we take as another vector.

Then, we have n naught, n1, n N minus 1. So, this is x plus another a noise vector n. So, this is an estimate of x it is corrupted only by noise. So, from here we can if you get y then, we can estimate x and then we will know we will know the vector. So, from there we can see what is m? So, to find which now this this there is also noise we cannot really extract this part alone; it is already added with noise. So, we have only this we do not have this.

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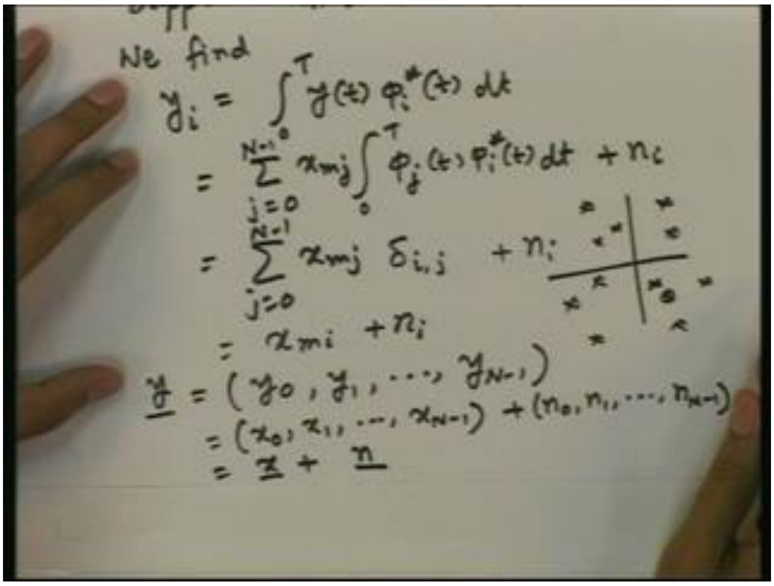

But. So, if we for example: take a constellation some with some points this will be these are the points, these are the transmitted points xm vectors. And then, what we receive here? Suppose, we transmitted this and what we receive here, is this vector plus some noise vector and the received vector will be somewhere here. Now, how do we find? If, we have received this vector that you have received after doing all these operations with each i we get all these components and we get this vector from there we plot it.

Now, how do you find? Which point was actually transmitted? This is not the transmitted point this is somewhere in between. So, to do that 1 can easily see that 1 way will be to find just the nearest point and that is the most probably probable transmitted point. So, for this we will take this. Because, this is the near this is the nearest point to this. So, we will say we assume that this was transmitted that is the that is our decision.

So, that is the way we will find we will see the distance of y from each xm vectors each xm vector like this. We have y and we take each xm vector and find the distance and take that m for which we get the minimum distance. That m will be assumed to be transmitted so, our decision will be that m.

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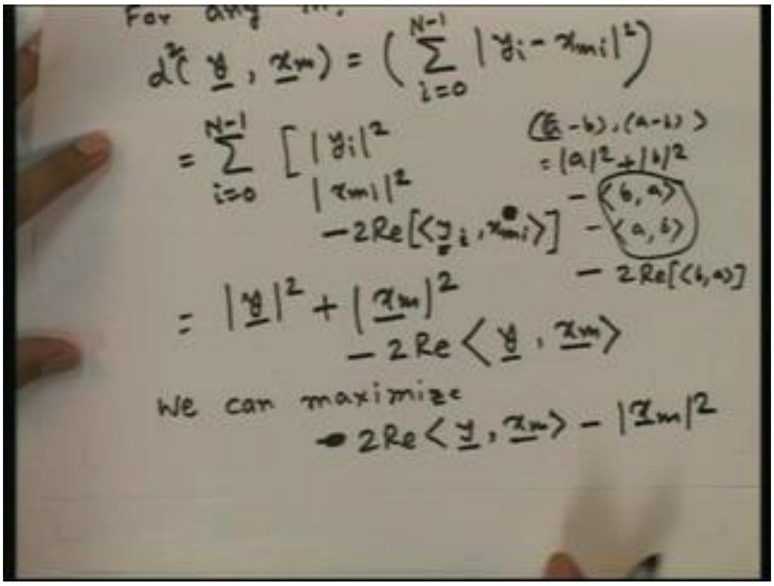

So, for any m now how do you compute the distance? For any m, distance between y and xm is let us, take the square. Because, we can to see which is the minimum distance we can take the minimum square distance also whichever distance is minimum that square also will be minimum among all the square distances. So, we do not want to have a power half here. So, I am taking it on that side.

So, yi minus xmi square is component wise is a Euclidian distance we have discussed it before. And now, we need to minimize this for different m we need to find which m gives us minimum square distance. So, what is this? So, this now can be written as i equal to 0 to N minus 1. Then, now what is this mod square? It is nothing, but the nothing, but the inner product of yi minus xmi with itself. We have seen that, this length square is nothing, but inner product of the vector with itself.

So, here we have mod yi square because first we take the inner product like a minus b; inner product a minus b can be written as, now we can do this multiplication a a minus a b inner product minus b a inner product plus b b inner product. So, a inner product is nothing, but again mod a square. So, we have this here and similarly, we have this term mod xmi square minus. Now let us, see what we get from this kind of expression?

We have mod a square plus mod b square minus inner product b, a minus inner product a, b. Now, from the definition of inner product we can see that, inner product of a, b will be conjugate of inner product of b, a. You see all the definitions of inner product for vectors we have seen that inner product is that way for complex numbers will be conjugate for (refer time: 25:01) it will be same because conjugate is nothing, but the same vector. So, here so this is conjugate of this. So, we will have their addition will be the 2 times the real part of this 1 or this 1 real part of them are same because, 1 is conjugate of other. So, this is nothing, but 2 times real b, a or a, b. So, here we have 2 times real y, yi, xmi this is not a star.

So, this now can be written as this summation is over each term. So, this summation can be written as mod y vector square. Then, this can be written as xm vector square. Then, this is 2 real part of the inner product of these vectors. Now, we want to minimize this quantity for different m, we want to compute this for different m for different xm and then find out which m gives us minimum here.

Now, for comparing that this is independent of m. So, we can we need not consider this. So, we can only compute this part and see which m gives us minimum. So, instead of that we can also do take the negative of that is we can maximize the negative of this quantity. That means maximize 2 real minus this. Now, we can also divide it by 2 and then maximize that.

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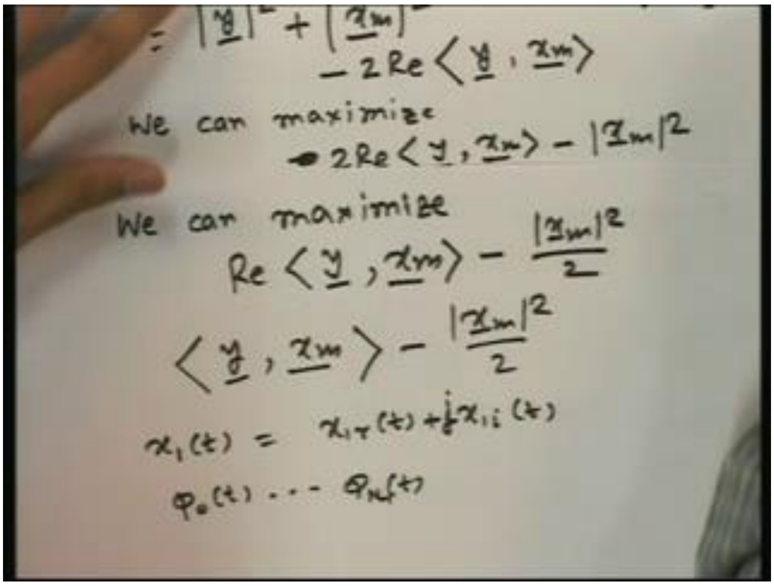

So, we can maximize real. Now, though we said these vectors and the all the signals in general complex signals. Now, we will specialize to real signals. The reason is that, first in practice the signals may be real. When, it is pass band or we are really transmitting through the low pass channel or even if the channel is complex and the signal set is complex. We can view the signal set as is if the signals space is of dimension of N over the complex field.

Then, we can assume that it is signals are actually real signals and the signal space is of dimension 2 times N. Because, we can separate the real part and imaginary part of the signals and then, we will get real signals 2 times m number of real signals. And the dimension of the signal space is will also increase and as a result we can assume that we are we have transmitted real signals. We have received real signals though actually we received complex signals and transmitted complex signal.

So, we can do the demodulation in terms of the real signals that is in terms of the real and imaginary parts of the signals without considering the signals as complex. So, we will assume that the signals are real signals. So, in practice what will happen is that, this y though the original signals are complex. We would have separated the real parts and imaginary parts and consider them separately and. So, each vector, each signal will be a point in the 2 n dimensional space.

So, we will assume that these vectors in the components also will be now the this vectors will be in a space of dimension 2 n or the real numbers. So, this will be real vectors of bigger length instead of complex vectors of length n. So, if the signal; if these vectors are real signals are complex, but we represent them as vectors over the real field then, we can assume these vectors to be real.

Then, we will have for real case we will have only not real part; real part of a real number is real itself only this. So, let me explain it again. Why we are taking it this way instead of complex vectors? Why are you taking real vectors? If, you have the signal x1 t it has some real part x1r t it has some imaginary part x1i t and we have some we can say that this is a vector this is times i,j. So, this is this can be considered as a point in 2 n dimensional space if, the dimension of the signal space over real numbers is 2 n.

So, then this can be expressed as, so this is they will have some basis phi naught t phi n t. Now, N minus 1 t now this N will be much larger if we consider a vector space as a vector space over real numbers. So, only thing what will happen is that, for considering it as a real a vector space we will have higher dimensional vectors here. So, with that we can maximize this quantity.

So, we have these vectors we can compute these vectors by taking this taking this and then. So, we can compute this this way and then find these quantities for different m; this quantity for different m and choose that m for which this is maximum. So, that is our demodulation principle. So, let us draw the diagram the way we will do it. So, this is the inner product this is also called correlation of 2 vectors or correlation of 2 signals. If they are signals, so inner product in terms of signals is also called correlation.

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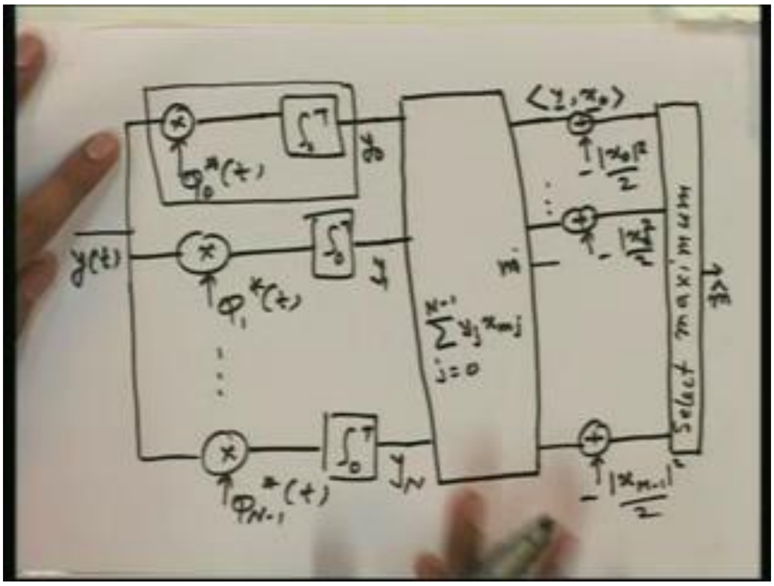

So, this receiver is called correlation what we going to draw in terms of this for decoding and using this principle. So, correlation receiver. So, what would we do first? First we will compute the yi's, this, we want to do these operations. So, yi yt is the received signal then, we will pass it through we will first multiply with this signal these signals. So, and we are assuming real; now we assuming that phi i's are real.

But these coefficients; these coefficients phi i's are complex, but these coefficients are real. So, phi i phi naught star t phi1 star t. So, we have this is not I drawn it as filter, but this is not a filter. We have to actually multiply this. So, we have product with phi naught star t phi1 star t phiN minus 1 star t. So, we have the products here now we want to do the integration from 0 to t.

So, this integration now 0 to t 0 to t 0 to t. So, we have got yi's here yi. So, y naught y1, yn this we have got. So, you have got this vector here. Now, what we want to do is? We want to compute this inner product this correlation. So, how do you find? xm's are known; these vectors are known. So, we have to take correlation with this vector with each xm.

So, here this column vector will multiply by a matrix. That will be such that so, here the output here; there will be the there are n inputs and there will be m outputs. So, the output here will be for output the i'th output, will be j the m'th output or m'th output. Let us, say will be j equal to 0 to N minus 1 yj, xmj this is basically the correlation. So, we are finding this correlation; this is defined as the summation of the product of the components. So, this is the m'th output. So, here we will have y, x0 and so on. So, there will be m number of such for each message there is 1 and then, we want to add x this term minus of this term to the m'th component. So, here we want to add minus mod x0 square by 2.

Second we want to add minus mod x 1 square by 2, the last 1 we want to add minus mod x m minus 1 square by 2. And then, these are our results here. So, we have computed these terms for different m for different m. And then, we want to pick the 1 which give which is maximum. So, what we need to do here is choose maximum. So, select maximum whichever, gives us the maximum that is the estimate of m. So, estimate of m is the that 1 if it is, this is maximum this output is maximum. Then, the m hat is 0 and so on.

So, this is called the correlation receiver because, we are finding the correlation here using this this is the correlation of y t with phi naught t and here again, we are finding the correlation of this vector with each signal vector. Now, this operation, finding this correlation of y t with phi naught t here and phi 1 t here and so on can be performed in terms of match filters. So, we will now see how it can be done using match filter?

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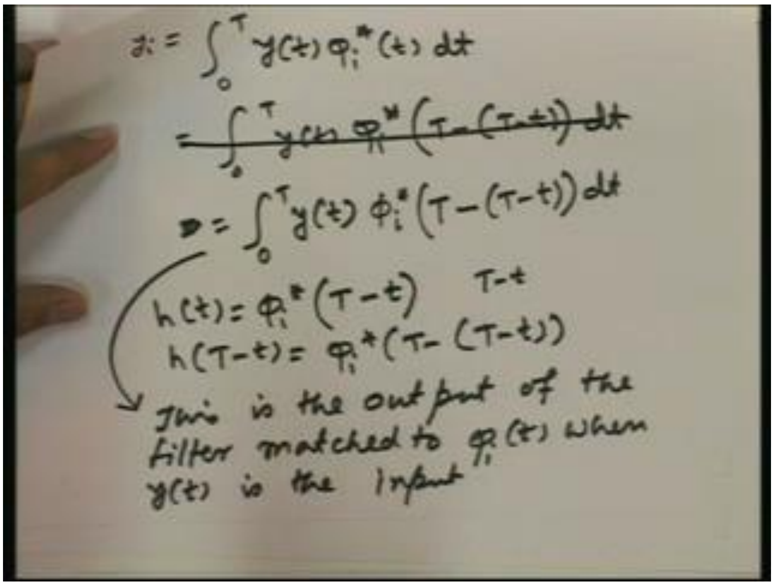

So, what is yi? We said it is 0 to t yt phi I star t dt. Now, this is nothing, but 0 to t y t phi i star I can write t as T minus T minus t dt. Now, what is this? If we see this is the convolution formula y t phi i star T T minus. So, this is this should be it is right. Now, if we take this filter and find the function this function at T minus t what will it be value? what will be the value of it? We have to put T minus t in place of t. So, if this is our function some say h t then, what is h T minus t; it is phi i star T minus in place of small t we will put this T minus t

So, this is we can say that this is the output of a match filter this filter with impulses upon this at T. So, we are evaluating the convolution of y t and h t at T. Then, we have this expression. So, this is nothing, but the output this is the output of the filter matched to phi i t when yt is the input. So, output of the filter matched to phi i t at T.

So, we have to we can evaluate this by passing yt through this filter and then sampling it at sampling the output at T, that will be the value of yi. So, we can say that we can implement the that operation in the following way. Instead of multiplying by phi i star t and then integrating we can pass it through this filter phi naught star T minus t phi 1 star T minus t.

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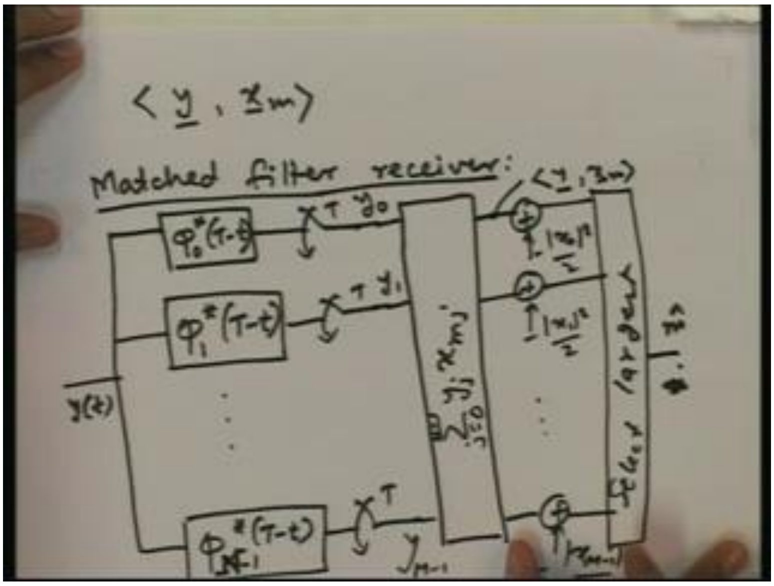

Then, we sample it at t; then here again, we have got y naught y 1 y M minus 1. Then, we do the same things as we did here, once we have got this we can do the same thing. So, we will here we will compute for different m; we will compute summation j equal to 0 to n minus 1 yj xmj. And then, we will add minus mod x1 square by 2 minus mod x naught square by 2 minus mod x M minus 1 square by 2 and then select the largest.

Select largest to get to estimate which message was transmitted. So, we have seen 2 kinds of receivers: 1 is they actually do the same things their performance will be exactly same, but 1 uses this multiplayer and integrator to find the correlation the other uses a filter matched filter and sampler to perform the same operation. So, this is called the matched filter receiver; so, matched filter receiver.

Now, instead of performing matched filtering at different basis signals with different basis signals. We can also, do matched filtering directly with the with each signal each transmitted a possible transmitted signal. So, the difference is that what is the use of having all these basis vectors? First of all we have seen that it had transmitted itself we need not have. So, many signal generators, separate signal generators as many as m we can have just capital N number of basis signals and then, use linear combination of them to generate the signals that are to be transmitted.

So, at the transmitter it helps to generate the signals in terms of these signals and at the receiver also if you do this way. Because then, we will this is N will need less number of matched filters; because, usually N will be smaller than M for most of the modulation skins not for orthogonal modulation skin or biorthogonal modulation skin of course, for biorthogonal also this will be half of M.

So, but we can still in principle do this matched filter receiver, reception using matched filters; matched to individual transmitted signals. So, how do we do this? So, we see that we want to compute m using matched filter. So, here we have computed that this is y, xm, but this can be computed directly by passing it through matched filter and sampling. But the matched filter has to be xm matched to xm t itself.

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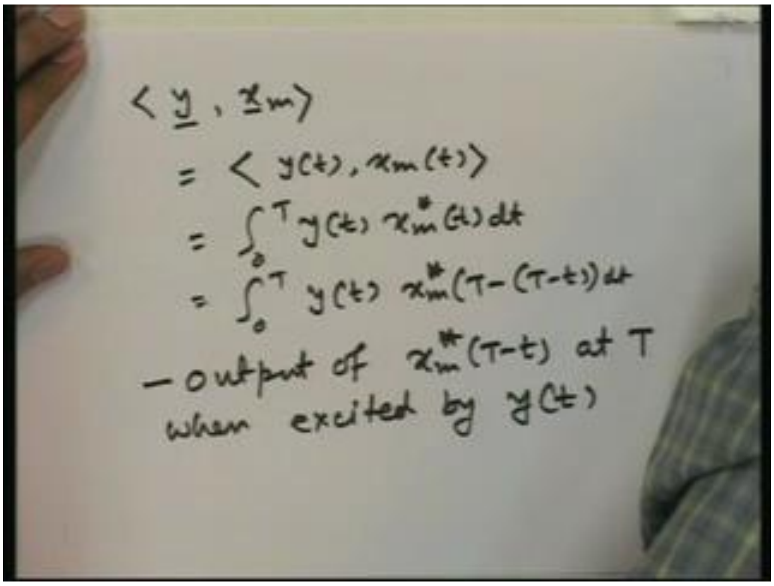

So, because this is let us see what this is? This is yt xm t we have seen that, we have we have discussed that once they are expressed both expressed in terms of orthonormal basis inner product of these 2 vectors will be same as inner product of these 2 signals. So, this is 0 to t y t xm t dt. So, star; so, this again just like before we can write this as xm T minus T minus t dt. This is again, the output of the filtered xm T minus t filter with impulse response this at T at T when excited by y t.

So, if we have this matched filter matched to xm t instead of those phi it is. And then, pass this yt through this filter then sample the output at T then we will get this thing; which is the output here for the m’th signal, m’th output here. Then, so we can now do the same operations as after this we can now do all these operations as before.

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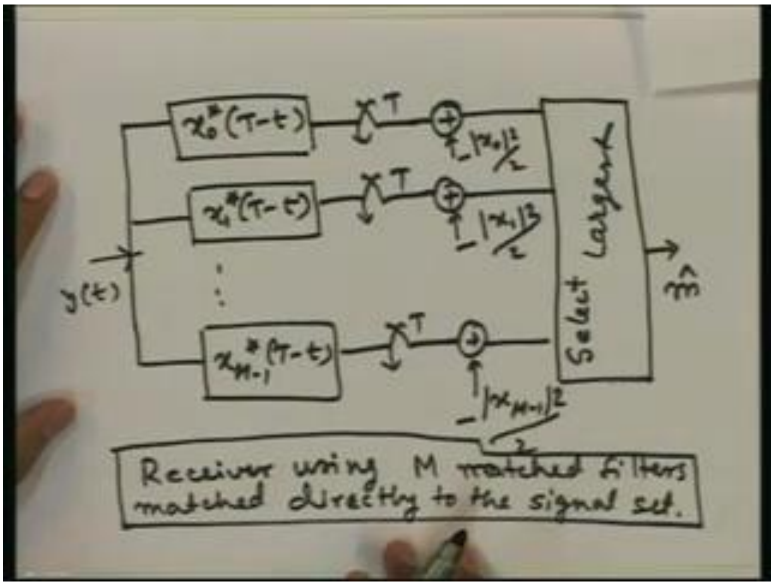

So, our receiver structure with this principle will be receive this y t and pass it through x naught star T minus t x1 star T minus t xm minus 1 star T minus t. And then, sample at T then as before add minus mod x naught square by 2. Minus mod x1 square by 2. Minus mod xm minus 1 square by 2; then, select the largest.

So, this is basically we have computed this here this quantities here by doing this matched filtering directly with the signal possible transmitted signals. So, this is the match this is also a matched filter receiver, but this is a receiver, but this is a receiver. Which uses M number of matched filters, M number of matched filters, matched directly to the signal set.

So, this is the and this both are matched filters, but this will possibly use less number of matched filters the number is N. Here we will use M number of matched filters which will so, we will need more number of matched filters. But both will perform exactly similarly, and there will not be any difference in the performance of the 2 the only difference will be possibly in the complexity of the implementation.

So, in this class we have considered another modulation technique called biorthogonal modulation technique. Biorthogonal is not a specific modulation technique, but it is a class of modulation techniques. So, it can be using FSK, it can be using PPM. We basically add more points, the negative points of the orthogonal signal set. So, that gives us a biorthogonal signal.

Then, we have discussed demodulation for different kind of modulation techniques as a in a generalized framework. So, we haven't assumed any particular kind of modulation. We have just discussed the demodulation techniques for any arbitrary signal set. So, if you are given a set of signals, if we are told that the transmitter uses these signals to transmit message. Then, we have seen how to see the demodulation at the receiver?

Now, in the next class we will see some specific modulation techniques. And see how the demodulation can be made simpler for specific modulation techniques? Like: FSK, PPM, PSK for these some of these modulation techniques; the demodulation some of these, some of the structure can be simplified at the receiver. So, that is all for today next class we will start the specific demodulation techniques for different type of modulations. See you next time.