Digital Communication
Prof. Bikash Kumar Dey
Electrical Engineering Department
Indian Institute of Technology, Bombay

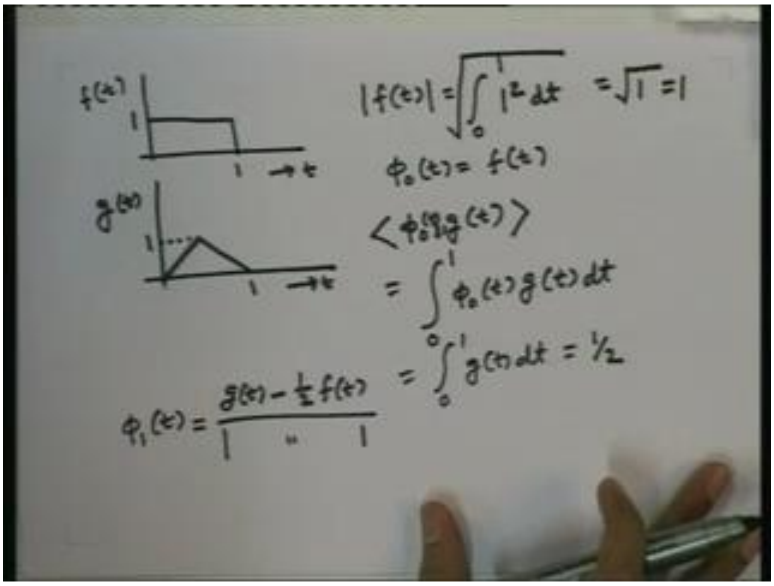
Lecture - 18
Digital Modulation Techniques (Part - 7)

Hello everyone, we have been discussing digital modulation techniques for some classes now and we will continue digital modulation techniques for some more classes. In the last class, we have finished some linear algebra background which could be required from the future classes. So, in particular we have in linear algebra we have seen, what is a linearly independent set of vectors. What is the generating set of a vector space? What is the span of a set of vectors. We have seen, that if the span of the set of vectors is the whole space then that set is a generating set of the vector space.

Then we have seen, what is basis a basis is a set of linearly independent vectors which also generate the vector space. Then we have introduced the notion of length and inner product in a vector space and we have seen the relation between them. We have seen that, inner product of a vector with itself is the square of its length. And we have also seen that if we are given a set of vectors and if we want to find an orthonormal basis of the span of the set of vectors. So, if you are given a set of vectors its span is the subspace of the whole vector space. And if you want to find an orthonormal basis of that vector space that span the subspace then, Gram-Schmidt orthonormalization gives a way to obtain that, a way to obtain an orthonormal basis of the span of a set of given vectors.

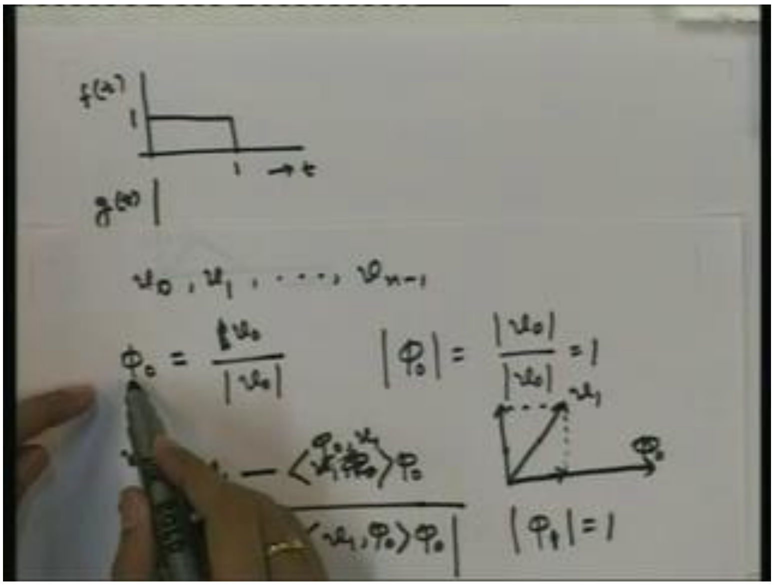
So, we have seen example for rn type of vector spaces, finite dimensional vector space over r and we will just conclude this topic this background of linear algebra with 1 example, with signals.

(Refer Slide Time: 03:18)



So, let us say we have suppose, we have we will just do it with 2 vectors. Suppose we are given these 2 vectors 1, 1 these 2 signals; say this is this can be v naught. So, this let us say this is a signal f t. And we have let us say, another signal 1, 1 g t. So, these 2 are the vectors given to us, we want to get a set of orthonormal vectors such that these 2 can be expressed a linear combination of those vectors.

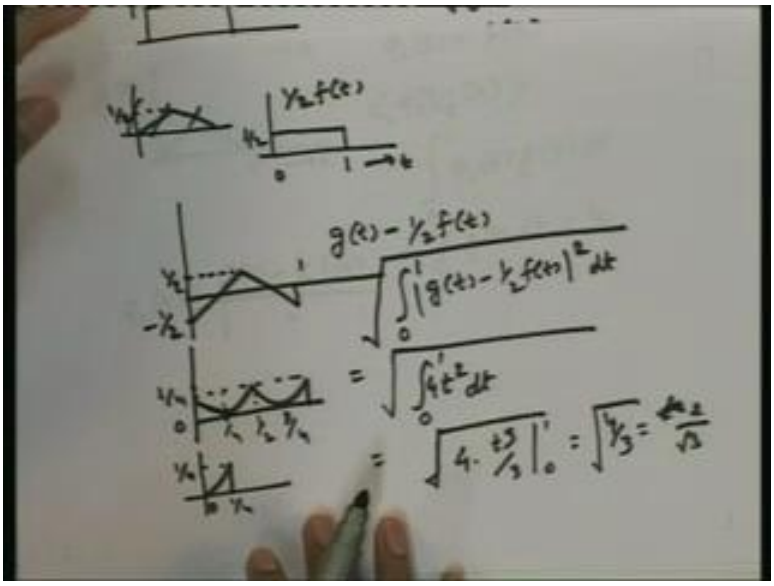
(Refer Slide Time: 04:12)



So, the first thing we have to do, we have seen is that, we have to compute the first vector by taking this scaling this first signals such that its length is one. So, let us compute its length and divide by that length. So, what is length of f t, it is integration 0 to 1. 1 square that is equal to 1 dt root over this. So, this is root over 1 that is 1. So, so our phi naught t is f t itself. Then we have to compute this. We have to take the second vector, first we have to compute this inner product inner product of phi naught with the second vector v1.

So, second vector is g t second signal. So, we have to compute the inner product of this with phi naught so, phi naught and g t. So, this is also function. So, this inner product is integration 0 to 1 phi naught t g t, dt. Now, this integration is nothing but, this is just one. So, this is it is this g t just g t phi naught is 1 in that interval. So, this integration g t, dt it is the area of g t. What is the area of g t? It is base 1, height 1. So, the area is half. So, the second vector we will get, by taking v 1 minus this inner product times phi naught. So, the second vector phi 1 t second signal will be. So, g t minus the inner product the inner product time's phi naught the inner product is half, half times phi naught is f t by the length.

(Refer Slide Time: 06:50)



Now, let us see what this is. So, g t minus half f t, half f t will basically it will be half f t will be like this, its height will be half here, this is half g t, half f t is this half this is half f t. Now, g t minus half f t. So, this is half everywhere the value is half from 0 to 1.

So, we have to subtract half from g t that so; that means, the axis will grow up to mid level. So, we will get this is 1, this is half, this is minus half. So, this is g t minus half f t.

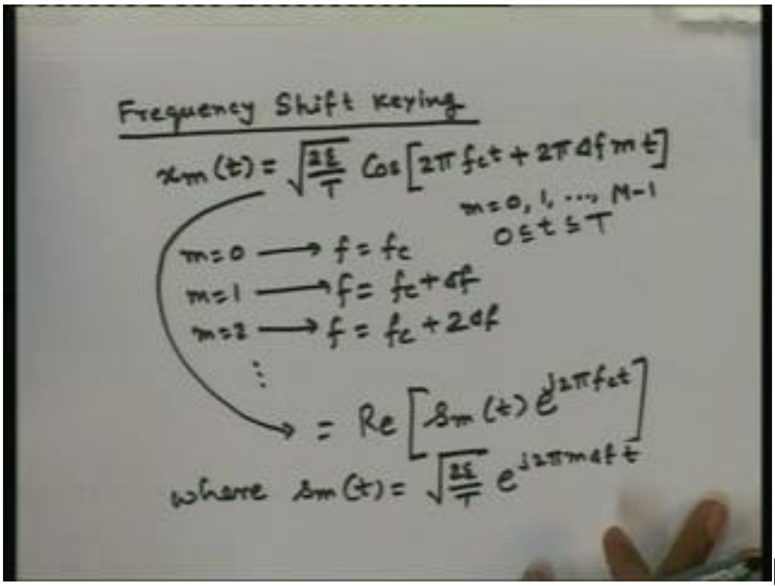
Now, we have to divide this by its length. So, what is the length of this vector this signal we have to compute its length. So, the length is that its integration 0 to 1, then g t minus half f t square dt, then root over is this is the length of the signal. So, what is the length let us compute the square of this and then integrate. So, what is the square of this signal square will have always positive altitude. So, here it will be one-fourth, it will be like this one-fourth now.

So you can see, that integration of this whole duration has 4 parts 1 is this is one-fourth, this is half, this is three-fourth and integration in each of these intervals is same. So, you can integrate in only 1 part. Let us say, from here to here we will integrate. So, in this part you say it is just like you can shift it here also and integrate it is like this you want to find the area 0 to 1 by 4 this is 1 by 4. So this is basically, it is the square of t which was actually linear square. So, 0 to 1 it is t square dt. So, t square dt in one-fourth it will go to half and that square. So, it is actually t square by integration of this will be 2 dt square one-fourth. So, t square at one-fourth will one-sixteenth, so, 4 dt.

So we will have, so root over 4 t cube by 3. So, this is now 4 by 3 that is root 2, that is 2 by root 3. So, now we have to basically divide this by this length to get the second vector. So, what we have is phi 1, phi 1 t is g t minus half f t by 2 by root 3 and that will be that will be divided by, this will be divided by 2 by 3. 2 by root 3 that is root 3 by multiplied by root 3 by 2. So, it will be root 3 by 2 here minus root 3 by 2 here and it will be like this.

Now; obviously, we can see that this itself and either is a this also is orthogonal to the first 1 phi naught t it is this. And we can see because we have divided by the length here the length of this signal will be 1. So, we have seen how to find orthonormal basis of the span of a given set of signals. This is a special case of the theory we did in the last class for orbitory vector space. So, we will basically use this particular technique later on and we have to keep that in mind in several discussions later. So, among the modulation techniques so far, we have discussed pam, psk and qm. Now, we will start another modulation technique called Frequency shift keying.

(Refer Slide Time: 13:16)

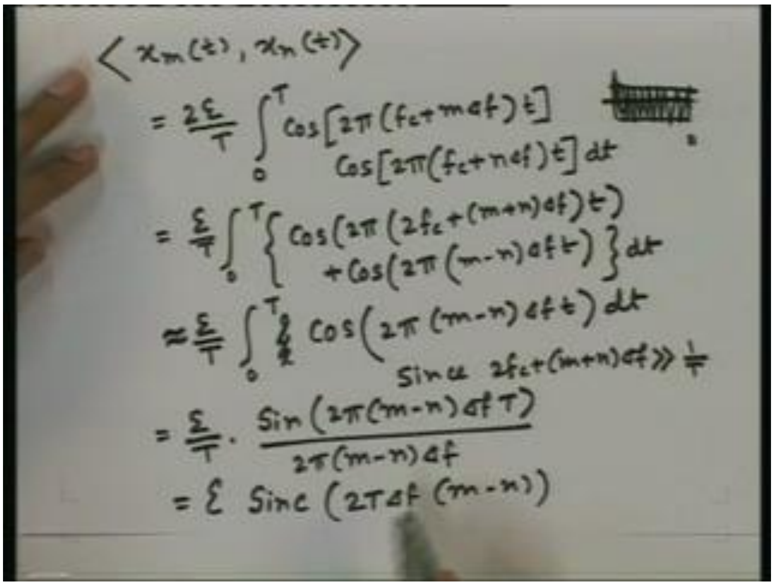


[

If frequency shift keying the signals that are transmitted are of this form a same sin square root with same amplitude and same phase, but of different frequency are transmitted. So, frequency is 2 pi delta f times m t for m, 0, 1 to m minus one. So, here for m equal to 0 we will transmit cos 2 pi f c t for m equal to 1 we will transmit cos 2 pi delta f plus f c t. So, the frequency we will transmit for m equal to 1 for m equal to 0, will be will transmit f c for m equal to 1 we will transmit f c plus delta f m equal to 2 will transmit f equal to f c plus 2 delta f and so on.

So, this signal can now be written also as real part of this is the lopus representation Sm t e to the power of j 2 pi f c t where Sm t is root over 2e by t, this factor and e to the power j 2pi m delta f t. So, and this is of course, a pulse which starts from tan 0 to time t. So, the lopus equivalent signal of the transmitted signal is basically this and it is different for different m. Now, we will see a very special property of this particular technique. See, what are the possible frequencies we can choose? Can we choose delta f as low as we wish? What would happen if we choose 2 small a frequency gap? So, we will see that if we have 2 small frequency gap we will. In fact, not do as good as if we have large frequency gap that is delta f.

(Refer Slide Time: 16:30)



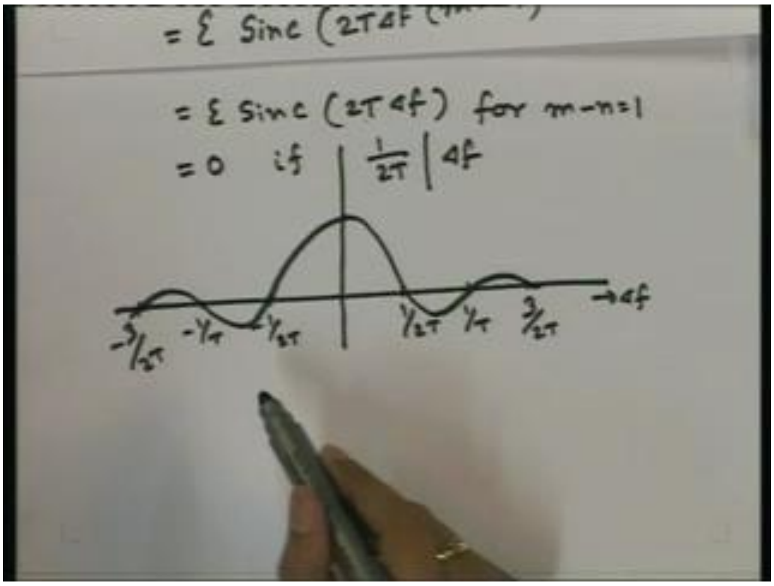
So let us see, what is the inner product of that m th signal and n th signal. This is we have discussed in the last class, that inner product of 2 signals is defined this way. So, product of this 2 we have to take now, product of this for different for m and n. So, we have square of this that is 2 e by t. So, we have 2 by t and then cos 2 pi f c plus m delta f t times cos 2pi f c plus n delta f t dt. Now, 2 times this can be written as addition of these 2 frequencies that is; 2 fc plus m plus n delta f t plus the same thing with the difference of these 2 frequencies cos 2pi difference of these 2 frequencies fc cancels and m minus n delta f.

Now, what we have here is 2 pi times this let us just multiply this whole thing and write as 1 cos 2pi is multiplied to both of this. So, even before multiplying we can see that the frequency of this signal this part is very high compared to this so; that means, the period of this signal, this is the periodic signal with period very small compared to capital T. So, this will have too many oscillations in 0 to t like this. So, its area will be almost 0 because every period will have a real 0 and then, at the end if any small part will remain and in that very small part which is less than 1 period we will have that is a very small area. So, this part will be almost 0 the integration of this part. So, we can write this is approximately same as the integration of only the second part.

So, what is the second part? So, only the second part is 2 pi m minus n delta f t dt. So, this is because 2 fc plus m plus n delta f because fc itself is very high this is much greater than 1 by t. So, this now is e by t and you can see it is sin of this by this quantity it is a t and then 0 sin of 0 is 0. So, we will have only for capital T. So, sin of this whole thing with small t replaced by capital T by this term except for t 2pi m minus n delta f. Now, we can take this capital T here. Then we can say, this is equal to e sinc, sinc of x is nothing but, sin pi x by pi x. So, you have to remove pi and write sinc of 2, 2t delta f m minus n. So we have seen here that, the inner product of any 2 signals of the fsk signal set is this.

Now, it depends on delta f we change delta f the inner product will change. So, for what all delta f the signals will be orthogonal? That is a very special question because we will see that for orthogonal signal set we can decode we can do the demodulation very easily. So we will see now, we will see now for what values of delta f this will be 0 for all possible m and n for any m and n this will be 0. So, that is true for what delta f that we want to see. So let us, take m minus n equal to 1 first the difference is 1, 1 or minus 1 will be same. So, we will take 1 or minus 1 whatever.

(Refer Slide Time: 22:57)



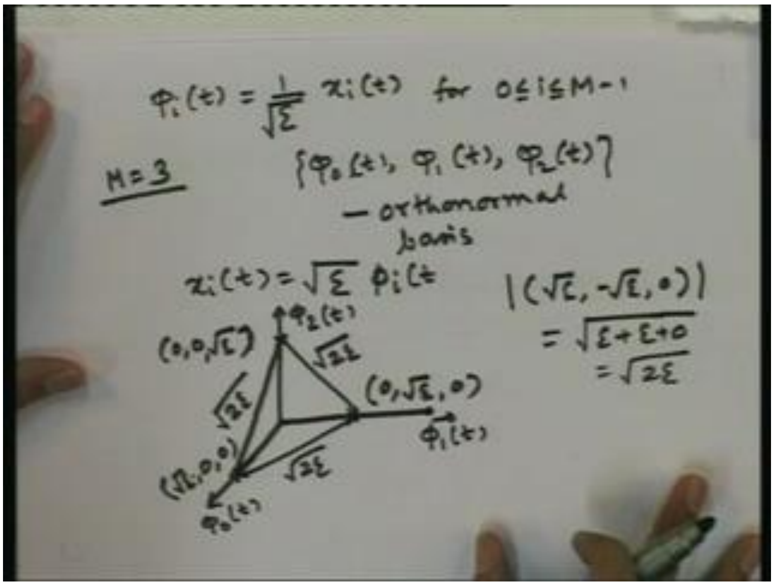
So, for that it is basically the co relation this inner product is basically e, e sinc 2t delta f for m minus n equal to one. Now, let us plot this we all know the well known shape of sinc functions. So its, it is 2 t times delta f is a function of delta f we are plotting and because it is 2t it will be 0 again 0 at 1 by 2t again at 2 times this. So, 1 by t then 3 by 2t and so on. So, minus 1 by 2t minus 1 by t minus 3 by 2t. So, we will have sinc function well this is symmetric, the way I have drawn does not look like symmetric, but it is actually symmetric. Now, we can see from this plot that this value will be 0 at these places. So, this is equal to 0 if delta f is a multiple of 1 by t 1 by 2t.

So, if 1 by 2t divides delta f this is a notation for divides. So one, so this is 0 if 1 by 2 t divides delta f; that means, delta f is a multiple of 1 by 2t and if 1 by 2t divides delta f, then it also divides m minus n time's delta f. So, as a result this also will be 0 for any m and n if delta f is a multiple of 1 by 2t. So, let us repeat that again if delta f is a multiple of 1 by 2t. Then, this will be 0 as you can see from the plot, but if delta f is a multiple of 1 by 2t here, then this is 0 we know and so, if delta f is a multiple of 2t 1 by 2t then delta f times m minus n this whole thing is also multiple of 0 multiple of 1 by 2t. So as a result, this itself will be 0 for any m and n.

So, we conclude that delta f that fsk signal set is orthogonal if 1 by 2t divides delta f. So, the step frequency delta f, if that is multiple of 1 by 2 t then we have seen that fsk signal set is an orthogonal signal set. So, all the signals are orthogonal to each other that is true, if the step frequency is multiple of 1 by 2t. So, let us draw some example orthogonal fsk consolations. So, to draw the consolation what should you do? We should find an orthonormal basis for the signal set generated the subspace generated by the signal set.

So, here we have all these orthogonal signals. So, only thing we need to do is we need to scale them. So, that the length of each signal is 1 then that will give us an orthonormal set of vectors because they are already orthogonal. So, we need to only have scale them appropriately to have length equal to 1 for each of the vectors.

(Refer Slide Time: 27:32)

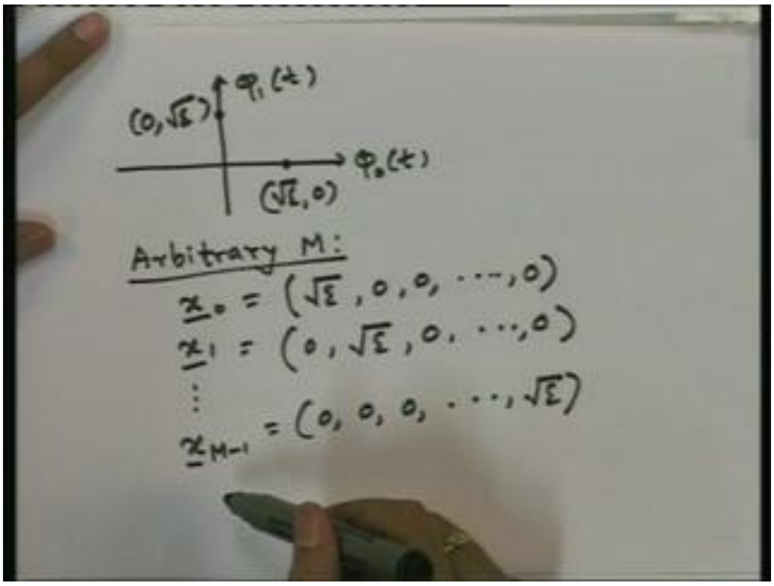


So, we can say that phi i t we will generate. So, that they are orthogonal orthonormal. So, phi i t can be taken as 1 by e xi t. So, xi t is given that we have constructed xm t basically. So, the mth ith vector here the orthonormal vector is we just divide by the, this is root e. So, that the length of this is 1 and they remain orthogonal if you scale they remain orthogonal this is for. Now, let us take 1 particular example and plot the consolation. So, what do we have phi i this phi equal to three phi naught t, phi 1 t and phi 2 t this is an orthonormal basis. And we will express xits in terms of this and then plot.

So, xit this in this case because we are considering orthogonal fsk signal set xit will be nothing but, root e times phi i t from here. So, let us draw the points. So, this is this point is somewhere say phi this say phi naught t this is phi 1 t and this is phi 2t. Now xit is just a scaling of that. So, here there will be some point here, there is some point here this is x naught t this is x 1t this is x 2t. So, if these are not the functions, but these are the components. So, we will draw only the scalars here. So, components of this will be you can say x naught t root e times phi not t plus 0 times, phi 1 t plus 0 times, phi 2 t. So, its coordinate is root e (0, 0) this point and this 1 is 0 root t 0, this point is 0 0 root t.

Now, what is the distance between these 2 points? Distance between these 2 points is the norm or the length of the difference vector. So, what is the difference between this and this minus this vector? It is root e minus root e 0. So, the lengths here is root e minus root e 0 length of this and that is; root over e plus e plus 0. So, this is root over 2e. So, this distance is root over 2e. Similarly, this distance also is root over 2e and this distance also is root over 2e. Similarly, for 2 dimensional case it is it will have, only 2 dimensions and for m equal 2 and remember here that the dimension of this dimension of the signal set signal space is same as number of points 3 here.

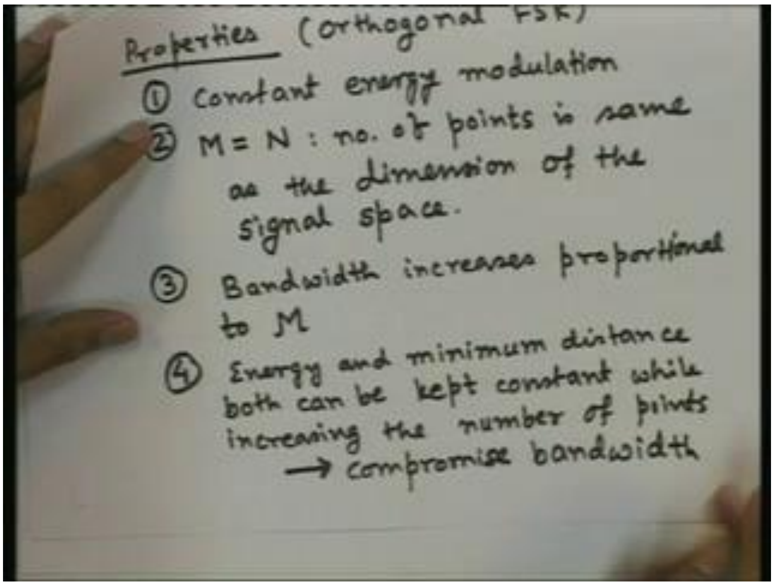
(Refer Slide Time: 31:35)



So, for 2 dimensional case this is say phi naught t, this is phi 1 t and here is 1 point single point that is root e 0 and this is another single point 0 root e and for arbitrary M. What will be the points with respect to the basis vectors? For arbitrary M, we will have the points when we plot them for 2 dimensional, 3 dimensional we are able to plot this way, but for higher dimensional vectors we will not be able to plot them, but we can easily see that their coordinates will be this.

The first 1 will be just first coordinate root e because x naught t is root e times phi naught t plus 0 times all the others. So, root t then 0, 0 all zeroes then x 1 will have only the first component 1, 0 and all others zeroes. Similarly, x m minus 1 will be all zeroes then root over e and you can see verify that the distance from any point to any other point is root over 2e for orthogonal fsk signal set.

(Refer Slide Time: 33:31)



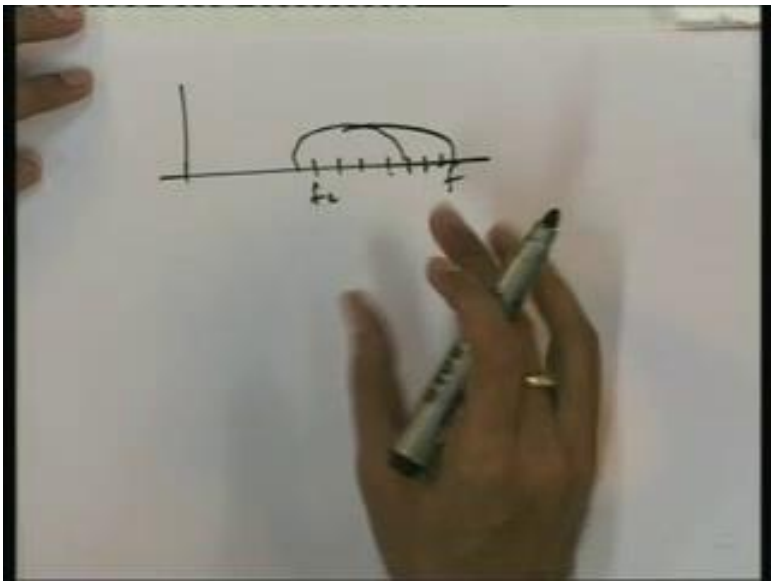
Now, what are the properties that are obvious from this discussion? So, the properties of orthogonal fsk, one is that all the signal points have same energy. We can see that, from these points we can see that the square of the length that is the energy of the signals have the same energy basically e. So, it is a constant energy constant energy modulation. Second, number of point's m is same as the dimension of the signal space that is the dimension of the span of the signals. So, that is; number of points signal points is same as the dimension of the signal space.

Third what happens to the bandwidth utilization? If we want to increase the number of points in fsk signal set, what will happen? We need to choose more and more frequency. So, as a result we will be using more and more and more bandwidth. So, as we increase m we need to increase the bandwidth also. So, bandwidth increases proportional to M. So, this is something very different from all the other modulation schemes we have used before.

Previously we have seen pam, qam then psk of all these modulations techniques if you want to increase the number of points we did not increase the bandwidth; bandwidth are same, but we are increasing either the energy of the signal set, average energy, average energy by putting more and more points far from origin or if you wanted to keep the energy average energy same we have to put more and more points near to each other. So, that the probability of error was sacrificed.

So, here to increase number of points, we are increasing the bandwidth this is another way. So, but the distance will remain same, energy will remain same, energy is still e and distance is still root over 2e the distance between end 2 points, but we are just increasing the band width that we are using. So, we are actually compromising something of the other. Why do you what to increase m. And so, you can say energy and minimum distance. Minimum distance is basically (( )) 1, 2e which is same for all pairs of points minimum distance both can be kept constant while increasing the number of points, but what do we compromise bandwidth as we said here you said here So, compromise.

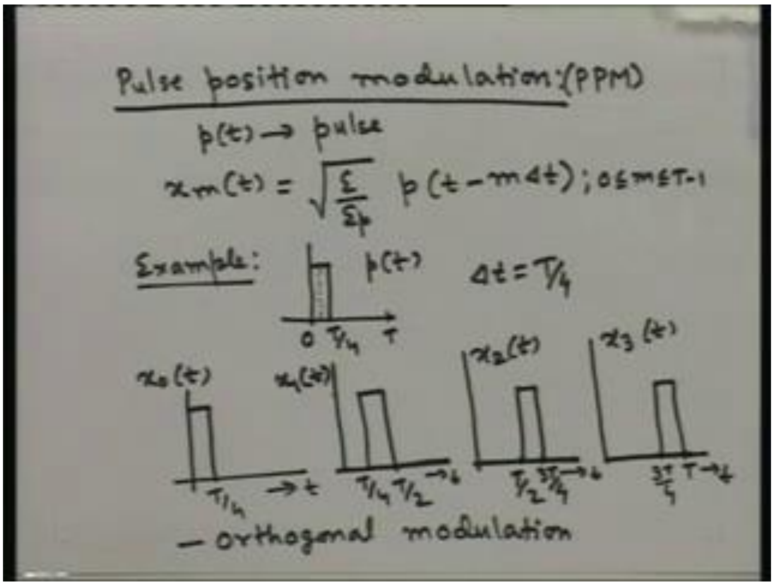
(Refer Slide Time: 38:20)



So, if you want to increase the number of points we will take more and more frequency if this is the frequency axis we first took 1 fc then, fc plus delta f another point another point from 4 fst we will have something like this band and then we increase number of points then we have to take lager band. So, we are taking more and more bandwidth to transmit, more and more bits per sigma that is the idea. So, difference between this fsk and all the pervious modulation techniques like pam, qam and psk is this the fundamental difference is this that energy and minimum distance did not be change to increased number of points it can be done by increasing the bandwidth.

So, we will discuss immediately another very similar modulation technique called pulse position modulation. So, here we have seen that in this case for the case of fsk we have taken different frequency for different symbol to transmit different symbol we have taken different frequency, but we can do the same thing in time also. We can take a pulse and transmitted different time depending on what we want to transmit.

(Refer Slide Time: 39:41)



So, this is called pulse position modulation. We will discuss the baseband version of that, but it can easily changed modified to take to have pass band modulation. So, it is basically the dual of fsk. In fsk we take different frequency here we take different time. So, if you have a pulse pt. So, if you have a baseband pulse it will baseband if you have pass band pulse you will have pass band pulse position modulations. Here, the signal set is this, what is the energy want of transmit that is e and ep is the energy of the pulse we have taken p t minus m delta t.

So, this is the signal set what are you doing? We are taking a fixed pulse, fixed amplitude. Amplitude is not changed whereas, we are for just changing the we are just shifting the pulse by different multiples of delta t. So, let us take an example, if we take this rectangular pulse say this is T by 4 we want to take 4 ppm, we want to have 4 points. So, this is t we want to transmit 4 symbols 4 possible symbols in t it is there are 2 bits in t seconds. So, is this is the pulse p t then for 0 m equal to 0 x 0 t will be simply m equal to 0 will give us simply p t times this, this constant. So, we will have T by 4 this is x naught t. We will have for m equal to 1 we will have t plus delta t. Now let us take, delta t equal to T by 4.

So, this pulse will be shifted by T by 4. This is our x1 t for m equal to 1 we will transmit this, then for m equal to 2 we shift it by 2 times delta t that is 2 time T by 4 that is T by 2. So, this is T by 2 to it will end at T by 4 plus T by 2 that is 3 T by 4 and this is x2 t this is x3 t for x3 t it will start from 3T by 4. 3T by 4 it will end at T. So, you can see that we are really just changing the position of the pulse to transmit the information. The information is content in the position of the pulse not in the amplitude if it is pass band not in the phase, not in the frequency phase, frequency or amplitude of the pulse is same for all the possible values of m only the position of the pulse is being changed.

So, this is just like frequency fsk where which is the frequency of the pulse here we are changing the time of the pulse. And you can see that, this signal set is also orthogonal you take any 2 there in different time. So, their product will be 0 functions, product of say x1 and x3. This is in 1, 0 only somewhere, this is in 1, 0 at somewhere else. So, product of these 2 functions will be zero. So, the inner product of these 2 will be 0 and as a result they are orthogonal. So, all these signals are orthogonal to each other.

So, this is another orthogonal modulation scheme. So, this is also orthogonal modulation and all the properties of fsk are still valid for ppm. Let us see all the properties of fsk still valid for fsk constant energy modulations; obviously, each has the same energy because 1 is the shifted version of the others. So, they have the same energy and energy can be computed easily take the integration of the square of this pulse. M equal to N this is again dimension as same as the number of points because there are all orthogonal to each other, you have to scale them appropriately to get the basis you will get 4 basis vectors orthogonal modulation vectors and constellation will be exactly same as just fsk modulation.

Then bandwidth increases proportional to M. Now, if you want to increase the number of points in ppm, how do you do we want to keep the same symbol interval? So, we cannot increase this time. So, that we put more and more time shifts we cannot do that we have to keep this t same, but then if you want to increase capital M to eight from 4 in this case 4 in this example, but if you want to have 8 we have to take a narrower pulse. So, the width of this pulse will be not T by 4, but T by 8 then we can have 8 shifts. So, it will be half width of this and then you can have 8 numbers of shifts of this pulse. Then we can, transmit 8 possible values in this symbol interval and what will that mean in terms of bandwidth?

If you take a narrower pulse the bandwidth of that pulse increases by it actually doubles. So, if we increase the number of points by 2, we by multiple of 2 then we actually increase the bandwidth used by 2 factor. So, we can see that bandwidth increases propositional to number of points N. And again energy and minimum distance both can be kept constant while increasing the number of points as we have seen we are actually put keeping a narrower pulse and we can still kept the energy same by amplifying it because we are taking narrower pulse we have to use a bigger amplitude pulse.

So, that its energy remains same; we can do that and its minimum distance of the signal set also will remain same, but the bandwidth will be compromised. Because, we are taking a narrower pulse the bandwidth will increase. So, we have discuss in this class frequency modulation, frequency shift keying and pulse position modulation and we have seen that they are a kind of dual to each other. 1 is in the frequency domain taking different frequency depending on M and another is in the time domain taking different time shift depending on M. So, they are dual to each other and they have exactly the same properties.

Bandwidth can be increased to increased number of points, while energy and minimum distance can be kept constant and bandwidth increases proportional to M as M increases and the number of points M is same as the dimension of the signal space. So, all these properties are true both for fsk and pulse position modulation which is also called ppm pulse position modulation ppm in short. So, in the next class we will see some more modulation techniques. That is all for this class.

Thank you.