Digital Communication
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Lecture - 17
Digital Modulation Techniques (Part-6)

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Hello everyone, we have been discussing digital modulation techniques for few classes now and we will continue digital modulation techniques for quite few more classes. So, far we have discussed: pulse amplitude modulation, phase shift key and curvature amplitude modulation; among the digital modulation techniques. We have also discussed: match filter receiver for pulse amplitude modulated signals and we have seen that, match filter receiver is optimum; in the sense that, it maximizes the s n r.

We have also seen; what are the criteria for having 0 ISI. So, that criteria is called the Nyquist criteria. So, Nyquist pulse shape; shaping Nyquist pulse shaping can be used to have 0 ISI and in that context we have also seen a very popular class of pulses, known as raised cosine pulse. In the last class, we have deviated from the modulation techniques to discuss, some elementary facts from linear algebra because, those will be useful in the later classes. And in this class we will start with the remaining part of linear algebra that you will need later and then if, time permits we will go to other modulation techniques.

So, in the last class we have seen what is a field. It has like real set of real numbers, set of complex numbers; they are fields. They have 2 operations 1 is addition and another is multiplication. Under addition and multiplication the sets satisfies certain properties like: every non 0 number has an inverse. So, you can divide any number by any non 0 number and there are other usual properties you know of. We will not discuss those in detail, but you know the properties of real numbers and complex numbers. So, such if such sets are called field.

Now, we have then discussed what is a vector space. It again a vector space over a field, like over a real set of real numbers over the set of complex numbers is, defined again to have certain properties with respect to 1 operation addition and another operation scalar multiplication. Addition is between 2 vectors, scalar multiplication is operation between 1 element from the field called a scalar and a vector from the vector space. And again those operations satisfy the usual rules we all know of; like scalar multiplication is distributive over vector addition and so on.

Then, we have discussed linearly independent vectors. We have discussed, what is a generating set of vectors of vector space. Then we have also discussed what a basis of a vector space is. We have said that linearly, a set of vectors will be called linearly independent, if no vector out of those can be expressed as the linear combination of the other vectors in that set. So that set in a sense; you cannot really exclude any element so that, that element is kind of redundant there. So, that is not possible for linearly independent vectors. So, no vector can be expressed in terms of the other vectors.

Now, what is a generating set of a vector space? A generating set is a set of vectors from which, you can get all the other vectors by linear combination. So, for example, if you have 2 vectors and you can get all the other vectors by taking linear combination of these 2 vectors then, those 2 vectors will be called a generating set of vectors and there are different ways of saying the same fact; another way is to say that those 2 vectors span the whole vector space. Now, we will see what is the span of a set of vectors?

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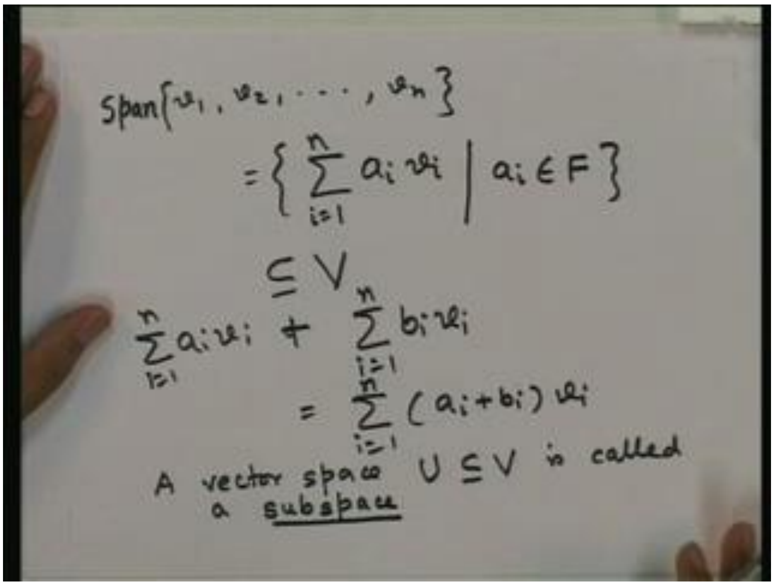

So, if we have a set of vectors v 1, v 2 till v n; they are all from the vector space v. Then, span of these denoted this way; span of these vectors is the set of all vectors which, can be obtained from these vectors by taking linear combinations. So, these are scalars from the field; so, where a i are from the field F. So, you can see here that, there is no, it does not make sense to talk about the vector space, without having a field in mind.

So, a vector space is always over a field. So, then span of a set of vectors is nothing, but the set of vectors which are obtained by taking linear combinations of these vectors. Now, when do we say that, these vectors set of these vectors is a generating set of the vector space v? When this span is, the whole space v.

So, this span in general even if this is; obviously, this is, this will be a set of this will be a sub set of the vector space because, this is a property of the vector space that, if you take 2 vectors, their linear combination will also be a vector; it will also be in the vector space. So, this linear combination because each v i is in V, this linear combination is also in V. So, this whole set is a subset of this V. But, we can in fact, say more. You can see that, if you take 2 elements of this type; 2 linear combinations 1 with a i's and another with v i's let us say. Then, if you add them, that will also be a linear combination of these vectors.

So, summation a i v i plus summation b i v i; another linear combination with different coefficients possibly can also be written as a linear combination of those vectors, where the scalars are, a i plus v i. So, take any 2 vectors from this set, you add them you get another vector in that set and similarly, if you multiply scalars to this vector say by c then you get, c a i everywhere and that will be also a linear combination of these vectors. So, we see that, this is such a set that you take 2 vectors from this set, you add them you get another vector in the same set; if you multiply by a scalar to any vector there, you get another vector in that set.

So, this set is also a vector space, may not be same as this, but will be a subset of this. Such a vector space which is a subset of this vector space is called a sub space of V. So, a vector space U which is a sub set of V is called a sub space. Just like sub set, you have a sub space. So, span of a few vectors is a sub space of V we know. Now, when this subspace is equal to V then; obviously, these vectors generate V that is, this set of vectors is a generating set of V.

So, we say that a set of vectors is a generating set of V if, the span of those vectors is V itself, but if it is not V, but it is a still subset of V of course, then this is not a generating set of V. Then, we have discussed we have said in the last class that, a basis is such a set

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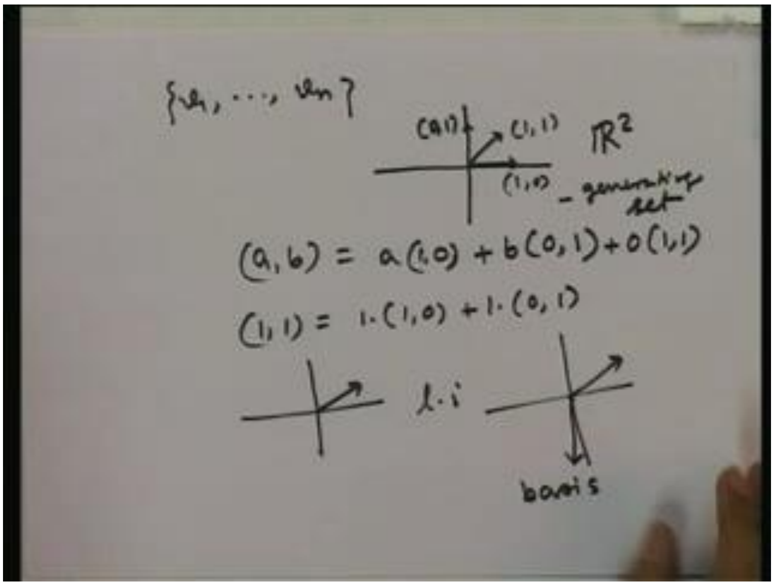

It generates V and also they are linearly independent. In general, if you have a generating set of vectors they may not be linearly independent. For example, if you have 1 0 1 0 1 and also 1 1 they generate the whole vector space R2. This we have taken R2; set of all pairs of real numbers. Now; obviously, these generate a the whole vector space because, any vector a b can be written as a times 0 a times 1 0 plus b times 0 1 plus 0 times 1 1.

So, this is a linear combination of these 3 vectors. So, any vector can be generated that way, this is a linear combination of these 3 vectors. So, these 3 vectors generate the whole space, but you can readily see that, this vector is kind of redundant. Even without this, these 2 vectors will generate the whole space. So, that is also reflected in a different way if you say that, this vector itself is a linear combination of these 2 vectors. So, we do not need this vector. So, these 3 vectors are non-linearly independent because, 1 1 can be expressed as a linear combination of these 2. 1 times 1 0 plus 1 times 0 1 these 2 are the scalars. So, this vector is linear combination of these 2. So, these 3 vectors are non-linearly independent.

So, these 3 vectors do not form a basis. They generate the vector space, but they are not linearly independent. We can have the other way around also; we can have a linearly independent set of vectors which are not a generating set. For example, if you simply have 1 vector here; this is linearly independent, but this does not generate the whole space. So, this is generating set. This is linearly independent. None of them is a basis, whereas, this is a basis. Now, let us see in terms of signals that is, functions.

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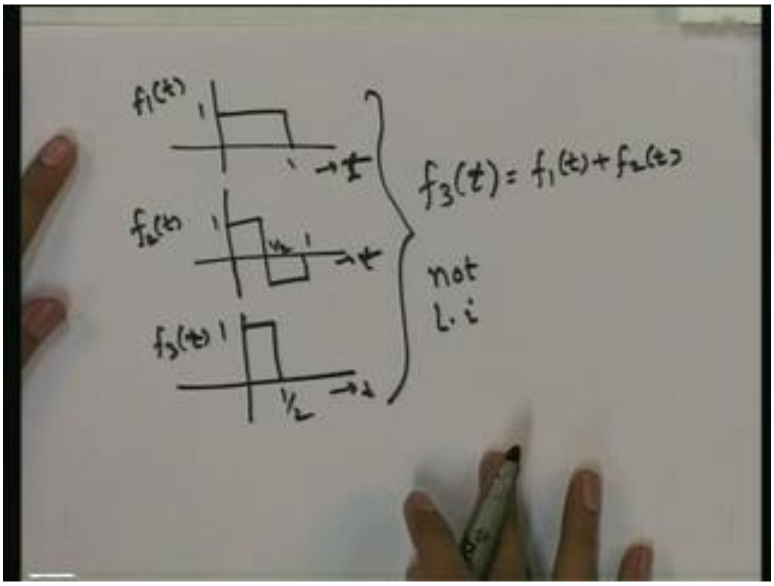

Now, if you have say, these 2 functions x or t; for signals it t is a better most appropriate variable. So, these 2 functions are linearly independent. You cannot get 1 function by taking linear combination, linear scaling of the other. They are linearly independent and for example, if you take again say this vector; say 1 itself this is, say this is; f3 x, this is f say t f2 t, this is f1 t. These are not linearly independent because, 1 of any of them can be obtained as linear combination of the others. Say take this 1: f3 x f3 t is just this plus this; f1 t plus f2 t. So, these are not linearly independent; they are not linearly independent, but they are linearly dependent.

Now, we will later see how to generate a set of vectors which are linearly independent, from a given set of vectors which are possibly linear linearly independent. Now, before going to that, we will introduce some more concepts 1 is length; length of a vector of a vector v is denoted by this symbol is o, it is defined as in somewhere differently for different vector spaces.

For example, for rn it is defined as if v is say v 1, v 2 let us say, x 1, x 2, x n; these are the components then, this is defined as root over x 1 square plus x 2 square plus x n square. There are different ways we can define this length, but this is the usual, the most natural length.

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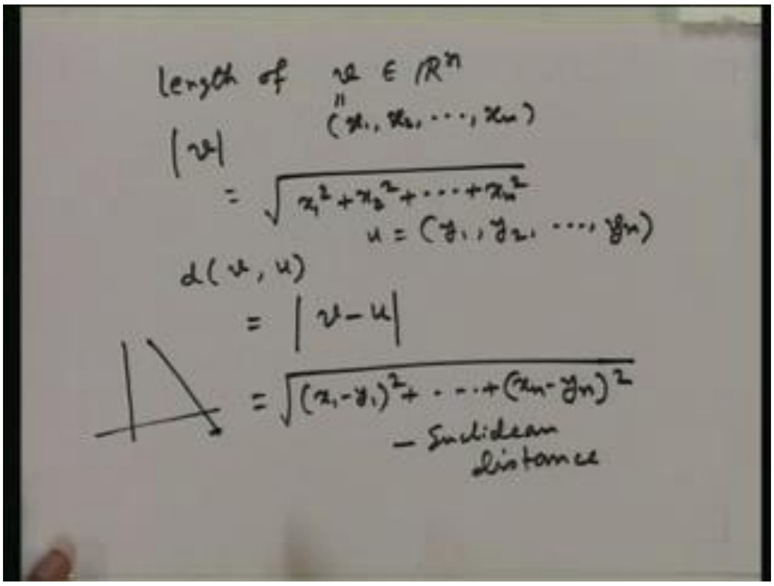

In terms of length you can also define, what is called the distance between 2 vectors v and u. Suppose, u is given by; u is y 1, y 2, y n, then, distance between them is nothing, but the length of v minus u or u minus v whatever. So, this is basically root over x 1 minus y 1 whole square plus x n minus y n whole square. This is the distance between 2 2 vectors. This length and this distance that usually called Euclidean distance because, there are other ways of defining distance. This is the most natural distance the you can measure this distance, with using a scale between this point and say this point. Simply you measure by scale that, that is the Euclidean distance between 2 vectors.

Now, our vectors space may be may not be rn type of vector space, but our vector space may contain functions that is; signals. So, how do we define distance between 2 signals and length of a particular signal? So, for that we have to define distance and length in the vector space of functions; real functions and complex functions. So, that is defined in the following way: suppose, we consider the complex functions; real functions will be a special case of that.

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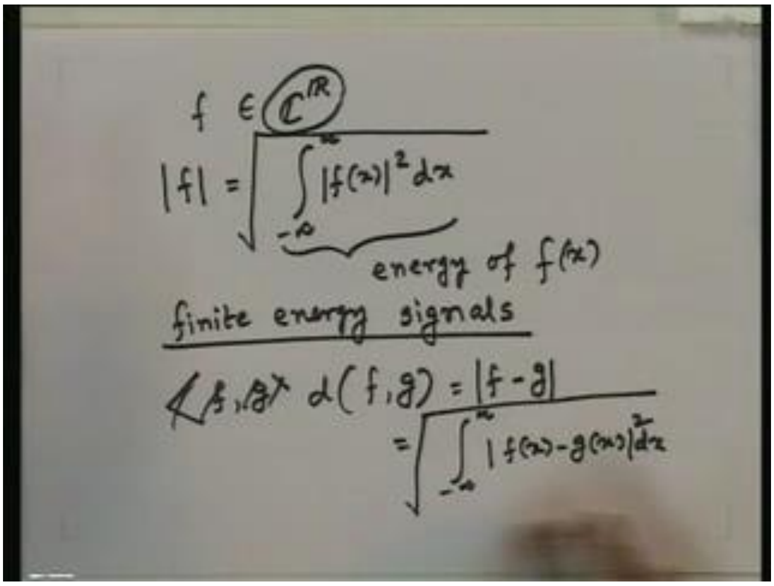

Suppose, we have f a function of complex; complex function of real numbers, this the set of such functions is denoted this way that is, set of functions; complex functions of real variable. So, let f be a such a function then, length a norm of f is defined as; remember f x may be complex. So, we take mod f x square dx this integration. Now, this integration may not converge, may not exist. So, this norm, this length is defined only for those functions whose absolute value square of absolute value is integrable. That means, what is this if we consider f x as signal, what is this quantity this integration? This is nothing, but energy of f. So, we are taking root over that. So, this is the, now this part inside root this part is the energy of f.

Now, we want to consider only the functions which have finite energy that is, for which the integration this integration exists. So, we consider only finite energy signals or functions. We can see we can it can be proved that, if you take only the finite energy signals from all these functions, all these signals they will form a sub space of this space. There are functions which are not finite energy. For example, the constant function 1 is not finite energy, but if you consider only the finite energy signals you, they will form a sub space meaning by they themselves will be a vector space.

So, if you add 2 finite energy signals, you will get another finite energy signals. That can be proved very easily. If you multiply by a scalar to any finite energy signal, you will get again a finite energy signal. So, finite energy signals from a sub space of this all possible signals and we will be considering only finite energy signals because, what we are going to transmit it, in each interval and what we are going to receive in each interval are finite energy that is what important to us.

So, for finite energy signals the in that vector space, the sub space of finite energy signals; we define this as the length and then this is the length of finite energy signals. Now this is basically the root over root over energy. Now, similarly you can define the distance between the 2 finite energy signals f and g, this is d of f and g; this is basically the length of f minus g that is this. So, this is the distance between 2 functions. Let us see an example; take 2 functions and try to compute their distance let us say, f x is

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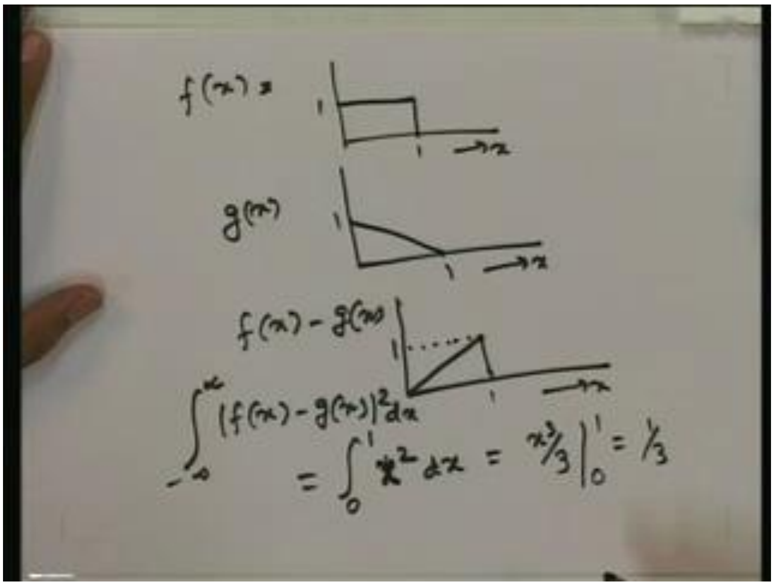

Let us take simple functions from which it can be computed very easily, this suppose 1 1 this is x this is f x. Then let us take g x say this is 1 1, this is g x. Let us compute first f x minus g x; f x minus g x as you can see; obviously, it is this 1, this value is also 1.

So, we want to find, we want to integrate the square of this from minus infinity to infinity, integration of this function as you can see, this is basically at 1 is going to 1 linearly. So, it is basically the function t from 0 to 1. So, this is 0 to 1 t square d, this is x x square dx. So, we have x cube by 3. So, 1 by 3 then we have to take root of that and that is the distance. So, we have the distance between f x and g x is; root over 1 by 3 that is, 1 over root 3. This is the distance between f and g.

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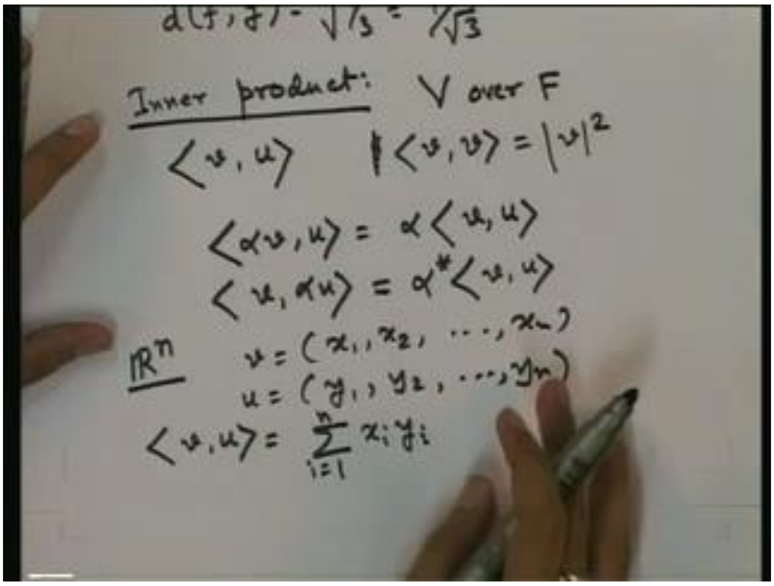

Now, we will define another concept here similar to that product. It is in fact, our product, but we call it inner product. So, inner product; this is the motivation for doing this is, to check orthogonality of vectors. How do we know if 2 vectors are orthogonal? So, to check that, we need inner product.

So, inner product of 2 vectors; 2 vectors v and u a considering a vector space let us say, V over a field F and 2 vectors v and u. Inner product of 2 vectors is defined in a certain way. We will not go into the definition, but it is defined in such a way that it satisfies again certain properties. Those properties will not list all of them, but for example, there are properties like inner product of v with itself, is always positive or 0. It cannot be, it is not negative always and it is same as the length of v square. So, inner product of v with itself is the length of v square.

Now, and there is a very famous and there are other properties like; if you multiply by a scalar to any of them, the inner product itself will be scaled by that same factor. So, if you take alpha v u is alpha times v u, whereas, if you take v alpha u it will be scaled by alpha star. This is for complex vector space 1; f is complex. So, alpha star v u, will verify this for the proper for the inner products we know. So, these are certain properties inner product should satisfy and let us see some examples now.

Take first Rn the vector space; Rn set of all integrals of real numbers or we can take Cn. So, for Rn if you have 2 vectors v say: x 1, x 2, x n and u: y 1, y 2, y n then, inner product of v and u is basically the, what we call the dot product that is, summation i equal to 1 to n x i y i. Now; obviously, you can see that, if you take if inner product of v with itself you will get x i square here. So, you get the root over of that is, length of v. So, we get square of the length. So, we take inner product of v with itself. Now, let us take to the next step; let us take Cn.

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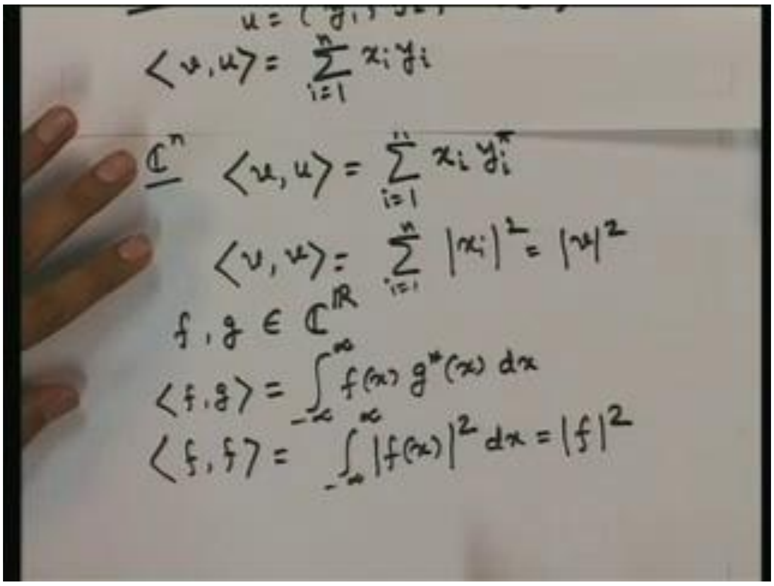

Inner product here is defined in the following way; again v is a x 1, x 2 to x n, u is y 1, y 2 to y n then, inner product of v and u is defined as i equal to 1 to n. Same as the real vectors space, but only difference is y i star. You take the conjugate of the second component instead of taking is as it is. So, this is the inner product for Cn and for Cn and here also you can see if, you take v here in place of u you get x i star x i times x i star is nothing, but mod x i square. x i times x i star is mod x i square.

So, summation of that; root over summation of that is, the so if, you take v v is you get mod x i square and that is the length of v square because, this is root over of this is, defined as the length of v. So, we have seen some particular examples, which we have we are already familiar with of it about inner product. Now, we will see some other examples, which are specially important for us that is, signals in terms of signals. Now, what how do we define the inner product of 2 signals that is 2 functions.

Let us say we have 2 complex functions f and g; 2 complex functions that is, they are in c power r. Then, what is the inner product between them? Inner product is defined as; f g is minus infinity to infinity and we will always take only finite energy signals then, this inner product we will also be defined will exist. So, f of x g star of x dx. This is the inner product. Now, what if we take inner product of f with itself?

We get minus infinite infinity f x f star x which is, if we do multiplication we will get mod f x star square dx. Now, root over this is basically the length of f that is how we define the length of the signal f. So, root over this. So, this is length of f square, again as we had here, we also have here that inner product of f with itself is the square of the length of f.

So, let us compute this inner product for some functions. Let us say, we have a function, again we take the same function that is, say we take f x and g x here. Let us take the inner product. So, what is this function? This function can be said to be 0 to 1; this function is basically 1 minus x. So, at 0 it is 1, at 1 it is 0 and it is linear; so 1 minus x. So, if you want to take inner product, we take f g integration. We are particularly taking here see, here we had complex functions, but if you take real functions; that is a special case of this. A real function is also complex function whose imaginary part is 0. So, this star will not be there, it will be simply multiplication because; star of a real function is same as itself.

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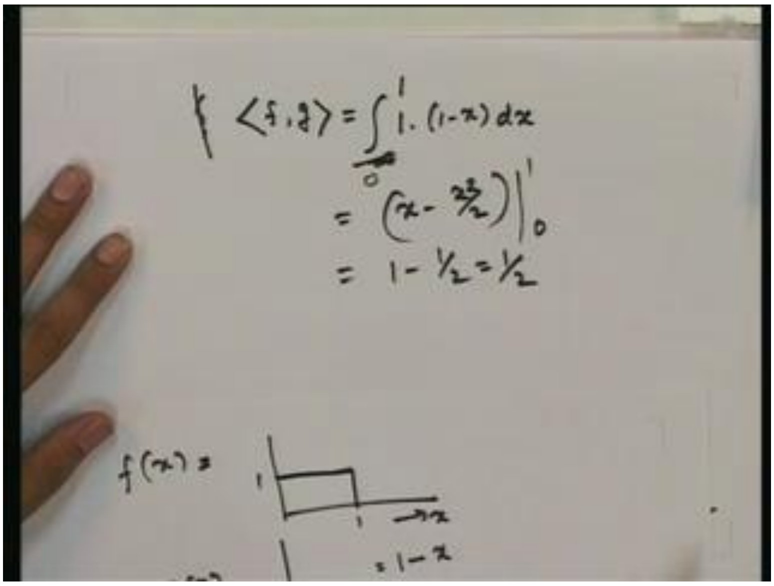

So, we have here f x that is, 1 so we will take the limit from 0 to 1 because it is non 0 in this interval and this function is just 1 times g of x; g star of x is same as go as we said as

1 said and so it is 1 minus x and dx. So, what do we have here? This is 1 minus x dx. So, it is x minus x square by 2; at 0 it is 0 at 1 it is 1 minus half that is, half. So, the inner product of these 2 functions is half.

Now, let us take inner product of g x with itself that will be; obviously, the length of g whole square. So, length of g square will be simply integration 1 minus x whole square integrated in this interval. So, that will not compute again that is quite obvious, how to do it is simple. Now, we say that inner product is; the purpose of defining inner product is to define orthogonality of vectors.

Now, we can have a particular as special case signal, signals; set of signals also can be considered as vector space, we have been all always doing it now to define orthogonality. You know orthogonality of 2 functions or 2 vectors.

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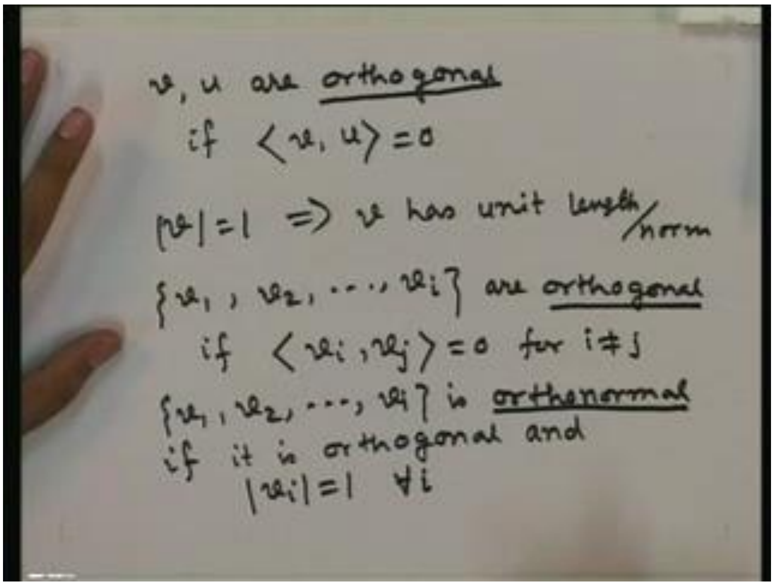

So, v and u are called orthogonal; if inner product of v and u is 0. This is definition of this is quite clear and we know this for a particular for the special case of Cn or Rn.

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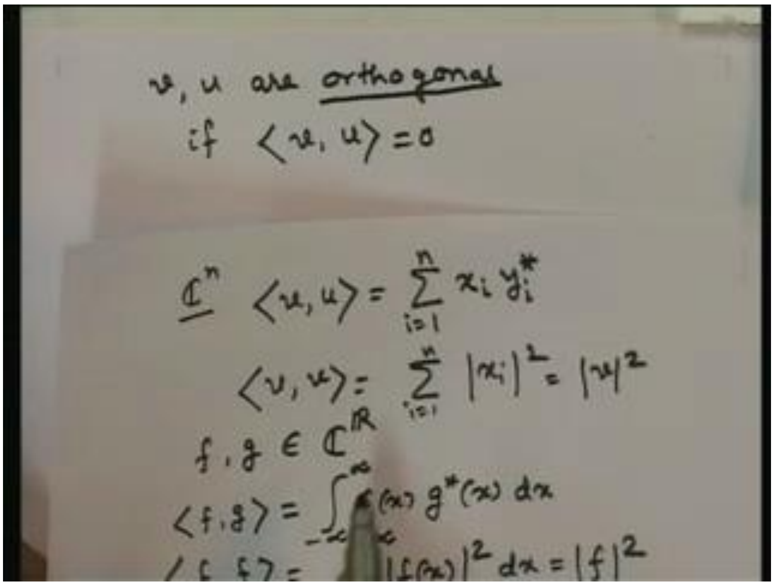

We know that, to check if 2 vectors are orthogonal, we have to compute the inner product and if the inner product is 0 the vectors are orthogonal. Here we defined this in this way so that, this can also be done for signals. We can check it to if the signals are orthogonal, by simply taking the inner product and seeing if it is 0.

Now, if a vector has length 1 then we say that, v has unit length or no. So if, now if we have a set of vectors v 1, v 2, v i are orthogonal if any 2 of them are orthogonal. You choose, so v1 is orthogonal with all of them, v2 is orthogonal with all of them all other vectors then, they are orthogonal. This set will be called orthogonal. If inner product of v i,v j is 0 for i not equal to j. So, you take any 2 their inner product will be 0 then this is this set is called orthogonal.

Now, this set is called orthonormal; is orthonormal if it is orthogonal and so if it is orthonormal if it is orthogonal and the length of each vector is 1 for all i; v i length of v i is 1 for all 1. Let us see examples.

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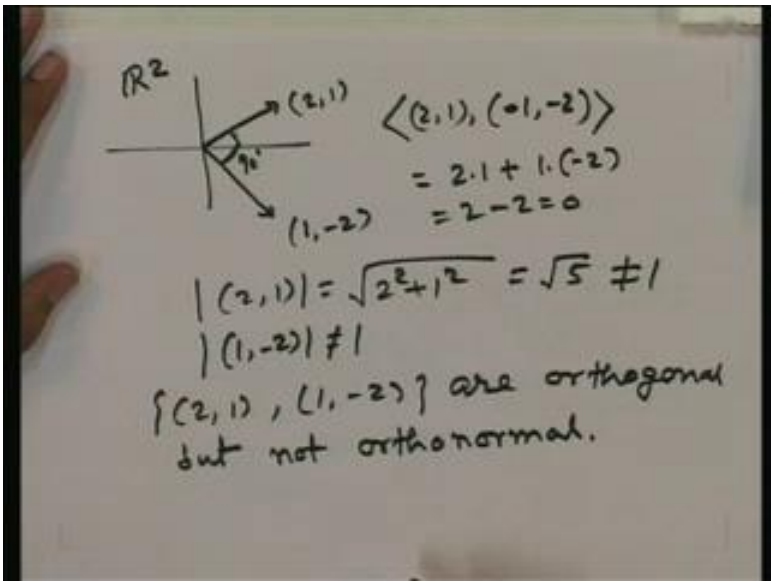

Take the 2 dimensional plane R2. If, we have say this vector and say this vector; this is orthogonal 90 degree. So, suppose this is 2 1 and this is 1 minus 2. They are orthogonal as you can see, but we will verify it here. Inner product of 2 1 and 1 minus 2 is 2 times 1 plus 1 times minus 2. So, 2 times 1 plus 1 time minus 2 that is, 2 minus 2 that is 0. So, these 2 are orthogonal, but are they orthonormal?

Let us see their lengths. So, length of 2 1 is root over 2 square plus 1 square that is, root of 5 root over 5 that is not equal to 1. Similarly, length of 1 minus 2 is not equal to 1. So, even if 1 of them is 1, we will not call them orthonormal. These 2 are orthogonal; so, 2 1 1 minus 2 are orthogonal, but not orthonormal. But if, we scale them down if we bring it down its length is root 5. If we divide by it by root 5 its lengths will be 1 and if you divide this also by its length that is root 5 it will become unit length, then those 2 vectors will be orthonormal. Let us see that so, suppose we have this, we want to scale them; take these 2 vectors.

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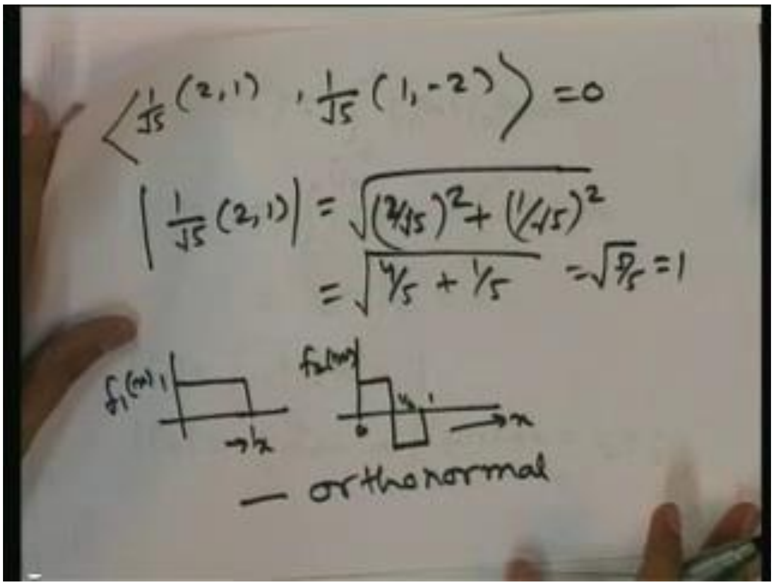

Now, 1 by root 5 2 1 and 1 by root 5 1 minus 2 and what is a inner product of them? This will be still 0, but what is a length of 1 by root 5 2 1? It is root over 2 by root 5 whole square plus 1 by root 5 whole square because, its components are 2 by root 5 and 1 by root 5. Then this is 4 by 4 by 5 plus 1 by 5 that is, root 5 by 5 that is 1. Similarly, length of this also will be 1.

So, we have seen that if we have 2 orthogonal vectors, we can scale them appropriately so that, they are orthonormal vectors. So, from a set of orthogonal vectors we can get a set of orthonormal vectors, by simply scaling each vector to bring it to unit length. Now, similarly let us do 1 example for functions now. Suppose, we have a function; suppose we have say function again say 1 1 and suppose, we have a function; this is suppose f1 x and suppose this is 1 f2 x. Then, we want to see whether they are orthogonal.

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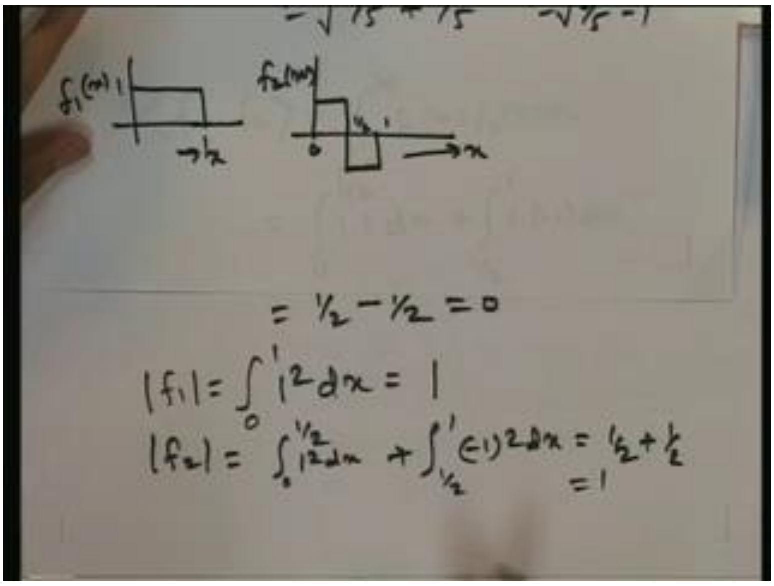

So, what is the inner product in this for functions? Inner product is integration minus infinity to infinity f1 x f2 star x, but it is real functions, so it is same as f2 x. Now, what is this; this are 0 non 0 only from 0 to 1 and f1 x is just 1 and f2 x is 1 from 0 to half and minus 1 from half to 1. So, we have to divide it into 2 parts; 0 to half it is 1 and then half to 1. This function is still is 1, but this function is minus 1. So, 1 times minus 1 dx. So, this will be half, this will be minus half. So, this will be 0.

So, these 2 functions are orthogonal, this 2 functions are orthogonal and let us see what their lengths are. So, length of this is 0 to 1 1 square dx and that is 1. So, its length is 1 and length of this also can be same to be. Take 0 to half and half to 1 and this interval it is 1 square dx and this interval it is minus 1 square dx, but this is half, this is also half; so 1. So, both these functions are of unit length and they are orthogonal to each other. So, these 2 functions are orthonormal.

Now, we have now seen how to check if a given set of vectors is orthogonal; if a given set of vectors is orthonormal, we have seen how to check that. But, suppose we are given a set of vectors which are not orthogonal, but we want to find a set of vectors which are orthogonal, but it will also we want the vectors in such a way that, the original vectors can be obtained as linear combination of the derived vectors.

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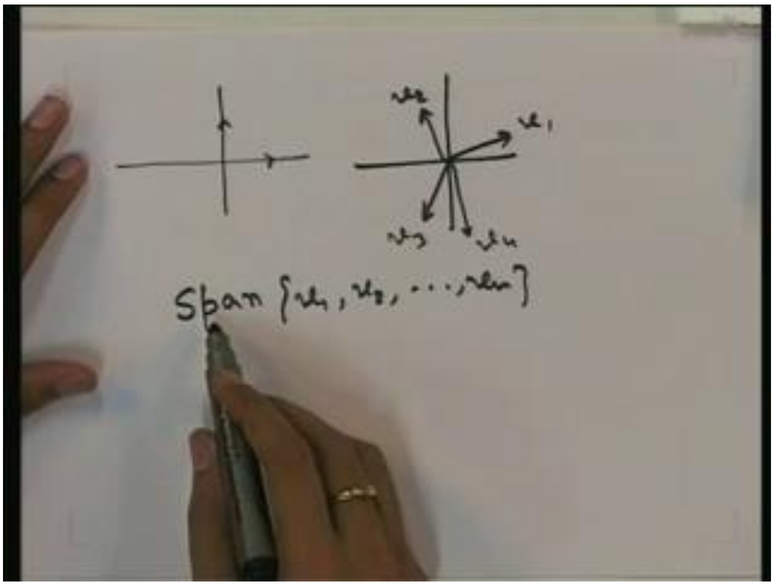

For example we know that for r2 we can take this 2 as a basis. So, that all the other vectors can be obtained as linear combination of these 2 vectors and these 2 vectors are orthonormal. Suppose we are given these vectors, say may be more. We want to find out 2 vectors such that, these vectors can be 2 vectors which are orthonormal; we want 2 vectors which are orthonormal such that these given vectors can be obtained as linear combination of those 2 vectors. That can be said in a different way also. It can we said that if we are given these v1 v2 all these vectors, then if we take the span that is, a vector sub space that is a vector in this case the whole r2. If, we take v 1. v 2, v n this given vector the span of that, there is sub space generated by these. We want an orthonormal basis of this vector space, this sub space.

For example here this is an orthonormal basis; this 2 form a basis because any linearly independent and any vector here can be obtained as linear combination of these 2. So, it can be also proved that, if a set of vectors is orthogonal then their linearly independent also. That can be also proved very easily, but we will not go into that now. So, we want to find a basis of these which is orthonormal. How do you find that? So, we want to find few vectors which are orthonormal, which are orthogonal to each other and unit length each and also these vectors can be obtained as linear combination of those vectors. So, to do that we can do in the following way

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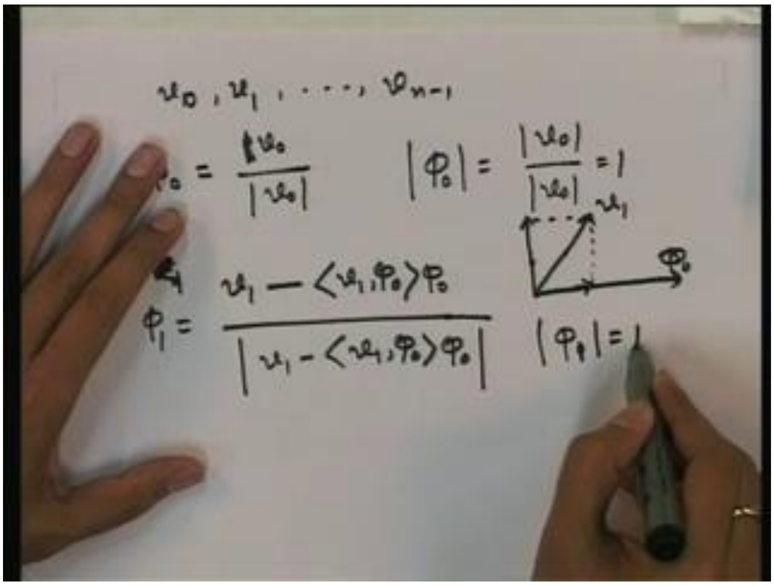

Suppose, we are given those vectors v 1, v 2. So, we will say v 0, we starts from v 0 and then it goes to v n minus 1. So, how do we do it? We will compute this vectors say phi 0; phi 0 first we will, we are taking v naught, but we do not want its length to be not equal to 1. So, we want the length to be equal to 1. So, we will divide v naught by its length. Then, this vector is basically a scaling of this. We are we are dividing by this length so that, the length of phi naught length of phi naught will be length of this vector by this scalar; which will we be 1.

So, we will take this vector, then we will take, then we have another vector v 1. Now v 1 if it is orthogonal to phi naught its fine, we can just scale it down, but if it may not orthogonal then what do we do, in intivitibly we know that, if we are given 2 vectors like this to find orthogonal; 2 orthogonal vectors we first project this. We get this vector. We subtract this vector from here. Then you will get this vector. So, this is sum of these 2 vectors.

So, this vector is nothing, but this minus this vector and this vector; new vector is orthogonal to the first vector. So, to get the first a vector orthogonal to first vector from here, we subtract the projection of this on this from the original; this vector. So, this is phi 1. We want to subtract from this vector that is, v 1 the new vector this is, say this is phi naught this is v 1. If, I project on phi naught and that projection is the inner product of v 1 phi naught times phi naught. It is the vector phi naught times the inner product of these 2 that is, the over 2 this is the projection of v 1 on phi naught.

So, we subtract that, this vector will be orthogonal to phi naught itself. That can be checked very easily and we will see later. Then here we want this vector to have unit length; not only that we want this to be orthogonal to phi naught, we also want this to have unit length. So, if its length is different from 1 we want to divide by that length. Now, we can see because we are dividing that length of this, we will get length of phi 1 equal to 1.

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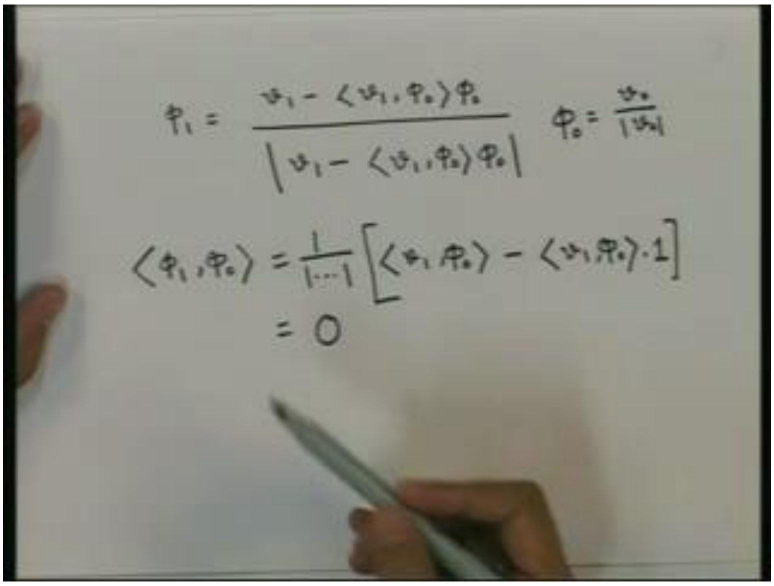

Now, we can check that, this 2 vectors phi 1 and phi naught that we have got, are orthogonal to each other. So, let us see their inner product and see. The inner product of phi 1 with phi naught; so, we can replace this phi 1 by this expression. So, this scalar will come out 1 by this quantity and then, the inner product of this with phi naught we have to take; that means, inner product of v 1 with phi naught minus this scalar times inner product of phi naught with itself that is 1. Because, as phi naught is constructed it is a unit norm vector and this 2 quantities are same. So, we have 0 here.

So, these 2 inner product of these 2 vectors is 0 and; that means, that phi 1 and phi naught are orthogonal to each other, but already we have taken phi naught to be v naught by mod v naught; the length of v naught. So, that is; this is a unit norm vector and this is also a unit norm vector and they are orthogonal to each other so; that means, they are orthonormal to each other.

From here we can see that v naught is a scalar multiple of phi naught and from here we can see that, this v 1 is a scalar multiple of phi 1 plus a scalar multiple of phi naught that is, v 1 is linear combination of phi naught and phi 1 and v naught is simply scalar multiple of phi naught. So, both v naught and v 1 are linear combinations of phi naught and phi 1. So, we have got phi naught and phi 1 as an orthonormal basis of the span of v 1 and v naught.

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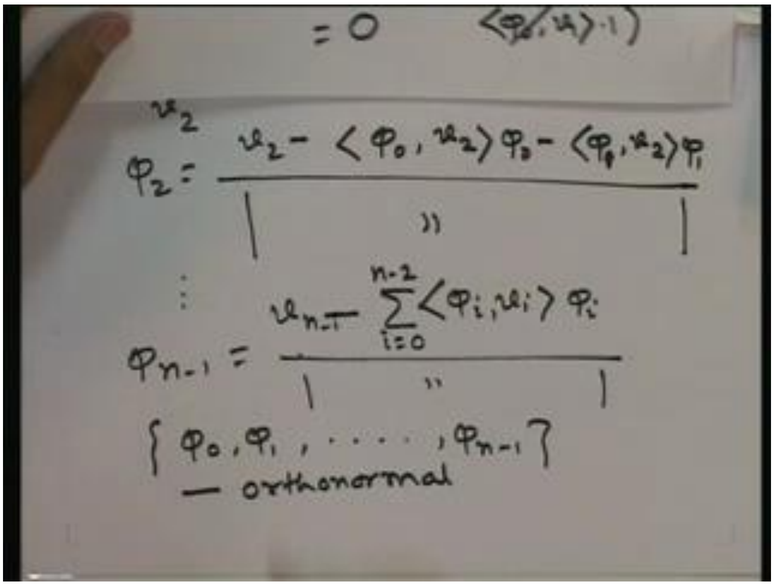

Now, we have 2 vectors phi naught and phi 1. We want to get an orthogonal vector to them from v 2. So, how do we get? Now phi 2 will take v 2 minus its projection on phi naught and phi 1. So, we will take phi naught v 2 phi naught minus phi naught v phi 1 v 2 phi 1. So, this is the projection on phi naught, this is the projection on phi 1. We have subtracted both of them from v naught v 2. So, this vector will be orthogonal to phi naught and phi 1.

Now, to make it unit length, we take the norm of the whole thing; the length of the whole vector here and divide by that. Then we will get an unit length vector phi 2 which is orthogonal to phi naught and phi 1 and; obviously, v 2 can be expressed as a linear combination of phi 2 phi naught and phi 1 because, multiply by this here and then this, you bring on the other side, you get v 2 as a linear combination of phi naught phi 1 and phi 2.

So, we keep doing that; this may be 0 of course. So, we will keep doing that and then when we are finished, we will get phi n minus 1 equal to at the a end. So, v n minus v n minus 1 minus summation phi i; so i equal to 0 to n minus 2 phi 1 v 1 phi 1 by the this. So, this will be the last. So, this way we will get some vectors, but some of them may be 0 also.

So, we will not take them, we will exclude them and then we will get some phi naught phi 1; we will take only the non 0 of them, then we will have this as the; an orthonormal vectors, set of orthonormal vectors such that all the vectors, all the original vectors these vectors can be expressed as the linear combination of these vectors Because, as we have seen v naught is a linear combination of phi naught itself, v 1 is a linear combination of phi naught and phi 1, v 2 is a linear combination of phi naught phi 1 phi 2 and so on, v n minus 1 is a linear combination of all the phi's.

So, we have got a set of vectors which is orthonormal and also it is the basis because, orthonormal a set of orthonormal vectors can be proved to be linearly independent. So, these vectors are linearly independent and all the original vectors can be expressed as a linear combination of these vectors. So, as a result any vector which is in the span of the original vectors can be expressed as a linear combination of these vectors. So, we can say that, these set of vectors is a is an orthonormal basis of the span of these vectors. Meaning by; so we will just summaries that, it means this set of vectors this is an orthonormal basis of span of v 1 v naught v 1 v n minus 1.

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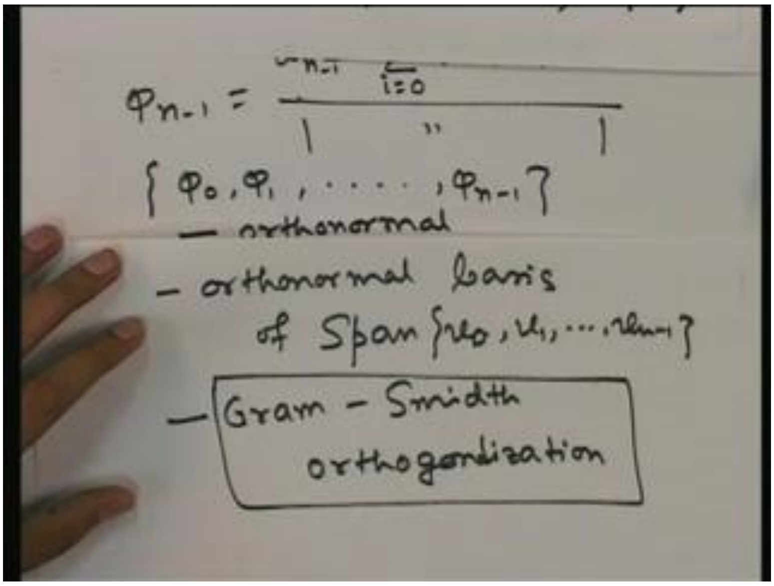

So, we have seen how to construct an orthonormal basis of the span of a given set of vectors. This procedure all is called Gram-Smidth orthogonalisation. This is very important very, useful technique. This gives a way of finding an orthonormal basis of the span of a few vectors given. So, if you are given a few vectors, we want to have a set of orthonormal vectors such that, all these vectors can be written as linear combination of these orthonormal vectors and these orthonormal vectors themselves are in the vector space spanned by them that is, they also can be written as linear combination of these. But, they are special because they are orthonormal.

So, they are linearly independent first of all and then they are also orthonormal. So, this will be smallest sufficient for sufficient linear algebra background, that will be required for the later classes and in the next class, we will start restart on digital modulation techniques.

Thank you.