Digital Communication
Prof. Bikash Kumar Dey
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture - 16
Digital Modulation Techniques (Part - 5)

(Refer Slide Time: 00:44)

For the last few classes, we have been discussing digital modulation techniques and we will continue in this class the same topic. In the last few class, we have first done or we have first discussed baseband representation of band pass signals. And similarly baseband representation of band pass systems and then we have discussed PAM modulation scheme which is the simplest digital modulation technique.

Then we have seen how to demodulate PAM modulated signal using merge filter we have shown that, merged filter is the optimum filter that gives us the highest SNR at the receiver and we have also seen with examples that, sampling at multiples of t seconds is the base thing to do that, is those are the best instances to sample that is because that would give as again maximum SNR. And in the last class, we have discussed PSK modulation that is phase shift keying and we have also introduced the term constellation which actually gives a pictorial representation of the signals that are transmitted.

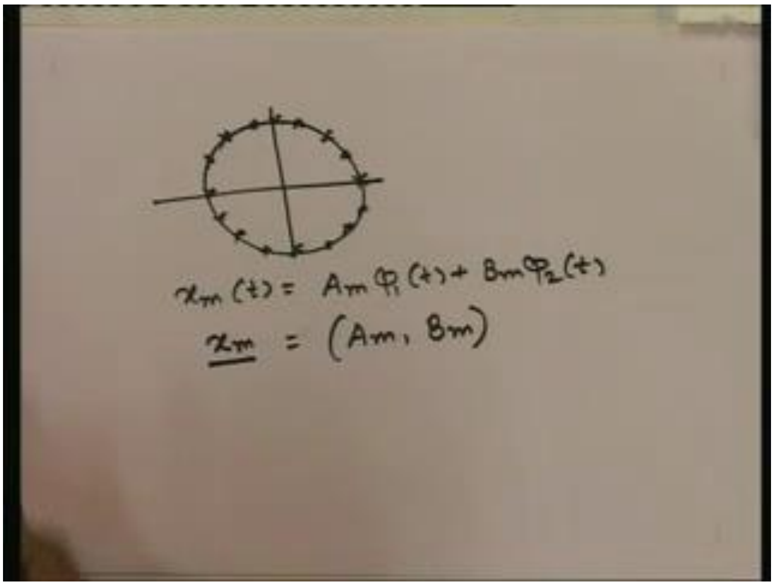
So, how to draw the constellation, we have seen that first we have 2 we have to take 2 signals not necessarily signals from the possible transmitted signals, but any 2 signals we have to have such that, the energy of each of those not necessarily 2 signals for, but for the modulation schemes we have discussed. So, far we need to have either only 1 or 2 signals. So, we have to take a few 1 or 2 signals such that all the transmitted signals can be expressed as linear combination of those signals.

If there is we need only 1 signal then all the transmitted signals are just scaled versions of that single signal and that is true for PAM constellation and that is why we get 1 dimensional constellation for PAM. So, those point, if we draw the scaling if the transmitted signal XMT is AMT times AM times phi t, phi t is that unit energy signal Then we draw that point AM on real axis then we have for AM for different M that would give us the constellation those points on the real line is the constellation.

Similarly, for PSK modulation scheme, we have seen that we have to have 2 signals we cannot represent all the signals as scaled versions of single signal. So, we have to have 2 signal 1 is cosign omega Ct times Pt the sub reasonably scaled. So, that suitably scaled such that, the energy is 1 that is rout over 2 by EP times Pt cos omega Ct and also root over 2 by EP Pt sin omega Ct

So, those 2 signals if we have we can express all the transmitted signals in terms of those 2 signals as linear constellations. So, then if we want to draw a point XMT we will first express it as a linear combination of the 2 signals phi 1 t and phi 2 t that is we will express it as AM phi 1 t and BM phi 2 t. Then we will draw the point AM BM on 2 dimensional plane AM BM point and that point is the representation of that particular signal XMT. Similarly, for different M we will get different points on the 2 dimensional plane.

(Refer Slide Time: 05:11)

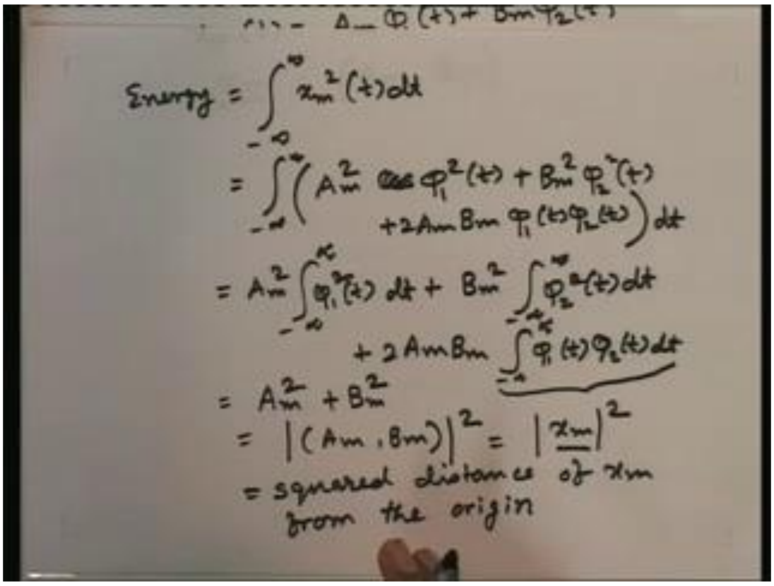


So, those points together is the constellation of the PSK signal set and we have seen that, for 4 PSK signal set the points are basically this points 8 PSK signal we have these points 16 PSK signal we have this point and so, on and we have also discussed how to increase the number of points in the PSK modulation scheme. So, if we want to keep the probability of error same we have to increase the radius of the circle that is we have to increase the energy that is transmitted because, we want to have the same distance, but we want more points on the circle.

So, but if we want do not, if we do not want to increase the energy we have to put more points on the same circle energy of the probability of error will increase because the distance between the points is decreasing. Now, we have same before that, the energy of the signal is nothing, but the square of this distance from 0 to that point, but we have not proved it. So, we first quickly show that actually the distance square of the distance gives us the energy of the signal.

So, let us see suppose, we have a signal xmt and it is represented as Am phi 1 t plus Bm phi 2 t. So, that the vector xm which we draw on the 2 dimensional plane for this signal is we will denote it t by this is basically Am Bm right. Now, we have said that the energy of this signal is nothing but, the square of the length from 0 to this vector. So, what is let us compute the energy of this first. So, what is the energy of this signal energy of this signal is, this is the energy.

(Refer Slide Time: 07:15)



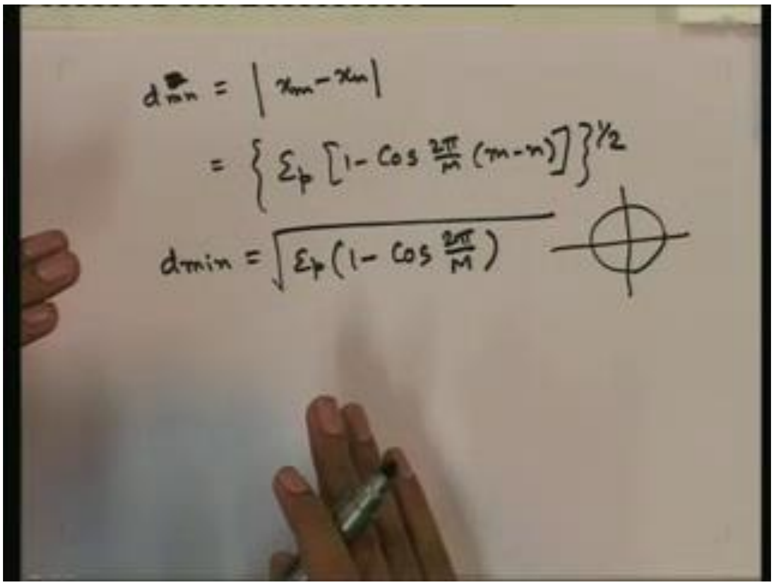
Now, using this we can see that, this can be written as 0 to t because at all other places it is the signal is 0 phi 1 square t plus Bm square phi 2 square t plus 2 Am Bm phi 1 t phi 2 t dt. Now, this is Am square Bm square then plus 2 Am Bm phi 1 t phi 2 t dt we are assuming the real signals phi 1 t phi 2 t. If they are complex we have take mod this square mod phi 2 square then phi 1 t phi 2 star t and so, on

So, what is this quantity, we have said that phi 1 t and phi 2 t are orthogonal if this quantity is zero. So, I am we have assumed that phi 1 t and phi 2 to t are orthogonal to each other. So, this quantity is 0. So, what we have here is Am square the and times energy of phi 1 t that is 1 plus Bm square times energy of this signal that is 1. So, we have Am square plus Bm square and that is, the square of the magnitude of the vector xm xm is this what is the length of this vector from 0 to this distance? That is the length that is root over Am square plus Bm square and this energy is the square of that.

So, this is this vector’s length square that can be written as xm this length square. So, this is the square distance of xm from the origin. So, we have seen that these points actually these seen what is the distance form 0 of this points we can actually say: what is the energy of the corresponding signals? So, energy of a signal in basically square of this distance. So, PSK so, this again give us tells us that PSK signals have equal energy. So, PSK signal is constant energy signal set and PSK modulation is called the constant energy modulation scheme.

Now, what is distance between these 2 points? We have said that, it will actually depend on this depend on this theta and can you express it exactly.

(Refer Slide Time: 11:21)



Let us see, the distance between the m'th and nth point let us say dmn is basically the magnitude of xm minus xn this is we can compute it, but we will not going to that you can verify it yourself that it will be Ep 1 minus cos 2 pi by m m minus n half. So, this is actually d min dmn square. So, what is the minimum distance? It will be minimum for mn n conjugative that is when m minus n is 1 or minus 1. So, that is m suppose 2 there is second point and third point third point and fourth point those will be the nearest.

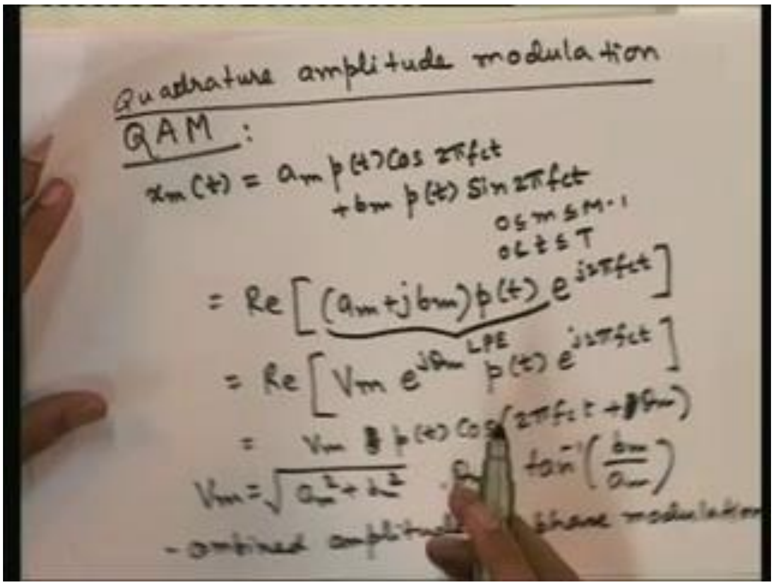
So, the d min we will put in place of this 1 and d min will be root over sorry this is half. So, it is dmn. So, this will be root over Ep 1 minus cos 2 pi by M this is the minimum distance and as you can see this also tells as that, if we increase N capital M this minimum distance will decrease because, if we increase capital M; that means, this angle is smaller and as this angle become smaller and smaller this increases cos of that value increases.

So, as a result this quantity decreases. So, as we increase M this minimum distance decreases and we can also see that from the figure we have to fit more points on the circle then we have to put them closer that, is the only way.

So, 8 PSK will have smaller minimum distance than 4 PSK 16 PSK will have smaller minimum distance than 8 PSK and. So, on if we keep the radius of the circle same. If we increase the radius of the circle then again we can have higher minimum distance. So, we have seen for PSK modulation scheme that, the constellation of the PSK modulation modulator signal is utilized on the on a circle and in now, we will start another modulation technique or quadrature amplitude modulation.

This is, this can be said to be a joint modulation of 2 different pulses, 2 different carrier signals PAM was just amplitude modulation of 1 particular carrier signal this can be said to be joint modulation of 2 different carrier signals. Actually PSK was also such a modulation technique because it could be expressed as linear convolution of 2 different carrier signal sign omega ct cos Pt and cos of omega ct pt that is phi 1 t and phi 2 t. So, but that was a special case all the points lied in the lay in the in a circle and quadrature amplitude modulation is again a generalization where they need not lie on a circle. So, quadrature, amplitude modulation or QAM in short.

(Refer Slide Time: 14:58)



So, this can be considered as, a joint amplitude modulation or 2 carriers signals and this expression will immediately say the same thing. So, previously for the PSK modulation also we have seen that, the modulated signal was actually of this type, but there there was some relation between Am and Bm Am was cos 2 pi by M capital M time small m and Bm was sin 2 pi by capital N time small m.

So, there was some relation. If I tell 1 you can tell the other, but here this can be arbitrary we can select those points in fact. So, this can be also expressed as real part of am plus jbm pt e to the power j 2 pi fct that is real.

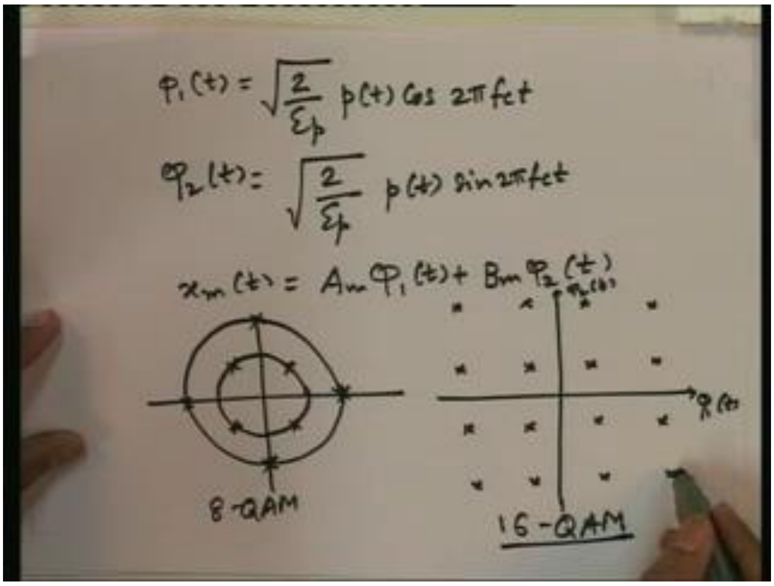
So, here this is basically this is the low pass equivalent this is this representation is in terms of the low pass equivalent signal. And this low pass equivalent signal is expressed in Cartesian coordinate system am plus jbm times pt, but this can be also expressed in the polar form and that is: let us say vm is the magnitude then e to the power j theta m is the phase then pt e to the power j 2 pi fct.

For PSK signal set the Vm was constant the magnitude of all the signals was said whereas, theta m varied here Vm and theta m both may be different for different m. So, this can be written as Vm gt Vm pt cos 2 pi fct plus j theta m not j theta m. So, this signal basically can be expressed in this form also. Now, here; obviously, Vm is the magnitude of am plus j bm and theta m is the phase.

So, this can also be said to be a joint or combined amplitude and phase modulation like we can say that, it is the joint amplitude modulation scheme it is modulating 2 carriers whereas, in the polar coordinate system we can also say that, it is and it is modulating the magnitude as well as the phase So, in the PSK scale PSK modulation it was only phase modulation Vm was constant here we can vary Vm as well as theta m. So, this is combined amplitude and phase modulation.

So, it can be viewed as joint amplitude modulation of 2 carrier signals whereas, it can be also viewed as joint amplitude and phase modulation of a carrier signal.

(Refer Slide Time: 19:49)



Now, let us draw the constellation for some specific cases we will draw, but let us see how to do it. So, here also just like PSK we can see that, because xmt is expressed in this form we can take the same basis same fundamental signals phi 1 t and phi 2 t. So, our phi 1 t is just like before 2 by Ep pt cos 2 pi fct and phi 2 t is root 2 by Ep pt sin 2 pi fct then we can say that, any signal xmt is Am phi 1 t plus Bm phi 2 t remember that for PSK signal sets again phi Am and Bm had some relation Am was cos 2 pi by capital M times small m and Bm was sin 2 pi by capital M times small m.

Whereas, here they need not have any relation we can choose the points and then we can say that, that is our QAM modulation So, let us see some typical constellations for QAM typical, QAM constellations. So, suppose we want to have 8 points on the constellation. So, we 1 way to get 8 points is take 2 PSK. So, 1 circle here and another circle here and then say take 4 points here this is like a shifted version of 4 PSK. So, this is also, In fact, called a 4 PSK signal set, if we if we shift all the points on the circle by some constant phase then those that is also a PSK signal because this is also changing the phase only then you take this extra point on a bigger circle.

So, this is 1 8 pm 8 QAM. So, we can have different 8 QAM modulations different 8 QAM signal set we can choose the point this way we can choose the point points in another way we can choose in many different ways. So, let us say we want 16 points we can have another circle we can put 8 points on that is 1 way to get again 16 QAM, but another a very standard signal set for 16 QAM is this. So, this is phi 1 t and this is phi 2 t and these are the points.

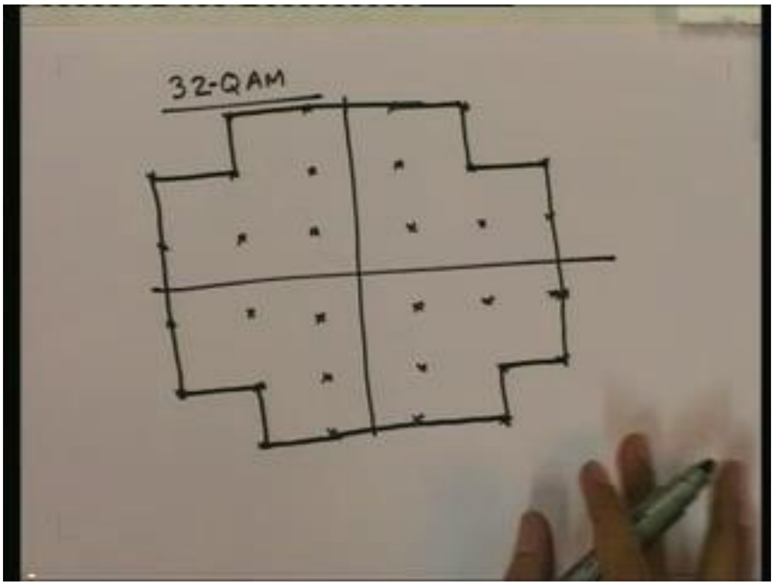
So, this is 16 QAM and this is a very popular 16 QAM constellation other 16 QAM constellations though the they can be used this is the stranded 16 KAM QAM constellation that is used in practice. So, this distance you can change this is symmetric all these distances are same, but this it can be amplified. If you want to the transmit more energy you can scale the whole signal set and then transmit. So, if we increase the energy.

So, that will increase the energy, if you have want to have more distance, more distance will result in less probability of error whereas, it will also result in more transmitted energy. So, you are compromising on transmitted energy to have less probability of error that is better reliability. So, this is 16 QAM constellation and there are some other very standard constellations used. So, using this constellation if you want to transmit then what do we do we take this phi 1 t phi 2 t and suppose, this is m equal to 0 1 m equal to 2 m equal to 3 4 5 6 7 8 I mean.

So, on till 15 because there are 16 points you simply do it this way that, if you want to transmit say m equal to 3. So, you can transmit 4bits in this because this is 16 QAM. And if you want to transmit the third signal that this point how do you do? You simply take this coordinate of this that is Am Bm and then multiply Am to phi 1 t multiply Bm to phi 2 t and then add them and transmit.

So, another advantage of this, modulation techniques is that you need not have signal generators for all these points separately you if you just have signal generators which generate phi 1 t and phi 2 t there is 1 generator which generates phi 1 t another signal generator which generates phi 2 t. Then you multiply this signal with some Am multiply this signal with Bm and add them and transmit you need not have separate signal generators for each point. And more over, if the phi 1 t is like this and phi 2 t is like this you can have phi 2 t from this itself by changing the phase. So, this is further advantage of having this kind of signal set.

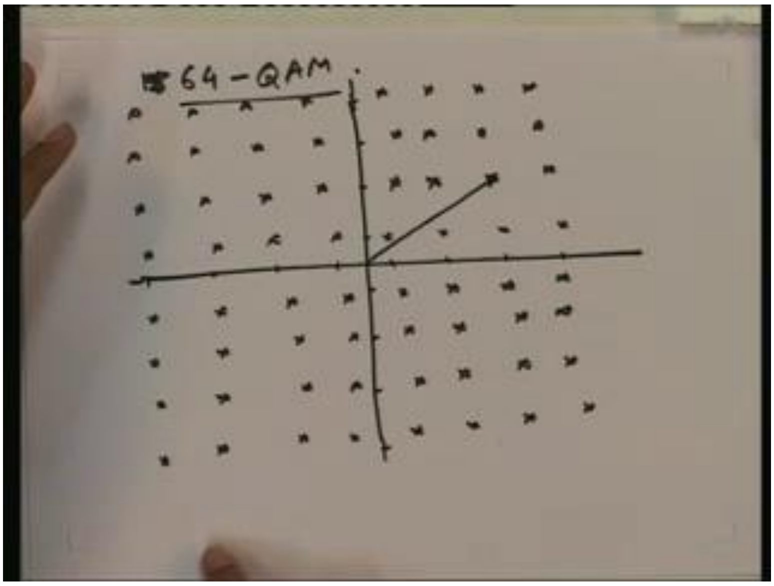
(Refer Slide Time: 25:34)



Now, let us draw other constellations which are also used in practice 32 QAM. So, it looks like this. So, you basically have 16 QAM constellation first then you we want to add 16 more points. So, we had 4 points on each side. Then on this side 4 points then on this side 4 points and on that side 4 points this is the 32 QAM constellation what is the boundary of this? Its boundary is like this, it is not a squire constellation like 16 QAM it is it is also called cross constellation because it looks like a cross. So, this is 32 QAM constellation.

Similarly, if you want to have 64 QAM constellation that is even easier that, will not be 16 or 32 cross constellation 1 very easy constellation to with 16 points Is 64 sorry 64 constellation is this 8 times 8 is 64 8 square.

(Refer Slide Time: 26:52)



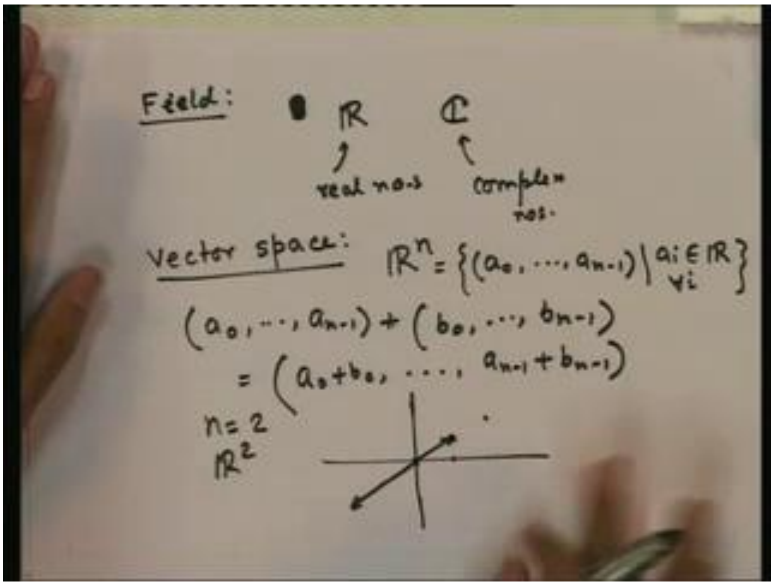
So, on each dimension there will be 8 levels. So, on this dimension there will be 8 so, 1 2 3 4 1 2 3 4 there will be 8 levels on this also there will be 8 levels 1 2 3 4 1 2 3 4. So, then you draw all the points like this. So, this is 64 QAM there are 64 points because on each dimension there are 8 levels. So, 8 times 8 is 64. So, this is not a cross constellation this is square constellation. So, that is 64 QAM constellation and we can have we can draw other constellations also, if we want say 129 QAM constellation we have to add more points.

So, again we have to remember that, these are not the only possible constellations for. So, many points for 64 QAM this is not the only possible constellation there are other ways of selecting the points, this is just 1 of them and similarly 32 QAM also this is just 1 of them. Now so, we have so, far done in this class we have seen that, if you draw a constellation then say this point the energy of the corresponding signal for this point is nothing but, the square of the distance from 0.

So, this distance square is the energy of the corresponding signal for this point, then we have introduced another modulation technique called quadrature amplitude modulation, it is called quadrature amplitude modulation because we are doing combined amplitude modulation of 2 quadrature signals 1 is cos another is sin. There are 2 correlation signals and we are doing combined amplitude modulation of both the signals and this can this modulation can also be viewed as joint amplitude and phase modulation of a single carrier signal.

If you represent simply this by in polar coordinate then it becomes amplitude and phase every point has an amplitude and phase. So, it is joint amplitude and phase modulation also. Now, in the from the next class onwards we will introduce other modulation techniques, but before that in this class we will introduce some basic linear algebra which will be used in the later classes and which will also make the later classes easier to explain other modulation schemes also.

(Refer Slide Time: 31:05)



So, we will quickly revise some of the linear algebra concepts 1 is that, first we will define something’s a field it is this field is a algebraic structure, this is basically a set this is like: say set of real numbers R set of real numbers are denoted by R real numbers, set of complex numbers is denoted by C this types C. So, these are fields they have 2 operations addition and multiplication then they are also they have addition satisfies at a nice condition multiplication satisfies certain nice conditions.

So, because of that those conditions these types of sets are called fields. So, what are the conditions? Typical conditions like, addition is commutative multiplication is multiplication is also commutative then there is negative element for every element then there is multiplicative inverse of every nonzero number in these fields in these sets. So, these properties because these sets satisfied those properties these sets are called fields and most of the times we will deal with only these 2 fields.

So, we will not give other examples though there all are other examples. So, then we will need vector space a vectors space is basically a vector space over R is again another set such that, it again satisfy certain conditions another set with another an operation. So, the operation is called basically the addition of vectors For example, Rn what is Rn? It is basically the set of all n triples of real numbers.

So, set of all n triples like; a naught to an minus 1 such that ai each ai is in R that is real numbers for all i this ai's are real number and then this is n triple this is 1 n triple take all such n triple that set is called R power n and we can define addition in that set in this set. How do we do addition? Addition of 2 vectors like this plus b naught bn minus 1 is simply defined as a naught plus b naught to an minus 1 plus bn minus 1 this is the addition of these 2 vectors and this addition and this addition are not same this is the real addition this is the addition of 2 vectors addition of 2 vectors is defined in terms of addition of real numbers.

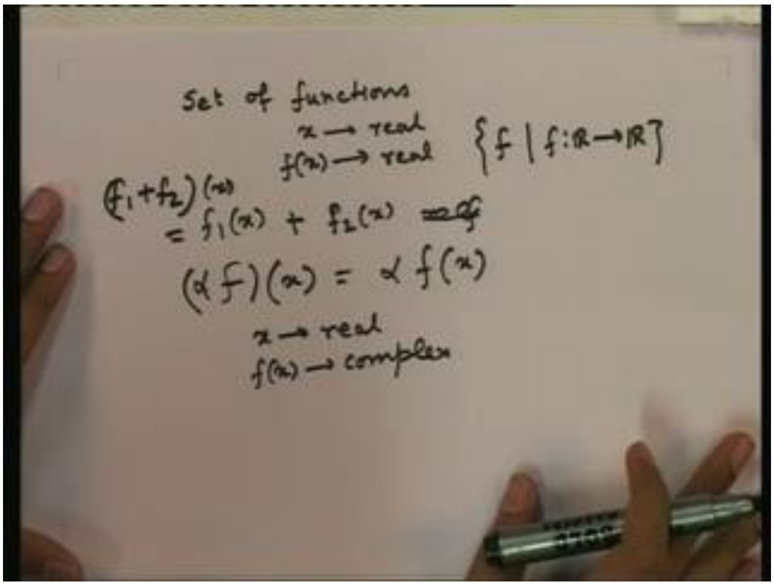
Similarly, if we have complex numbers C power n will be defined as n triples of complex numbers similarly, you can define the vector addition in terms of the complex additions this, so this set is a vector space over R and there is another thing we can do in this set that is multiply by any real number, if we have 1 vector we can multiply by another real number 1 real number to this vector and we will get another real vector in Rn. So, this is called scalar multiplication we are multiplying a vector by a scalar.

So, if a vector space should have 1 addition we should be able to add 2 vectors and we should be able to multiply by a scalar that, is scalar multiplication. So, we should be able to multiply a vector by a scalar and that then get another vector these 2 properties also should satisfy certain conditions these are all natural conditions like: scalar multiplication of the addition is alpha times this vector plus this vector is nothing but alpha times this vector plus alpha times this vector.

So, all such certain natural conditions should be satisfied also by a vector space. So, let us see some examples of vector spaces this is 1 example there are other examples also. So, 1 special case of this for example, take n equal to 2 what is R power 2 it is basically the 2 dimensional vector space it is a you can represent the points this elements in R 2 as points on a plane. So, 1 0 is 1 (refer time: 36:02) hence R 2 is basically all of real numbers.

So, 1 0 root 2 5 all these are points all these are vectors in R 2 you can multiply a vector by any real number this is 1 vector, you will get if you multiply by 2, you will get another vector, if you multiplied by minus 3, you will get another vector and so, on. So, you can multiplied by scalar to any vector here. So, this is 1 example there are other examples which are not like this and this will be useful for us and that example is let us take, all functions set of functions take a real functions, of a real numbers.

(Refer Slide Time: 36:45)



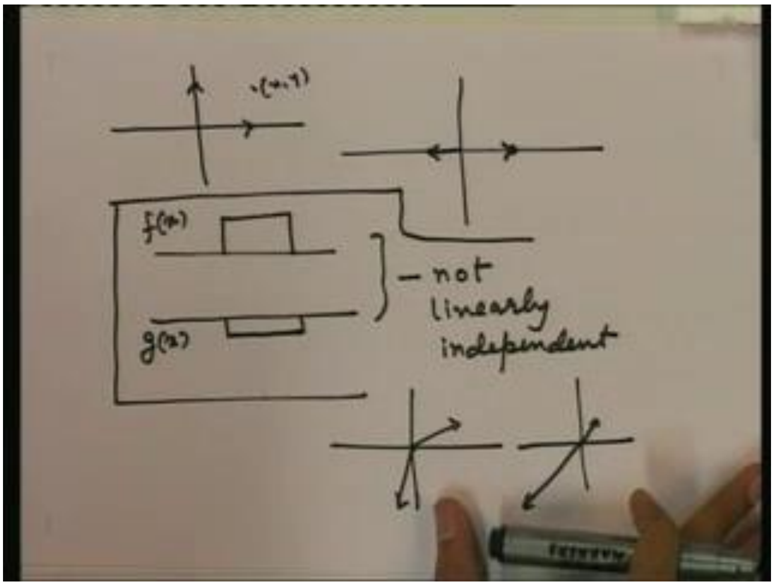
So, x is real and fx is also real. So, fx is a function and take all such functions. So, set of all functions. So, f is a functions of a real number and its value is also a real number take this. So, this is also a vector space now, what should we have in a vector space? We should have addition of vector. So, the vectors here are functions and if you have 2 functions f1 x and f2 x their addition is also a function. So, it is a vector in this vector space.

So, how do we do the addition this plus it is defined as. So, it is f1 plus f2 this is a function how do you define f1 plus f2 of x? It is f1 x plus f2 x that, is the way it is define this addition. Addition of 2 functions we define this way the each value is added f1 x value and f2 x value are added to get the value of f1 plus f2 at x and similarly you can multiply by a real number to any function.

So, if you have a function f alpha times f is also a function that, function at x the value is nothing but, alpha times the value of f at x this is the way this scalar multiplication is defined, if we multiplied by a scalar to a function; that means, multiply each value. That is the way scalar multiplication of function is defined. Similarly, we can take complex functions of real numbers. So, x is still real fx is complex these particular examples will be useful for us because signals are nothing but functions.

So, set of all is real signals continuous time real signals are basically all these functions and. So, set of all real functions of real set of all real signals actually formed a vector space and similarly set of all complex signals of this kind form anther vector space. So, we can again add 2 functions we can multiply by a complex number to any function, we will get another complex function. So, this will also give as anther vector space. Now, we will define some more concepts these are some examples of vector spaces. Now, we will define some more concepts regarding vector space.

(Refer Slide Time: 40:06)



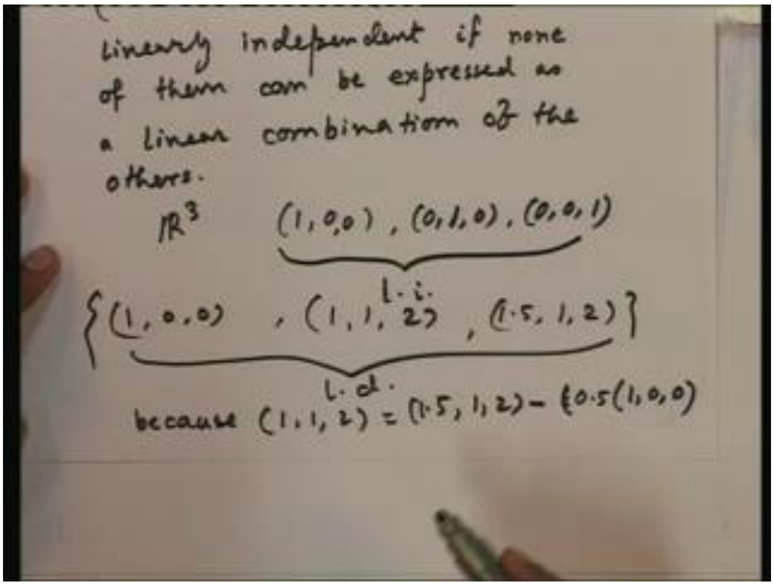
Now, if we have few vectors. Let us say, if we have a vector like this and a vector like this we know these 2 vectors are something special and 1 special thing about these 2 vectors is that, any other vector on this plane can be expressed as a linear combination of these 2 vectors. So, any point here is, if it is x y then this is basically x times this vector plus y times this vector. So, these 2 vectors this vectors then cannot be expressed as linear combination of just a scalar multiple of this vector.

So, that is 1 nice property and we can see that, that property is missing from this these 2 vectors if we take 1 vectors this and another vector. Let us say, this we can express 1 vector as a scalar multiple of anther vector here. So, here these 2 vectors are not linearly independent similarly, we can take signals if we have, say 1 signal this, this pulse this is our f of x and if we have another signal this, this is g of x we know that gx is nothing but a scalar multiple of fx it is minus something times fx.

So, these 2 vectors these 2 functions are not linearly independent. So, these are some examples which actually will come to very good use in digital modulation and other common examples of linearly independent vectors are like this. And we have. In fact, on 2 dimensional plane we can have any 2 vectors which are not on a line these 2 vectors are linearly independent. But, if we take 2 vectors which are on the same line this and this is on the same line.

So, this these 2 vectors are not linearly independent because 1 can be expressed as linear combination of the other Now so, we have introduced linearly independent vectors, if we have now this. So, for all the examples where, 2 vectors we can have. In fact, more number of vectors.

(Refer Slide Time: 42:36)



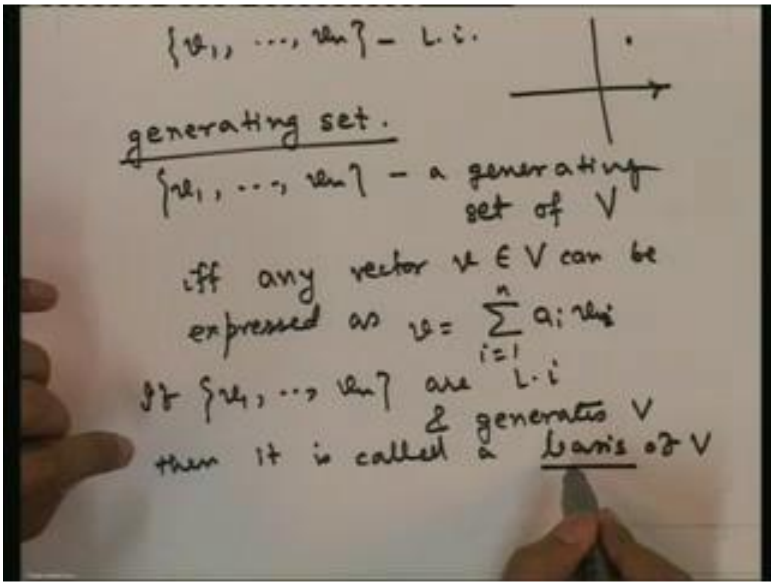
Suppose, we have a collection of vectors we always deal with finite number of vectors in this course. So, suppose we have vector v1 vector v2 and so, on till vector vn these vectors remember can be signals also all set of all signals is also a vector space. So, we can have n number of signals. So, they will be n number of vectors. So, these vectors are called linearly independent, if no way no none of them can be expressed as a linear combination of the others. So, we have seen in this case for example, that this vector can be written as a linear combination of this vector that, is basically a scaling of this vector whereas, these 2 cannot be written that way. So, these are linearly independent.

So, if these vectors are such that no vector here can be expressed as a linear combination of the other vectors then these set of vectors will be called linearly independent. So, for example, take the 3 dimensional space, if we have say that, is basically R3 set of all 3 triples you know that, all any point in the 3 dimensional space can be expressed as a 3 triple is a triple of real numbers.

So, if it we have say 1 0 0 0 1 0 and then 0 0 1 these 3 points we know that, none of them can be expressed as a linear combination of the others. So, these 3 points all linearly independent, linearly independent whereas, if you take 1 0 0 then 1 1 2 and then say 1.512 these 3 are linearly independent linearly dependent because these 3 are linearly dependent because this vector can be expressed, as a linear combination of these 2. Similarly, this also can be expressed as a linear combination of these 2.

So, let us just write that, let us see 1 1 2 this vector is nothing but, this vector 1.5 1 2 minus this vector minus half times this vector. So, this vector is linear combination of these 2 vectors. So, this, this set of vectors is not linearly independent they are linearly dependent. Now, if we have a set of vector such that, they are linearly independent.

(Refer Slide Time: 46:14)



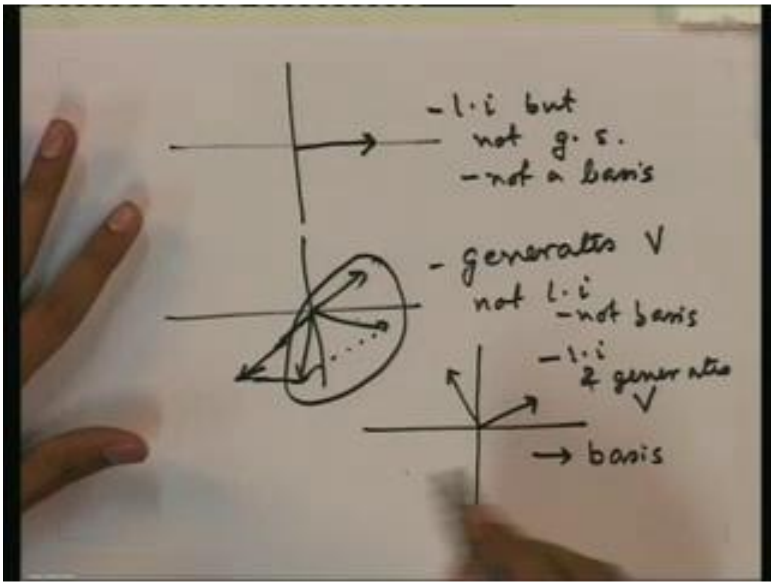
Now, if we have for example, like this 1 point here this singles, this is the single point is a nonzero point. So, this is; obviously, linearly independent, but certainly this does not generate the wholes space whole plane because this point cannot be written as a linear combination of just this vector; that means, it cannot be expressed as a scaling of this vector. So, there is something more we need to have some vectors such that they will generate all the vectors, but they will also be linearly independent.

So, this requires a generating set. So, if v1 vn are linear or is a set of vectors this is called a generating set of the vector space V, if any vector v in this vector space can be expressed as a linear combination like this of these vectors. So, if these sets is such that, any vector here can be expressed as a linear combination of these vectors then the set of vectors will be called a generating set of this vector space; that means, all the vectors in these vector space are generated by these vectors.

So, in we say that, in different ways we say that this vector is generated, this vector space is generated by this vector these vectors or these vectors generate this vector space. Now, if we have, if v1 to vn are linearly independent and it generates v then it is called a basis of V. So, this is a new term again. So, we have seen what is a linearly independent set of vectors? Just now we have also seen what is a generating set of a vector space? And then we have seen that, we have defined a basis to be a set of vectors such that, it is linearly independent as well as it generates the vectors space.

So, let us see some examples suppose, we take simply these vector this is linearly independent, but not generating set

(Refer Slide Time: 49:25)



Now, let us take 3 points like this, this generates because we can express any point. In fact, as linear combination of just these 2 points because these 2 we can take linear combination scale it down scale it up or scale it scale this vector. And then add them in such a way that you get this point any point can be obtained that way, but we cannot say that these are linearly independent because this vector can be expressed as a linear combination of these 2 it is simply like this for example, it is if we have.

So, this is the scaled version of this and this is a this is just as it is, if we add these 2 we get this So, this vector is a linear combination of these 2 vectors. So, this set of vectors is not linearly independent. So, this is generates V, but it is not linearly independent whereas, if we take just these 2 vectors. Let us say then these 2 are linearly independent and it generates V. So, this is a basis.

So, in this class we have defined a vector space particularly we have seen that, set of all functions that is set of all signals real or complex of real variables. Real variable is a vector space over real numbers or complex numbers respectively and then we have introduced 2 notions 1 is linearly independent vectors a set of vectors is linearly independent, if any vector in that set cannot be expressed as a linear combination of the other vectors.

We have seen some examples which are linearly independent we have seen some examples however, the vectors are linear not linearly independent they are linearly dependent. Then we have defined what is called generating set of a vector space? It is a set of vectors such that any vector in the vector space can be expressed as the linear combination these vectors then we call that, we say that set of vectors generates the vector space V.

We have seen examples, of sets of vectors which generate the vector space and we have also seen examples which do not generate the vector space and then we have defined a basis to be a set of vectors, which are linearly independent as well as it generates V and we have seen just now examples, of such sets which this set is a basis it is linearly independent as well as a generating set of V and we have also seen examples where the set of vectors generates V, but it is not linearly independent.

So, it is not a basis and we have seen example of a vectors of a set of vectors this vector single vector, which is linearly independent, but is not a generating set of V and. So, it is again not a basis. In the next class, we will continue this some more preliminary notions in linear algebra and then again come back to modulation techniques in this which the, which is the central part of this course.

Thank you.